

# The Book of Knowledge

Ver 1.3

## Navigator

Ver 3.0



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# **The Book of Knowledge**

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# Quality

The Pride &  
The Commitment

## *Six Sigma Quality Overview*





# *Objectives*

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**GE's Quality goals**

**What is Six Sigma?**

**What is Design for Six Sigma  
(DFSS)**

**Six Sigma and DFSS successes**

**Focus on the Customer**

**Going Forward with Six Sigma**



# Quality Challenges in 2000+



**Six Sigma and the Customer - Despite real progress, our focus on the customer must intensify. E-business, which is all about the customer, will help. But we have to make customer satisfaction a GE value and reward those who demonstrate this value in the same way we reward boundaryless behavior.**

## Focus

**Six Sigma  
@ the  
Customer**

**Enabling  
e-  
Business**

**As always ... incremental moves won't get you there!**



## *Striving Towards Six Sigma*

<b>S</b>	<b>PPM</b>
2	308,537
3	66,807
4	6,210
5	233
6	3.4

**Process Capability**      **Defects per Million Opportunities**

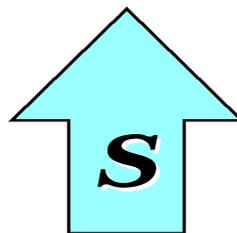
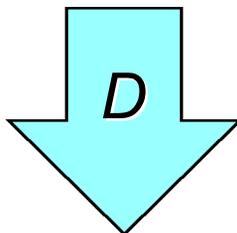


## Two Meanings of “Sigma”



- The term “**sigma**” is used to designate the distribution or spread about the mean (average) of any process or procedure.
- For a business or manufacturing process, the **sigma capability (z-value)** is a metric that indicates how well that process is performing. The higher the sigma capability, the better. Sigma capability measures the capability of the process to perform **defect-free** work. A defect is anything that results in customer dissatisfaction.

As defects  
go down...



the Sigma Capability  
goes up



# Quality

## **The Classical View of Quality**

**“99% Good” (3.8s)**

-  • **20,000 lost articles of mail per hour**
-  **Unsafe drinking water almost 15 minutes each day**
-  **5,000 incorrect surgical operations per week**
-  **2 short or long landings at most major airports daily**
-  **200,000 wrong drug prescriptions each year**
-  **No electricity for almost 7 hours each month**

## **The Six Sigma View of Quality**

**“99.99966% Good” (6s)**

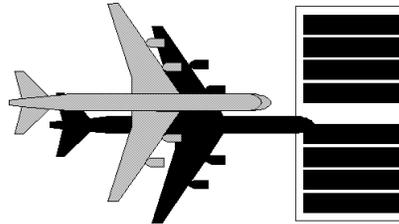
-  **Seven lost articles of mail per hour**
-  **One minute of unsafe drinking water every seven months**
-  **1.7 incorrect surgical operations per week**
-  **One short or long landing at most major airports every five years**
-  **68 wrong drug prescriptions each year**
-  **One hour without electricity every 34 years**



## *An Example Driven by the Customer*

**Compare:**

**6s**



**Less than  $\frac{1}{2}$  failure per million flights**

**~ 35,000-50,000 lost items per million**



**3.5s**



## *GE's Quality Goal*

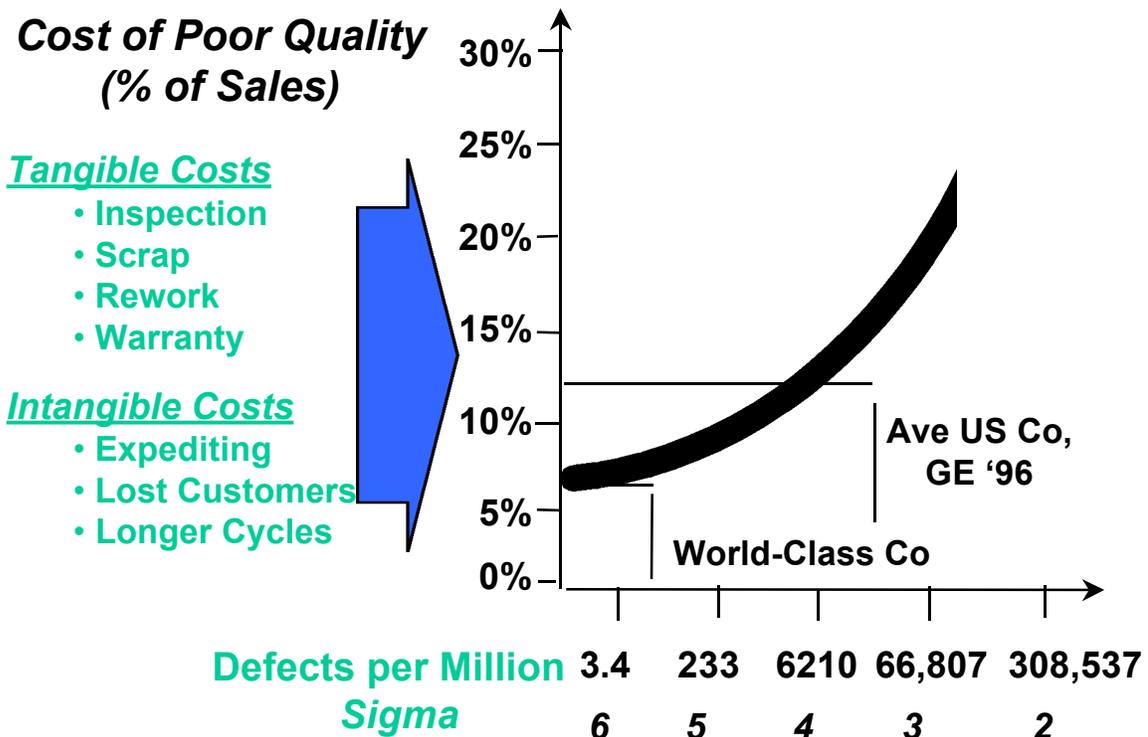
- Although GE's quality levels were equal to or better than those of its competitors, in late 1995  
*GE set a stretch goal to reach 6s quality by 2000*
- **GE 1996: 3<sup>1</sup>/<sub>2</sub>s**
  - 35,000 defects per million opportunities
  - 96.5% good results
- **GE goal: 6s**
  - 3.4 defects per million opportunities
  - 99.9997% good results - *nearly flawless performance*
- To reach goal, must reduce defect rate by factor of 10,000!

**Enormous challenge**



## Why Does GE Need a Quality Initiative?

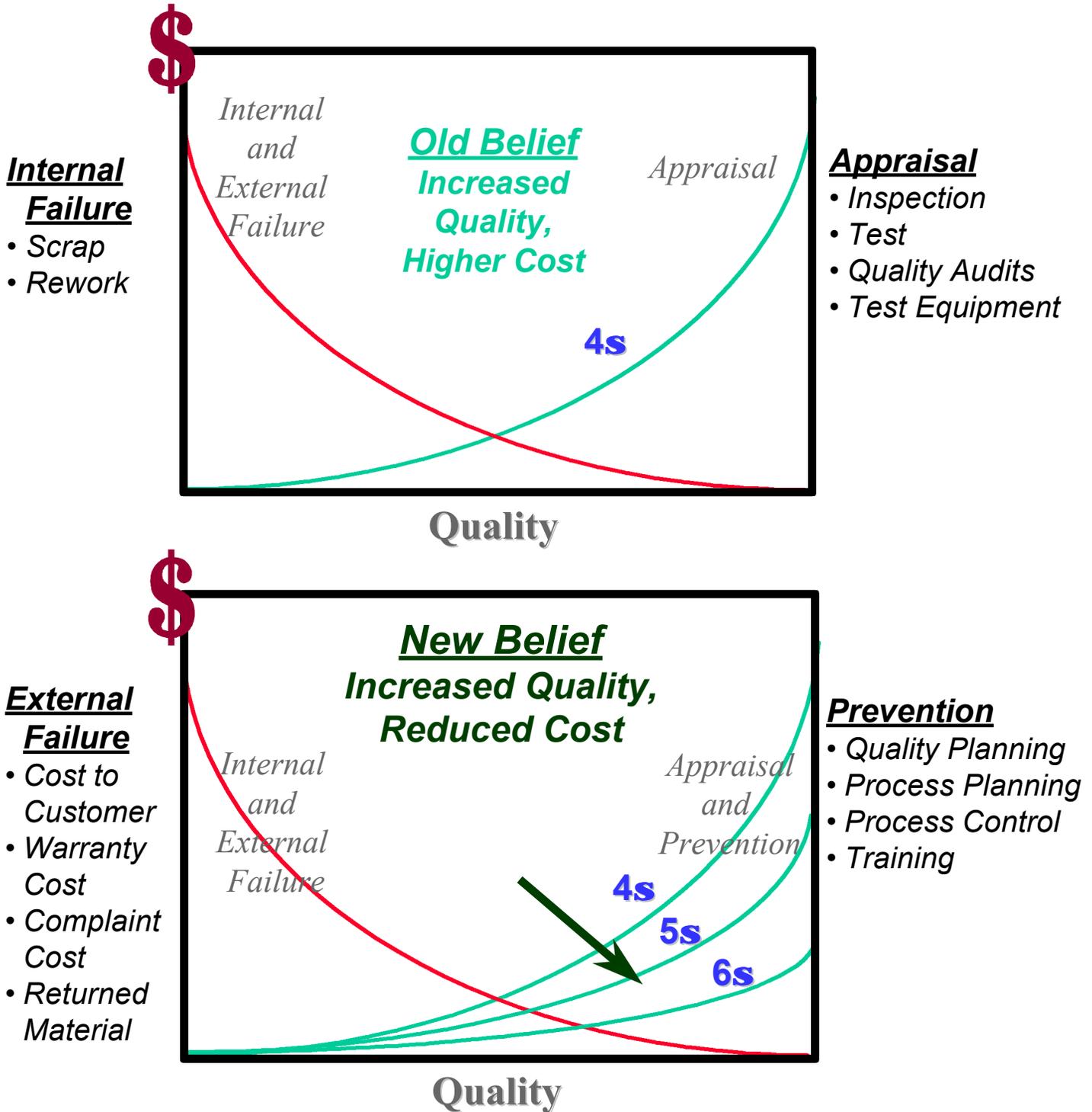
- Meet customer expectations for higher quality
- Provide a competitive differentiator in the market
- Build greater pride and satisfaction in the GE team
- Drive other key goals: productivity and growth



**Enormous opportunity**



# Cost of Quality—A Change in Mindset





## *Achieving GE's Quality Goal: Six Sigma*

- **Quantitative, data-driven “DMAIC” methodology - Define, Measure, Analyze, Improve, Control - based on statistics, process understanding and process control**
- **Developed by Motorola; used successfully by TI, AlliedSignal, ...**
- **Internal focus: *improve existing processes* - manufacturing, business transactions, ...**
- **Uses trained teams**
  - ***Champions: business leaders*, provide resources and support implementation**
  - ***Master Black Belts: experts and culture-changers*, train and mentor Black Belts/Green Belts**
  - ***Black Belts: lead Six Sigma project teams***
  - ***Green Belts: carry out Six Sigma projects related to their jobs***

**Driver for cost savings**



## Definitions

- **CTQ** *Critical-to-Quality attribute, An attribute important to the customer - A “Y” Response*
- **Opportunity** *Any measurable event that provides a chance of not meeting specification limits of a CTQ*
- **Defect** *Anything that results in customer dissatisfaction. Anything that results in a non-conformance.*
- **DPMO** *Defects Per Million Opportunities*
- **Sigma Capability (z-value)** *The probability of defect, a measure of process capability, measured in units of standard deviations*
-  **MBB** *Master Black Belt - A Full-Time Teacher and Mentor of Black Belts.*
-  **BB** *Black Belt - Full-Time, Trained Resource who Completes 5-10 Projects to Reduce Defects*
-  **GB** *Green Belt - Trained Resource who Completes 2 Projects to Reduce Defects*



# *Who Gets the Benefits of Six Sigma? The View at the End of '98*

To Shareholders   To Customers

*What they have  
been  
( '95 - '98)*

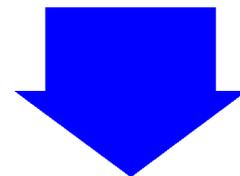
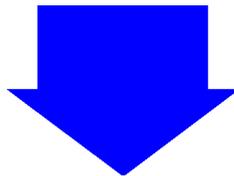
**90% on  
internal  
productivity**

**10%  
fixing  
customer  
issues**

*Going forward  
( '99 - )*

**50-70% on  
cost  
reduction**

**30-50% on  
delighting  
customers**



***Objective:***

***Cost  
reduction***

***Revenue  
growth***

- **Need to make customers feel the benefits**
- **A key element for GE's growth**



## '99 - : *Delighting Customers*

- DFSS-based new products
- Making customers successful
  - Customer-centric metrics
  - GE's fulfillment process
  - Six Sigma *with* the customer
- e-business

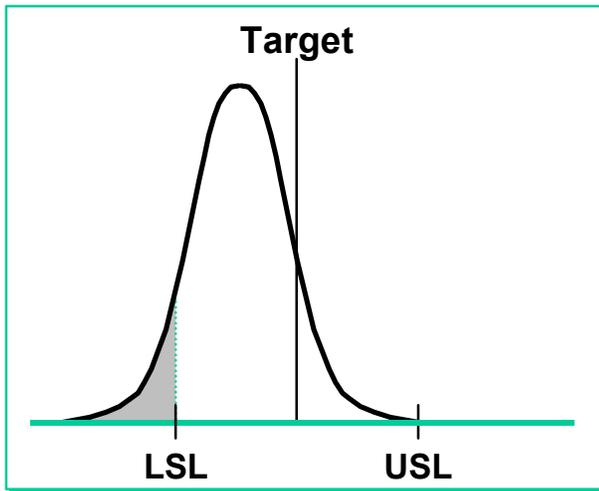


**Make customers feel Six Sigma**

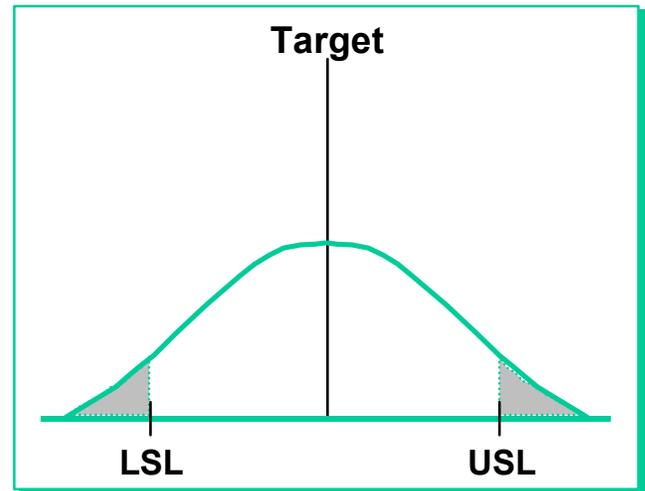


# The Statistical Objective of Six Sigma

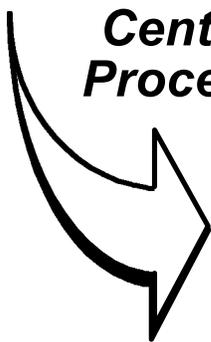
**Process Off Target**



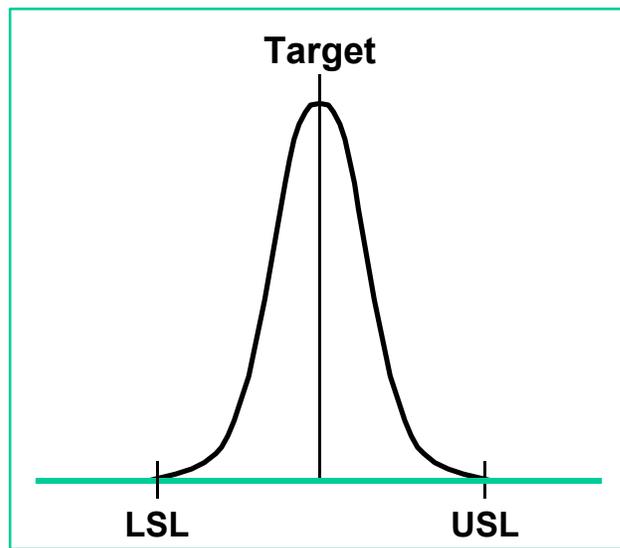
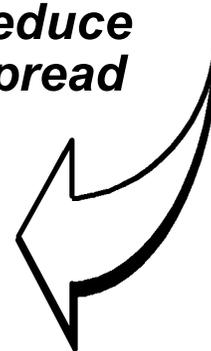
**Excessive Variation**



**Center Process**



**Reduce Spread**

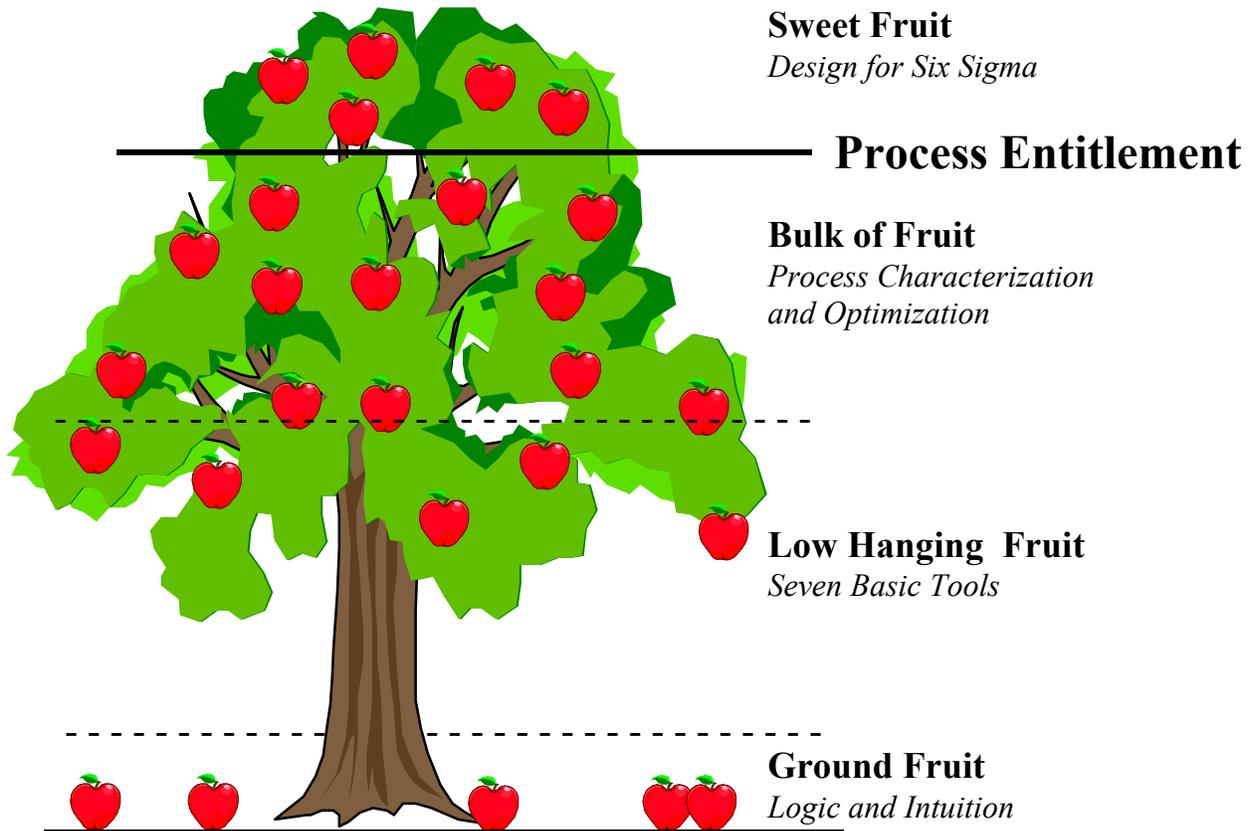


 Defects

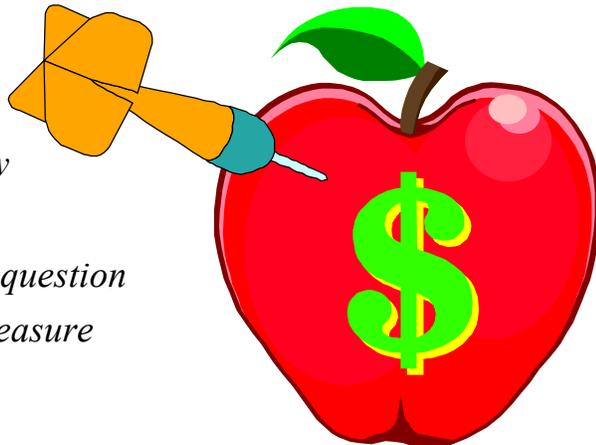
**Reduce Variation & Center Process—Customers feel the variation more than the mean**



# Harvesting the Fruit of Six Sigma

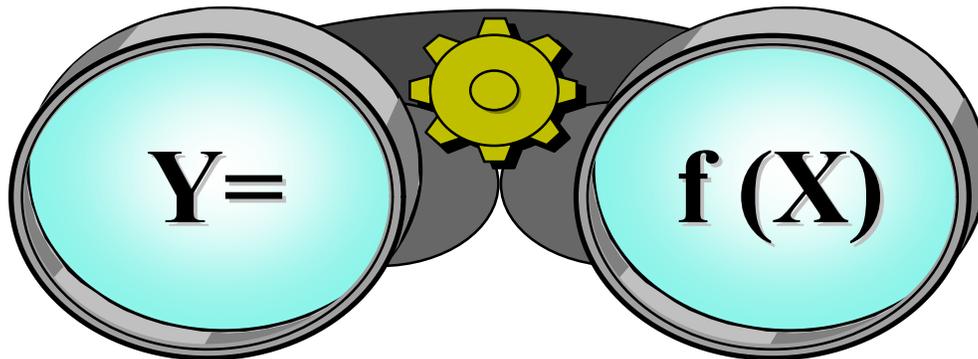


*We don't know what we don't know  
We can't act on what we don't know  
We won't know until we search  
We won't search for what we don't question  
We don't question what we don't measure  
Hence, We just don't know*





## *The Focus of Six Sigma*



*To get results, should we focus our behavior on the Y or X?*

- Y
- Dependent
- Output
- Effect
- Symptom
- Monitor

- $X_1 \dots X_n$
- Independent
- Input-Process
- Cause
- Problem
- Control

*Historically the Y, ... with Six Sigma the Xs*



## ***D-M-A-I-C***

***For Each Product or Process CTQ –  
Define, Measure, Analyze, Improve, & Control***

$$Y = f(X)$$

***Define*** 1. Customer expectations of the process?

***Measure*** 2. What is the frequency of defects?

***Analyze*** 3. Why, when, and where do defects occur?

***Improve*** 4. How can we fix the process?

***Control*** 5. How can we make the process stay fixed?



# *GE Design for Six Sigma (DFSS): Product Quality Methodology*

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## **Define**

Step 1. Identify Product/Process Performance and Reliability CTQ's and Set Quality Goals.

## **Measure**

Step 2. Perform CTQ flowdown to subsystems and components.

Step 3. Measurement System Analysis / Capability.

## **Analyze**

Step 4. Develop Conceptual Designs (Benchmarking, Tradeoff Analysis).

Step 5. Statistical Analysis of any relevant data to assess capability of conceptual designs.

Step 6. Build Scorecard with initial product/process performance and reliability estimates.

Step 7. Develop Risk Assessment.

## **Design**

Step 8. Generate and Verify system & subsystem models, allocations and transfer functions.

Step 9. Capability Flow Up for all subsystems and gap identification

- low Zst
- Lack of Transfer function
- Unknown process capability.

## **Optimize**

Step 10. Optimize Design

- statistical analysis of variance drivers
- robustness
- error proofing.

Step 11. Generate purchasing and manufacturing specifications and verify measurement system on X's.

## **Verify**

Step 12. Statistically confirm that product process matches predictions.

Step 13. Develop manufacturing and supplier control plans.

Step 14. Document and transition.



## *Design for Six Sigma - DFSS* *Changing the Game for GE*

**Create products that have 6s Quality  
“designed in”:**

- **Wow our customers with 6s performance on their CTQs**
- **Have 6s reliability**
- **Have 6s manufacturability**
- **Have high performance/cost ratios**

- **Outside-In: Focus on meeting customer's CTQs**
- **Insight through variance: Statistical design to reduce performance variability**



## *Some GE DFSS Products*

### 1997

Performix™ X-ray Tube,  
GEMS

### 1998

LightSpeed™ CT, GEMS

Small Motor products,  
GEIndS

Ultem 1285 Tupperware  
material, GEP

Spectra™ Gas Range, GEA

### 1999

Spectra™ Electric Range,  
GEA

Advantium™ Speed Oven,  
GEA

Triton Dishwasher, GEA

ConstantColor™ Ceramic  
Metal Halide  
Lamps, GEL

T5 Fluorescent Lamp, GEL

Signa OpenSpeed™ MR,  
GEMS

OQ 1050C Lexan, GEP

Locomotive Upgrades &  
Services, GETS

### 2000 or later New Designs

Innova™ 2000 CV Digital X-Ray,  
GEMS

Senographe<sup>â</sup> 2000D Mammo  
Digital X-Ray, GEMS

Launcelot™ Breaker, GEIndS

119 mm GEHI Dishwasher Motor,  
GEIndS

Arctica™ SxS Refrigerator GEA

Thermo™ Electric Range, GEA

Ecolux™ Low-Hg Fluorescent  
Lamp, GEL

Locomotive Emissions System,  
GETS

Dense Pack Steam Turbine  
Upgrade, GEPS

FB Gas Turbine, GEPS

H Gas Turbine, GEPS

CF34-80/E Engine, GEAE

CFM56-E5 (“Tech56”) Engine,  
GEAE

GE90-115B Engine, GEAE

...



# LightSpeed<sup>TM</sup> CT Scanner

## GEs First DFSS System ('98): Full Use of Six Sigma/DFSS Tools

- Key customer CTQs identified
  - Image quality
  - Speed
  - Software reliability
  - Patient comfort
- Disciplined systems approach: 90 system CTQs
- 33 Six Sigma (DMAIC) or DFSS projects
- Part CTQs verified before systems integration



## Leading-Edge Technology

- World's first 16-row CT detector
- Multi-slice data acquisition
- 64-bit RISC computer architecture



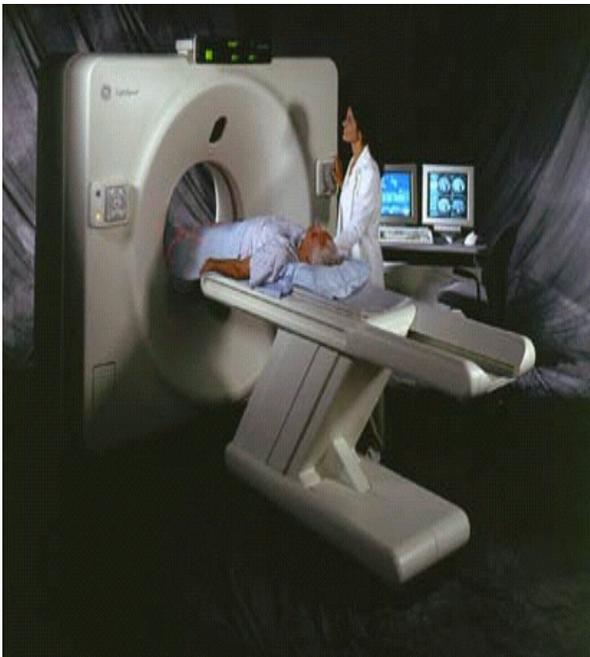
Head



Abdomen

## Results

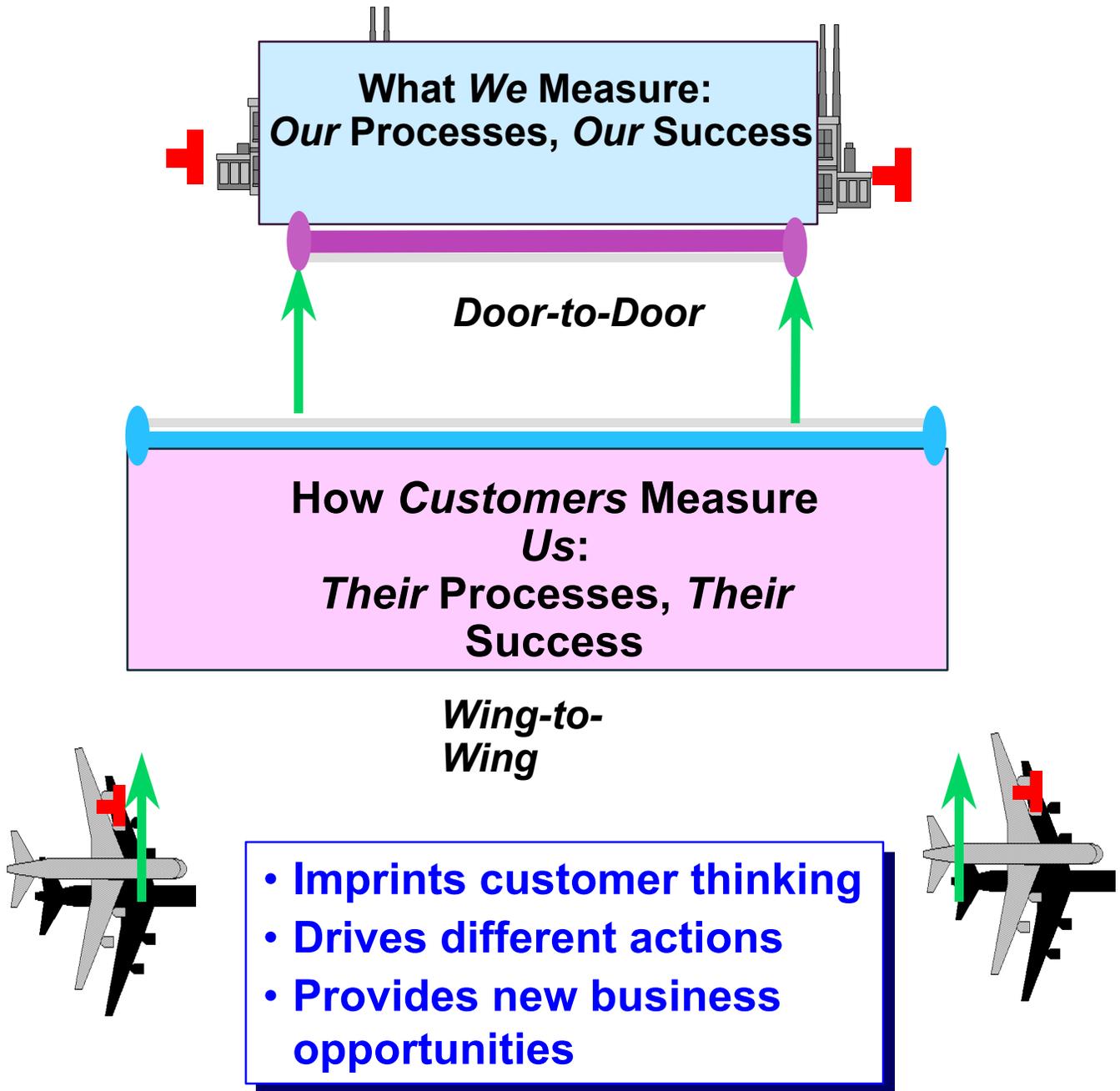
- Better image quality
  - Earlier, more reliable diagnoses
  - New applications: vascular imaging, pulmonary embolism, multi-phase liver studies, ...
- Much faster scanning:
  - Head: from 1 min to 19 sec (9 million/yr)
  - Chest/abdomen: from 3 min to 17 sec (4 million/yr)
- Clinical productivity up 50%
- 10x improvement in software reliability
- Patient comfort improved - shorter exam time
- Development time shortened by 2 years
- High market share; significant margin increase



**“Biggest breakthrough in CT in a decade,” Gary Glazer, Stanford**



# *Customer-Centric Metrics: Making Customers Successful*

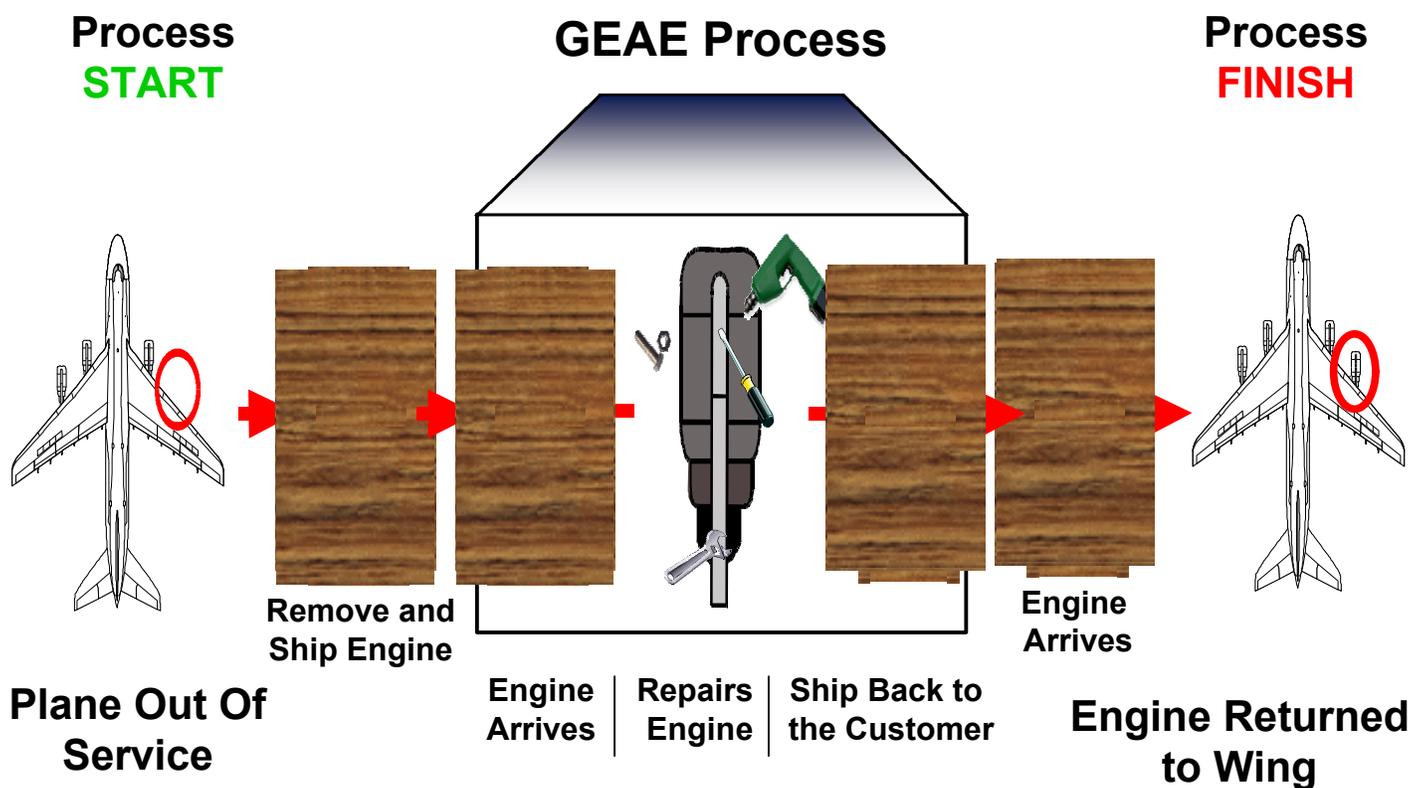




# Case Study: GE Aircraft Engines

Did AE Have A Customer-Focused View?

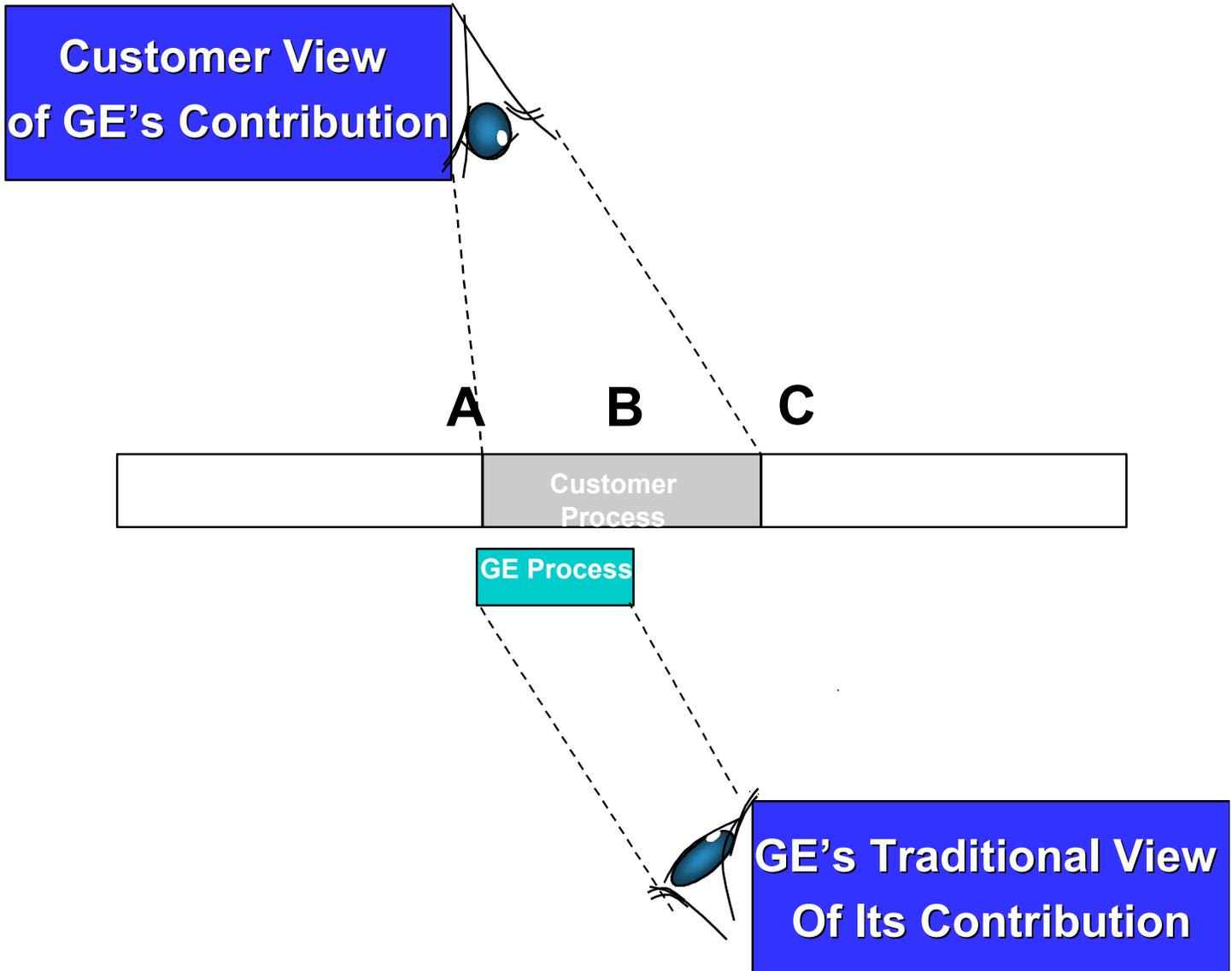
## Customer Process:





# Road Map To Customer Impact

## The Eye Of The Beholder

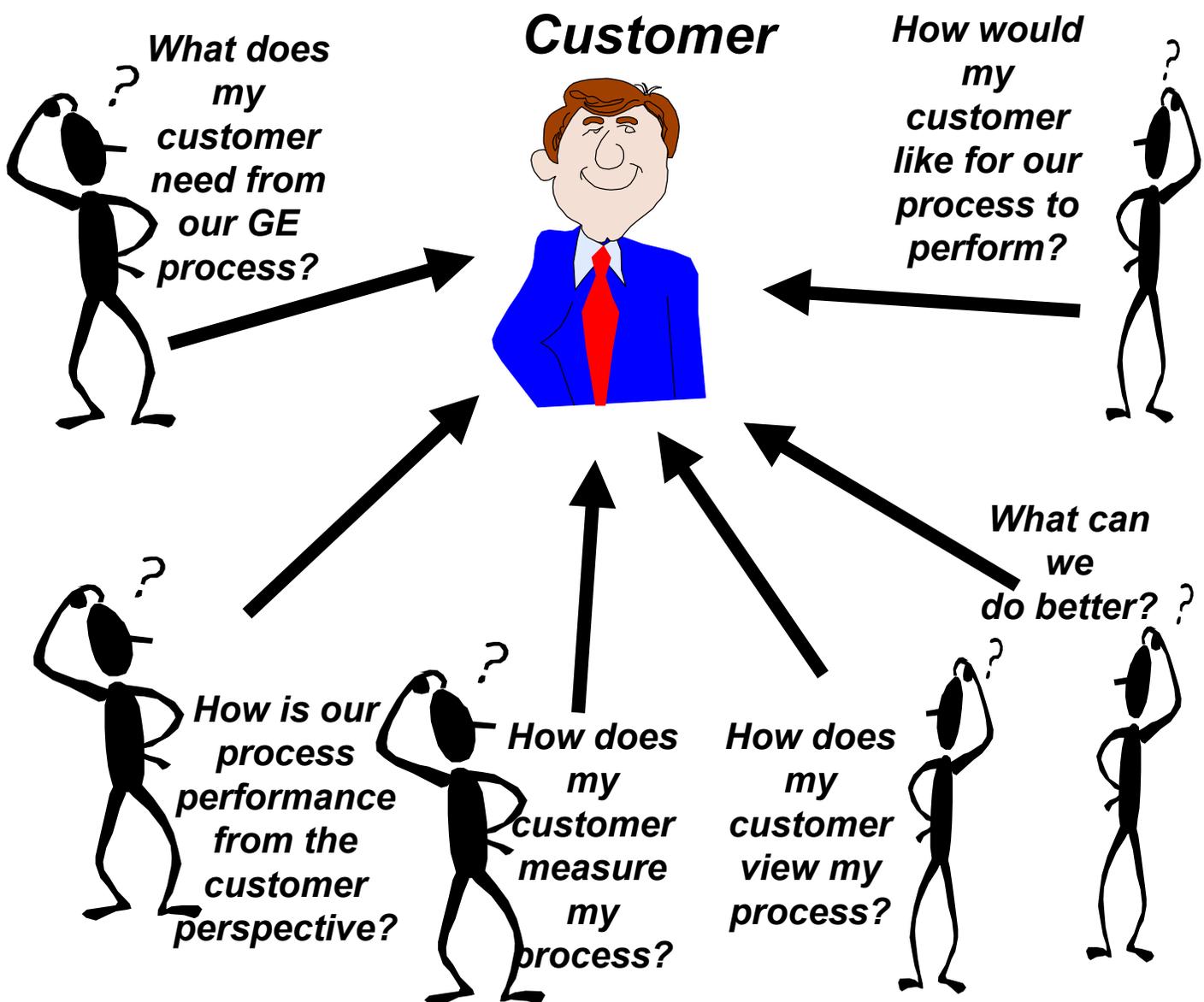


1. GE Fully Met Its Contractual Obligations (AB)
2. Customer's View Determined By Their Process Performance (AC)



# *Design For Customer Impact<sup>SM</sup> (DFCI)*

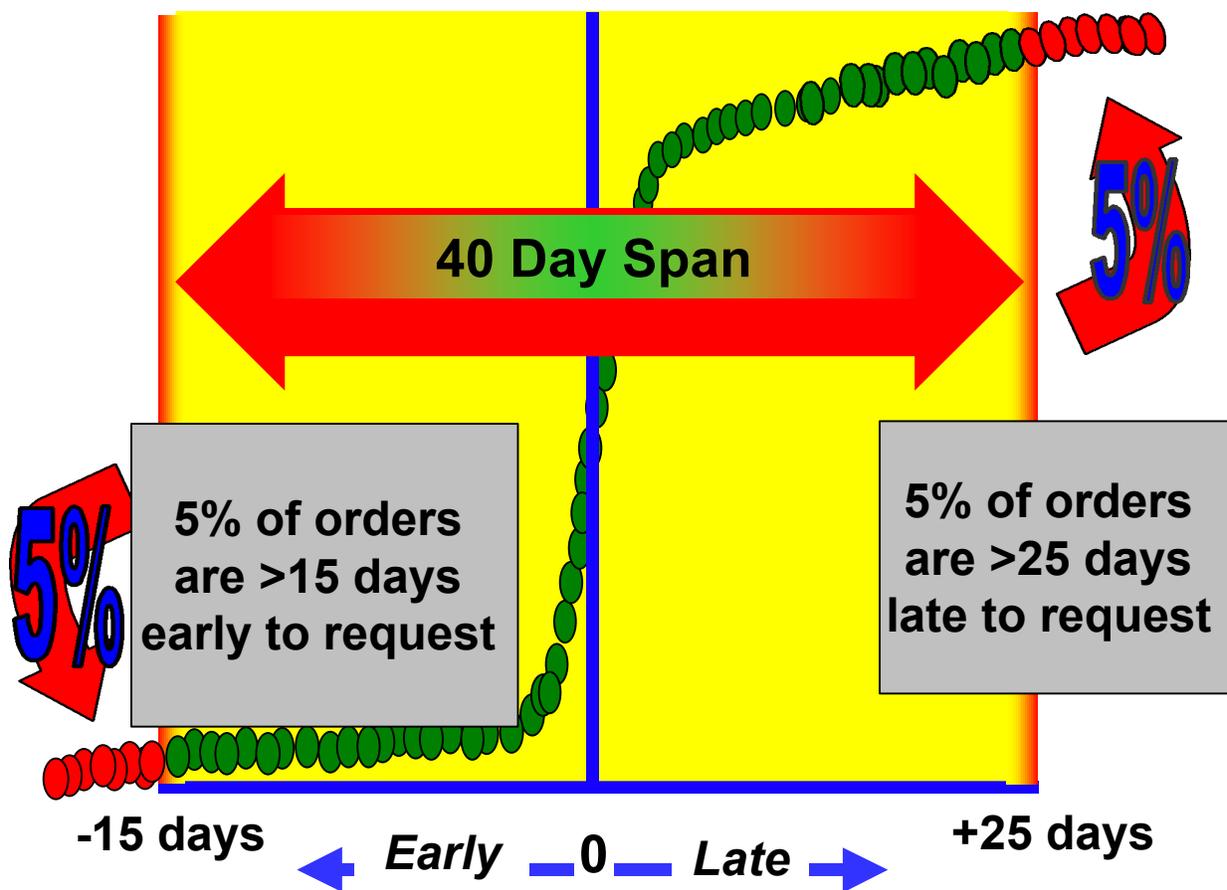
■ **The focus for 6s quality is characterized by a continuous and thorough understanding of our customer. We need to ensure our customers feel and see the benefits of 6s quality.**





## Redesigning the Fulfillment Process

*Give customers what they want, when they want it*



**Customer Want Date**

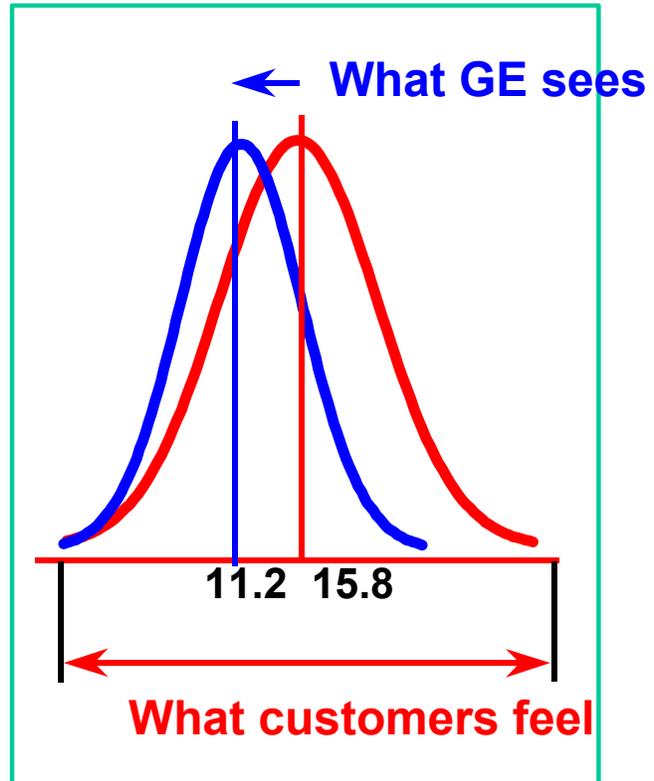
**The reality, early '99: Not meeting customer wants**



# Insight Through Variance

Delivery cycle time (days)

Baseline	Improved?
12	27
24	7
13	15
7	4
16	18
8	6
20	23
25	6
14	2
10	24
11	2
30	6
16	5
Mean 15.8	→ 11.2
Std Dev 7.0	→ 9.0



- **Using mean-based thinking, we improve average performance by 29%, and break out the champagne ...**
- **But our customer only feels the variance and cancels the next order!**

**Customers feel variance, not the mean**



## *Company-Wide Fulfillment Push*

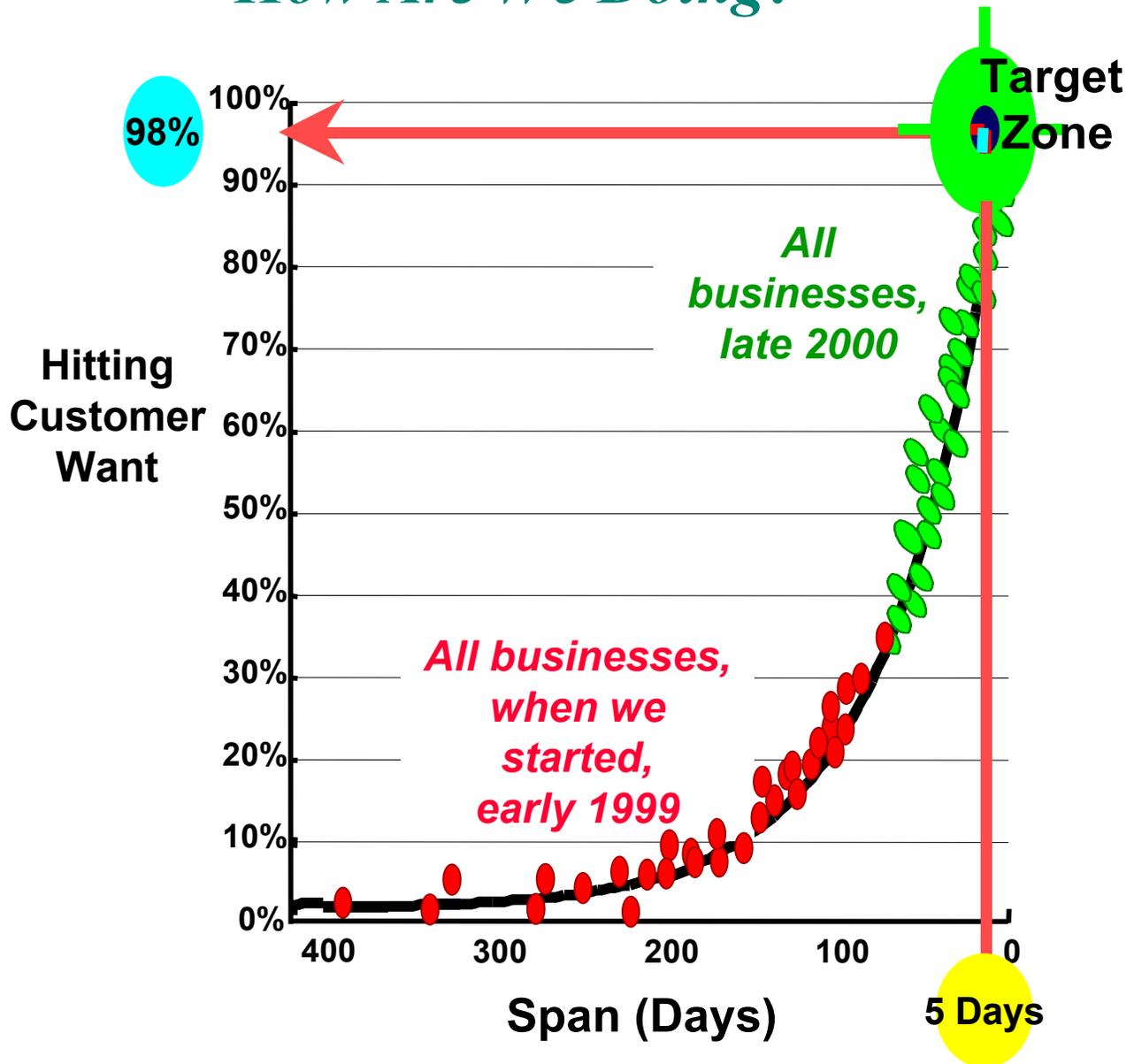
- **Goals:**
  - **Median delivery at or before customer request (i.e.,  $< 0$ )**
  - **5 day spans (or less) on all major product deliveries**
- **Use Six Sigma and DFSS**
  - **Map the order-to-delivery process**
  - **Analyze the data to find root causes of misses**
  - **Understand underlying GE behaviors which lead to misses**
  - **Redesign the process to eliminate variance**

**Most tangible 6s accomplishment for our customers**



# Redesigning the Fulfillment Process

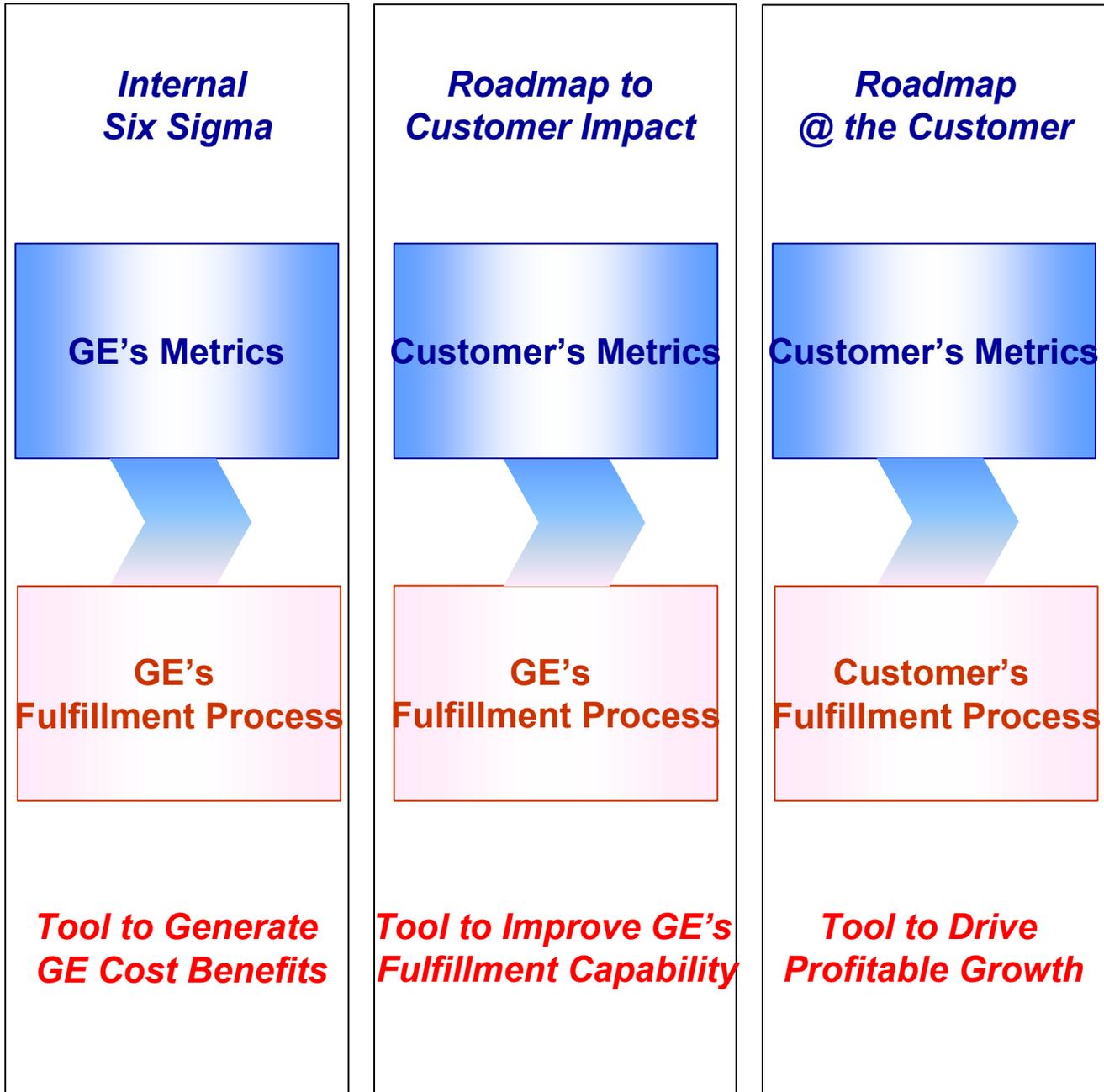
## How Are We Doing?



**Huge progress ... 5x more customers get their want!**



# *Six Sigma and the Customer*



**You need all three to win the game!**



# Six Sigma @ the Customer

## What?

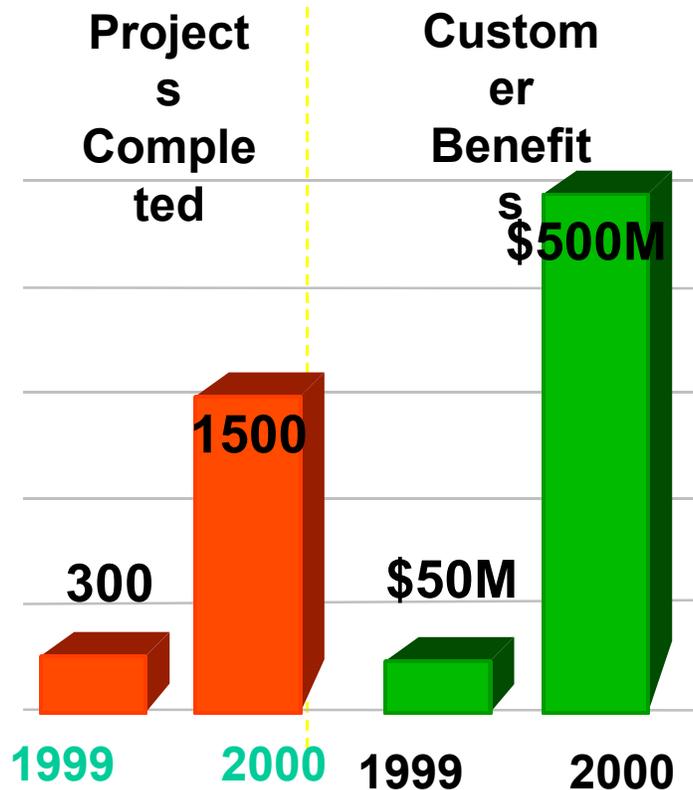
- Projects done by
  - GE BB/GBs or
  - Customer BB/GBs, trained or mentored by GE BB/GBs
- Address customer Y's
- Address customer or GE X's

## Why?

- Build key customer relationships
- Grow share on GE sales
- Identify new opportunities

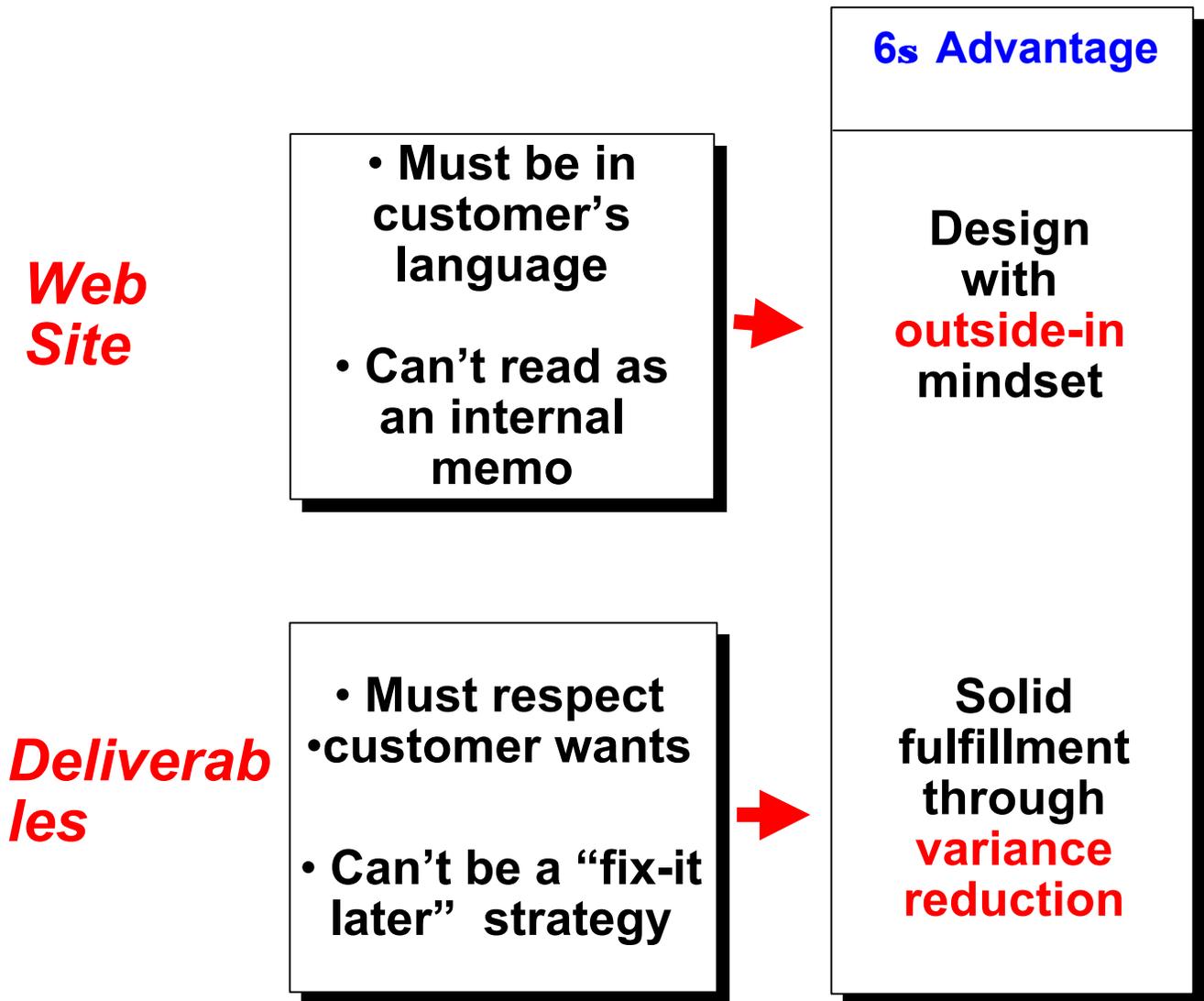
	Customer	GE
Y's	@ the Customer	GE DMAIC/DFSS Projects
X's	@ the Customer	@ the Customer

## 6s Projects @ the Customer





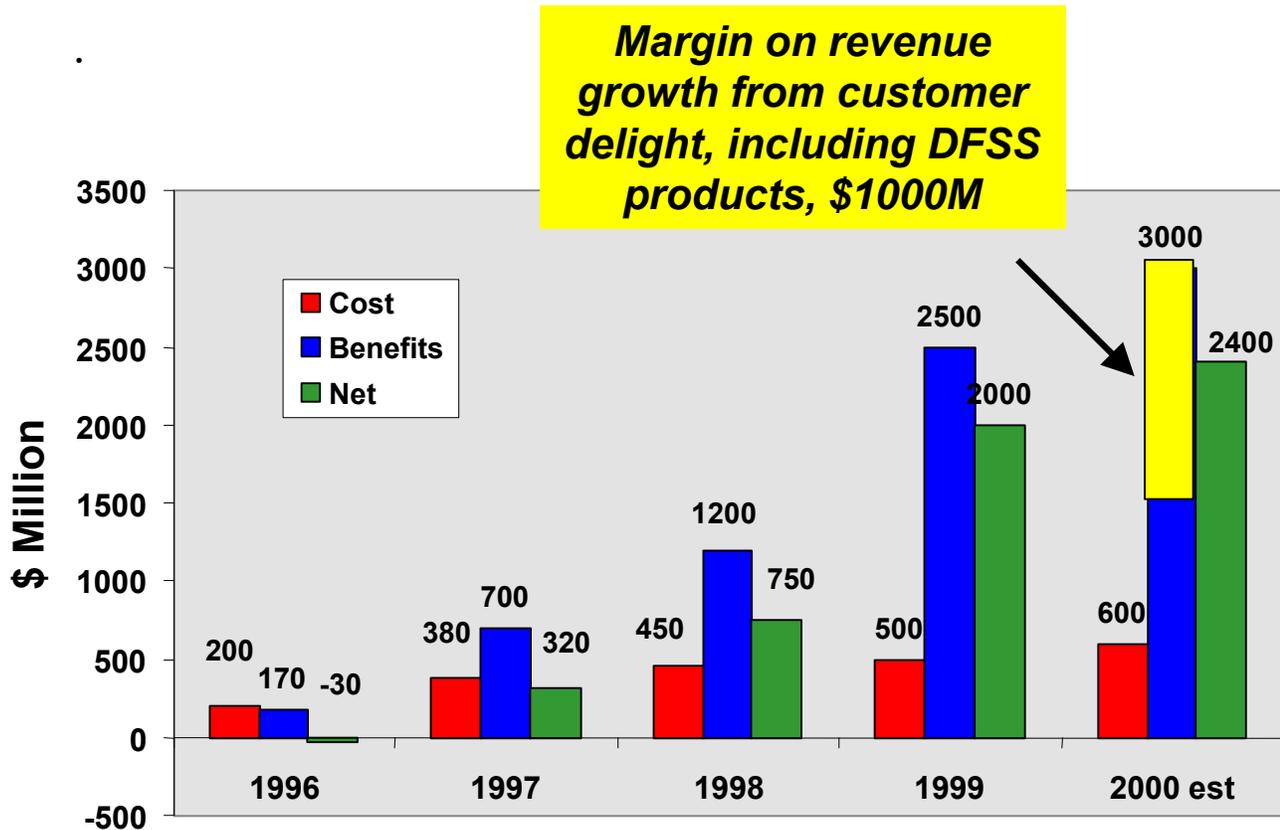
## Six Sigma: Enabling e-Business



**Six Sigma: A competitive differentiator in e-business!**



## Six Sigma Financial Benefits, 2000



**Now getting significant benefits from customer delight, including DFSS**

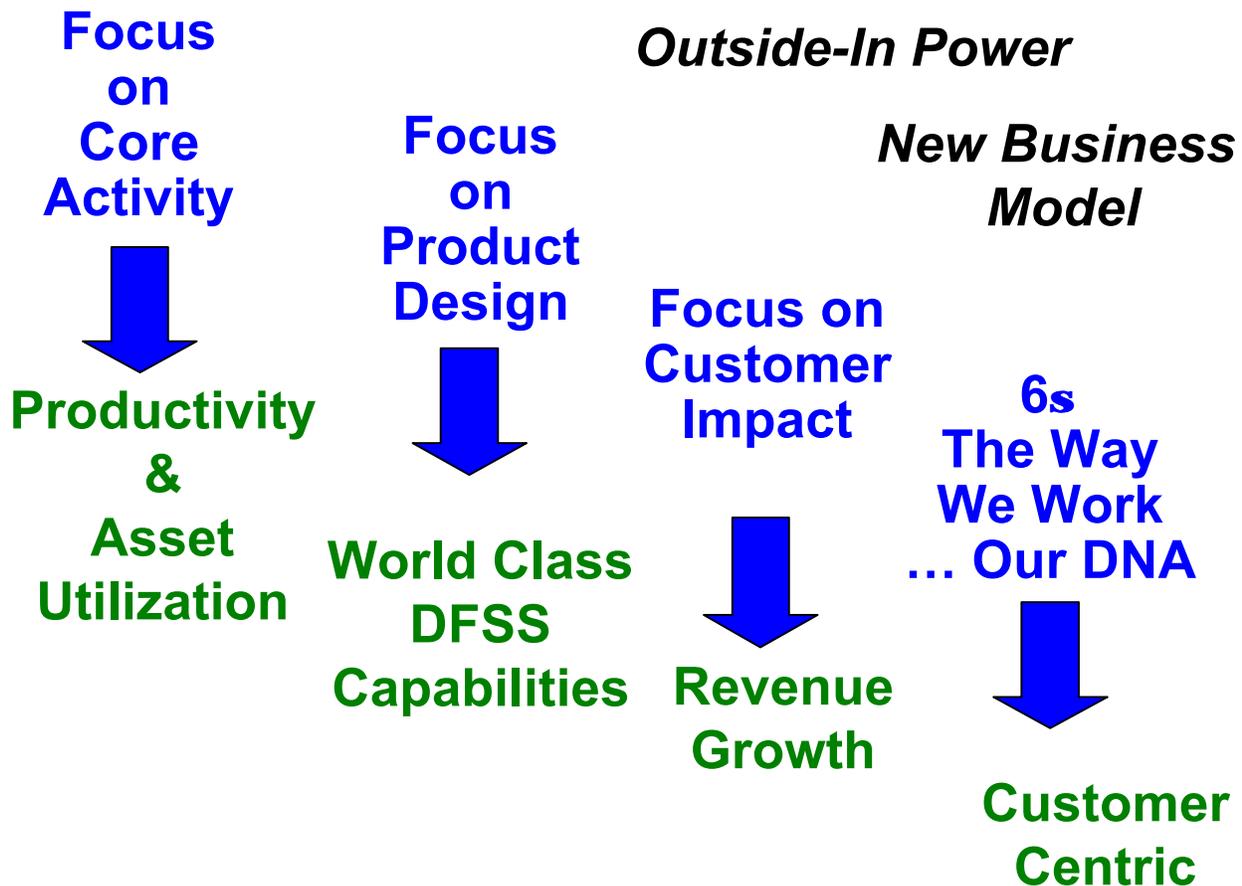


# The Changing Focus of Six Sigma

'95      '96      '97      '98      '99      '00      '00+

## Getting Started

### Expanding Horizons



**Six Sigma - a dynamic, living initiative**



## Summary & “Take-Aways”

**Six Sigma: From...** tools to improve  
Quality  
**To...** business Game  
Changer

- Cost reduction from productivity
- Knowing what matters to customer
- Growth from DFSS products and customer satisfaction
- Fulfillment for e-business
- A new leadership paradigm
- *And ...* record financial returns

“This is the most important initiative this Company has ever undertaken. (It) will fundamentally change our Company forever.  
John F. Welch, Jr. Letter to GE Officers, May 18, 1996



## *Take Aways—Course Overview*

- *Six Sigma focuses on:*
  - *the customer - critical to quality characteristics*
  - *data driven improvements*
  - *the inputs (Xs) of the process*
  - *reducing or eliminating defects*
  - *reducing variation*
  - *increasing process capability*
- *To efficiently drive improvements, the focus must be on the inputs (Xs) to the process.*
- *The word “sigma” is used in two ways, to describe capability and to describe variation.*
- *As DPMO goes down, process capability goes up.*



## Course Objectives

- *Discuss the steps of the 12 Step Process: Define, Measure, Analyze, Improve, & Control*
- *Recognize how statistics can be applied to the problem solving process*
- *Illustrate the problem solving flow*
  - *Practical Problem => Statistical Problem => Statistical Solution => Practical Solution*
- *Learn how to apply the Six Sigma tools and methodology to your project*
  - *Process Mapping*
  - *Fishbone*
  - *FMEA*
  - *Gage R&R*
  - *Hypothesis Testing*
  - *Design of Experiments*
  - *Mistake Proofing*
  - *Control Charts*
- *Understand process capability and its impact on quality*



# The 12 Step Process

Step	Description	Focus	Tools	SSQC Deliverables
<b>Define</b>				
A	Identify Project CTQs			Project CTQs (1)
B	Develop Team Charter			Approved Charter (2)
C	Define Process Map			High Level Process Map (3)
<b>Measure</b>				
1	Select CTQ Characteristics	Y	Customer, QFD, FMEA	Project Y (4)
2	Define Performance Standards	Y	Customer, Blueprints	Performance Standard for Project Y (5)
3	Measurement System Analysis	Y	Continuous Gage R&R, Test/Retest, Attribute R&R	Data Collection Plan & MSA (6), Data for Project Y (7)
<b>Analyze</b>				
4	Establish Process Capability	Y	Capability Indices	Process Capability for Project Y (8)
5	Define Performance Objectives	Y	Team, Benchmarking	Improvement Goal for Project Y (9)
6	Identify Variation Sources	X	Process Analysis, Graphical Analysis, Hypothesis Tests	Prioritized List of all Xs (10)
<b>Improve</b>				
7	Screen Potential Causes	X	DOE-Screening	List of Vital Few Xs (11)
8	Discover Variable Relationships	X	Factorial Designs	Proposed Solution (13)
9	Establish Operating Tolerances	Y, X	Simulation	Piloted Solution (14)
<b>Control</b>				
10	Define & Validate Measurement System on X's in Actual Application	Y, X	Continuous Gage R&R, Test/Retest, Attribute R&R	MSA
11	Determine Process Capability	Y, X	Capability Indices	Process Capability Y, X (15)
12	Implement Process Control	X	Control Charts, Mistake Proof, FMEA	Sustained Solution (15), Documentation (16),



## *Define Objectives*

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- *To identify the process or product for improvement.*
- *Explain and show examples of VOC tools and VOC data techniques.*
- *To identify customers and translate the customer needs into CTQs.*
- *To develop a team charter.*
  - *Problem/goal statement, project scope, business case, team roles, and milestones*
- *To develop a high-level process map for the most significant four to five steps of the process.*
- *To obtain formal project approval.*



## *Define - Beginning With an Idea*



- *Who's the customer?*
- *What does he/she think is critical to quality?*
- *Who speaks for the customer?*
- *What's the business strategy?*
- *Who in the business holds a stake in this?*
- *Who can help define the issues?*
- *What are the processes involved?*



## *A Great Project Should...*

- *Be clearly bound with defined goals*
  - *If it looks too big, it is*
- *Be aligned with critical business issues and initiatives*
  - *It enables full support of business*
- *Be felt by the customer*
  - *There should be a significant impact*
- *Work with other projects for combined effect*
  - *Global or local “Beta Themes”*
- *Show improvement that is locally actionable*
  - *Difficult to manage improvements in Schenectady from the field*
- *Relate to your day job*





## *Sources of Project Ideas*

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- *Quality Function Deployment (QFD)*
- *Customer dashboards*
- *Surveys and scorecards*
- *Active beta themes*
- *Other projects available for leverage*
- *Brainstorming*
- *Analysis of critical processes*
- *Six Sigma Quality Project Tracking Database*
- *Discussions with customer*
- *Financial analysis*
- *Internal problems*



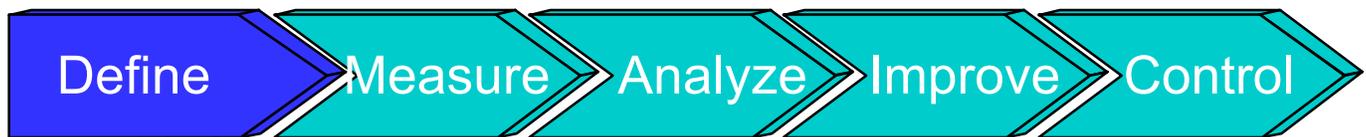
## *Project Selection*

- *Success Factors*
  - *Project scope is manageable*
  - *Project has identifiable defect*
  - *Project has identifiable impact*
  - *Adequate buy-in from key stakeholders*
- *To be successful ¼*
  - *Set up project scope charter and have it reviewed*
  - *Measure where defects occur in the process*
  - *Assess and quantify potential impact up-front*
  - *Perform stakeholder analysis*
- *Common Pitfalls*
  - *Resourcing of project is inadequate*
  - *Duplicating another project*
  - *Losing project momentum*
  - *Picking the easy X, not the critical X*
- *Avoiding Pitfalls ¼*
  - *Identify and get committed resources up-front*
  - *Research database and translate where possible*
  - *Set up milestones and communications plan*

**Optimize on the Success Factors to Maximize 6 Sigma Project Benefits**



## *Define Phase - Identify Project CTQs*



### **A. Identify Project CTQs**

*Deliverable: Identify Customer(s) and Project CTQs*

### **B. Develop Team Charter**

*Deliverables:*

- 1) Develop The Business Case*
- 2) Develop The Problem and Goal Statements*
- 3) Determine Project Scope*
- 4) Select Team & Define Roles*
- 5) Set Project Milestones*

### **C. Define Process Map**

*Deliverable: High Level Process Map Connecting the Customer to the Process*



## Who Is the Customer?



- **Customer** - Whoever receives the output of your process.
  - Internal Customer Vs. External Customer
- **Output** - The material or data that results from the operation of a process.
- **Process** - The activities you must perform to satisfy your customer's requirements.
- **Input** - The material or data that a process does something to or with.
- **Supplier** - Whoever provides the input to your process.

**What is critical to the quality of the process?  
...according to your customer!**

# CTQ



## *Sources of Existing Customer Data*

- *Business Goals*
- *Customer Surveys*
- *Complaints*
- *Benchmarking Data*
- *Executive Level Discussions*
- *Job Specific Discussions*
- *Market Strategies*
- *Scorecards & Dashboards*
- *Focus Groups*

*If we can measure it, we can develop strategies to meet customer needs ..*



## *Voice of the Customer (VOC)*

**Definition: What is critical to the quality of the process ....**

**According to your customer !**

**A key method to do this: VOC tools  
The next section will focus on these tools and VOC data.**

### **3 Key VOC tools:**

- **Surveys**
- **Focus Groups**
- **Interviews**



## Research Method Pros / Cons

### Surveys

#### Pros:

- Lower Cost approach
- Phone response rate 70-90%
- Mail surveys require least amount of trained resources for execution
- Can produce faster results

#### Cons:

- Mail surveys...can get incomplete results, skipped questions, unclear understanding
- Mail surveys...20-30% response rate
- Phone surveys... interviewer has influential role, can lead interviewee producing undesirable

### Focus Groups

#### Pros:

- Group interaction generates information
- More in-depth responses
- Excellent for getting CTQ definitions
- Can cover more complex questions or qualitative data

#### Cons:

- Learning's only apply to those asked...difficult to generalize
- Data collected typically qualitative vs. quantitative
- Can generate too much anecdotal information

### Interviews

#### Pros:

- Can tackle complex questions and a wide range of information
- Allows use of visual aids
- Good choice when people won't respond willingly and/or accurately by phone/mail

#### Cons:

- Long cycle time to complete
- Requires trained, experienced interviewers



## *Customer Information Issues*

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### **Real Needs vs. Stated Needs**

- Xerox: Focus On Copiers Or Documents?

### **Perceived Needs**

- A Hershey Bar or Godiva Chocolates?

### **Intended vs. Actual Usage**

- Is A Screwdriver Also A Hammer?



### **Internal Customers**

- Turf Wars And “Not Invented Here”

### **Effectiveness vs. Efficiency Needs**

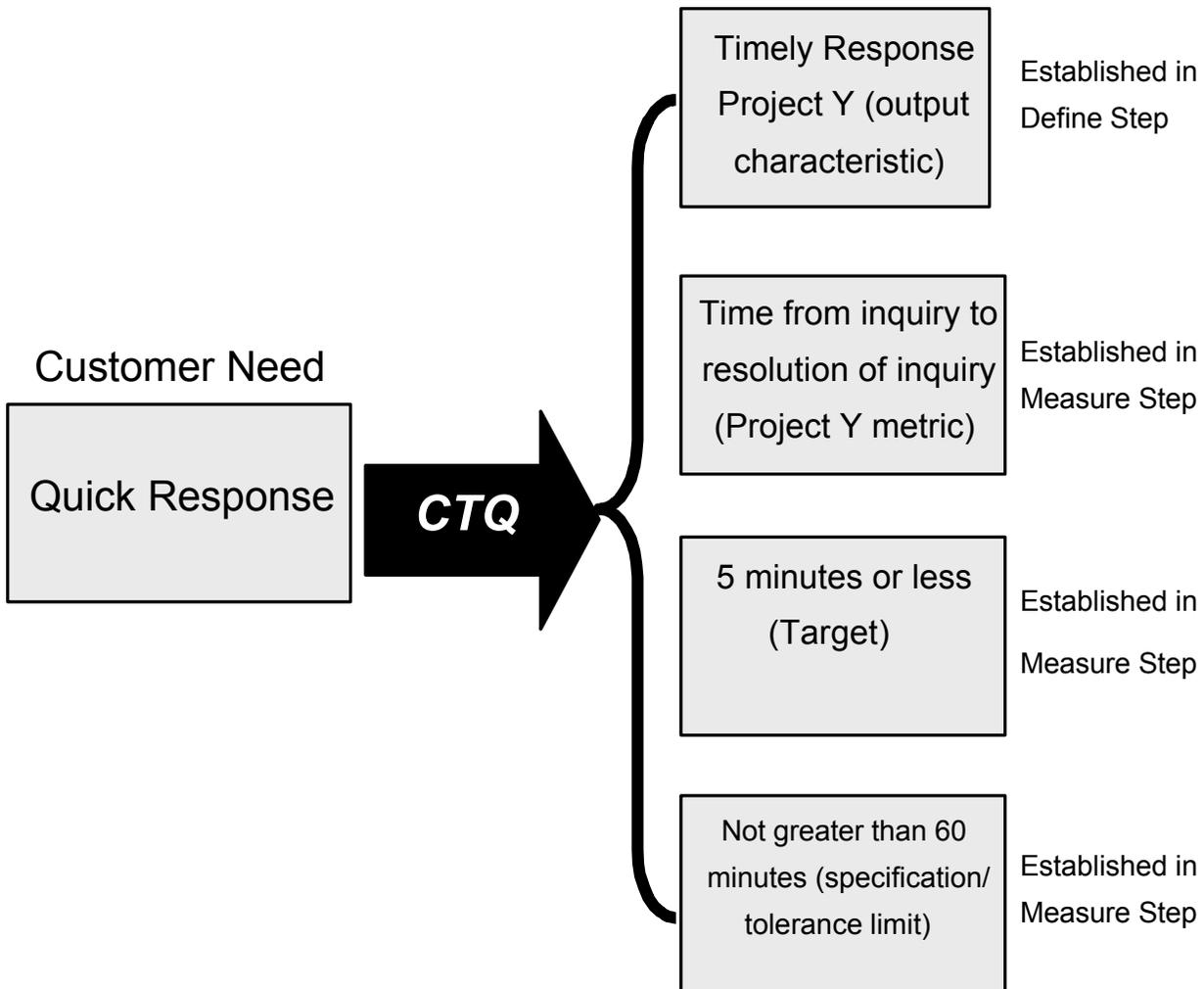
- “You Want It Right Or You Want It Fast?”



# Determine Priority CTQs (continued)

## CTQ Definition And CTQ Elements

Once the specific needs statement has been written, the actual CTQ can be developed. The first element to be identified is the output characteristic.



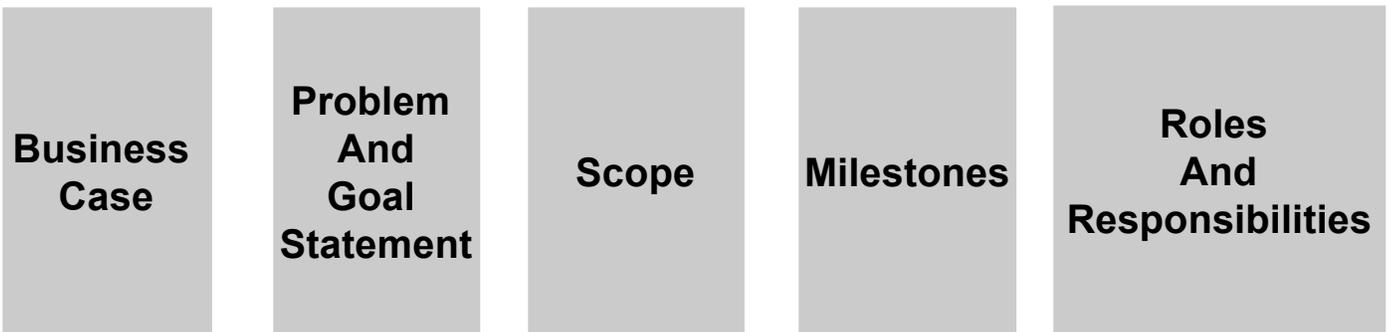
**CTQs Are The Bridge Between Our Process Output And Customer Satisfaction**



## *Return To Charter*

---

- *After you have identified your CTQs, refer back to the charter and revise as necessary*
  - *Does your CTQ relate to the problem statement?*
  - *Do you need to rewrite or revise any of the sections?*

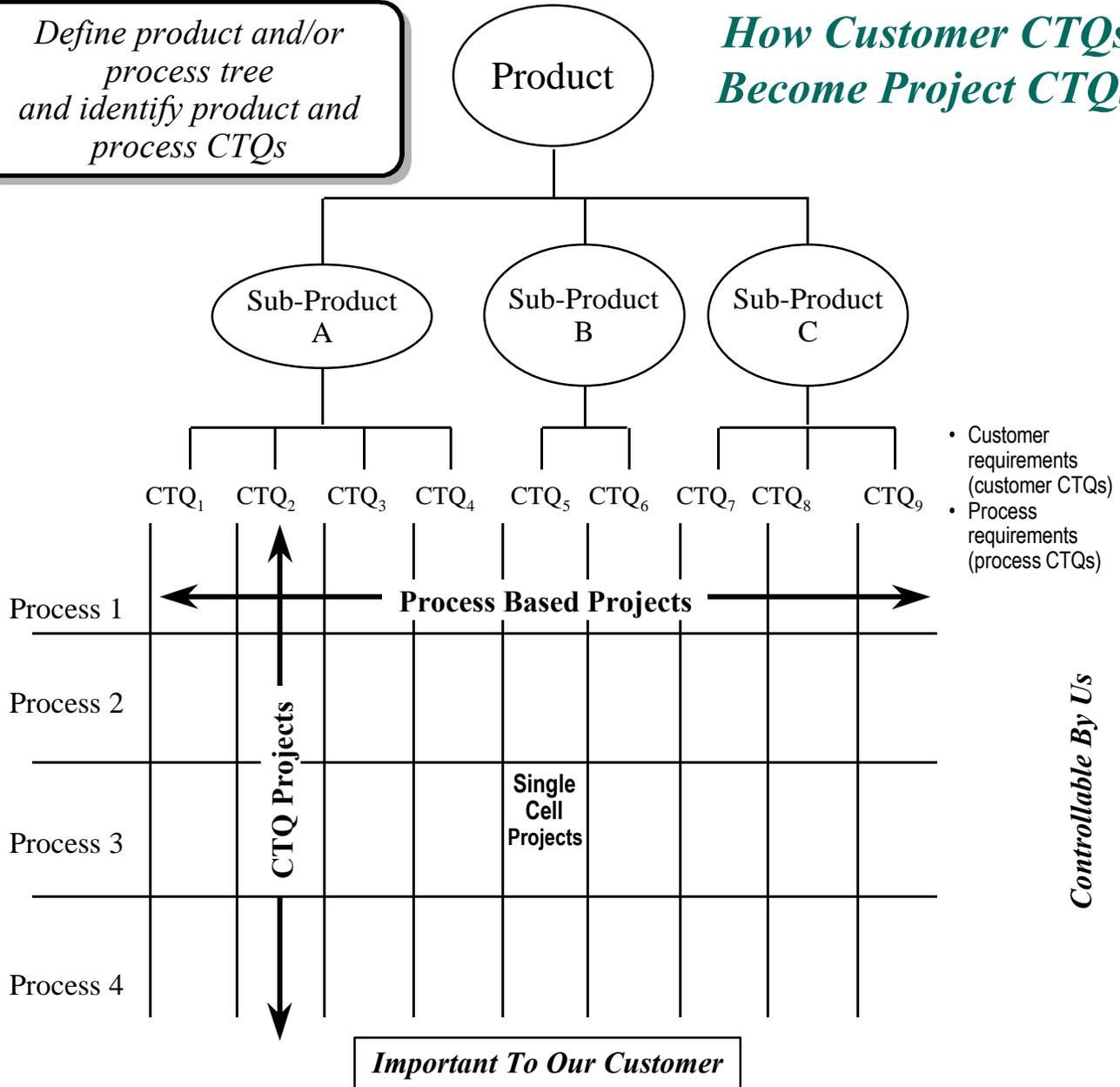




# Process/Product Drill-Down Tree

*Define product and/or process tree and identify product and process CTQs*

*How Customer CTQs Become Project CTQs*

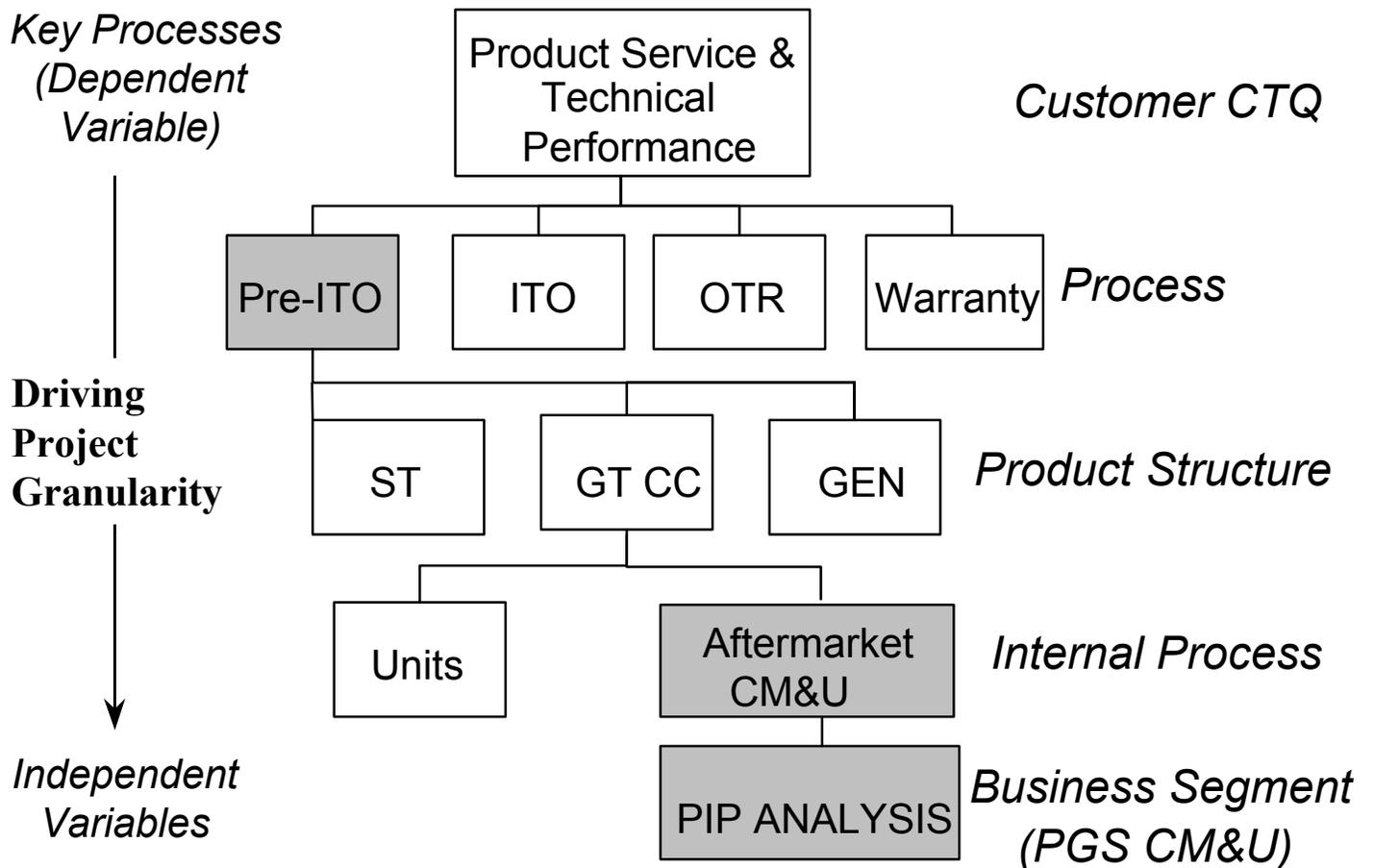


*The Black/Green Belt is assigned to work on removing defects on the selected CTQs by improving processes.*



# S171.1 Drill-Down Tree-Example

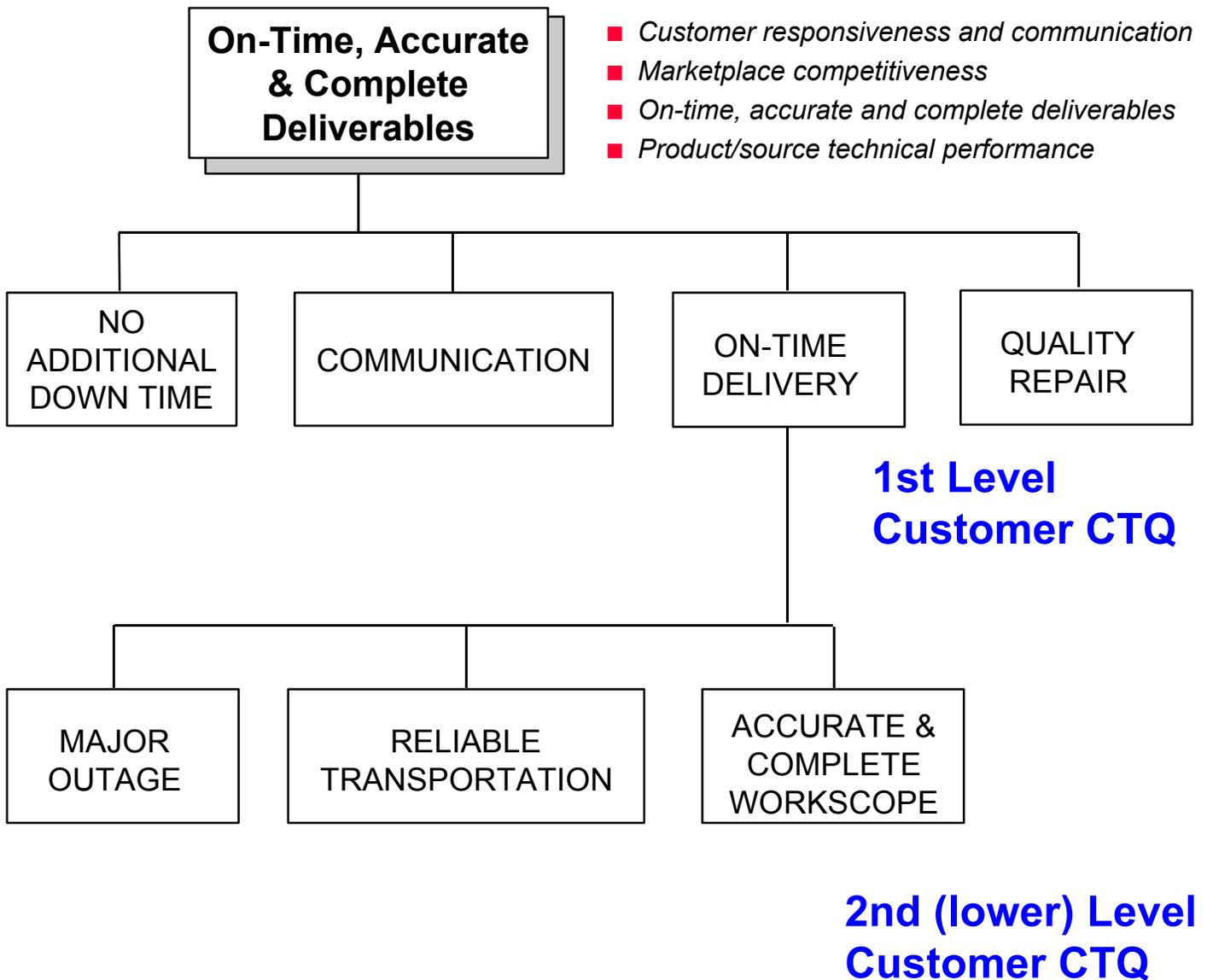
## Payback analysis of combined-cycle gas turbine uprates





# B3095.1 Drill-Down Tree-Example

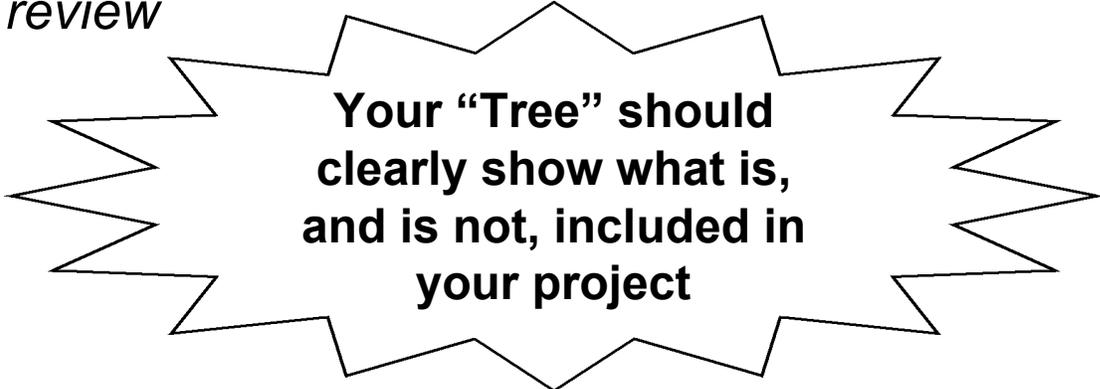
## Reducing Rotor Blasting Time High-Level CTQ





## *Class Exercise: Project/Product Tree*

- *Work individually on your projects; compare with others at your table*
- *Your task:*
  - *Based on all previous Define work, draw a process/product Drill-Down tree for your project*
  - *The instructor will select individuals to present their definitions to the class for review*

A starburst graphic with a jagged, multi-pointed border, containing the following text:

**Your “Tree” should clearly show what is, and is not, included in your project**

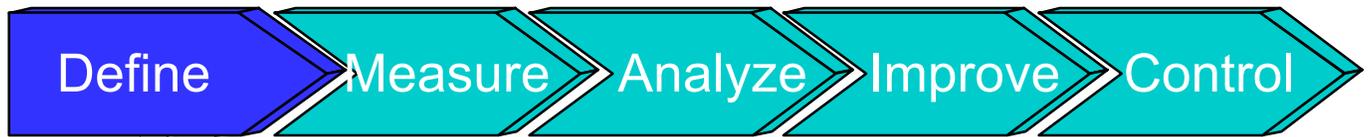


## *Take Aways—Identify Project CTQs*

- *A successful project is focused on the customer and is clearly bound with defined goals.*
- *To determine project CTQs the customer and their wants must be determined. Critical to Quality characteristics (CTQs) are determined by the customer.*
- *A successful project is related to one or more of the four Vital Customer CTQs:*
  - *Customer Responsiveness/Communication*
  - *Market Place Competitiveness - Product/Price/Value*
  - *On-Time, Accurate, and Complete Customer Deliverables*
  - *Product/Service Technical Performance*
- *Project CTQs are integrated with the business strategy through the product/process drill-down tree.*



# Define Phase



**A. Identify Project CTQs**

*Deliverable: Identify Customer(s) and Project CTQs*

**B. Develop Team Charter**

*Deliverables:*

- 1) Develop The Business Case*
- 2) Develop The Problem and Goal Statements*
- 3) Determine Project Scope*
- 4) Select Team & Define Roles*
- 5) Set Project Milestones*

**C. Define Process Map**

*Deliverable: High Level Process Map Connecting the Customer to the Process*



## *Team Chartering*

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### ■ *A Charter:*

- *Clarifies what is expected of the team*
- *Keeps the team focused*
- *Keeps the team aligned with organizational priorities*
- *Transfers the project from the champion to the improvement team*



## *Five Major Elements of a Charter*

---

- ***Business Case***
  - *Explanation of why to do the project*
  
- ***Problem and Goal Statements***
  - *Description of the problem/opportunity or objective in clear, concise, measurable terms*
  
- ***Project Scope***
  - *Process dimensions, available resources*
  
- ***Milestones***
  - *Key steps and dates to achieve goal*
  
- ***Roles***
  - *People, expectations, responsibilities*



## *The Business Case*

---

- *Why is the project worth doing?*
- *Why is it important to do it now?*
- *What are the consequences of NOT doing the project?*
- *What activities have higher or equal priority?*
- *How does it fit with business initiatives and target?*



## *Problem and Goal Statements*

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*The purpose of the Problem Statement is to describe what is wrong*

---

*The Goal Statement then defines the team's improvement objective*

***Together they provide focus and purpose for the team***



## *Problem Statement*

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- *What is wrong or not meeting our customer's needs?*
- *When and where do the problems occur?*
- *How big is the problem?*
- *What is the impact of the problem?*

***Description of  
the "Pain"***



## *Problem Statement Example*

---

### **Poor Example:**

*Our customers are angry with us and late in paying their bill.*

### **Improved Example:**

*In the last 6 months (**when**), 20% of our repeat customers - not first-timers - are over 60 days late (**what**) paying our invoices. The current rate of late payments is up from 10% in 1990 and represents 30% of our outstanding receivables (**magnitude**). This negatively affects our operating cash flow (**impact or consequence**).*



# *The Problem Statement*

## **Key Considerations/Potential Pitfalls**

- *Is the problem based on observation (**fact**) or assumption (**guess**)?*
- *Does the problem statement prejudge a root cause?*
- *Can data be collected by the team to verify and analyze the problem?*
- *Is the problem statement too narrowly or broadly defined?*
- *Is a solution included or implied in the statement?*
- *Would customers be happy if they knew we were working on this?*



# *The Goal Statement*

---

## ***Project Objective***

- *Definition of the improvement the team is seeking to accomplish?*
- *Starts with a verb (**reduce, eliminate, control, increase**).*
- *Tends to start broadly - eventually should include measurable target and completion date.*
- *Must not assign blame, presume cause, or prescribe solution!*



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# *SMART Problem and Goal Statements*

---

**Specific**

**Measurable**

**Attainable**

**Relevant**

**Time Bound**



## *Project Scope*

---

- *What process will the team focus on?*
- *What are the boundaries of the process we are to improve? Start point? Stop point?*
- *What resources are available to the team?*
- *What (if anything) is out-of-bounds for the team?*
- *What (if any) constraints must the team work under?*
- *What is the time commitment expected of team members? What are the advantages to each team member for the time commitment?*



## *8 Steps to Bound a Project*

---

*These steps work best when used in a Project Bounding Workout Session with the project team. Plan a minimum of 1-2 hours for the session, depending on the complexity of the project.*

### *1. Identify the customer*

- Who receives the process output?*
- May be an internal or external customer*

### *2. Define customer's expectations and needs*

- Ask the customer*
- Think like the customer*
- Rank or prioritize the expectations*

### *3. Clearly specify your deliverables tied to those expectations*

- What are the process outputs?*
- Tangible and intangible deliverables*
- Rank or prioritize the deliverables*
- Rank your confidence in meeting each deliverable*

### *4. Identify CTQs for those deliverables*

- What are the specific, measurable attributes that are most critical in the deliverables?*
  - Select those that have the greatest impact on customer satisfaction*
-



## *8 Steps to Bound a Project*

---

### *5. Map your process*

- The process of producing the deliverables*
- The process as it is working prior to the project*
- If you are delivering something, there is a process, even if it has not been formalized*

### *6. Determine where in the process the CTQs can be most seriously affected*

- Use a detailed flowchart*
- Estimate which steps contain the most variability*

### *7. Evaluate which CTQs have the greatest opportunity for improvement*

- Consider available resources*
- Compare variation in the processes with the various CTQs*
- Emphasize process steps which are under the control of the team conducting the project*

### *8. Define the project to improve the CTQs you have selected*

- Define the defect to be attacked*
-



## Milestones

- *A preliminary, high-level project plan with dates*
- *Tied to phases of DMAIC process*
- *Should be aggressive (don't miss "window of opportunity")*
- *Should be realistic (Don't force yourselves into "band-aid" solution)*

	Week:	1	2	3	4
Review charter with Champion			X		
Collect VOC			X	X	
Complete Map			X	X	
Validate Map					X
Collect Data					X



## *Team Roles*

---

- *How do you want the champion to work with the team?*
- *Is the team's role to implement or recommend?*
- *When must the team go to the champion for approval? What authority does the team have to act independently?*
- *What and how do you want to inform the champion about the team's progress?*
- *What is the role of the team leader (Black/Green Belt) and the team coach (Master Black Belt)?*
- *Are the right members on the team? Functionally? Hierarchically?*



## *Problem/Goal Statements Exercise*

Objective

- *Practice drafting key components of team charter.*

Instructions

- *Divide your team into two subgroups.*
- *Each subgroup is to identify one real problem.*
- *Describe the business case--an explanation of why to do the project.*
- *Determine the problem and goal statements. Provide a description of the problem/opportunity or objective in clear, concise, measurable terms.*
- *Project scope--include process dimensions, available resources, what is in and out of bounds.*
- *Milestones--key steps and dates to achieve goal.*
- *Roles--people, expectations, responsibilities.*
- *Review and critique each other's statements.*
- *Prepare to share learnings.*



## *A Good Project*

---

### ■ **A good project:**

- *Problem & goal statement clearly stated*
- *Defect & opportunity definition is clearly understood*
- *Does not presuppose a solution*
- *Clearly relates to the customer and customer requirements*
- *Aligns to the business strategy*
- *Uses the tools effectively*
- *Data Driven*

### ■ **A bad project:**

- *Project is not focused—scope is too broad*
- *Not clear on what you are trying to fix*
- *Solution is already known/mandated without proper investigation*
- *Difficult to see linkage to customer needs*
- *Working on a project that will not move the needles*
- *Little or no use of tools*
- *Anecdotal--not data driven*

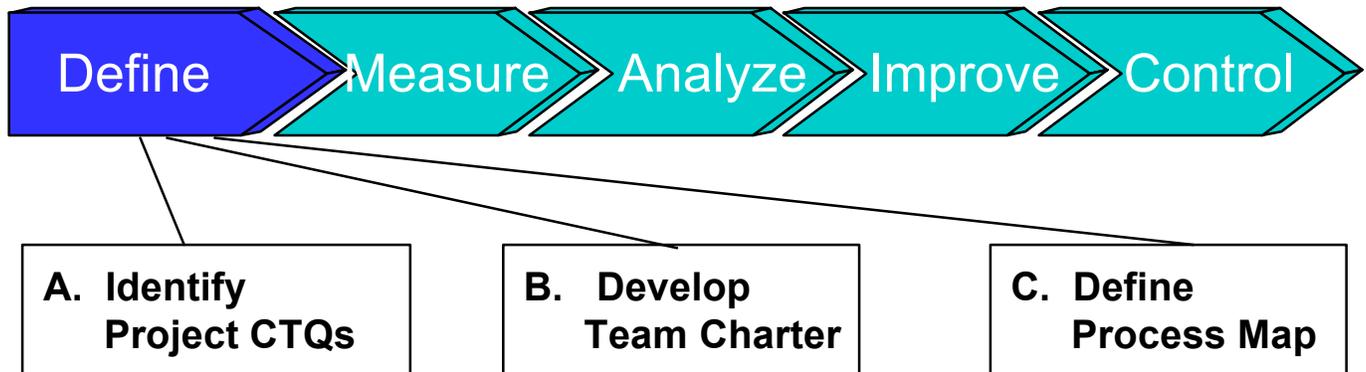


## *Take Aways—Develop Team Charter*

- *Key elements of a charter include: Business Case, Problem and Goal Statements, Project Scope, Milestones, and Roles.*
- *The team charter is a vital part of the project's overall success. It communicates the project direction to all members of the team.*
- *A Problem Statement describes what is wrong while a Goal Statement defines the improvement objective.*
- *A charter clarifies what is expected of the project team, keeps the team focused, keeps the team aligned with organizational priorities, and transfers the project from the champion to the improvement team.*



## Define Phase - Define Process Map



*Deliverable: Identify Customer(s) and Project CTQs*

*Deliverables:*

- 1) *Develop The Business Case*
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- 3) *Determine Project Scope*
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- 5) *Set Project Milestones*

*Deliverable: High Level Process Map Connecting the Customer to the Process*



## Selecting the Right Projects

<b>Delivered CTQ Importance</b>	High Impact	Low	Medium	High
	Medium Impact	Low	Medium	High
	Low Impact	Low	Medium	High
		Low	Medium	High

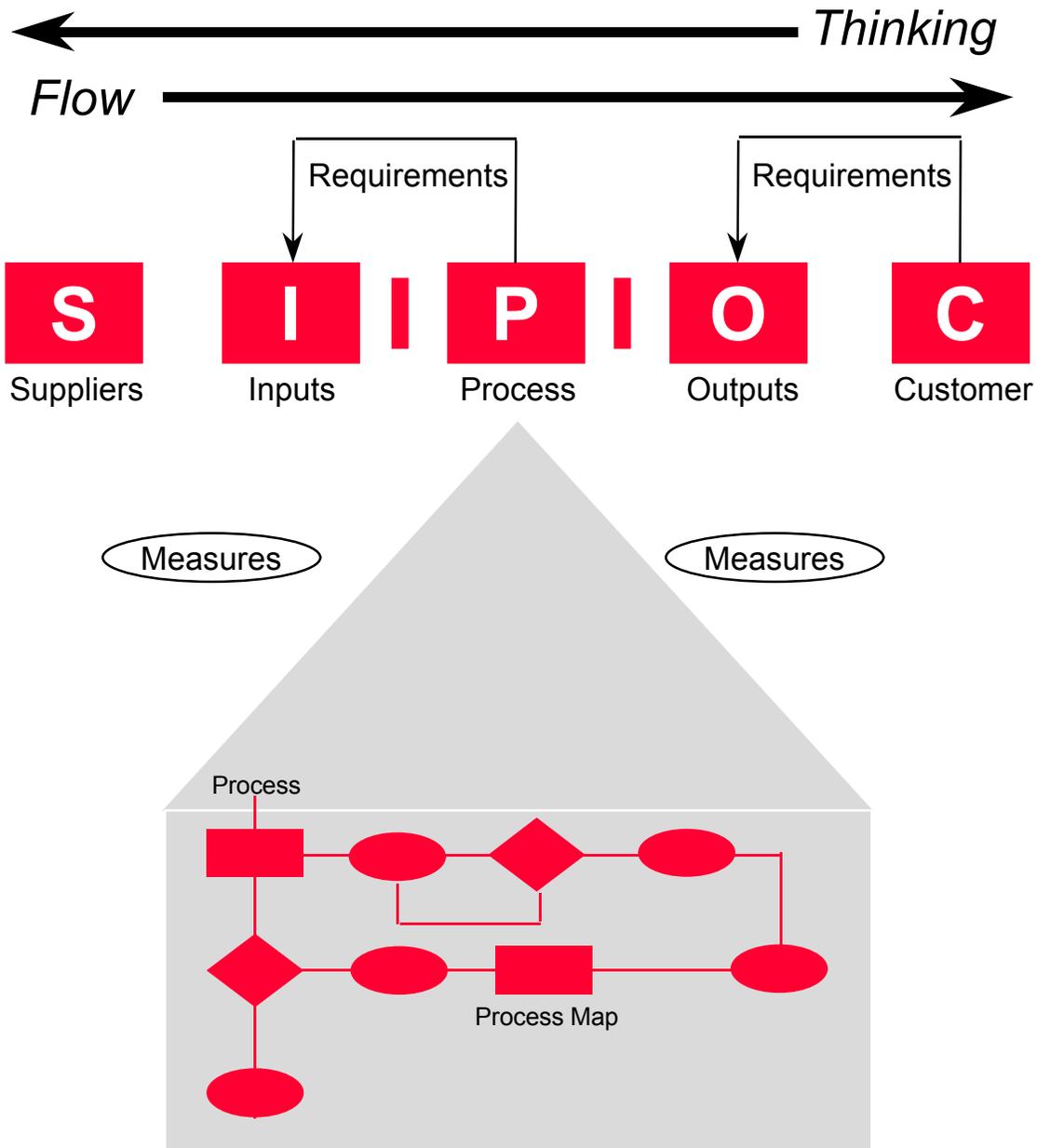
**Performance**

**Top Priorities Based On Impact & Performance:  
Strategic Issues**

- **Six issues in selecting a project:**
  - *Process*
  - *Feasibility (Is it doable?)*
  - *Measurable impact*
  - *Potential for improvement*
  - *Resource support within the organization*
  - *Project interactions*



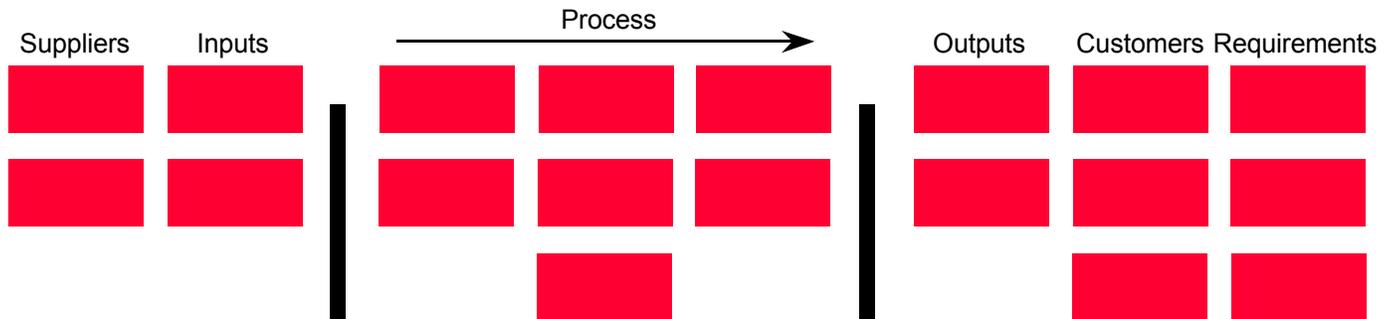
# Business Process Mapping



*The Focus is on the Customer: COPIS*



## *Steps of Process Mapping*



- *Name the process*
- *Identify the outputs, customers, suppliers, & inputs*
- *Identify customer requirements for primary outputs*
- *Identify process steps*



## *Take Aways—Define Process Map*

- *A process map for a project includes:*
  - *Customers and their key requirements*
  - *Outputs*
  - *Process Steps*
  - *Inputs*
  - *Suppliers*
  
- *A process map connects the customer to the process and helps to identify key inputs and requirements.*
  
- *The process map at this stage of your project should be at a high level.*



# *CAP Tools*



## *Tools to Get Started: ARMI Model*

Key Stakeholders	PROJECT PHASE		
	Startup/Planning	Implementation	Evaluation

**What:** *A tool to determine individuals and/or groups whose commitment is essential for project success*

**Why:** *To ensure that the project leader has identified Key Stakeholders*

**How:** *List individuals/groups involved in the process and identify project function*



## ARMI Model

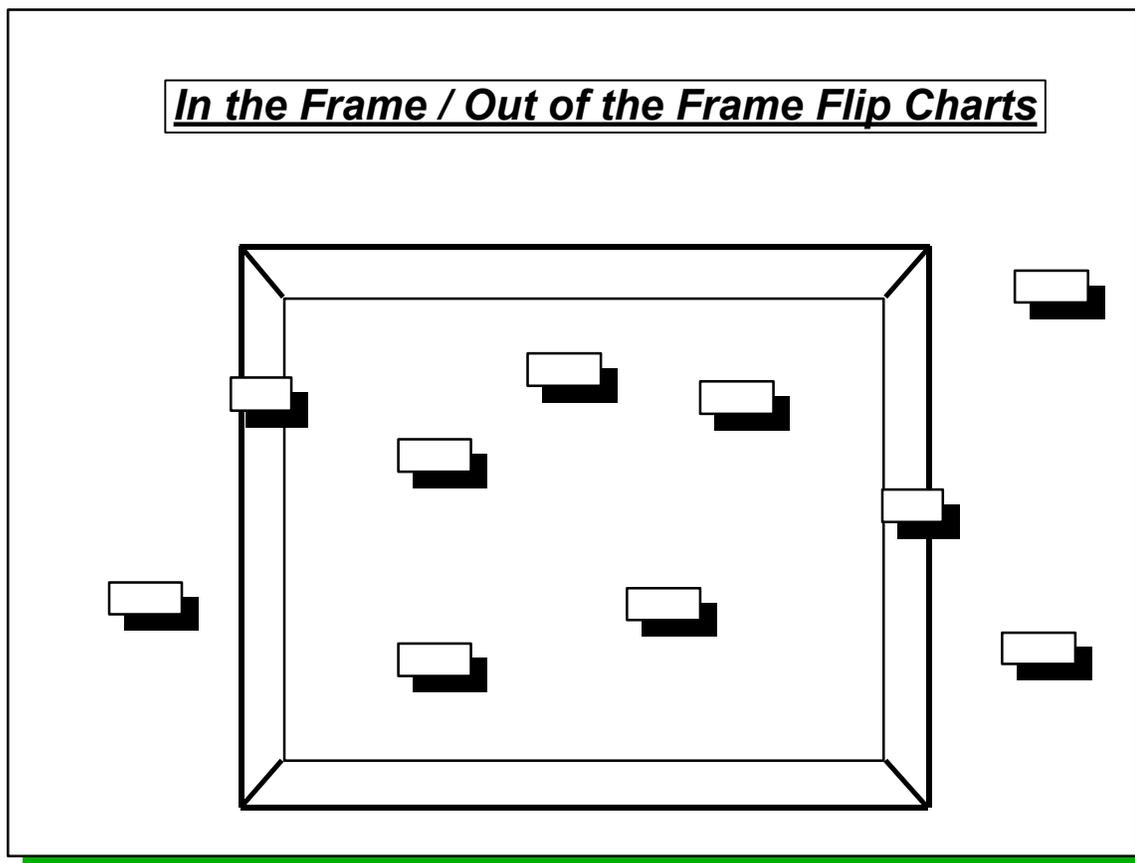
Key Stakeholders	PROJECT PHASE		
	Startup/Planning	Implementation	Evaluation

- A** Approval of team decisions outside their charter authorities, i.e., sponsor, business leader
- R** Resource to the team, one whose expertise, skills, or clout may be needed on an ad hoc basis
- M** Member of team, with the authorities and boundaries of the charter
- I** Interested party, one who will need to be kept informed on direction, findings, if later support is to be forthcoming



## *In/Out of the Frame*

**Tool:** **In/Out of the Frame** - This is a visual tool based on the analogy of a picture frame. It challenges the team to identify those aspects of the project (the type and extent of end results or deliverables, the people impacted, timing, product lines impacted, sites involved, etc.) which are “in the frame” (meaning clearly within the scope of work), “out of the frame,” or “half-in-half-out” (meaning this is either up for debate, or some aspects are in the scope of work but only in a partial way.)



**Uses:** *Not as complex as SIPOC, but useful when you feel there are many “boundary issues” facing the team (differences of opinion as to what is and isn’t in the scope of work).*



## *In/Out of the Frame*

---

- Steps:
- 1. Gather all storyboard materials and find a wall space large enough to accommodate the completed chart.*
  - 2. Help the team get organized to complete the chart. A hint here: encourage team members to use the location of cards they place on the chart to indicate how strongly they feel about a particular aspect of the project (a card placed in the middle of the frame signifies a strong sense that this aspect is clearly within the scope of work, while one placed near the border refers to an aspect that a person is a bit suspicious about).*
  - 3. Draw a large square "picture frame" on a flip chart (or use tape on a wall) and use this metaphor to help the team identify what falls inside the picture of their project and what falls out. This may be in terms of type and extent of end results, people impacted, time frame, product lines, sites, etc.*
  - 3. Discuss with Champion/Functional Leader and other key stakeholders and resolve differences.*
  - 4. OPTION: You might want to instruct the team to place their cards on the chart without talking to one another and then to have a 5-10 minute silent team huddle to resolve differences. That is, in silence, team members begin to move the cards others have placed on the chart to "get it right" from their perspective. If a card is moved and the originator of it disagrees, he/she can move it back, but then a dot is placed on it to signify that some discussion is warranted. Having to work in silence is challenging for a talkative team and can be both fun and a nice change of pace, and, they are really ready to talk when the time is up.*



# Threat vs. Opportunity Matrix

**Tool:** **Threat vs. Opportunity Matrix** - "Best Practice" organizations know how to frame the need for change as more than a short-term threat. They work to find ways to frame the need as a threat and opportunity over both the short and long term. By doing so, they begin to get the attention of key stakeholders in a fashion that ensures their involvement beyond what can be gained from a short-term sense of urgency.

	<i>Threat</i>	<i>Opportunity</i>
<i>Short Term</i>	1	3
<i>Long Term</i>	2	4

**Uses:** *Building the "case for change" is one of the first and most important tasks of the team. This simple tool helps the team discover how to frame the need for change more broadly and perhaps break some old habits about change only as it applies to a short-term threat.*



## *Threat vs. Opportunity Matrix*

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- Steps:**
- 1. Working individually, team members pick which of the four quadrants "fits" the need for change for this change initiative. Team members share their perceptions and then debate and discuss similarities and differences.*
  - 2. Individuals then write a 3-4 sentence statement of the need for change using language that speaks to as many of the four quadrants as possible.*
  - 3. Team members read their statements and the team debates and discusses each to create a statement that encompasses the best of each individual effort. This statement is then modified to appeal to key constituent groups (manufacturing, marketing, engineering, etc.).*
  - 4. OPTION: Though most teams find this discussion fairly straightforward, some struggle with the degree of specificity required to really frame the need for change along both dimensions. Therefore, it may be useful to begin this discussion and then table it for additional work once the vision has been articulated and the key stakeholders have been identified. It is not unusual to find a team finally ready to use this tool after they have worked on the vision and begun to do a stakeholder analysis.*



# G.R.P.I. Check List

**Tool:** **G.R.P.I. Check List** - This tool is based on a simple model for team formation. It challenges the team to consider four critical and interrelated aspects of teamwork: **G**oals, **R**oles, **P**rocesses and **I**nterpersonal relationships. It is invaluable in helping a group become a team.

## G. R. P. I. Team Model

<b>G. R. P. I. Checklist</b>	Low				High
<b>GOALS</b> - How clear and in agreement are we on the mission and goals of our team/projects?	1	2	3	4	5
<b>ROLES</b> - How well do we understand, agree on, and fulfill the roles and responsibilities for our team?	1	2	3	4	5
<b>PROCESSES</b> - To what degree do we understand and agree on the way we'll approach our project AND our team? (Procedures and approaches for getting our project work done? For running our team?)	1	2	3	4	5
<b>INTERPERSONAL</b> - Are the relationships on our team working well so far? How is our level of openness, trust, and acceptance?	1	2	3	4	5

**Uses:** *An excellent organizing tool for newly-formed teams or for teams that have been underway for a while, but who have never taken time to look at their teamwork. Ideally, this tool should be used at one of the first team meetings. It can and should be updated as the project unfolds.*



## ***G.R.P.I. Check List***

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- Steps:**
- 1. Distribute copies of the check list to all team members prior to a team meeting. Invite team members to add details/examples on each of the four dimensions of the check list. Ask each team member to bring his/her completed checklist to the team meeting.*
  - 2. At the team meeting discuss and resolve issues related to the check list.*
  - 3. Share certain aspects with Champion/Functional Leader if appropriate.*
  - 4. OPTION: When there is considerable disagreement or tension within the team environment, team members can choose to complete the questionnaire individually and turn it in to a neutral party who will collate the data and give it back to the team in an aggregate fashion (thus protecting the anonymity of individual team members).*



# G.R.P.I. Check List

**Expanded Version of the Tool:** Useful when a more detailed look at team elements is required.

## Assessing Project Team Status

*How would you rate the degree to which your team presently has CLARITY, AGREEMENT, and EFFECTIVENESS on the following GRPI-related elements?*

		0 %	25 %	50 %	100 %
<b>G</b>	<b>• Purposes &amp; Outcomes</b> We understand and agree on our project mission and the desired outcome (vision).				
	<b>• Customer &amp; Needs</b> We know who the project stakeholders are, what they require, and why this project is really needed.				
	<b>• Goals &amp; Deliverables</b> We have identified specific, measurable & prioritized project goals & deliverables linked to our business goals.				
	<b>• Project Scope Definition</b> We understand/agree on what in/out of our project scope & tasks. The project scope is “set”.				
<b>R</b>	<b>• Roles &amp; Responsibilities</b> We have defined & agreed on our roles, responsibilities, required skills, and resources for our project team				
	<b>• Authority &amp; Autonomy</b> Our team is clear on the degree of authority / empowerment we have to meet our project mission.				



# G.R.P.I. Check List

## Assessing Project Team Status (Cont.)

		0 %	25 %	50 %	100 %
<b>P</b>	<ul style="list-style-type: none"><li>● <b>Critical Success Factors</b> We know and are focusing on the key factors needed to meet the project goals and mission,</li><li>● <b>Plans &amp; Activities</b> We have an effective game plan to follow that includes the right tasks; clearly defined/assigned.</li><li>● <b>Monitoring &amp; Measures</b> We have an effective monitoring process and specific metrics linked to progress and goals</li><li>● <b>Schedule / Milestones</b> We have defined our project schedule and know what the key phases and milestones are.</li></ul>				
<b>I</b>	<ul style="list-style-type: none"><li>● <b>Team Operating Agreement</b> We have shared expectations, agreed and followed guidelines for how our team works together</li><li>● <b>Interpersonal / Team</b> We have the necessary relationships, trust, openness, participation and behaviors for a healthy &amp; productive team.</li></ul>				



## *Define Deliverables*

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- *Identify project through business theme and personal issue*
- *Review Six Sigma Quality Project Tracking database for similar projects*
- *Identify internal/external CTQs*
- *Create high-level process map*
- *Create defect definition and opportunity*
- *Create drill-down tree*
- *Identify potential data sources*
- *Identify team members and business functions required*
- *Identify Information Technology (IT) requirements*
- *Identify financial impact*
- *Open project in Six Sigma Quality Project Tracking database*



## *Now That We Have Defined the Project...*

---

*Define - Measure - Analyze - Improve - Control*

- *The next phase is Measure:*
  - *Select one or more CTQ characteristics; i.e., dependent variables,*
  - *Map the respective process,*
  - *Define performance standards,*
  - *Measurement Systems Analysis*



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# *Introduction to the Measurement Phase*

- *Using Statistics to Solve Problems*
- *The 12 Step Process*

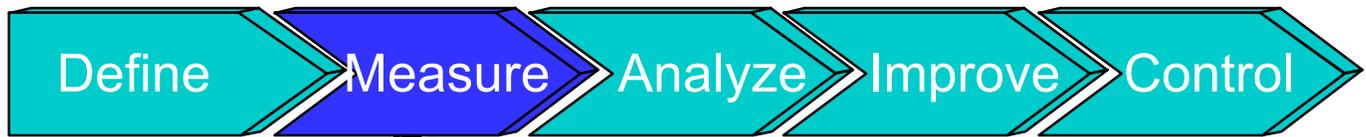


# The 12 Step Process

Step	Description	Focus	Tools	SSQC Deliverables
<b>Define</b>				
A	Identify Project CTQs			Project CTQs (1)
B	Develop Team Charter			Approved Charter (2)
C	Define Process Map			High Level Process Map (3)
<b>Measure</b>				
1	Select CTQ Characteristics	Y	Customer, QFD, FMEA	Project Y (4)
2	Define Performance Standards	Y	Customer, Blueprints	Performance Standard for Project Y (5)
3	Measurement System Analysis	Y	Continuous Gage R&R, Test/Retest, Attribute R&R	Data Collection Plan & MSA (6), Data for Project Y (7)
<b>Analyze</b>				
4	Establish Process Capability	Y	Capability Indices	Process Capability for Project Y (8)
5	Define Performance Objectives	Y	Team, Benchmarking	Improvement Goal for Project Y (9)
6	Identify Variation Sources	X	Process Analysis, Graphical Analysis, Hypothesis Tests	Prioritized List of all Xs (10)
<b>Improve</b>				
7	Screen Potential Causes	X	DOE-Screening	List of Vital Few Xs (11)
8	Discover Variable Relationships	X	Factorial Designs	Proposed Solution (13)
9	Establish Operating Tolerances	Y, X	Simulation	Piloted Solution (14)
<b>Control</b>				
10	Define & Validate Measurement System on X's in Actual Application	Y, X	Continuous Gage R&R, Test/Retest, Attribute R&R	MSA
11	Determine Process Capability	Y, X	Capability Indices	Process Capability Y, X (15)
12	Implement Process Control	X	Control Charts, Mistake Proof, FMEA	Sustained Solution (15), Documentation (16),



# Measure Phase



**1. Select CTQ Characteristics**

**2. Define Performance Standards**

**3. Measurement System Analysis**

*Deliverable: Identify Measurable CTQ That Will Be Improved*

*Tools:*

- QFD
- Process Mapping
- FMEA
- Discrete vs. Continuous Data

*Deliverable: Determine and Confirm Specification Limits For Your Y.*

*Deliverable: Measurement System Adequate to Measure Y.*

*Tools:*

- Continuous Gage R&R
- Test/Retest
- Attribute Gage R&R



## *Overview*

---

- *In the Measure phase we will select one or more product or process characteristics to address, we will map the respective process to show us what it actually looks like, we will validate our measurement system and then we will take our measurements.*



## *Objectives*

---

- *By the end of this section, the participant will be able to:*
  - *Identify the Project Y*
  - *Define the performance standards for Y including specification limits as well as defect and opportunity definitions*
  - *Validate the measurement system*
  - *Collect the data*
  - *Characterize the data using mean and standard deviation*



# *The Phases of the 12 Step Process*

---

**Phase 1 (Define).** This phase defines the project. It identifies customer CTQs and ties them to business needs. Further, it defines a project charter and the business process bounded by the project.

**Phase 2 (Measurement).** This phase is concerned with selecting one or more product characteristics; i.e., dependent variables, mapping the respective process, making sure the measurement system is valid, making the necessary measurements and recording the results.

**Phase 3 (Analysis).** This phase entails estimating the short- and long-term process capability and benchmarking the key product performance metrics. Following this, a gap analysis is often undertaken to identify the common factors of successful performance; i.e., what factors explain best-in-class performance. In some cases, it is necessary to redesign the product and/or process.

**Phase 4 (Improvement).** This phase is usually initiated by selecting those product performance characteristics which must be improved to achieve the goal. Once this is done, the characteristics are diagnosed to reveal the major sources of variation. Next, the key process variables are identified by way of statistically designed experiments. For each process variable which proves to be significant, performance specifications are established.

**Phase 5 (Control).** This phase is related to ensuring that the new process conditions are documented and monitored via statistical process control methods. After a "settling in" period, the process capability would be reassessed. Depending upon the outcomes of such a follow-on analysis, it may be necessary to revisit one or more of the preceding phases.

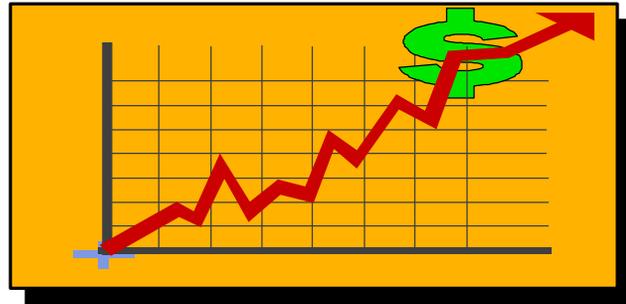


# The Components of Breakthrough

## Product

### Characterization

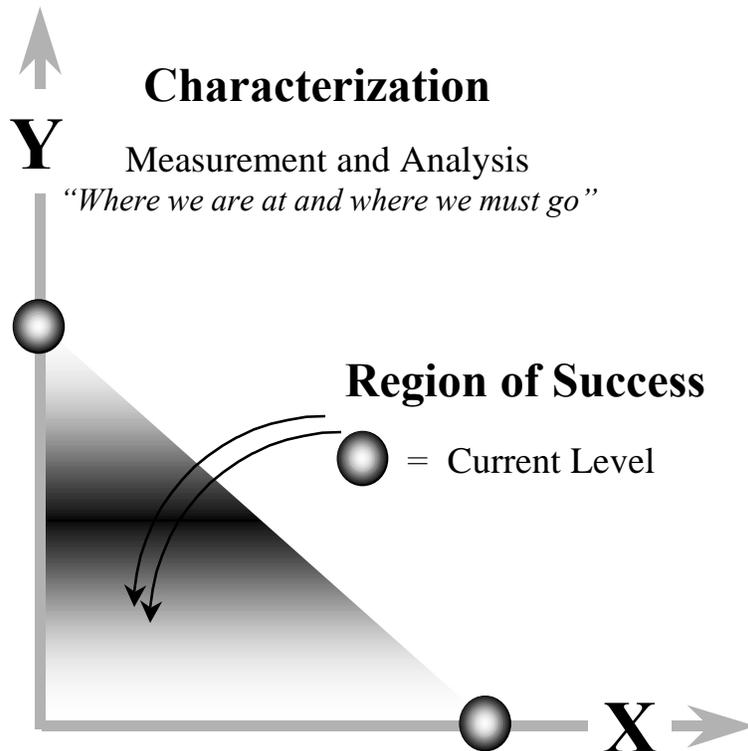
is concerned with the identification and benchmarking of key product characteristics. By way of a gap analysis, common success factors are identified.



## Process

### Optimization

is aimed at the identification and containment of those process variables which exert undue influence over the key product characteristics.



## Characterization

Measurement and Analysis

*"Where we are at and where we must go"*

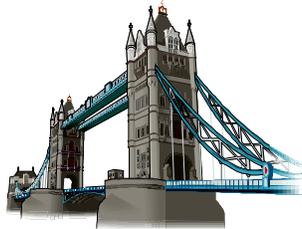
## Optimization

Improvement and Control

*"What action we must take to get and stay there"*



# Using Statistics to Solve Problems



How would your project fit?

**Practical Problem**

**Statistical Problem**

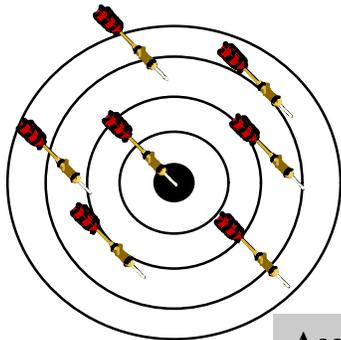
**Statistical Solution**

**Practical Solution**

Practical Problem: Low Yield  
Statistical Problem: Mean Off Target  
Statistical Solution: Isolate Key Variables  
Practical Solution: Install Automatic Controller

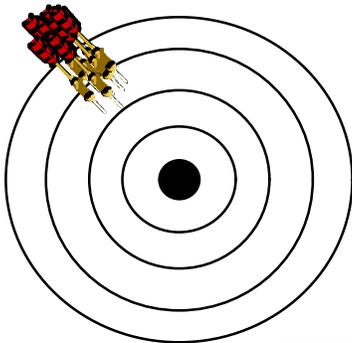
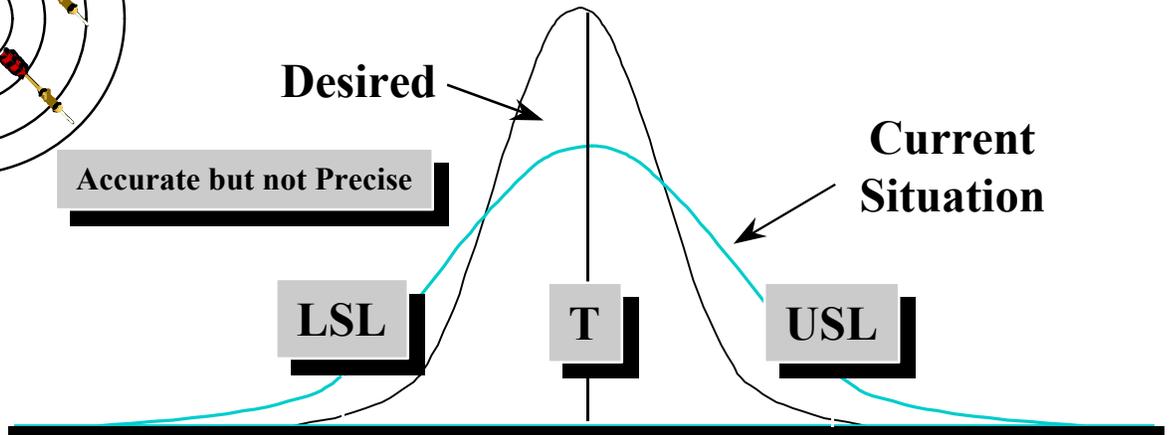


# The Nature of Statistical Problems



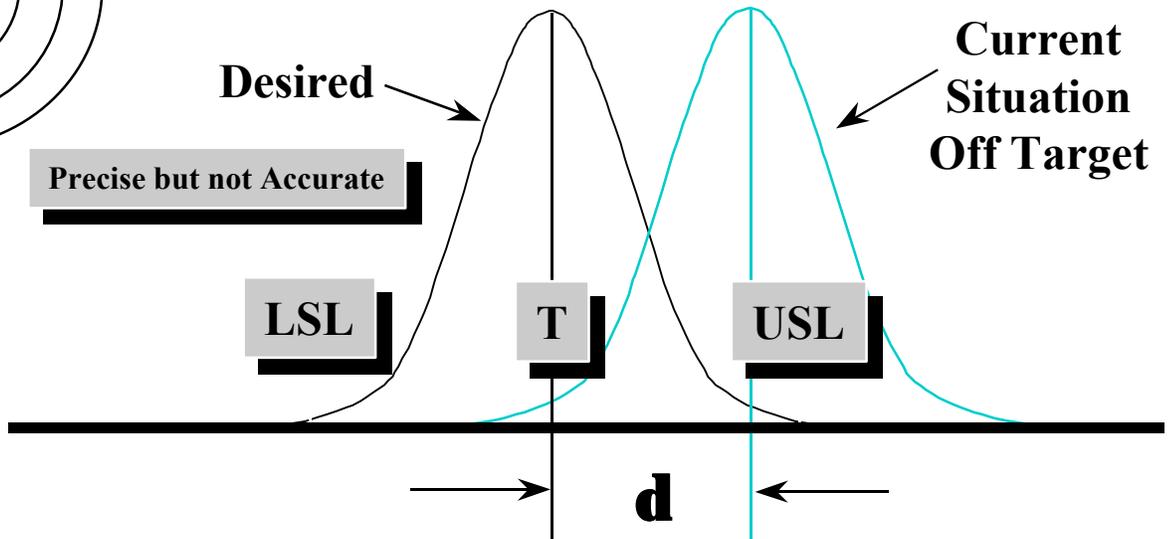
Accurate but not Precise

### Problem with Spread



Precise but not Accurate

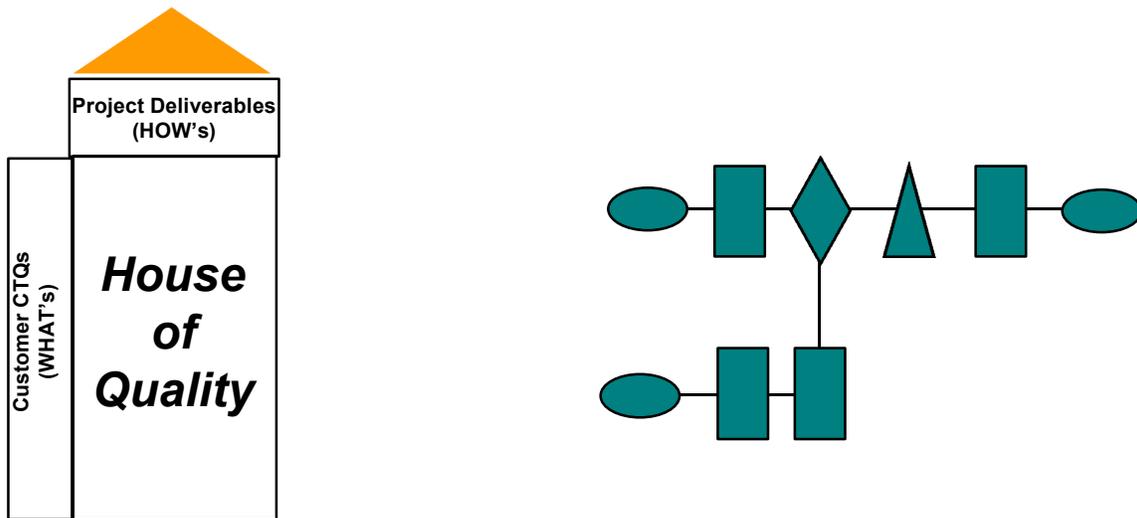
### Problem with Centering





# Select a CTQ Characteristic

## ■ Selecting the project Y



Process/Product Failure Modes and Effects Analysis (FMEA)								
Process or Product Name:	REDUCTION IN NDT PREP TIME ON GT COMPRESSOR ROTORS					Prepared by:	STALEY EDWARDS	
Responsible:	STALEY EDWARDS					FMEA Date (Orig)	JUNE , 25, 1998 (Rev) _ _ _	
Process Step/Part Number	Potential Failure Mode	Potential Failure Effects	SEV	Potential Causes	OCC	Current Controls	DET	RPN
PREP FOR BLAST	IMPROPER TAPING	BLAST MEDIA GETS IN BETWEEN ROTOR WHEEL	8	WORKMANSHIP / MATERIAL	2	PLANNING ROTOR AND VISUAL	2	32
**BLAST CLEAN	TAPE COMES OFF	BLAST MEDIA GETS IN BETWEEN ROTOR WHEEL	8	MATERIAL / OVERBLASTING	3	VISUAL INSPECTION DURING BLAST PROCESS	3	72
**PREP FOR NDT	TAPE RESIDUE ON WHEELS	EXCESSIVE HOURS TO CLEAN FOR NDT	7	MATERIAL / OVERBLASTING	9	NONE	10	630
NDT	INSUFFICIENT CLEANING CAUSING QUESTIONABLE	POSSIBLITY OF NOT SEEING RELATIVE INDICATIONS DURING NDT	10	BLASTING , CLEANING AND MATERIAL	2	MANUAL CLEANING PRIOR TO NDT	3	60



## *Define Performance Standards*

---

*It answers the questions:*

*What does the customer want?*

*What is a good product/process?*

*What is a defect?*

*Examples:*

*Cycle time for drawings*

*from order to proceed to delivery of  
drawings < 15 weeks*

*Part dimensions-*

*15.5 ± 0.03 inches*

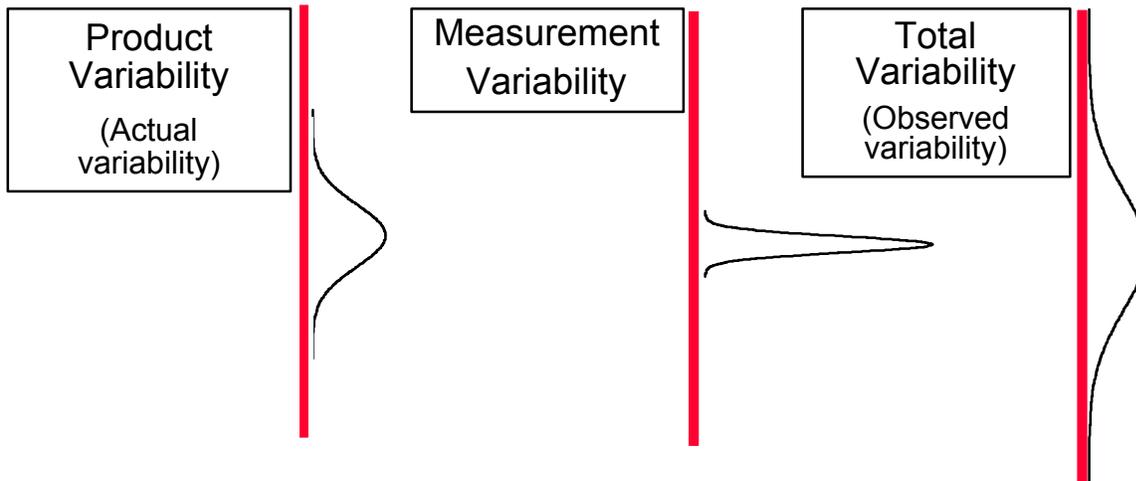
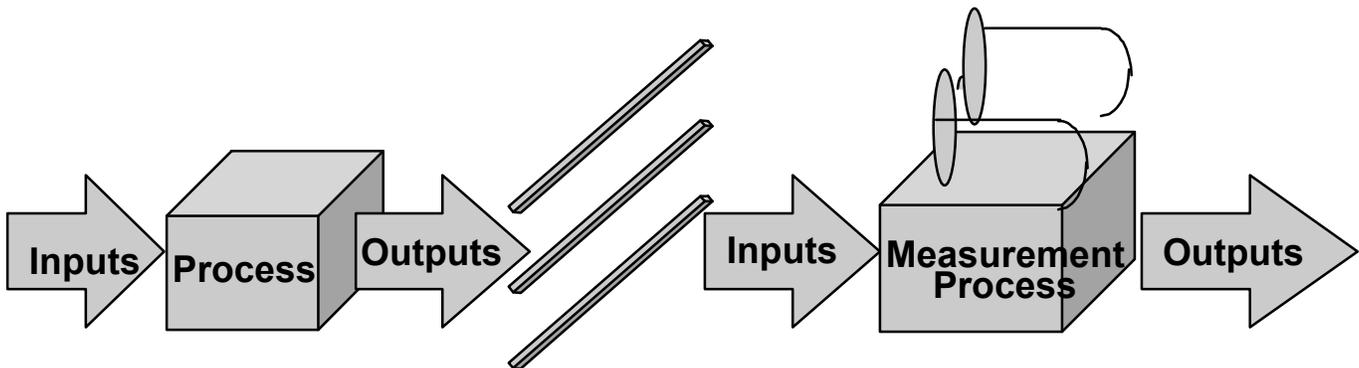
*Computer leasing cost*

*< \$100 per unit per month*



# Measurement Systems Analysis

- Observations
- Measurements
- Data

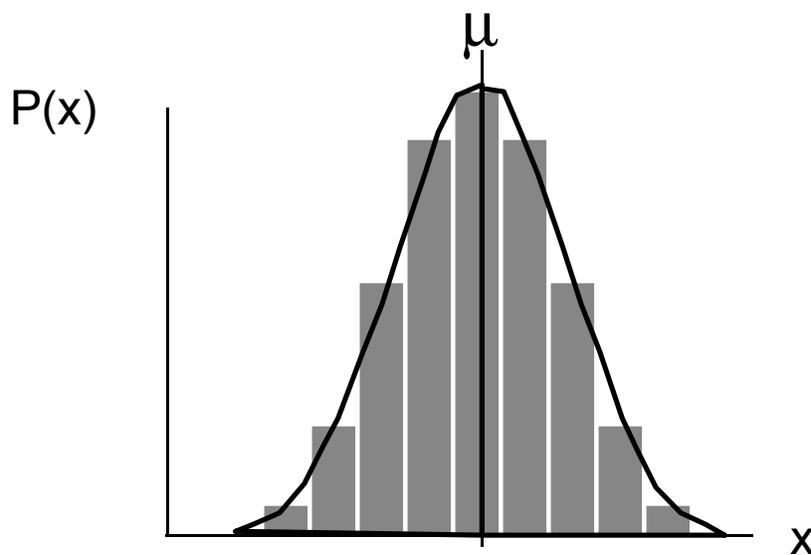




# *Using Statistics to Solve Problems*

*Goal: To find the relationship*

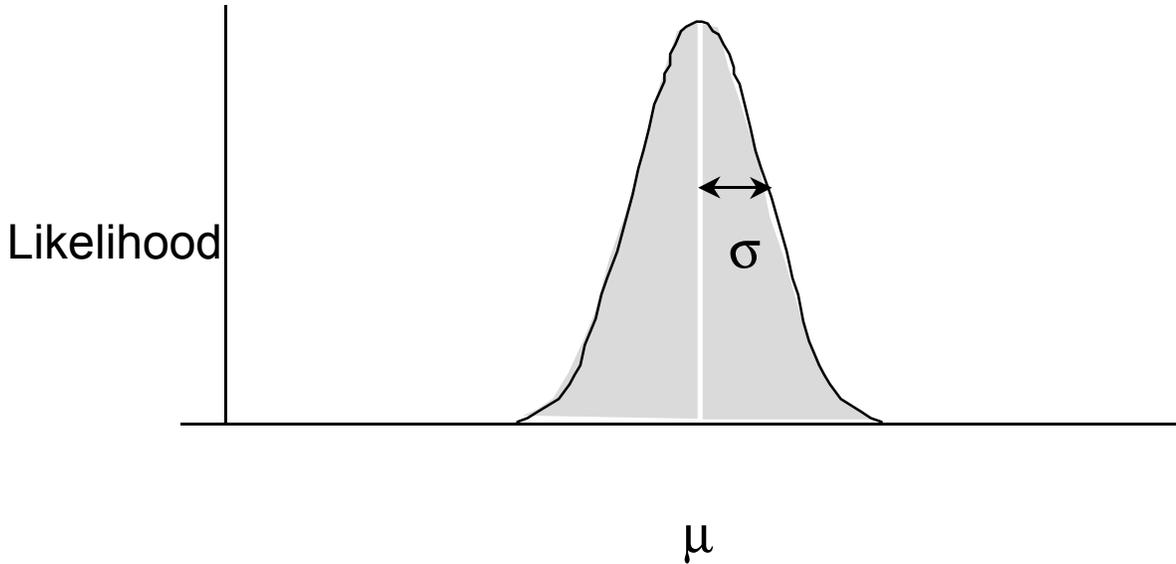
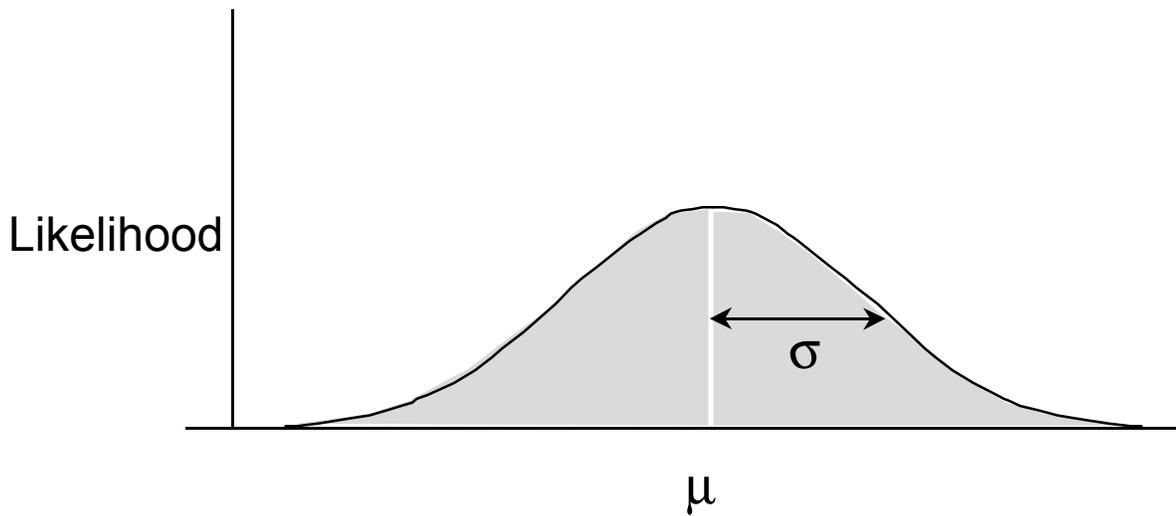
$$Y = f(X_1, \dots, X_n)$$



*Data Driven Analysis*



# Using Statistics to Characterize Processes





## *Measure Deliverables*

---

- *Review internal/external CTQs*
- *Review competition/customer requirements to establish defect definition*
- *Validate that defect definition can be measured (validate data source)*
- *Establish data type (discrete vs. continuous), validate measurement system*
- *Start data gathering process*
- *Insure that team members provide input and understanding of project*
- *Develop process map with team members*



## *Take Aways— Introduction to Measure*

---

- *The variation of any **dependent** variable **Y** is determined by the variation of each of the **independent** variables **Xs**.*
- *The shape, **mean** and **standard deviation**, of a distribution curve characterizes a process.*
- ***Precision** is the consistency of a process as measured by the **standard deviation**. **Accuracy** is the ability to stay on target (on goal) as measured by the **mean**. The goal is to have a process that is both accurate (on target) and precise (small variation).*



## *Take Aways— Introduction to Measure*

---

- *Process **characterization** describes the **distribution** of the data.*
- *We will be using the five phases: **D - M - A - I - C** as a systematic approach for our improvement efforts.*



# *McDonald's Case Study*

---

## *PREVIEW of Define-Measure-Analyze (DMA)*

This is an optional example.



## *Objectives of this Preview*

---

### **McDonald's Case:**

- *Give preview of what is to come*
- *Show use of data to drive improvement*
- *Set performance standards (specifications) using customer survey data*
- *Identify sources of variation (SOVs) = total, within & between variation (SST = SSW + SSB)*
- *Determine process capability—subgrouping, control vs. technology*
- *Provide hands-on introduction to Minitab tools*



# The 12 Step Process

Step	Description	Focus	Tools	SSQC Deliverables
<b>Define</b>				
A	Identify Project CTQs			Project CTQs (1)
B	Develop Team Charter			Approved Charter (2)
C	Define Process Map			High Level Process Map (3)
<b>Measure</b>				
1	Select CTQ Characteristics	Y	Customer, QFD, FMEA	Project Y (4)
2	Define Performance Standards	Y	Customer, Blueprints	Performance Standard for Project Y (5)
3	Measurement System Analysis	Y	Continuous Gage R&R, Test/Retest, Attribute R&R	Data Collection Plan & MSA (6), Data for Project Y (7)
<b>Analyze</b>				
4	Establish Process Capability	Y	Capability Indices	Process Capability for Project Y (8)
5	Define Performance Objectives	Y	Team, Benchmarking	Improvement Goal for Project Y (9)
6	Identify Variation Sources	X	Process Analysis, Graphical Analysis, Hypothesis Tests	Prioritized List of all Xs (10)
<b>Improve</b>				
7	Screen Potential Causes	X	DOE-Screening	List of Vital Few Xs (11)
8	Discover Variable Relationships	X	Factorial Designs	Proposed Solution (13)
9	Establish Operating Tolerances	Y, X	Simulation	Piloted Solution (14)
<b>Control</b>				
10	Define & Validate Measurement System on X's in Actual Application	Y, X	Continuous Gage R&R, Test/Retest, Attribute R&R	MSA
11	Determine Process Capability	Y, X	Capability Indices	Process Capability Y, X (15)
12	Implement Process Control	X	Control Charts, Mistake Proof, FMEA	Sustained Solution (15), Documentation (16),



## *Scenario: McDonald's Case Study*

---

### **Define & Measure**

**Define :**     **Step A: Identify Project CTQs**  
                  **Step B: Develop Team Charter**  
                  **Step C: Define Process Map**

### **Measure Step 1: Select CTQ Characteristic**

You are McDonald's Green Belt Project team for this market area. To increase customer satisfaction and capture more market share, you performed customer surveys and a **Quality Function Deployment** (QFD) analysis. The survey and analysis indicate that "service time" is a major **Critical to Quality Characteristic (CTQ)** for the customer.

You separate "service time" into two parts:

1. wait time in line
2. order time (time from when the customer is greeted and begins to order until they are given their food and change)



## *Scenario: McDonald's Case Study*

---

### **Measure Step 2: Define Performance Standards**

*Order Time is the **measure of performance** selected for the CTQ of minimal service time. To determine what the performance standard should be, you survey 100 randomly selected customers, asking them:*

*“To ensure repeat business, how much time are you willing to spend from when you start to order your food until you are given your food and change?”*



*The survey data is contained in Minitab file `McD_stp2.mtw`—note that there are  $n = 100$  responses. Each response is a customer's reply, in seconds, to the above question.*

*Based on this data, determine the performance standard, or “specification” for order time that you would recommend.*



## *Scenario: Continued*

---

### **Measure Step 3: Establish Data Collection Plan, Validate Measurement System & Collect Data**

*The survey response is in seconds, based on the customer's reply. The Green Belt team's next concern is:*

*"...can the actual order time be measured in seconds (or better, say to 1/2 second, or ...)?"*

*To keep our story moving, let's assume the answer is YES—the actual order time can be reliably measured in seconds.\**

*Your first analysis effort will focus on order time. Team discussion and "brain storming" results in some preliminary opinions—order time may differ for:*

- *store location*
- *service type = drive-thru versus inside counter service*
- *order method = by numbers(fixed) vs. menu (ala carte)*
- *time of day = breakfast, lunch, dinner*
- *etc.*

*\*[the section on Measurement Systems Analysis addresses this issue in detail]*



## *Scenario: Continued*

*Two teams of Six Sigma Green Belts were sent to gather data at two different locations—one to the Schenectady (Union St.) McDonald’s and one to Clifton Park. Half of each team was asked to order at the drive-thru and half at the inside counter—for each, some to order “by number” and some “off the menu.”*

*The following data was collected:*

*Order Time [seconds]*

*Location [1 = Schenectady, 2 = Clifton Park]*

*Service [1 = Drive-thru, 2 = Counter]*

*Order Method [1 = By Number, 2 = Off Menu]*

*Time of Day [hh.mm]*

**[Actual order time data is in file McD\_st4.mtw]**

## **Analyze Step 4: Establish Process Capability**

*What is the process actually doing?*

*Examine how store location, service type, order method and time of day impact order time.*



# Example: Part I Performance Standard

## Measure Step 2: Define Performance Standard

Data File = McD\_stp2.mtw

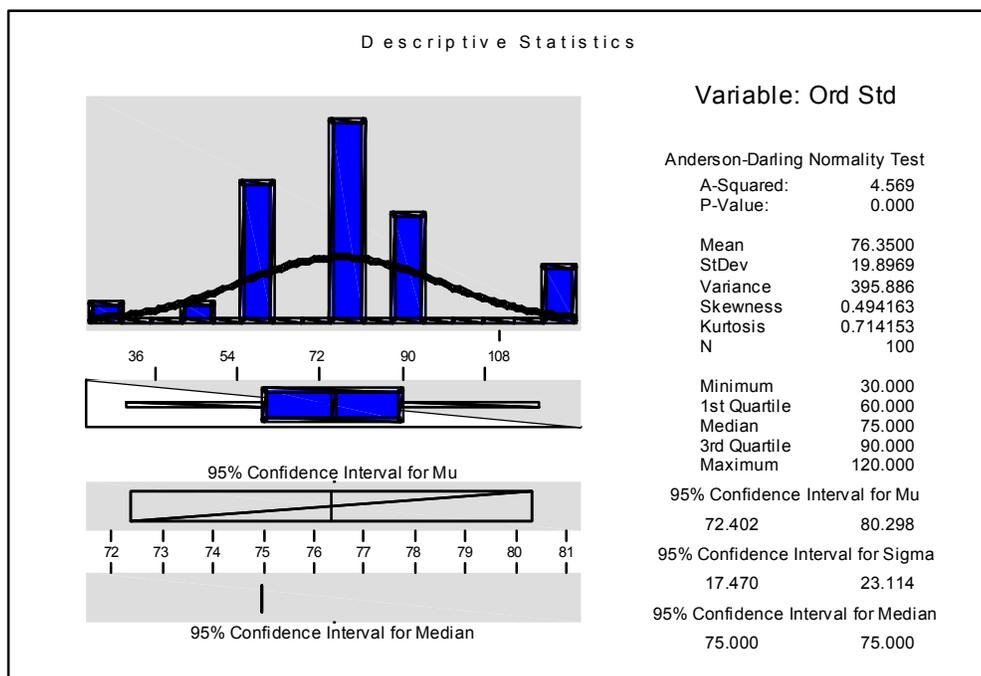
Survey Results:

- What order time is acceptable to the customer?
- Based on this, what is our “specification” for order time?

### Descriptive Statistics

Variable	N	Mean	Median	Tr Mean	StDev	SE Mean
Ord Std	100	76.35	75.00	76.17	19.90	1.99

Variable	Min	Max	Q1	Q3
Ord Std	30.00	120.00	60.00	90.00





## Example: Part I Performance Standard

### Measure Step 2: Define Performance Standard

Data File = McD\_stp2.mtw

#### DESCRIPTIVE STATISTICS:

“Center” or “Location” = the 4 Ms

- Mean =  $\bar{X}$
- Median
- Mode
- Mid-Range =  $(\text{Max} - \text{Min})/2$

“Dispersion” or “Spread”:

- Range =  $(\text{Max} - \text{Min})$
- Variance =  $SD^2 = s^2$  [based on SST]
- Std Dev =  $SD = s$
- Coeff. of Variation =  $CV = (SD/\bar{X}) \times 100\%$

“Normal Curve” ?? :

4 Ms are » equal  $\Rightarrow$  Mean = Median = Mode = Mid-Range

Skew » 0 [Skew »  $3(\text{Mean} - \text{Median})/SD$  (Pearson)]

Kurtosis » 0



## *Example: Part I Performance Standard*

### **Measure Step 2: Define Performance Standard**

Data File = McD\_stp2.mtw

*Review: based on the Customer survey data, what is the recommended performance standard?*

*Mean = 76 seconds*

*Median = 75*

*Min = 30*

*Max = 120*

*Range = 120-30 = 90*

*StdDev = 20*

*Also, McDonald's desires to:*

*“be better than average,” “delight the customer”*

*“set realistic stretch goals,” “make fair profits”*



#### **Performance Standard ... or Specifications**

**Desire**

**Data**

**LSL = 30**

**Min = 30**

**Target = 60** vs.

**Mean = 76**

**USL = 90**

**Max = 120**

**Data + Desire = Standard**



## *Example: Part II Process Performance*

### **Analyze Step 4: Establish Process Capability**

Data File = McD\_stp4.mtw

Review: We now look at the actual Order Time data gathered by the Green Belt teams—there are  $k = 54$  items.

Look at data—column orientation of Minitab

Order Time =  $Y = f(\text{Location, Service, } \dots, \text{etc.})$

or

Order Time =  $Y = f(X_1, X_2, X_3, \dots, X_n)$

- the  $X$ s are factors that may influence the output  $Y$
- they are the basis for “SubGrouping” the data:
  - by Location
  - by Service
  - by Order Method
  - by Team
  - etc.

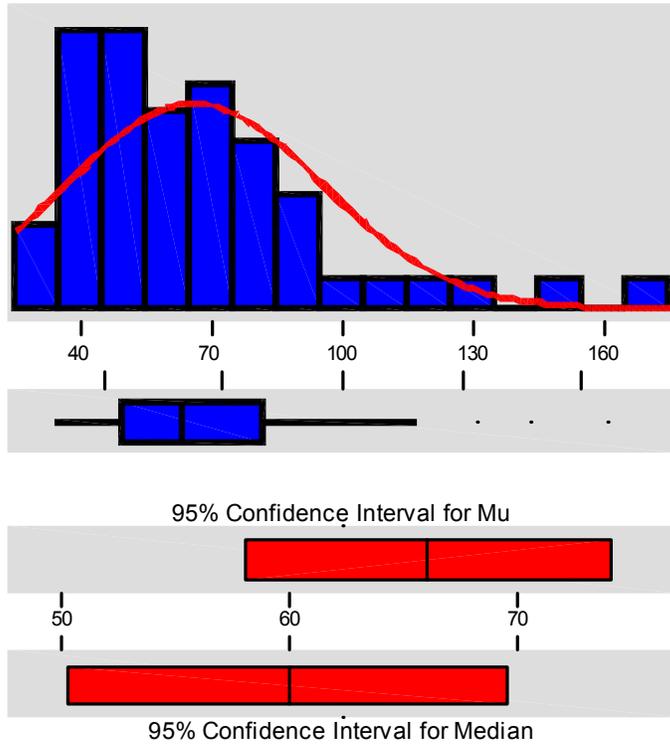
What questions can this data answer?

Plots or graphs help raise questions about relation between  $Y$  and the  $X$ s



# Example: Part II Process Performance

## Descriptive Statistics



Variable: Ord Time

Anderson-Darling Normality Test

A-Squared: 1.493  
P-Value: 0.001

Mean 66.0185  
StDev 29.3241  
Variance 859.905  
Skewness 1.42494  
Kurtosis 2.43366  
N 54

Minimum 28.000  
1st Quartile 44.750  
Median 60.000  
3rd Quartile 80.000  
Maximum 167.000

95% Confidence Interval for Mu  
58.015 74.022

95% Confidence Interval for Sigma  
24.651 36.201

95% Confidence Interval for Median  
50.357 69.643

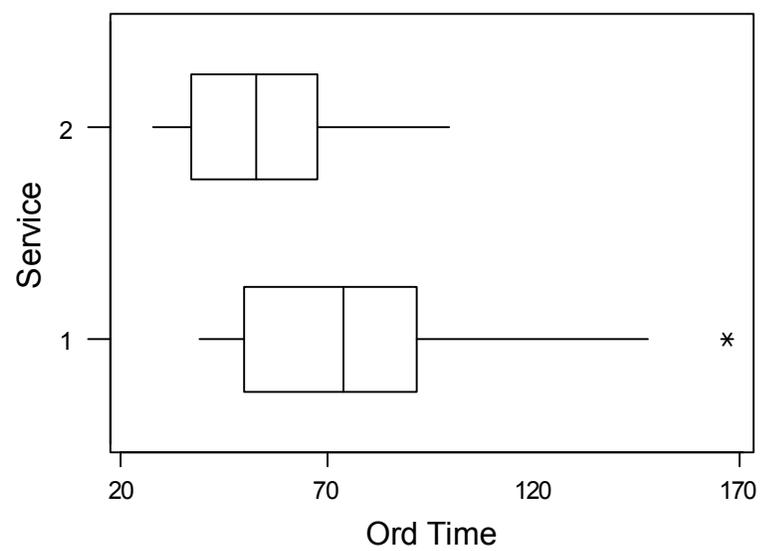
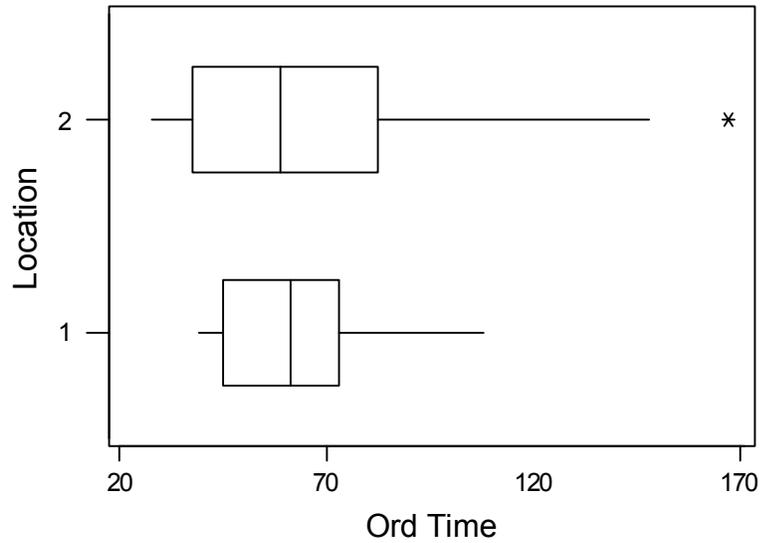
### Descriptive Statistics

Variable	N	Mean	Median	Tr Mean	St Dev	SE Mean
Ord Time	54	66.02	60.00	63.02	29.32	3.99

Variable	Minimum	Maximum	Q1	Q3
Ord Time	28.00	167.00	44.75	80.00

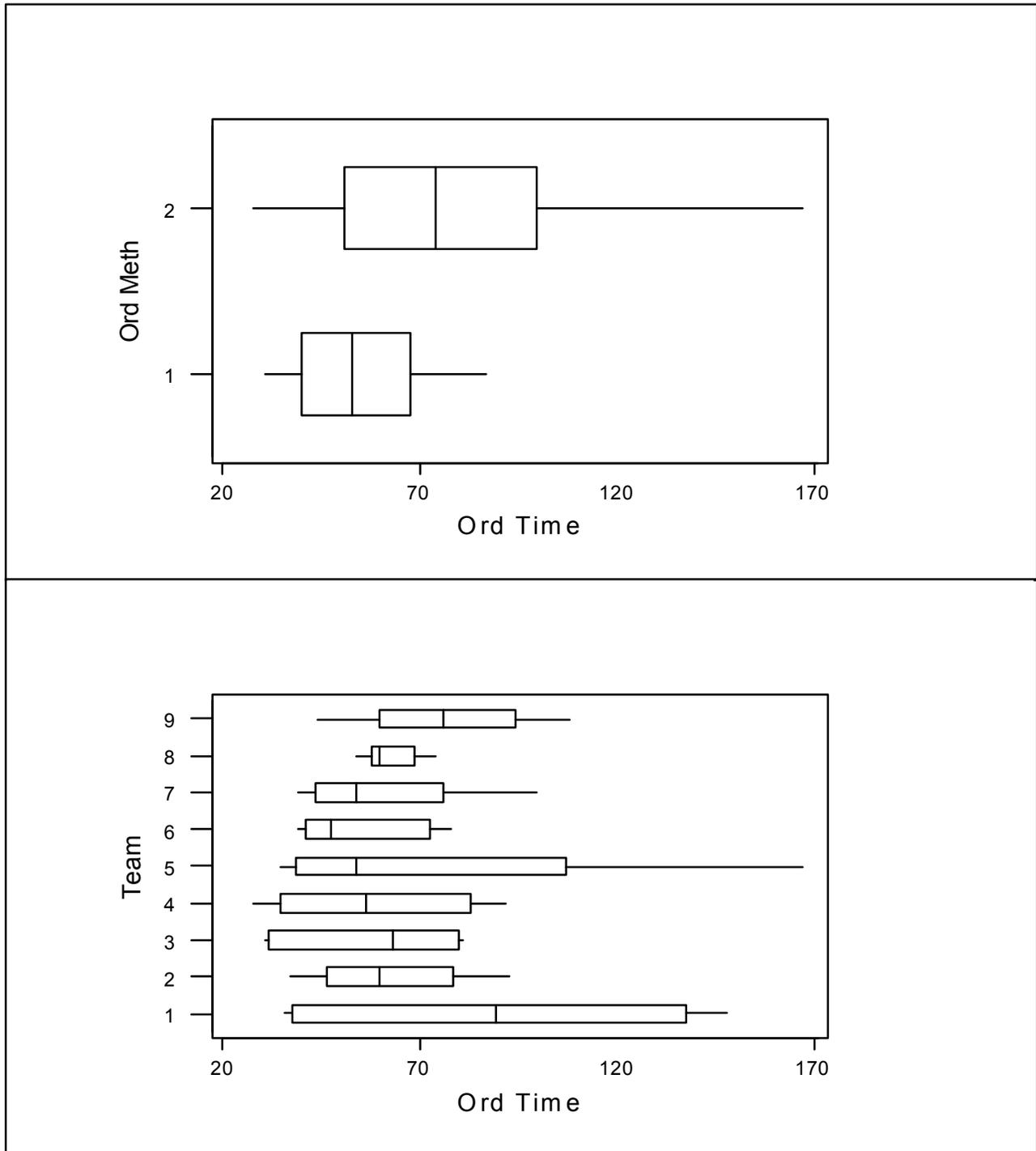


## Example: Part II Process Performance



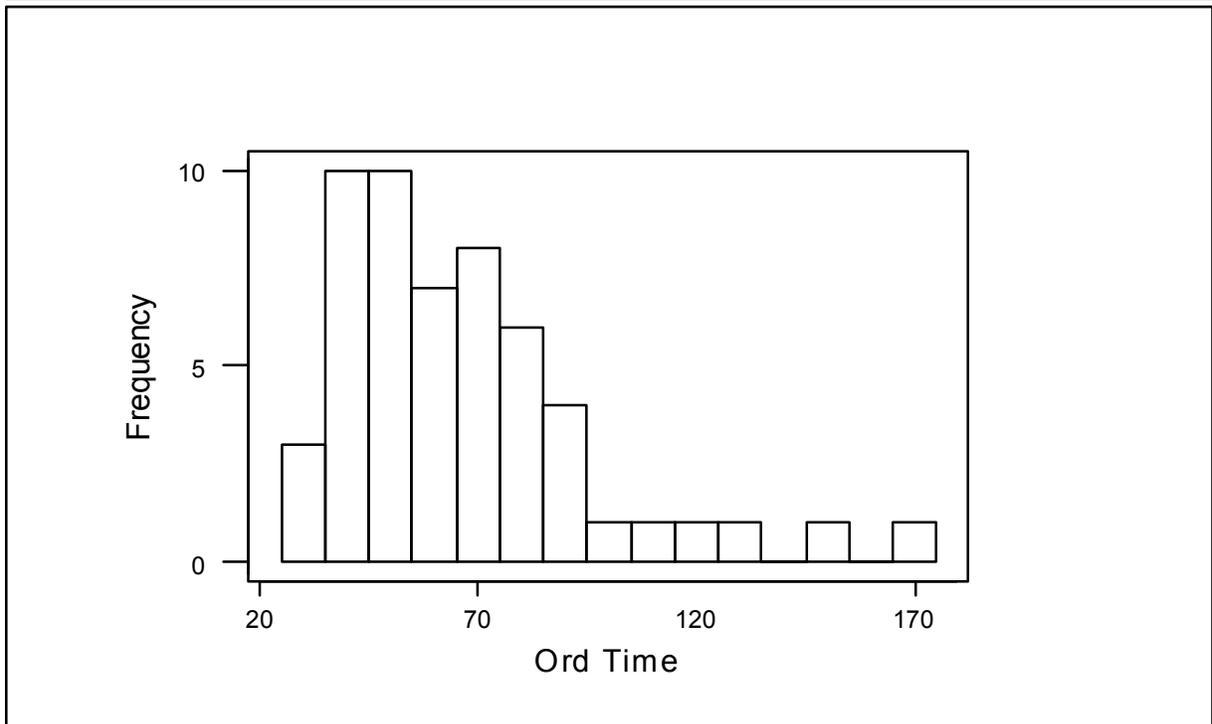
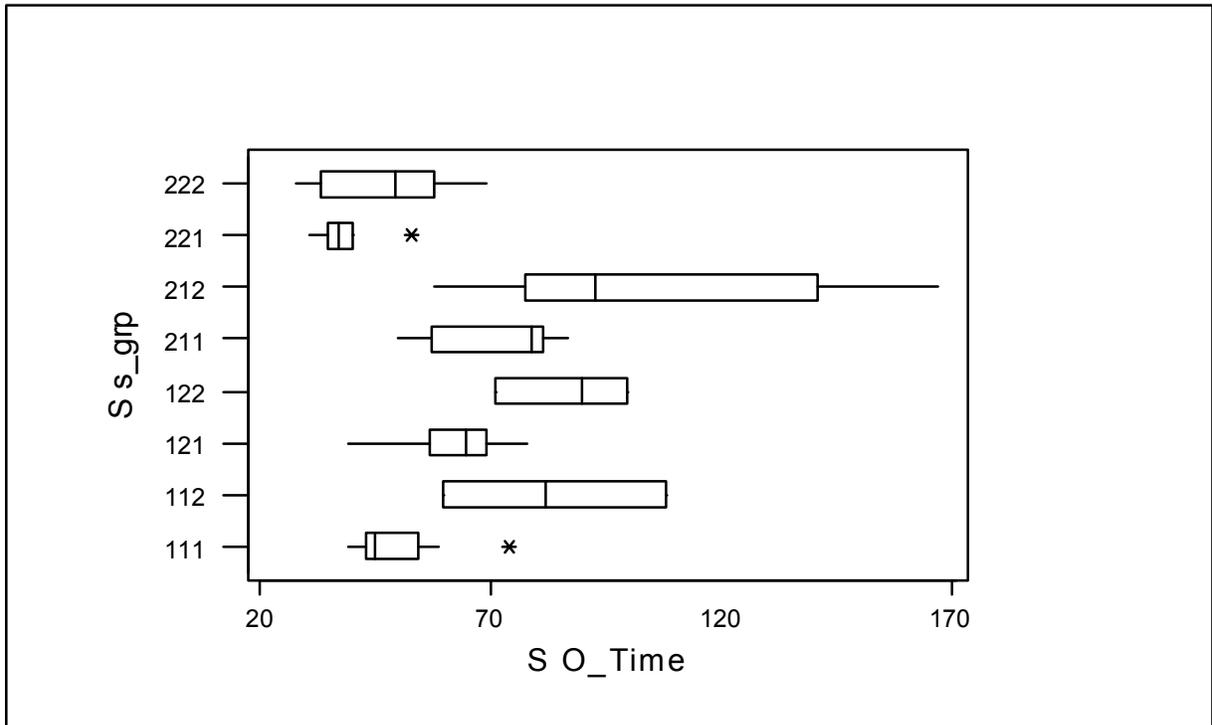


## Example: Part II Process Performance



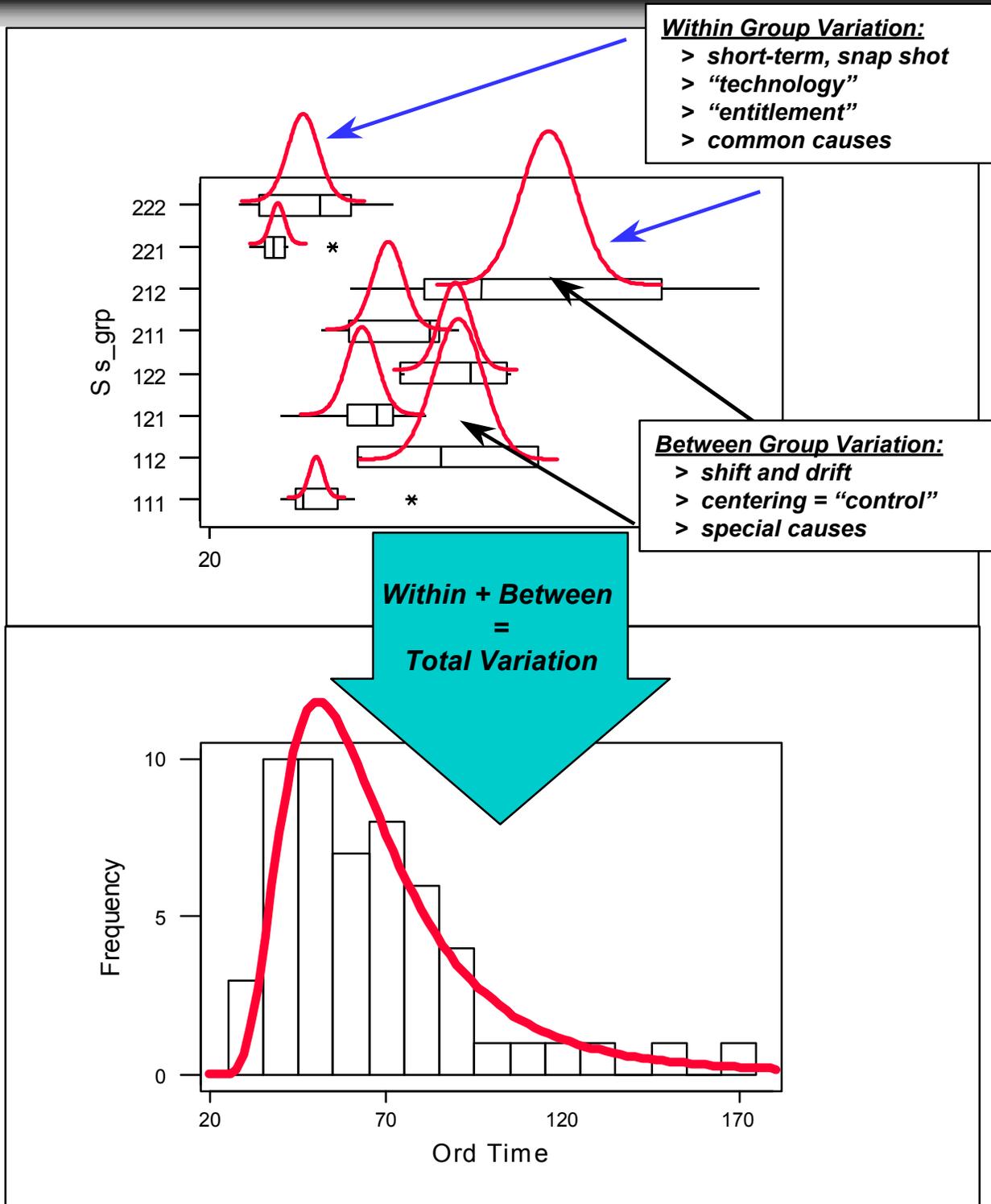


## Example: Part II Process Performance





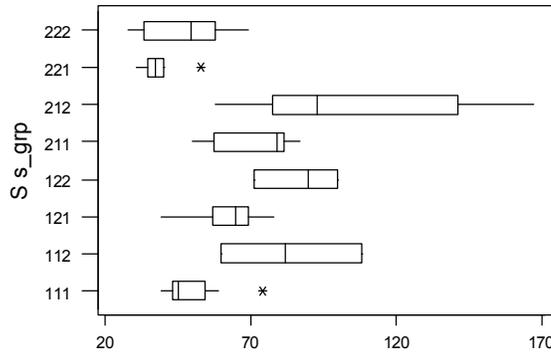
# Example: Part II Process Performance



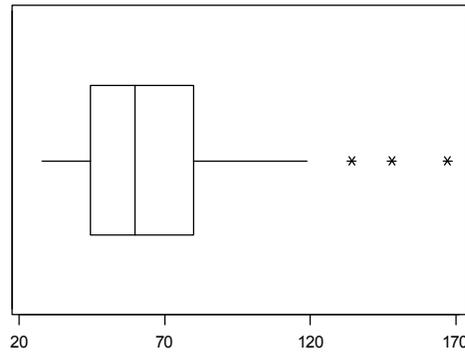


# Example: Part II Process Performance

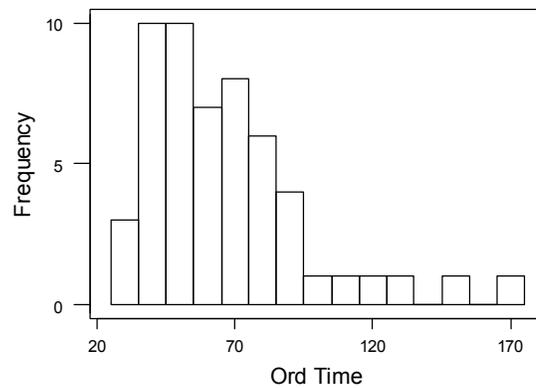
**3 Views:**



**Subgrouped—  
shows “components  
-of-variation”**



**Overall  
BoxPlot**



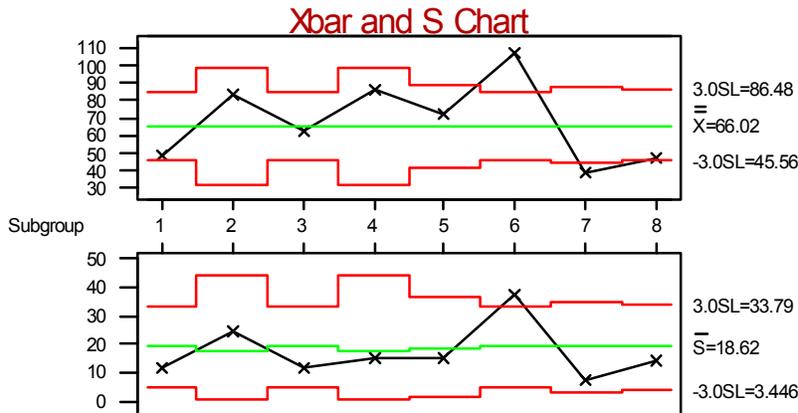
**Overall  
Histogram**



# Example: Part II Process Performance

**Minitab >Six Sigma >Process Report**  
 [or >Stat >Quality Tools >Six Sigma Process Report]  
 >Data = Single column = 'S O\_Time'  
 >Subgroup size = 'S s\_grp' > enter LSL = 30, TARGET = 60, USL = 90

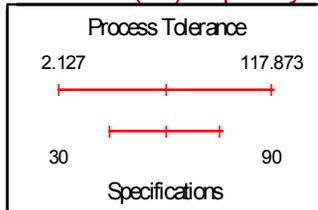
## Report 2: Process Capability for S O\_Time



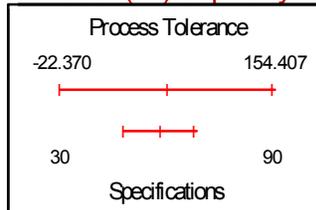
### Capability Indices

	ST	LT
Mean	60.0000	66.0185
StDev	19.1864	29.3241
Z.USL	1.5636	0.8178
Z.LSL	1.5636	1.2283
Z.Bench	1.1855	0.4778
Z.Shift	0.7077	0.7077
P.USL	0.058955	0.206734
P.LSL	0.058955	0.109669
P.Total	0.117910	0.316403
Yield	88.2090	68.3597
PPM	117910	316403
Cp	0.52	
Cpk	0.41	
Pp		0.34
Ppk		0.27

### Potential (ST) Capability



### Actual (LT) Capability



Data Source:  
 Time Span:  
 Data Trace:

## BOTTOM LINE:

**VARIATION (SPREAD) = too much**  
**CENTER (Mean) = not on target**



## *Example: Part II Summary*

### **Summary:**

Performance Standard = what the customer wants & McDonald's desires:

Survey data:      Mean = 76 seconds  
                            Min = 30                      Max = 120  
                            Range = 90                      StdDev = 20

McDonald's desire: "Better than average," "Fair profit," ...

Specification:    LSL = 30    Target = 60    USL = 90

Process Performance = what is actually happening:

Study data:      Mean = 66 sec. [vs. Target = 60]  
                            Min = 28                      Max = 167  
                            Range = 139                      StdDev = 29

#### **BOTTOM LINE—replay:**

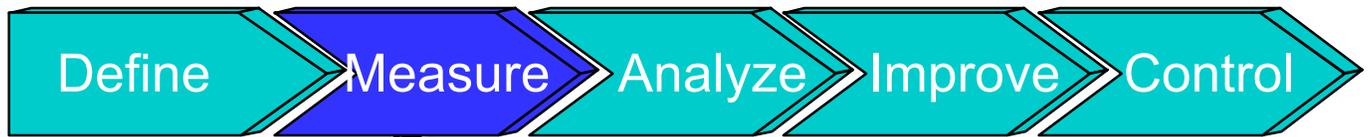
**VARIATION (SPREAD) = too much**

**[StdDev = 29 relative to USL = 90 and LSL = 30]**

**CENTER (Mean) = not on target [66 vs. 60]**



# Measure Phase



**1. Select CTQ Characteristics**

**2. Define Performance Standards**

**3. Measurement System Analysis**

*Deliverable: Identify Measurable CTQ That Will Be Improved*

*Tools:*

- QFD
- Process Mapping
- FMEA
- Discrete vs. Continuous Data

*Deliverable: Determine and Confirm Specification Limits For Your Y.*

*Deliverable: Measurement System Adequate to Measure Y.*

*Tools:*

- Continuous Gage R&R
- Test/Retest
- Attribute Gage R&R



## *Select CTQ Characteristic Objectives*

---

- *By the end of Step 1, the BB/GB will have:*
  - *Selected the Critical to Quality (CTQ) characteristic to be improved in his/her project.*
  - *Narrowed the focus of his/her project to an actionable level.*
  - *Established the project team and gained consensus on the project definition.*



---

# *Quality Function Deployment (QFD)*



## *QFD Objectives*

---

- *By the end of the training program, the participant will be able to:*
  - *Relate the Quality Function Deployment (QFD) process to the Six Sigma process.*
  - *Analyze when QFD is appropriate to use.*
  - *Describe the phases of QFD.*



## ***THE QFD “OPPORTUNITY”***

- *QFD Is An Opportunity to Really Listen*
  - *The Customer Knows What They Want*
  - *They Often Don’t Directly Verbalize*
  - *Lots of “OTHER” Stuff Tends to Surface*
  - *Watch for What They Say They Don’ Want*
- *Understand The Types Of Needs Of Customers*
  - *What Does the Customer Say*
- *“Voice Of The Customer” Is Also A Process*
  - *VOC Is The Independent Process*
  - *QFD Is The Dependent Process*

**The Customer Knows What They  
Want  
How We Extract It Also Matters**



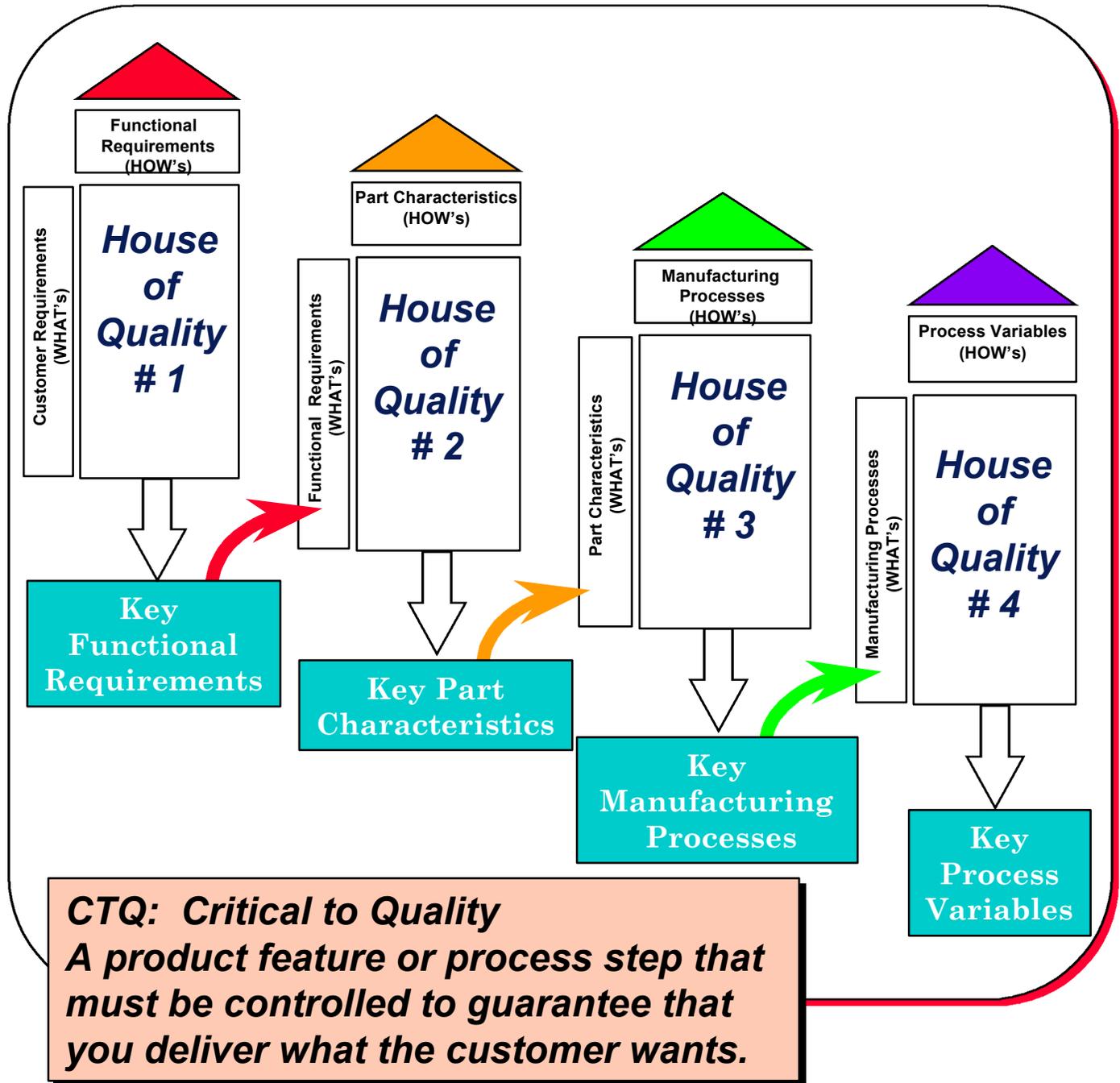
## *Definition of QFD*

---

- *Structured methodology to identify and translate customer needs and wants into technical requirements and measurable features and characteristics:*
  - *From marketing and sales*
  - *To research and product development*
  - *To engineering and manufacturing*
  - *To distribution and services*
  
- *Used to identify Critical to Quality Characteristics (CTQs).*

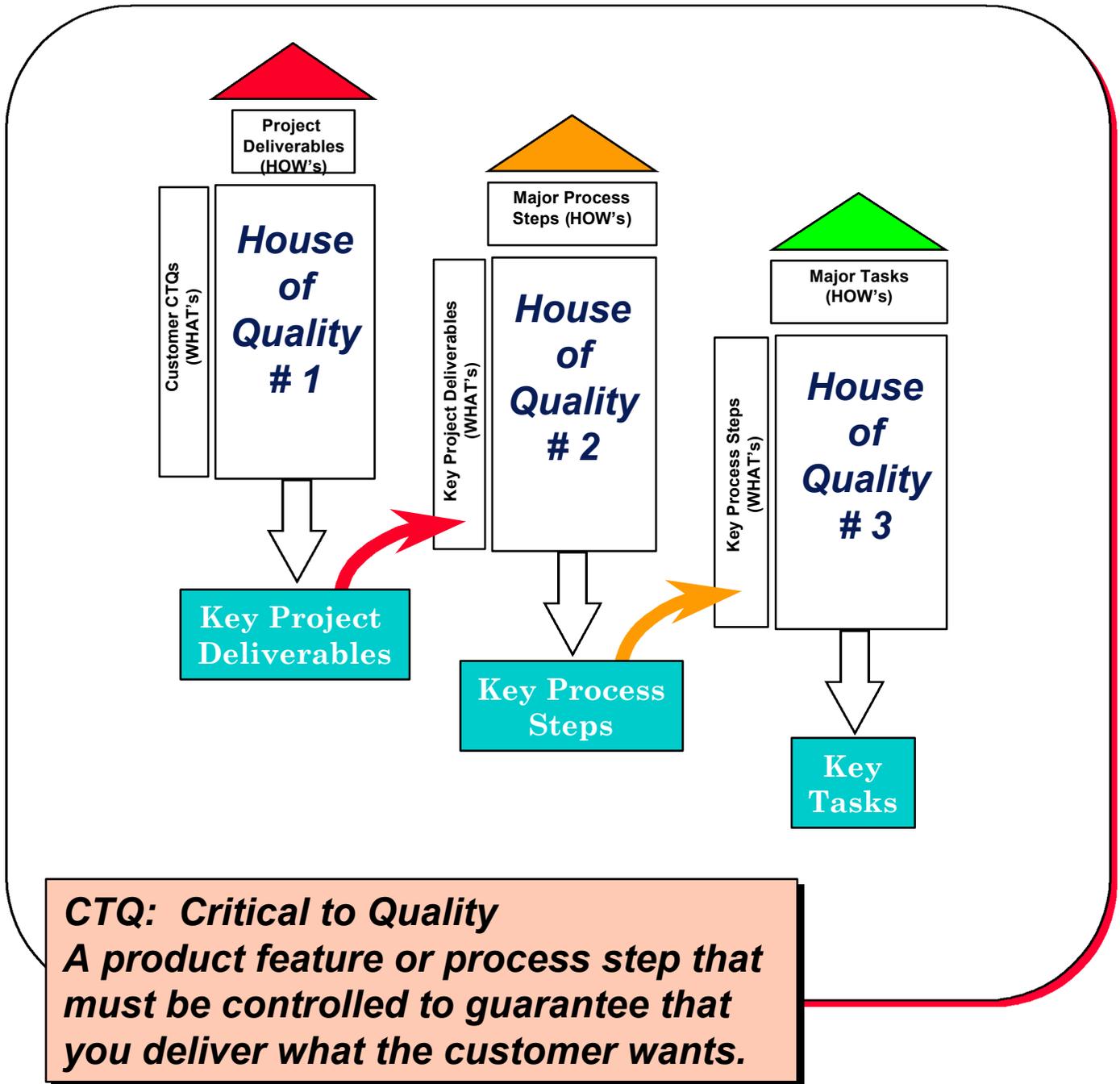


# QFD Flowdown (Product Application)





# QFD Flowdown (Services Application)





---

# *Building a House of Quality*

## *A Product Quality Example*



# Generator House of Quality

		↑	↑	↓	↑	↓	↓	○
		Kilowatt Ratings	Total Available Running Hours	Total Forced Outage Time	Component Life Cycles	Fuel Cost/Kilowatt	Total Repair/Maintenance Time	Rotor Burst Speed
Max Power Output	3	◎						
Availability	4		◎	○	◎		△	
Reliability	5			◎	◎		△	
Long Component Life	2		△		◎			
Efficiency	4	◎				◎		
Maintainability	2						◎	
Easy to Troubleshoot	1							
		A Kilowatts	B Hours/Yr	C Hours/Yr	D Years	E \$/Kilowatt	F Hours	G RPM
		63	38	57	99	36	27	



## What Does the Customer Want?

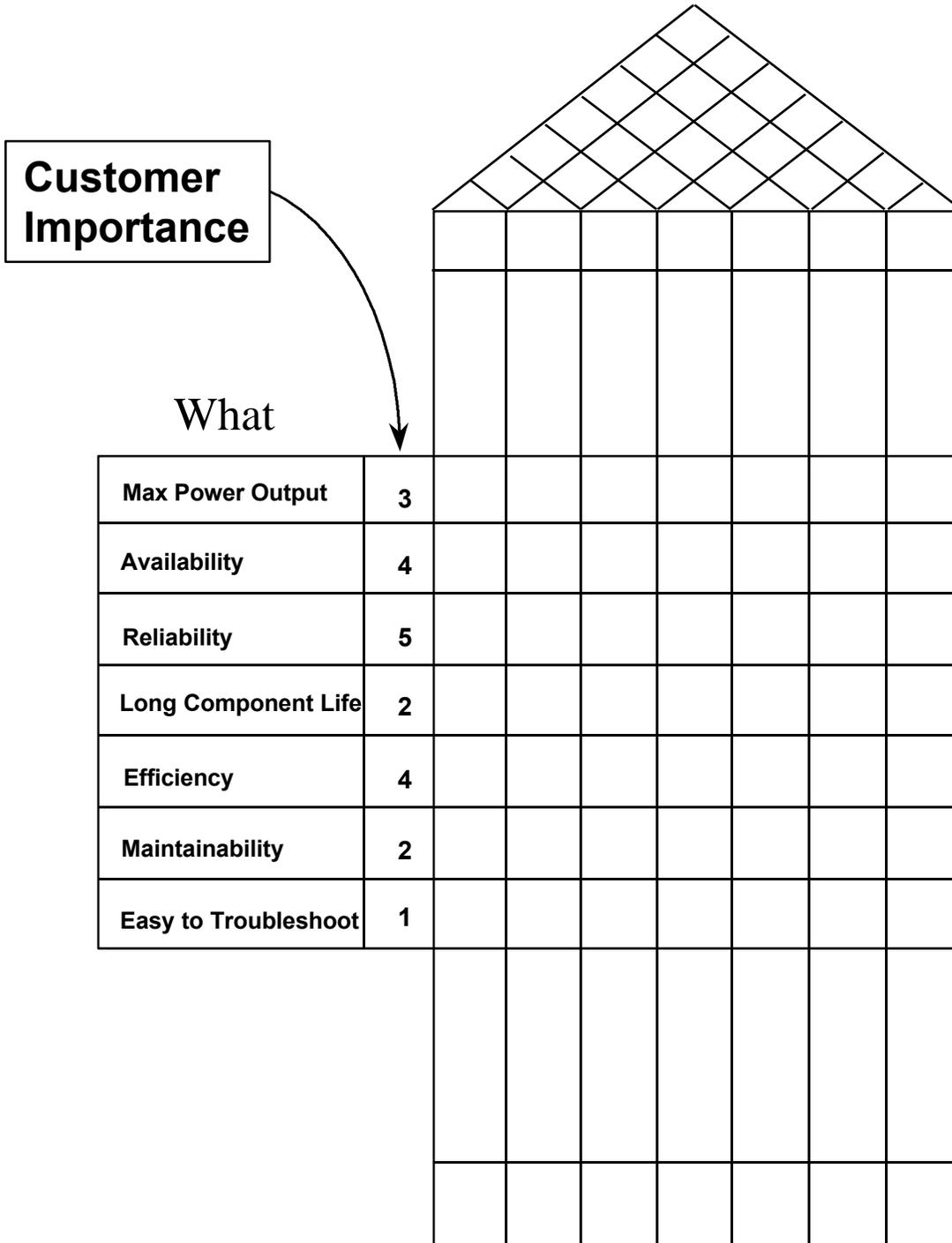
### **What**

Max Power Output
Availability
Reliability
Long Component Life
Efficiency
Maintainability
Easy to Troubleshoot

***The Voice of the Customer***

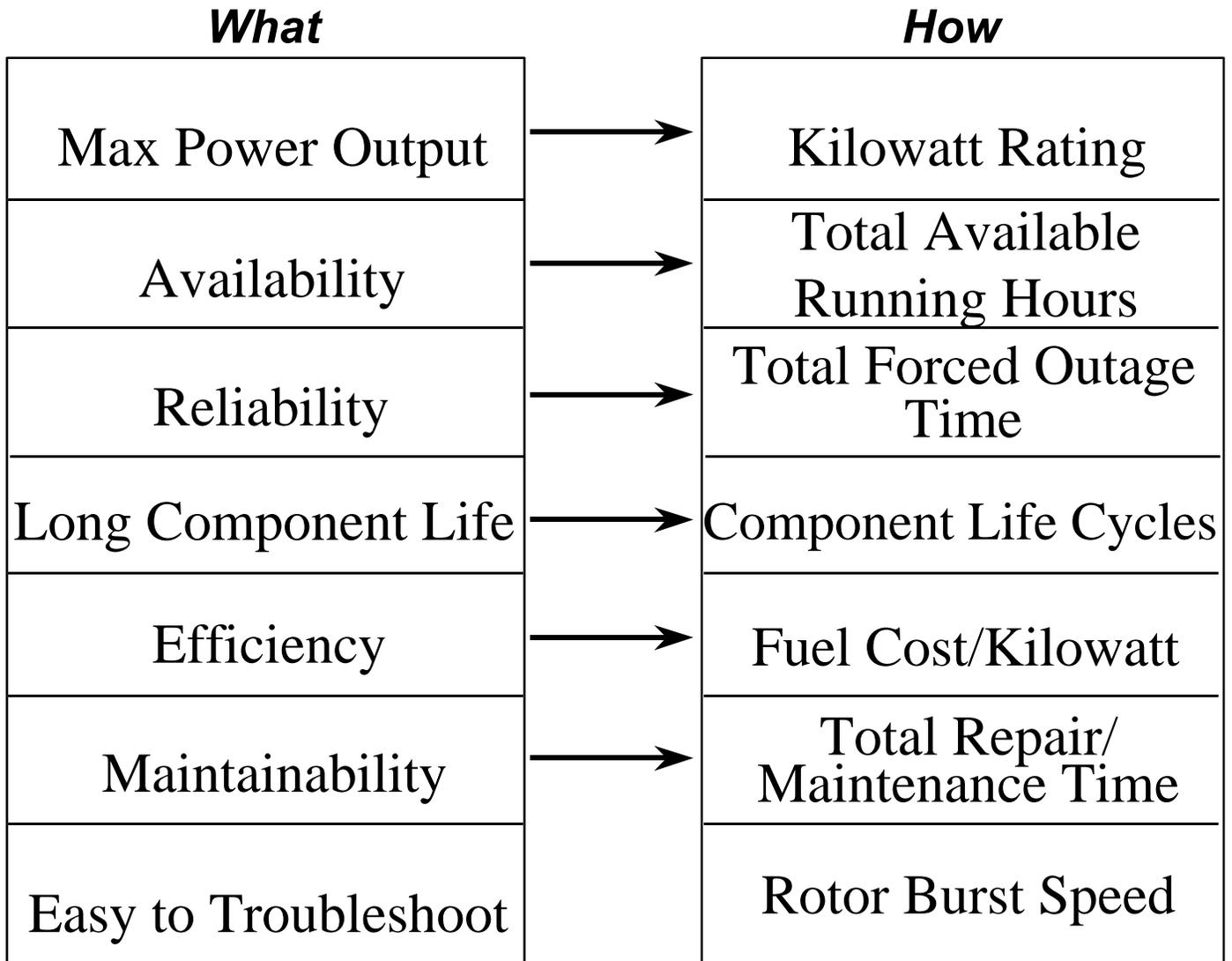


## What Does the Customer Want (cont.)





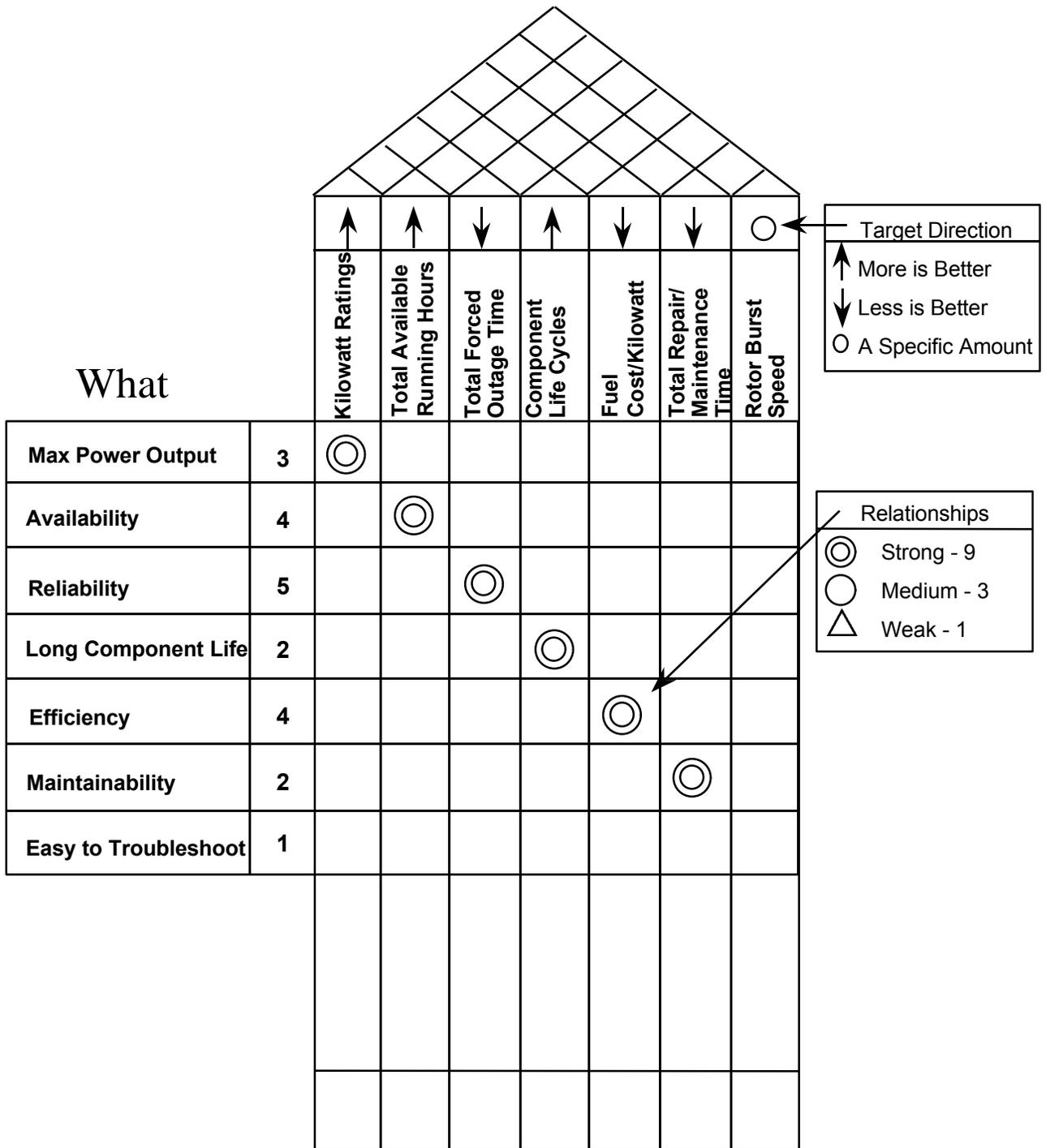
## *Translating for Action*



***Danger: There is not necessarily a one-to-one relationship***

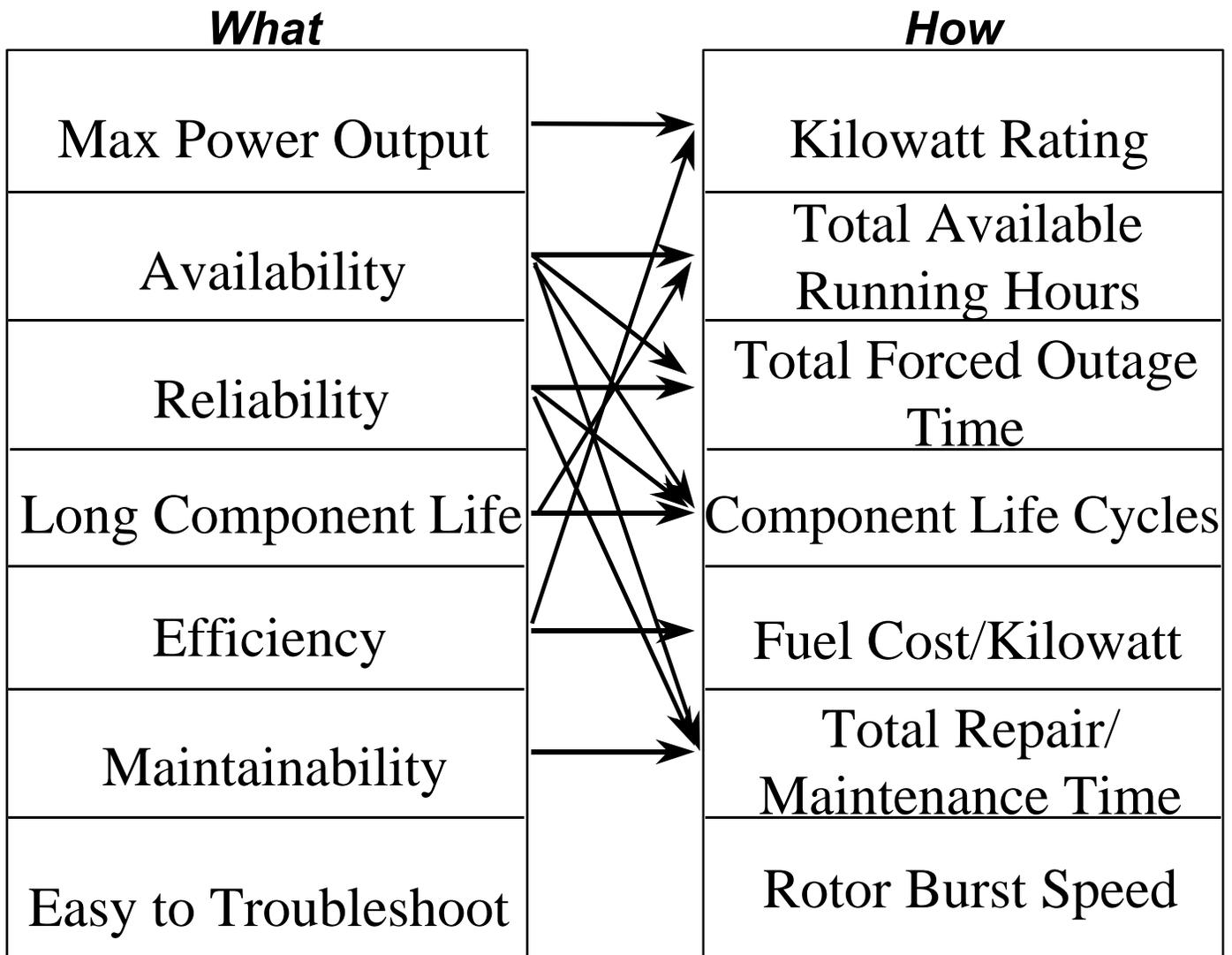


# How Do You Satisfy the Wants?





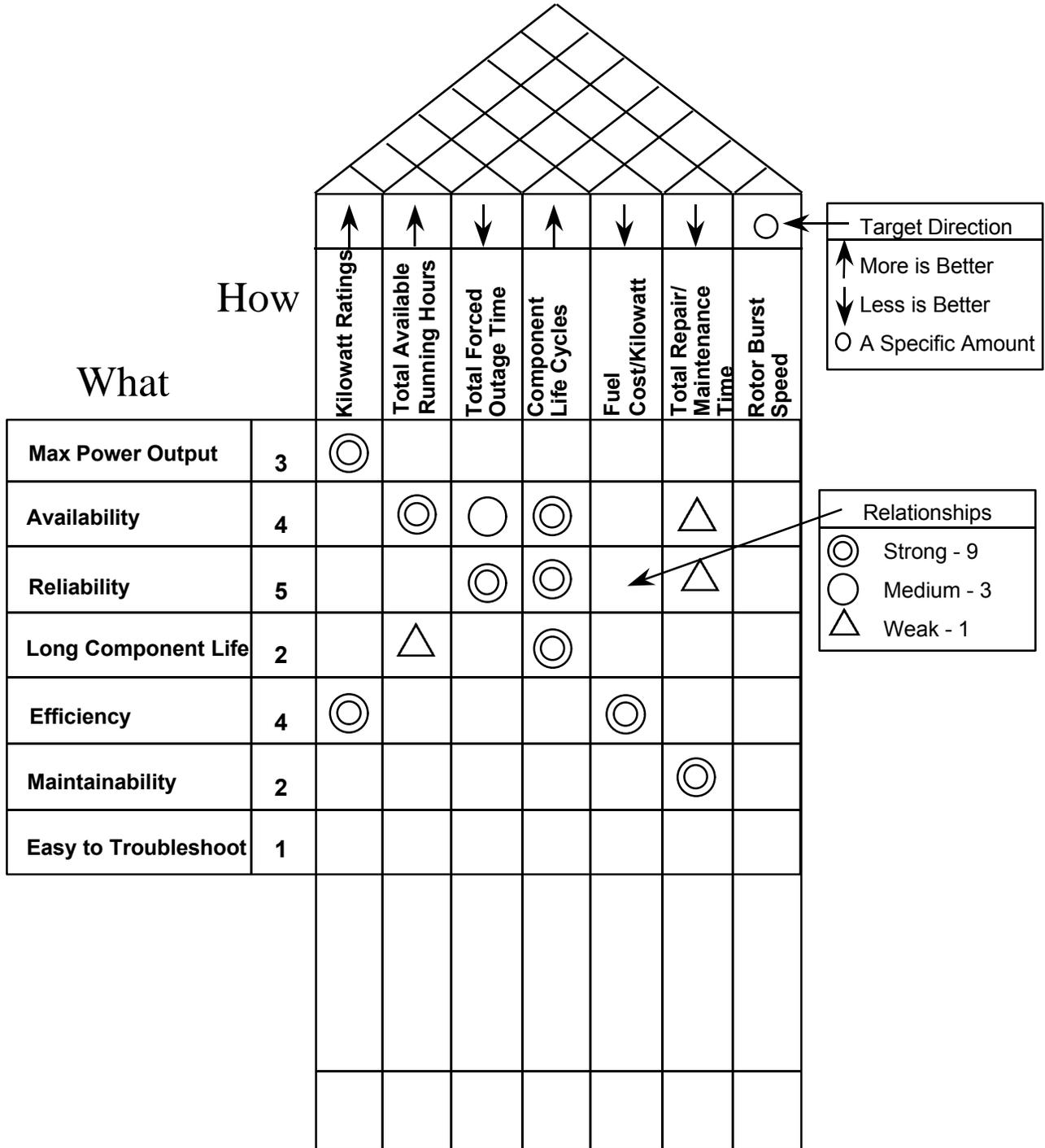
## *Complex Relationships*



***There are usually interrelationships with no single solution***

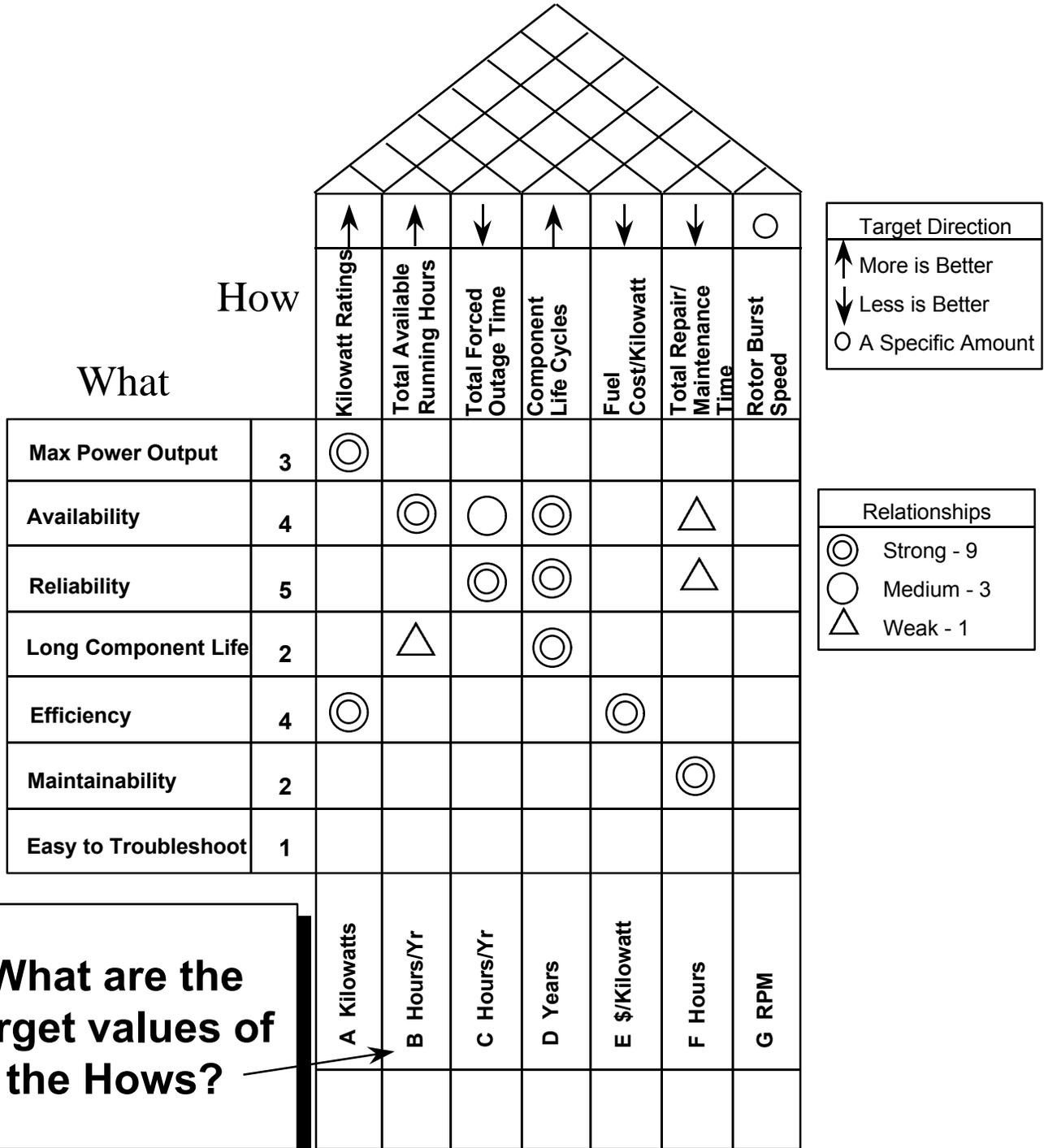


# Untangling the Web



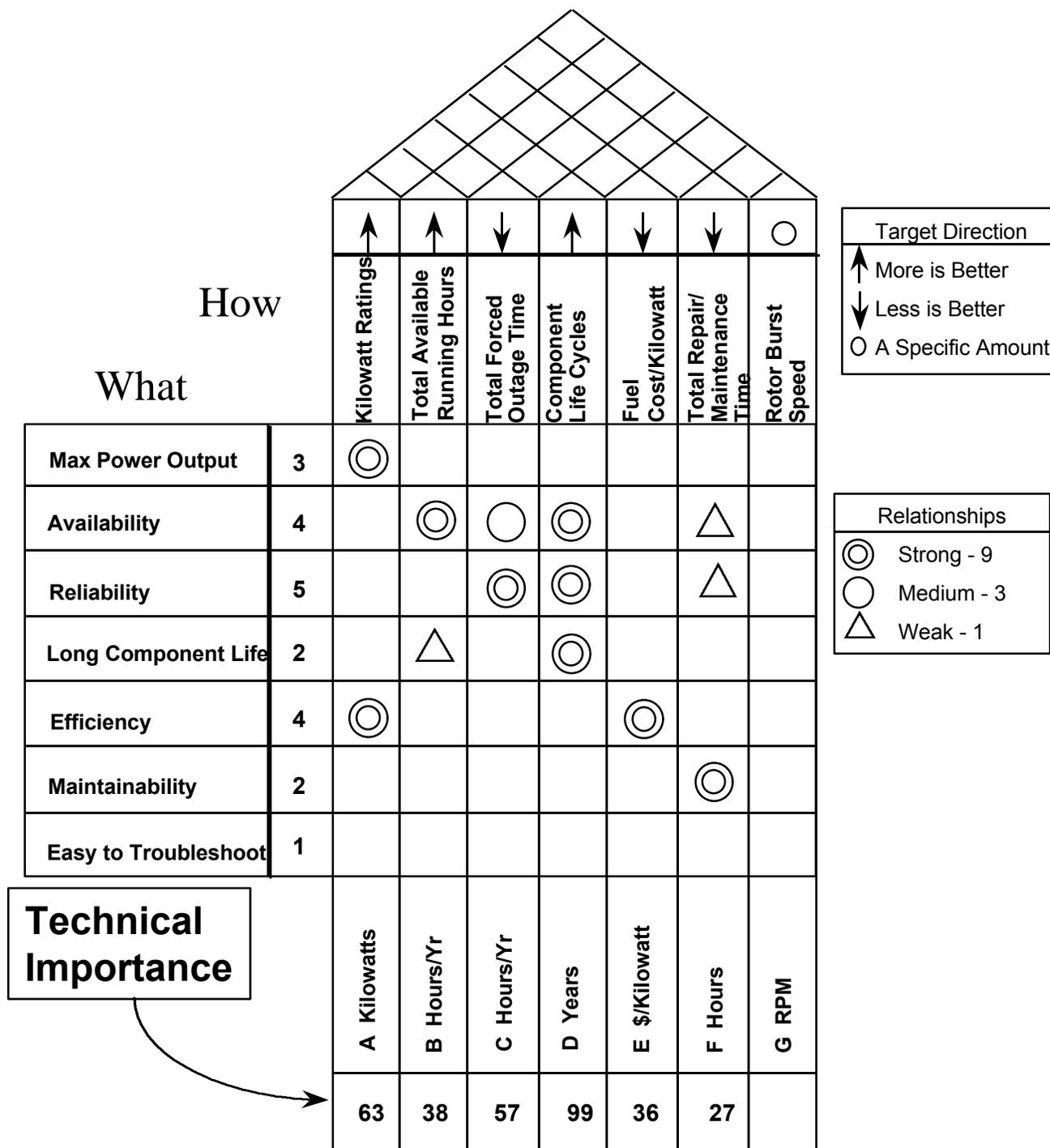


# How Much?





# How Important are the Customer Wants?





# Correlation Matrix

Strong Positive  
 Positive  
 Negative  
 Strong Negative

How

What

		↑	↑	↓	↑	↓	↓	○
		Kilowatt Ratings	Total Available Running Hours	Total Forced Outage Time	Component Life Cycles	Fuel Cost/Kilowatt	Total Repair/Maintenance Time	Rotor Burst Speed
What	How	A Kilowatts	B Hours/Yr	C Hours/Yr	D Years	E \$/Kilowatt	F Hours	G RPM
Max Power Output	3	⊙						
Availability	4		⊙	○	⊙		△	
Reliability	5			⊙	⊙		△	
Long Component Life	2		△		⊙			
Efficiency	4	⊙				⊙		
Maintainability	2						⊙	
Easy to Troubleshoot	1							
		63	38	57	99	36	27	

Target Direction

↑ More is Better  
 ↓ Less is Better  
 ○ A Specific Amount

Relationships

⊙ Strong - 9  
 ○ Medium - 3  
 △ Weak - 1



---

## *Analyzing & Diagnosing the QFD*

---

1. *Blank rows*
2. *Blank columns*
3. *No design constraints in hows*
4. *Resolve negative correlations*
5. *Finalize target values*
6. *What technical requirements should be deployed to phase II (Design Deployment)?*



---

# *Building a House of Quality*

## *A Transactional Quality Example*



# QFD...Begin With the Customer

**What Does the Customer Want?**



What

<b>Responsiveness to the Customer</b>									
<b>Price &amp; Product Competitiveness</b>									
<b>Hardware Quality</b>									
<b>Hardware On Time Delivery</b>									
<b>Software Quality</b>									
<b>Software On Time Delivery</b>									
<b>Contract Understanding</b>									
<b>Product Performance</b>									




# Begin With the Customer

**How Important is It?**

What

Responsiveness to the Customer	5								
Price & Product Competitiveness	3								
Hardware Quality	5								
Hardware On Time Delivery	4								
Software Quality	3								
Software On Time Delivery	4								
Contract Understanding	3								
Product Performance	4								



## Translating Whats to Hows

**Identify the Functions or Processes that Impact Customer Wants**



		Sales	Project Management	Engineering	Manufacturing	Sourcing	Partners	Field Engineer
Responsiveness to the Customer	5							
Price & Product Competitiveness	3							
Hardware Quality	5							
Hardware On Time Delivery	4							
Software Quality	3							
Software On Time Delivery	4							
Contract Understanding	3							
Product Performance	4							



# The Relationship Between What & How

**Evaluate the Impact of Each Function/Process on the Customer Wants**

What		Hows						
		Sales	Project Management	Engineering	Manufacturing	Sourcing	Partners	Field Engineer
Responsiveness to the Customer	5	9	9	9	3	1	3	9
Price & Product Competitiveness	3	9		9	9			
Hardware Quality	5			3	9	9	3	9
Hardware On Time Delivery	4	1	3	3	9	9	3	
Software Quality	3			9	3	3		3
Software On Time Delivery	4		3	9		3	3	1
Contract Understanding	3	9	9	9			3	1
Product Performance	4	3		9	3		3	9

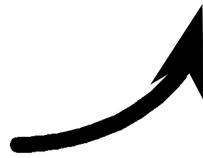
**Relationships**  
 Direct & Strong = 9  
 Direct = 3  
 Indirect = 1



# Qualifying Importance

What	Functions							
	Sales	Project Management	Engineering	Manufacturing	Sourcing	Partners	Field Engineer	
Responsiveness to the Customer	5	9	9	9	3	1	3	9
Price & Product Competitiveness	3	9		9	9			
Hardware Quality	5			3	9	9	3	9
Hardware On Time Delivery	4	1	3	3	9	9	3	
Software Quality	3			9	3	3		3
Software On Time Delivery	4		3	9		3	3	1
Contract Understanding	3	9	9	9			3	1
Product Performance	4	3		9	3		3	9
	115	96	225	144	107	75	142	

**Calculate the overall magnitude of the impact each function/process has on the customer wants**

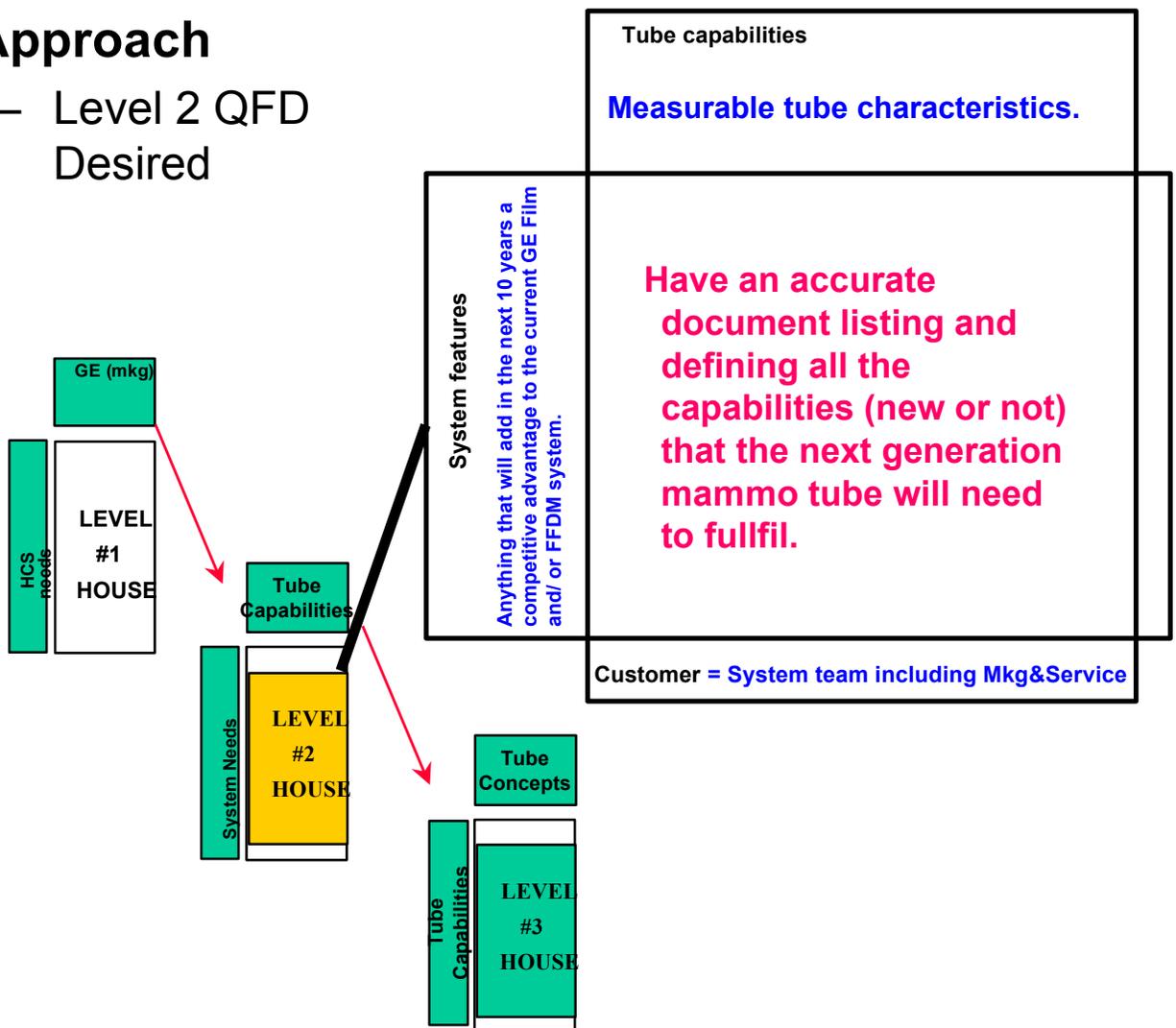




# GEMS Tube Example - optional

- **Project Goal**
  - Develop QFD to Identify Capabilities of the Next Generation Mammo Tube

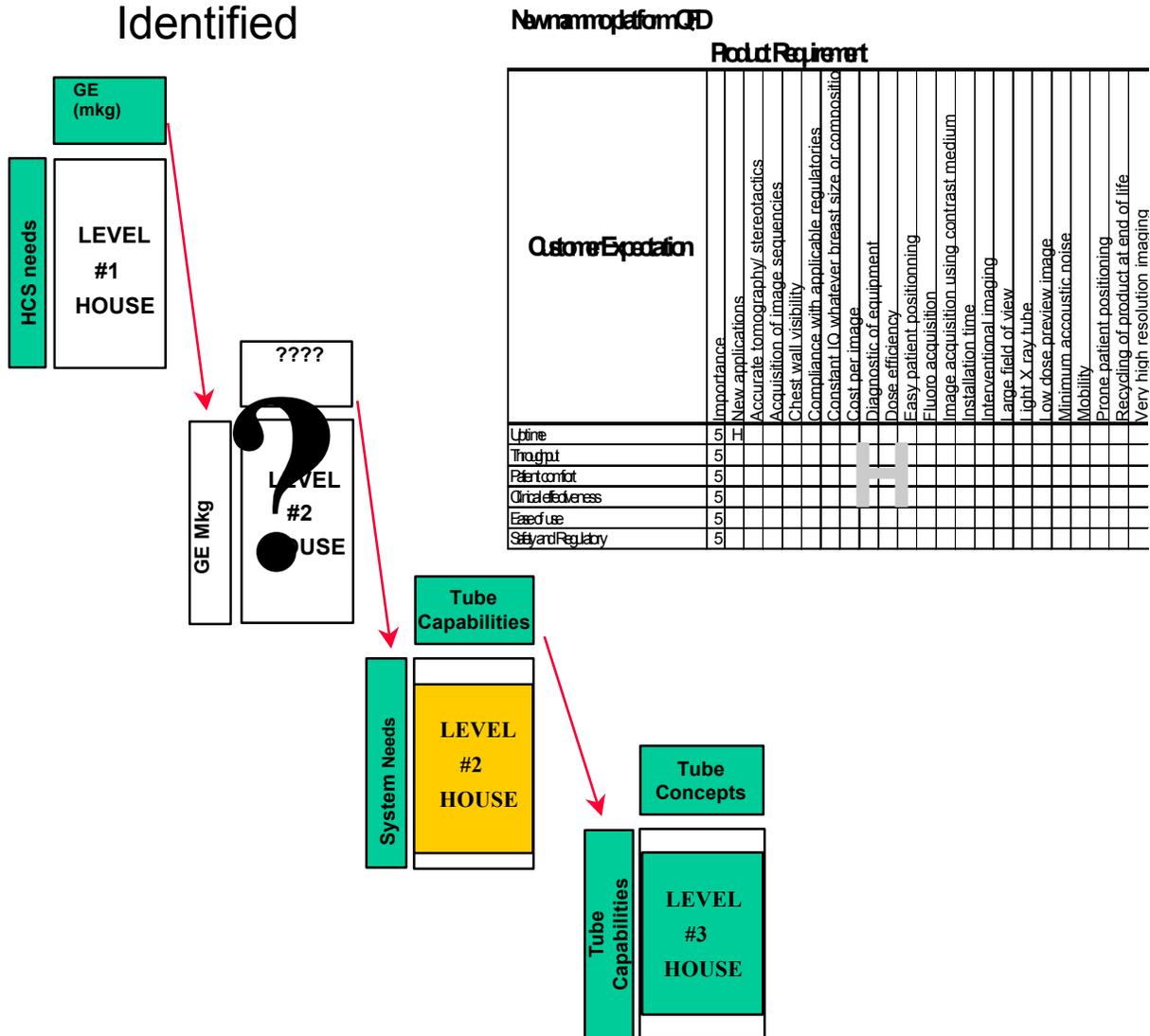
- **Approach**
  - Level 2 QFD Desired





# GEMS Tube Example

- **After First Day**
  - Level 1 Strawman -- Needs Verification
  - Possible Missing House Identified



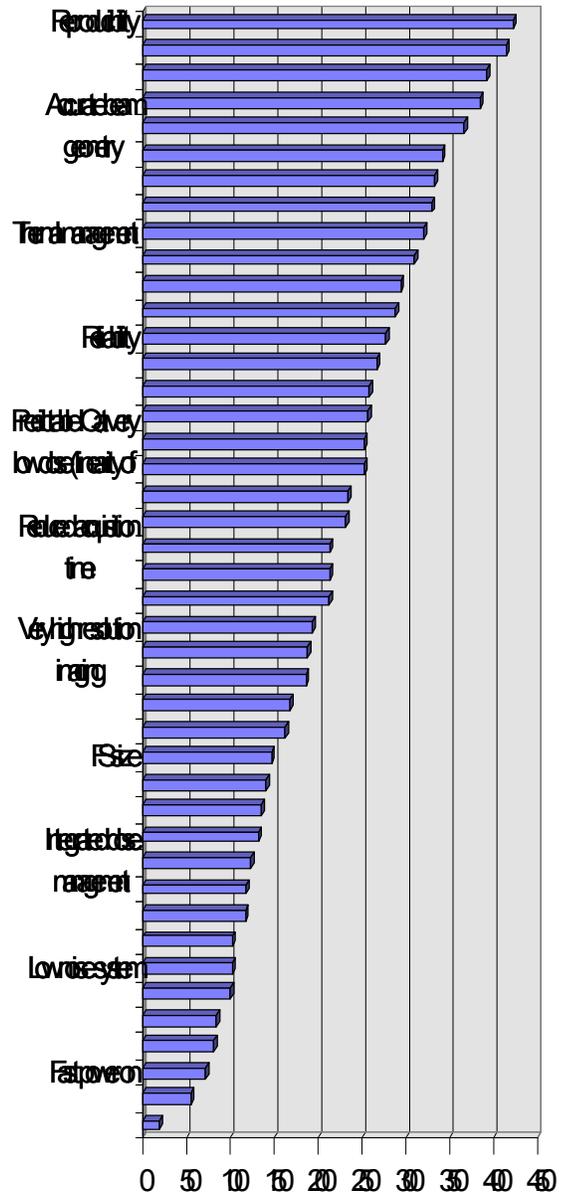


# GEMS Tube Example

- Level 2 House
  - Customer Importance Levels Set By Dots Voting Method

Tube related system features

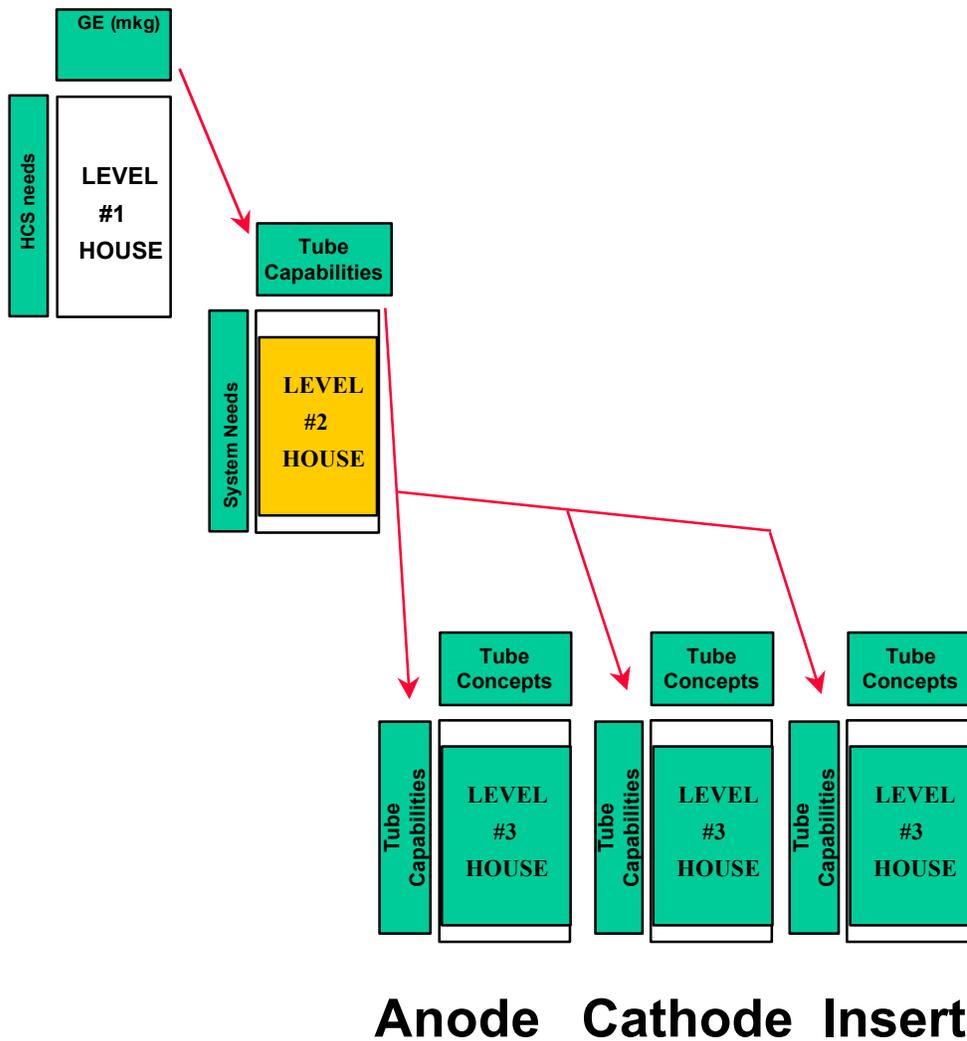
Product Requirement	Importance	Reproducibility	Increase SID	Stable operation	Short exposure time	Accurate beam geometry	Reduced tube head	Easy image quality check	AOP	Thermal management	No Blurring	Homogeneity of detectability, over field of view	No exposure abort	Reliability	No Artifact	X ray beam Spectrum	Compensating filter	Predictable IQ at very low dose (linearity of dose VS mAs)	Reduced access time	Vibration	Light and Collimator integrated to tube	Reduced acquisition time		
Nav applications	0																							
Accurate tomography/stereotaxis	5	H	H	M	H	H	H	M	H	H	H	H		H	H			H	H	H				
Acquisition of image sequences	3	H	H	H	H				H			H	H						M	H	L		M	
Chest wall visibility	5	H	H	H	M	H	H	H				H								M			H	M
Compliance with applicable regulations	1	H	H	H	H	H		H	H	H	H	H	H	H	H	H				H			H	
Constant IQ whatever breast size or composition	5	H		H	H	M	L	M	H	H	H	H	H	H	H	H	H	H	M	M			H	
Cost per image	2																							
Diagnostic equipment	1	M		M				H	H					L			M			M	H		H	
Dose efficiency	4	H	H	H	H	M		H	H	H	M	M	H	M	M	H	H	H			M		L	
Easy patient positioning	5		H			M	H																H	
Fluoro acquisition	3	H	H	H	H		H	H	H				H	H		H	H	H	H	H	H		H	
Image acquisition using contrast medium	4	H	M	H	H			H	H	H	H	H	H	H	H	H	M	H	H	M			H	
Installation time	3	H		H	M	H	M	H	H				M		H	H							H	
Interventional imaging	4	H	H	H	H	H	M	H	H	H	M	H	H	H	H	M	H	H	M	H	M		H	
Large field of view	5		H		H	H	M			H	H			H	H								H	
Light Xray tube	1		M			H	H		H														H	M
Low dose preview image	2	H		H			H	H					H	H					H	H			H	
Minimum acoustic noise	3	H	L	H	M		L				M		H							M	H		M	
Modularity	4	H	H	H		H	H	H	H					H	M		H						H	H
Prone patient positioning	3		H			H	H	H	M	H														
Recycling of product at end of life	1					H								M		H	H						H	
Very high resolution imaging	5	L	H	L	H	H		L	L	L	H	H				M	L		M	H			L	
Total		422	414	392	384	366	341	332	329	320	309	294	288	277	267	258	257	252	252	234	231	213		





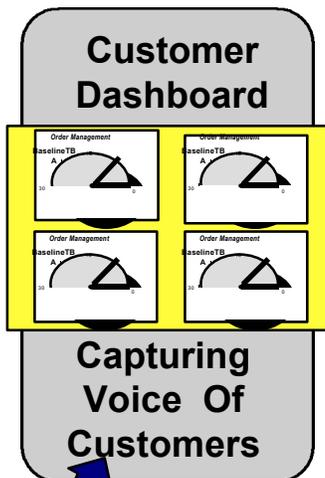
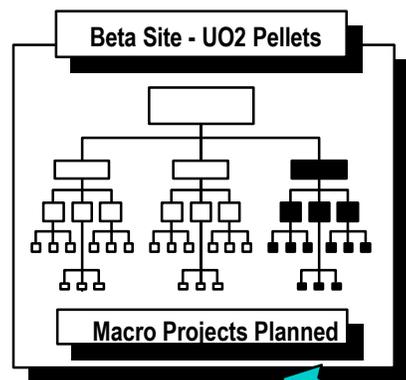
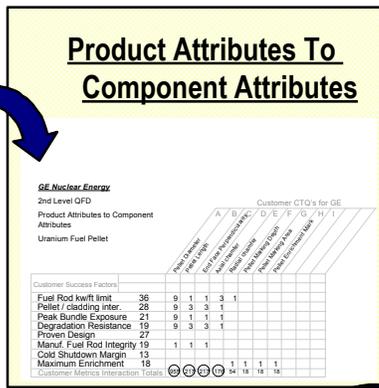
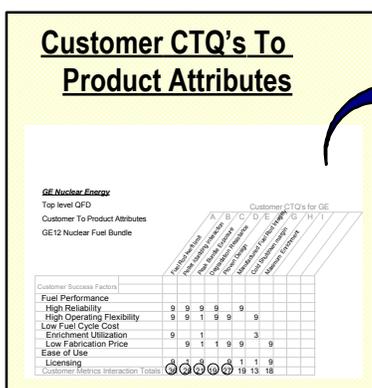
# GEMS Tube Example

- **Level 3 House**
  - Identified Need To Subdivide

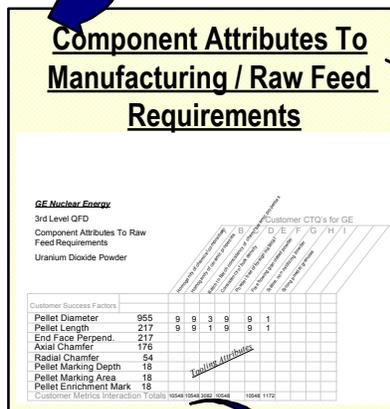




# Multiple House QFDs Evolve into Beta Site



- Key Issues & Highlights**
- Team Work Is The Key To Success & Control
  - Links Customer CTQs To Projects On Factory Floor
  - Beta Site To Affect Customer Dashboard / Surveys



$$Y = f(x_1 + x_2 + x_3 + \dots)$$

C79.1	DCP powder and process CTQs
C145.1	MICS Accuracy in Powder Production
C218.1	Utilization of UO2 Powder from ULBA Metallurgical in BWR Fuel
C573.1	Qualification of JCC Line 1 conversion and homogenization processes
C573.2	Qualification of JCC Line 1 for preparation of iso-enrichment powder blends
C574.1	Specification Requirements for Blending of U-235 Enrichments in UO2
C631.1	DCP Extended Monitoring
C671.1	Dry Recycle Blender Qualification
C675.1	DCP + UCON Qualification
C698.1	Upgrade of Stabilized Assay Meters (SAMII) Units



**QFDs... They Work & Are Worthwhile**



# QFDs Drive Customer Dashboards & Projects

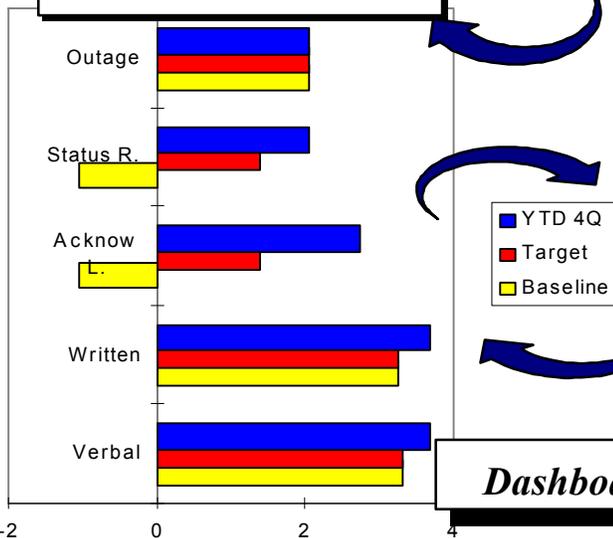
## Account Level QFD



## Establishing Metrics

Customer Success Factor	Baseline	Target	YTD 4Q	YTD 1Q	YTD 2Q	YTD 3Q
<b>Availability</b> %	93.5%	98.0%				
Customer CTQ: Responsiveness						
GE Metric: # of written request return in 2 w/days	3.25	3.25	3.67			
GE Metric: # of personal contact return in a day	3.3	3.3	3.62			
Customer Success Factor	Baseline	Target	YTD 4Q	YTD 1Q	YTD 2Q	YTD 3Q
<b>Output</b> GW/yr	2,928	3,050				
Customer CTQ: Post order Support						
GE Metric: # of order acknowledgment	-1.02	1.39	2.73			
GE Metric: # of order status report delivered	-1.02	1.39	2.04			
Customer Success Factor	Baseline	Target	YTD 4Q	YTD 1Q	YTD 2Q	YTD 3Q
<b>Lost Energy Due to outage</b> GW/yr	0.278	0.156				
Customer CTQ: Installation						
GE Metric: Outage duration delay	2.04	2.04	2.04			

## Dashboards Provide Feedback



## Projects to Move Dashboards

- S90.1--Verbal Responsiveness
- S90.2--Written Responsiveness
- S89.1--Parts Order Acknowledgement
- S89.2--Parts Order Status
- Multiple--Outage Cycle Time

**Dashboard will continue to Drive Future Projects**



## *COMMON QFD PITFALLS*

- *QFD On Everything*
  - *Set the “Right” Granularity*
  - *Don’t Apply To Every Last Project*
- *Inadequate Priorities*
- *Lack of Teamwork*
  - *Wrong Participants*
  - *Lack of Team Skills*
  - *Lack of Support or Commitment*
- *Too Much “Chart Focus”*
- *“Hurry up and Get Done”*
- *Failure to Integrate and Implement QFD*



# THE "STATIC" QFD

- **Review Current Status**
  - **At Least Quarterly**
  - **Monthly on 1 Yr Project**
  - **Weekly on Small Projects**

		↑	↓	↓	↑	↓	↓	○		
		HOW 1	HOW 2	HOW 3	HOW 4	HOW 5	HOW 6	HOW 7		
Need 1	5	H	L			L		M	65	
Need 2	5			H					45	
Need 3	3				M	M	L		21	
Need 4	4		H						36	
Need 5	2			L				M	8	
Need 6	4	M			L	H			52	
Need 7	1			L			M		4	
		3 lbs	12 in.	3 mils	40 psi	3	8 atm	1 mm		
		57	41	48	13	50	6	21		



**QFD IS A VALUABLE MANAGEMENT TOOL**



## *Points to Remember*

---

- *The process may look simple, but requires effort.*
- *Many of the entries look obvious—after they are written down.*
- *If there aren't some "tough spots" the first time, it probably isn't being done right!*
- *Focus on the end-user customer.*
- *Charts are not the objective.*
- *Charts are the means for achieving the objective.*
- *Find reasons to succeed, not excuses for failure.*
- *QFD is a Valuable Decision Support Tool, Not a Decision Maker*



## *Other QFD Applications*

- *It is a flexible tool for focusing business resources and mitigating risk*
  - *Customer Project Specific*
  - *Service Applications*
  - *Business Process Analysis*
  - *Customer Dashboards*
  - *Proposal Strategy*



---

# *Understanding Processes*

- ① *Process Mapping*
- ② *Failure Modes & Effects Analysis (FMEA)*



## *Understanding Processes Objectives*

---

- *By the end of the training program, the participant will be able to:*
  - *Use Process Mapping, and Failure Modes and Effects Analysis to narrow the focus of his/her project*
  - *Understand when each of these tools are appropriate to use*



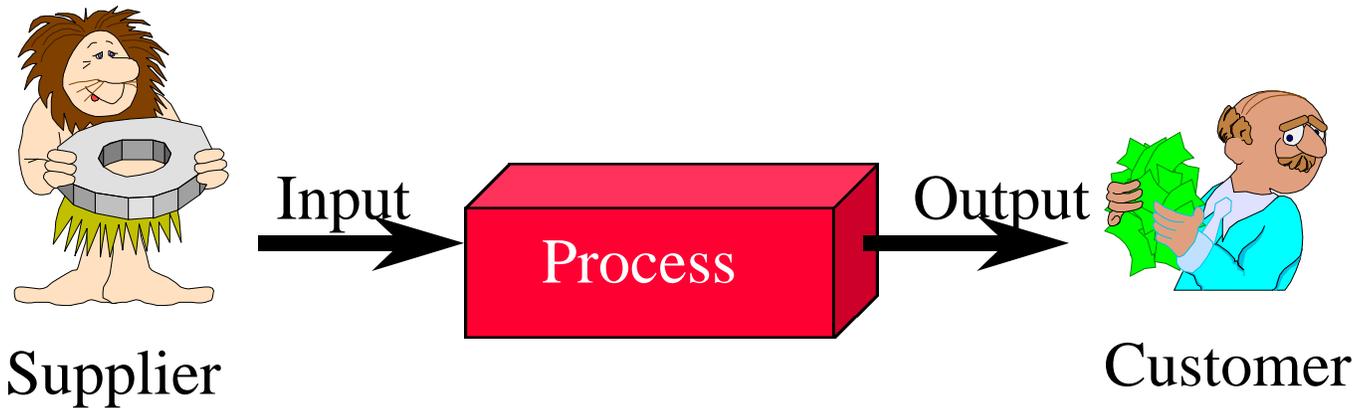
## *I. Process Mapping*

---

- *A graphical representation of steps, events, operations, and relationships of resources within a process.*



## *Elements of a Process*

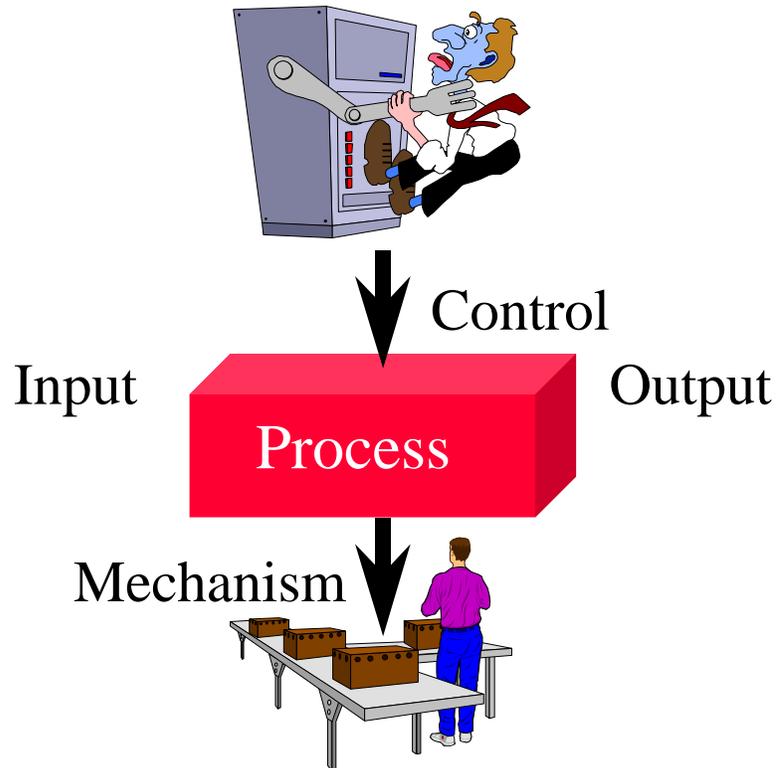


- **Customer** - Whoever receives the output of your process.
- **Output** - The material or data that results from the operation of a process.
- **Process** - The activities you must perform to satisfy your customer's requirements.
- **Input** - The material or data that a process does something to or with.
- **Supplier** - Whoever provides the input to your process.

**C.O.P.I.S. Focus:**  
**Start with the customer and work backwards**



## *Elements of a Process*

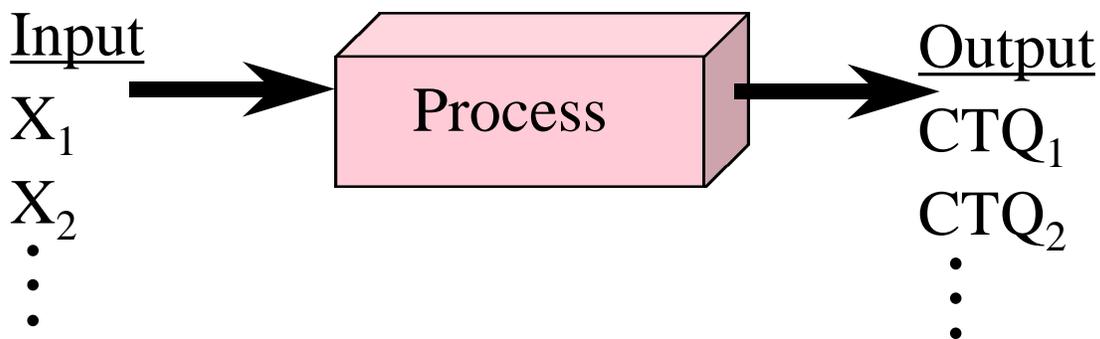


- **Control** - *The material or data that is used to tell a process what it can or should do next.*
- **Mechanism** - *The resources (people, machines, etc.) that come to bear on a process to change the input to an output.*
- **Process Boundary** - *The limits of the process, usually identified by the inputs, outputs and external controls that separate what is within the process from its environment.*



## Process Mapping

*A means of systematically diagnosing activity and information flow*



- *To prepare:*
  - *Establish the process boundaries*
  - *Observe the process in operation*
  - *List the outputs, customers, and their key requirements*
  - *List the inputs, suppliers, and your key requirements*



## *Benefits of Process Mapping*

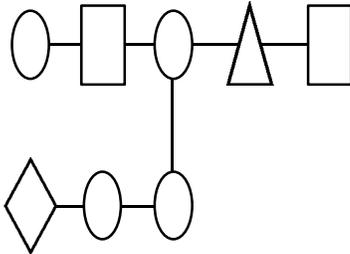
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- *Can reveal unnecessary, complex, and redundant steps in a process. This makes it possible to simplify & troubleshoot.*
- *Can compare actual processes against the ideal. You can see what went wrong and where.*
- *Can identify steps where additional data can be collected.*

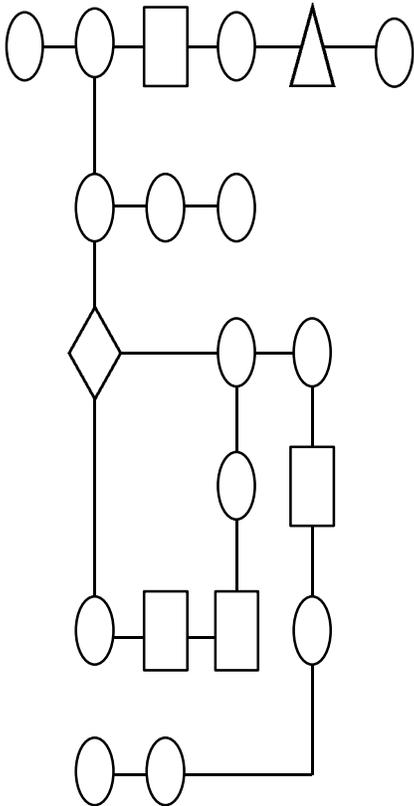


# Perceptions of a Process

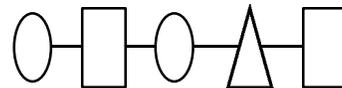
What we think it looks like:



What it actually looks like:



What we wish it would look like:



**There are usually three versions of every process**

**Do not jump to “What we wish it would look like”**



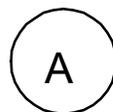
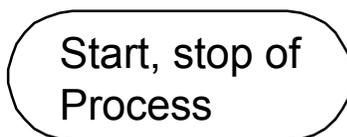
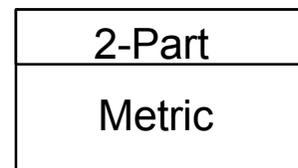
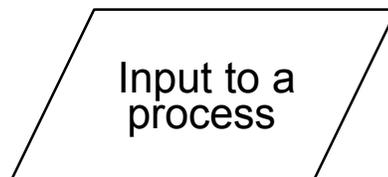
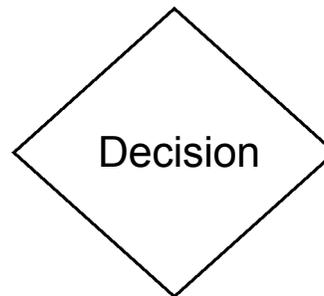
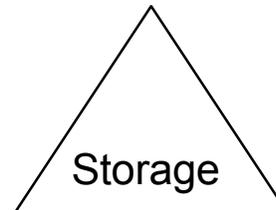
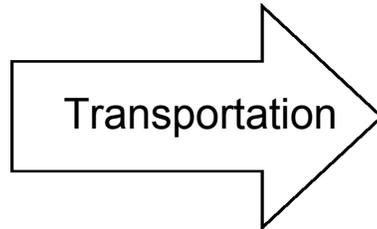
## *Building a Map*

---

- *Determine the scope*
  - *How complex and detailed a map do you need to give you what you want?*
- *Determine the steps in the process*
  - *Don't worry about order*
  - *Don't worry about priorities*
  - *Just list them!*
- *Arrange the steps in order*
- *Assign a symbol (See next page)*



# Process Mapping Symbols



On page  
connector



Off page  
connector



Process  
flow  
connector



# *Process Mapping*

---

## ■ *Testing the Flow*

- *Are the process steps identified correctly?*
- *Is every feedback loop closed?*
- *Does every arrow have a beginning and ending point?*
- *Is there more than one arrow from an activity box? Perhaps it should be a diamond.*
- *Are all the steps covered?*



## *Validate Process Map*

- *Walk the Process Again*
- *Ask the Questions*
  - *What happens if...?*
  - *What could go wrong?*
  - *Who...?*
  - *How...?*
  - *When...?*
- *Update Map*





## *Evaluating a Process Map*

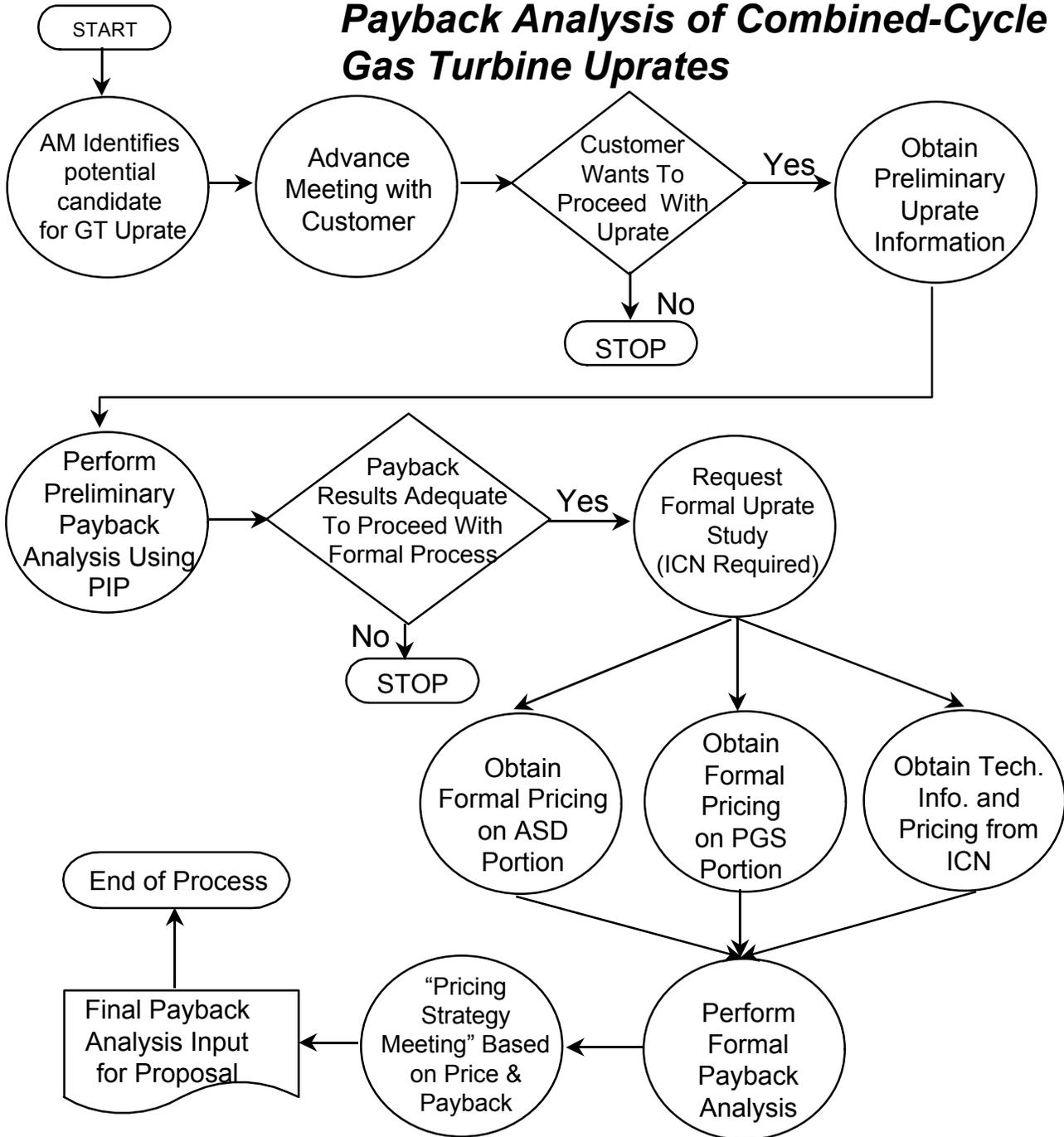
- *Does each step add value?*
- *Are controls and measurement criteria in place?*
- *Are “Re’s” occurring?*
  - *Rework*
  - *Revise*
  - *Repeat*
  - *Review*
- *Is the step necessary?*





# Process Map Example

## Process Mapping Example: S171.1 Payback Analysis of Combined-Cycle Gas Turbine Uprates





## *Process Mapping Exercise: 20 mins.*

---

- *For one or more projects in your group, construct a process map.*
  - *Is there unnecessary rework within the current process?*
  - *Are there unnecessary process steps?*
  - *Can you identify areas in your process needing improvement?*



---

# *Failure Modes and Effects Analysis*



## *Objective*

---

- *To understand the use of FMEA within the context of Six Sigma methodology.*
- *To learn the steps to developing and using an FMEA.*



# What & How of FMEA

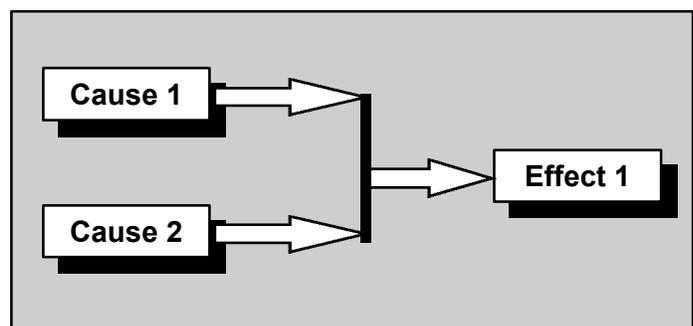
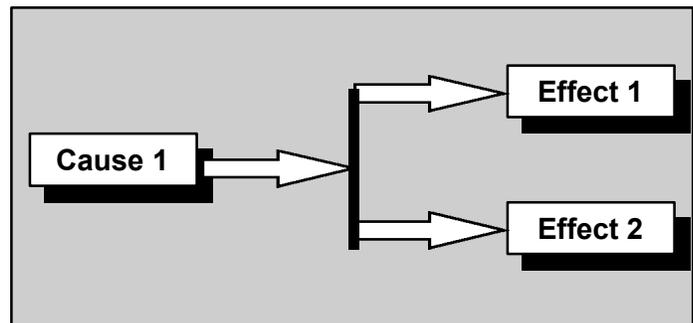
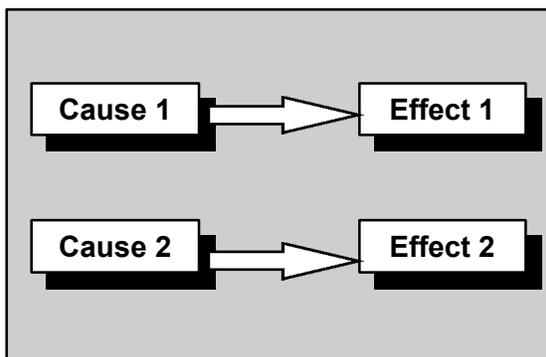
## What is failure modes and effects analysis?

- Identify ways the product or process can fail
- Plan how to prevent those failures

## How Does FMEA Work?

- Identify potential failure modes and rate the severity of their effect
- Evaluate objectively the probability of occurrence of causes and the ability to detect the cause when it occurs
- Rank order potential product and process deficiencies
- Focus on eliminating product and process concerns and help prevent problems from occurring

## FMEA Links Cause To Effect:





## *Definition - FMEA*

- *A structured approach to:*
  - *identifying the ways in which a process can fail to meet critical customer requirements*
  - *estimating the risk of specific causes with regard to these failures*
  - *evaluating the current control plan for preventing these failures from occurring*
  - *prioritizing the actions that should be taken to improve the process*

***Identify ways the product or process can fail. Then plan to prevent those failures.***



## *Purposes & Benefits of FMEA*

- *Improves the quality, reliability, and safety of products.*
- *Helps to increase customer satisfaction.*
- *Reduces product development timing and cost.*
- *Documents and tracks actions taken to reduce risk.*

### **Types of FMEA**

- **System FMEA:** *is used to analyze systems and subsystems in the early concept and design stages. Focuses on potential failure modes associated with the functions of a system caused by design.*
- **Design FMEA:** *is used to analyze products before they are released to production.*
- **Process FMEA:** *is used to analyze manufacturing, assembly and transactional processes.*



# Steps in the FMEA Process



1. Select Process Team
2. Develop Process Map & Identify Process Steps
3. List Key Process Outputs To Satisfy Internal And External Customer Requirements
4. List Key Process Inputs For Each Process Step
5. Define Matrix Relating Product Outputs To Process Variables
6. Rank Inputs According To Importance

7. List Ways Process Inputs Can Vary (*Causes*) and identify associated Failure Modes and Effects
8. List Other Causes (Sources of Variability) And Associated FM&Es
9. Assign Severity, Occurrence And Detection Rating To Each Cause
10. Calculate Risk Priority Number (*RPN*) For Each Potential Failure Mode Scenario

11. Determine Recommended Actions To Reduce RPNs
12. Establish Timeframes For Corrective Actions
13. Create "Waterfall" Graph To Forecast Risk Reductions
14. Take Appropriate Actions
15. Re-calculate All RPNs
16. Put controls into place

Failure Modes and Effects Analysis (FMEA)										
Process or Product Name:										
Responsible:										
Process Step/Part Number	Potential Failure Mode	Potential Failure Effects	S E V	Potential Causes	O C C	Current Controls	D E T	R P N	Actions Recommended	Resp.



## *Definition of Terms*

---

### ■ **Failure Mode**

- *The manner in which a part or process can fail to meet specification*
- *Usually associated with a **Defect** or nonconformance*

### ■ **Cause**

- *A deficiency that results in a Failure Mode*
- *Causes are sources of **Variability** associated with Key Process Input Variables*

### ■ **Effect**

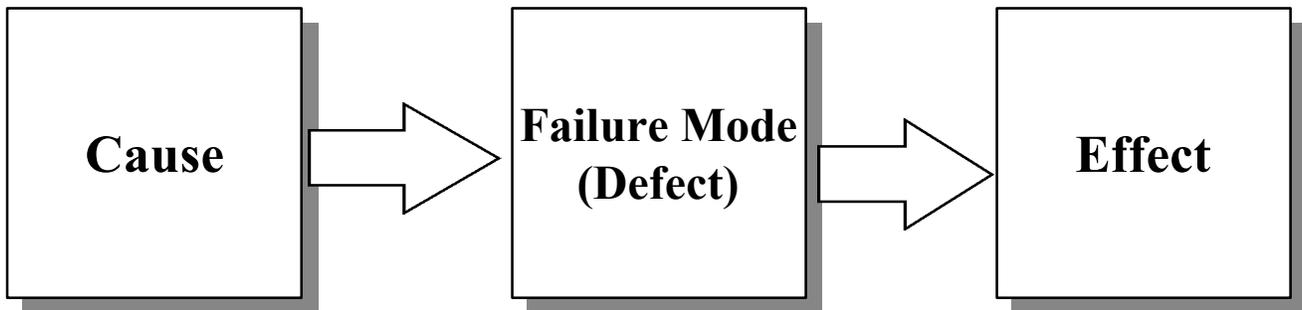
- *Impact on **Customer** if Failure Mode is not prevented or corrected*
- *Customer can be downstream or the ultimate customer*

*The Failure Mode can be thought of as the “in-process” defect, whereas an Effect is the impact on the customer requirements.*



# *FMEA Model*

**Linking Failure Mode to Cause and Effect**



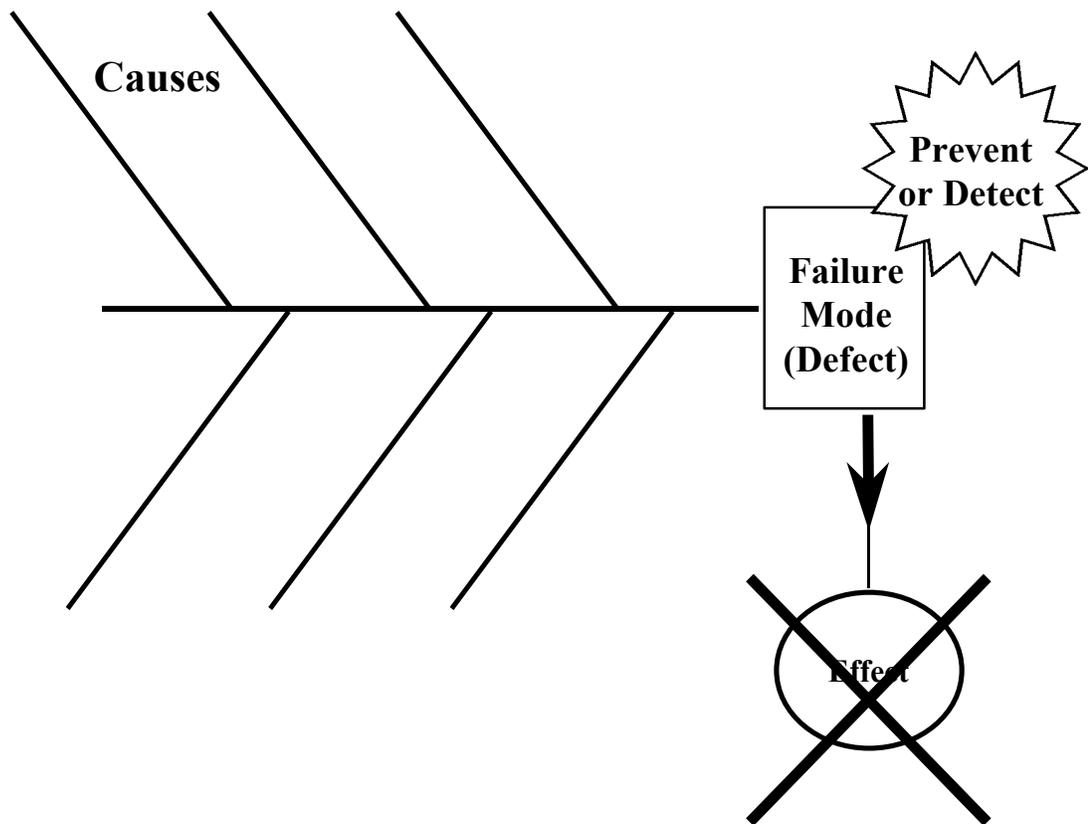
**TIME**





# *Failure Mode Fishbone Model*

Goal of FMEA





# *FMEA Calculations*

---

## **Risk Ratings:**

Scale: 1 (Best) to 10 (Worst)

### **Severity (SEV)**

*How significant is the impact of the **Effect** to the customer (internal or external)?*

### **Occurrence (OCC)**

*How likely is the **Cause** of the Failure Mode to occur?*

### **Detection (DET)**

*How likely will the current system detect the **Cause or Failure Mode** if it occurs?*

## **Risk Priority Number:**

- *A numerical calculation of the relative risk of a particular Failure Mode.*
- $RPN = SEV \times OCC \times DET$
- *This number is used to place priority on which items need additional quality planning.*



# Standardization of Ratings

RATING	DEGREE OF SEVERITY	LIKELIHOOD OF OCCURRENCE	ABILITY TO DETECT
1	Customer will not notice the adverse effect or it is insignificant	Likelihood of occurrence is remote	Sure that the potential failure will be found or prevented before reaching the next customer
2	Customer will probably experience slight annoyance	Low failure rate with supporting documentation	Almost certain that the potential failure will be found or prevented before reaching the next customer
3	Customer will experience annoyance due to the slight degradation of performance	Low failure rate without supporting documentation	Low likelihood that the potential failure will reach the next customer undetected
4	Customer dissatisfaction due to reduced performance	Occasional failures	Controls may detect or prevent the potential failure from reaching the next customer
5	Customer is made uncomfortable or their productivity is reduced by the continued degradation of the effect	Relatively moderate failure rate with supporting documentation	Moderate likelihood that the potential failure will reach the next customer
6	Warranty repair or significant manufacturing or assembly complaint	Moderate failure rate without supporting documentation	Controls are unlikely to detect or prevent the potential failure from reaching the next customer
7	High degree of customer dissatisfaction due to component failure without complete loss of function. Productivity impacted by high scrap or rework levels.	Relatively high failure rate with supporting documentation	Poor likelihood that the potential failure will be detected or prevented before reaching the next customer
8	Very high degree of dissatisfaction due to the loss of function without a negative impact on safety or governmental regulations	High failure rate without supporting documentation	Very poor likelihood that the potential failure will be detected or prevented before reaching the next customer
9	Customer endangered due to the adverse effect on safe system performance with warning before failure or violation of governmental regulations	Failure is almost certain based on warranty data or significant DV testing	Current controls probably will not even detect the potential failure
10	Customer endangered due to the adverse effect on safe system performance without warning before failure or violation of governmental regulations	Assured of failure based on warranty data or significant DV testing	Absolute certainty that the current controls will not detect the potential failure



# *Numerical Ratings Analyzing Risk Potential*

<b>Numerical Ranking</b>	<b>OCCURRENCE Likelihood</b>	<b>DETECTION Certainty</b>
1	1 in 10 <sup>6</sup>	100%
2	1 in 20,000	99%
3	1 in 5,000	95%
4	1 in 2,000	90%
5	1 in 500	85%
6	1 in 100	80%
7	1 in 50	70%
8	1 in 20	60%
9	1 in 10	50%
10	1 in 2	<50%

***Capability link into FMEA***



# FMEA Form

**Failure Modes and Effects Analysis (FMEA)**

Prepared by: \_\_\_\_\_ Page \_\_\_\_ of \_\_\_\_  
 FMEA Date (Orig) \_\_\_\_\_ (Rev) \_\_\_\_\_

Process Step/Part Number	Potential Failure Mode	Potential Failure Effects: S E V	Potential Causes	O C C	Current Controls	D E T	R P N	Actions Recommended
							0	
							0	
							0	
							0	
							0	
							0	
							0	

**Annotations:**

- List of Process Steps or Product Parts
- Lists Failure Modes for each Process Step
- Rates the Severity of the Effect to the Customer on a 1 to 10 Scale
- Lists the Effects of each Failure Mode
- Lists the Causes for each Failure Mode: Each Cause is Associated with a process input out-of-Spec
- Rates how often a particular Cause or Failure Mode Occurs: 1 = Not Often, 10 = Very Often



# FMEA Form (cont.)

Risk Priority Number (RPN) is:  
 $Sev * Occ * Det$

Documents how the Cause is Currently being controlled in the Process

Documents actions recommended based on RPN Pareto

RPN number is recalculated when action is completed

Prepared by: \_\_\_\_\_ of \_\_\_\_\_  
 FMEA Date (Original): \_\_\_\_\_

Potential Failure Effects	SEV	Potential Causes	OC C	Current Controls	DET	RPN	Actions Recommended	Resp.	SEV	OC C	DET	RPN
						0						0
												0
												0
												0
												0
												0

Rates how well the Cause or the Failure Mode can be detected.  
 1 = Detect Every Time  
 10 = Cannot Detect

Designates who is responsible for Action and projected completion data

An Improvement Plan is required when the RPN number exceeds 120.



# Failure Modes and Effects Analysis

## Reducing Rotor Blasting Process Time B3095.1

### Process/Product Failure Modes and Effects Analysis (FMEA)

Process or Product Name:	REDUCTION IN NDT PREP TIME ON GT COMPRESSOR ROTORS
Responsible:	STALEY EDWARDS

Prepared by: STALEY EDWARDS
FMEA Date (Orig) JUNE , 25, 1998 (Rev) ____

Process Step/Part Number	Potential Failure Mode	Potential Failure Effects	S E V	Potential Causes	O C C	Current Controls	D E T	R P N
PREP FOR BLAST	IMPROPER TAPING	BLAST MEDIA GETS IN BETWEEN ROTOR WHEEL	8	WORKMANSHIP / MATERIAL	2	PLANNING ROUTER AND VISUAL	2	32
**BLAST CLEAN	TAPE COMES OFF	BLAST MEDIA GETS IN BETWEEN ROTOR WHEEL	8	MATERIAL / OVERBLASTING	3	VISUAL INSPECTION DURING BLAST PROCESS	3	72
**PREP FOR NDT	TAPE RESIDUE ON WHEELS	EXCESSIVE HOURS TO CLEAN FOR NDT	7	MATERIAL / OVERBLASTING	9	NONE	10	630
NDT	INSUFFICIENT CLEANING CAUSING QUESTIONABLE	POSSIBLITY OF NOT SEEING RELATIVE INDICATIONS DURING NDT	10	BLASTING , CLEANING AND MATERIAL	2	MANUAL CLEANING PRIOR TO NDT	3	60



## *When is an FMEA Started?*

- *When new systems, products, and processes are being designed.*
- *When existing designs or processes are being changed.*
- *When carryover designs/processes will be used in new applications, or new environments.*
- *After completing a Problem Solving Study (to prevent recurrence of problem).*
- *For a System FMEA, after System functions are defined, but before specific hardware is selected.*
- *For a Design FMEA, after product functions are defined, but before the design is approved and released to manufacturing.*
- *For a Process FMEA, when preliminary drawings of the product are available.*



## *Who Prepares an FMEA?*

---

- *The team approach to preparing FMEAs is **recommended**.*
- *The responsible system or product leads the FMEA team.*
- *The responsible design is expected to involve representatives from all affected activities. Team members should include design, manufacturing, assembly, quality, reliability, service, purchasing, testing, supplier, and other subject matter experts as appropriate.*

## *When is an FMEA Updated?*

---

- *Whenever a change is being considered to a product's design, application, environment, material, or to any process.*



## *Who Updates an FMEA?*

---

- *The individual responsible for the system or product is responsible for keeping the FMEA up to date.*
- *Suppliers keep their own FMEAs up to date.*

## *When is a FMEA Completed?*

---

- *A Design FMEA is considered completed when the design is released for production.*
- *A Process FMEA is never completed unless the process is removed from the product line.*



## *FMEA Exercise: 20 mins.*

---

- *For one or more projects on your team, construct an FMEA to identify possible areas of improvement for your process.*
- *Is there one or more potential vital Xs that can be identified?*
- *What if any, are possible improvement actions to mitigate the failure mode?*



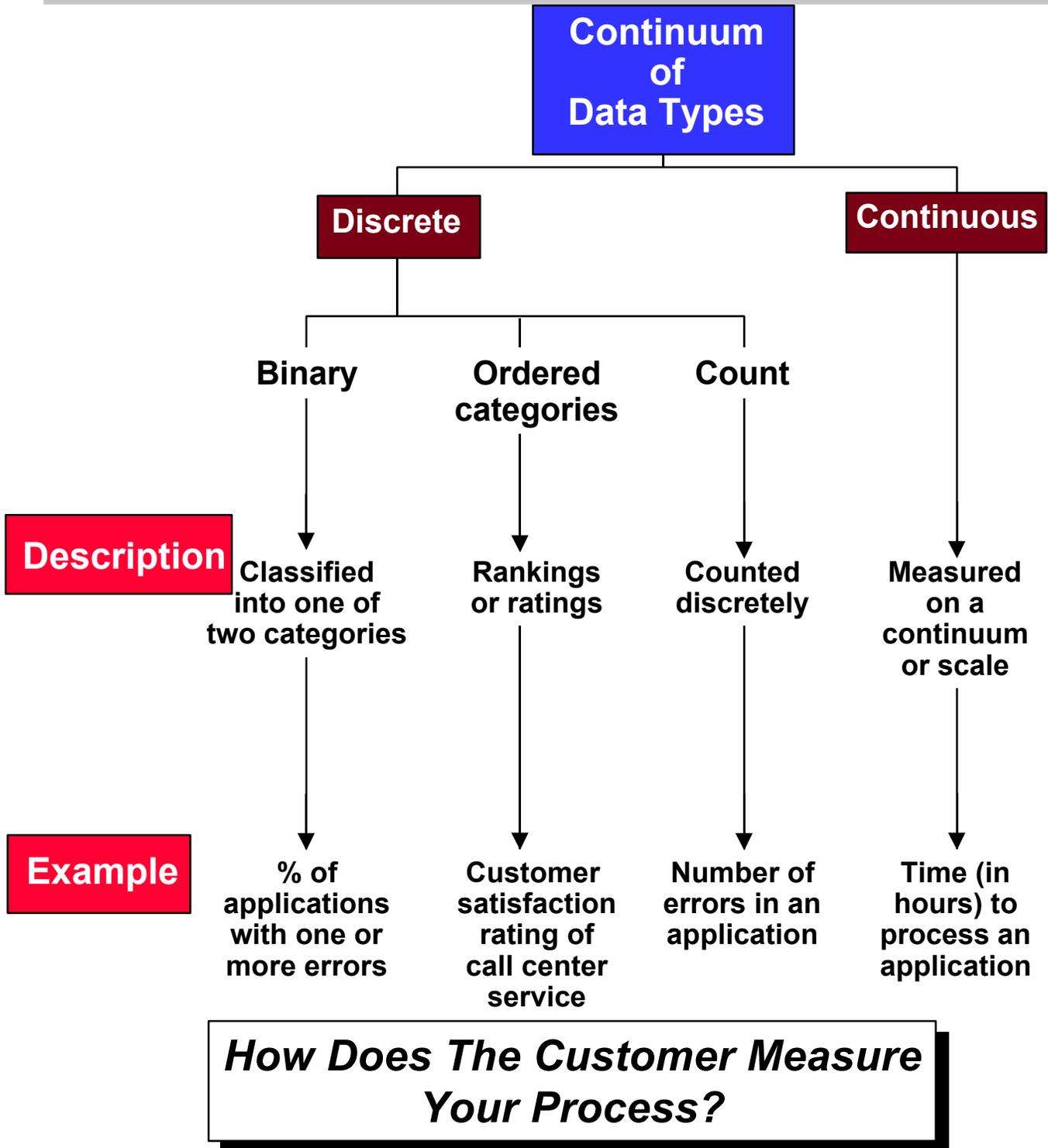
## *Data Types Objective*

---

*Extend the concept of data type beyond continuous vs. discrete and understand examples of each of four data types.*



# Overview: Types Of Data





## *Why Is Type Of Data Important?*

- Choice of data display and analysis tools*
- Amount of data required: continuous data often requires a smaller sample size than discrete data*
- Information about current and historical process performance*

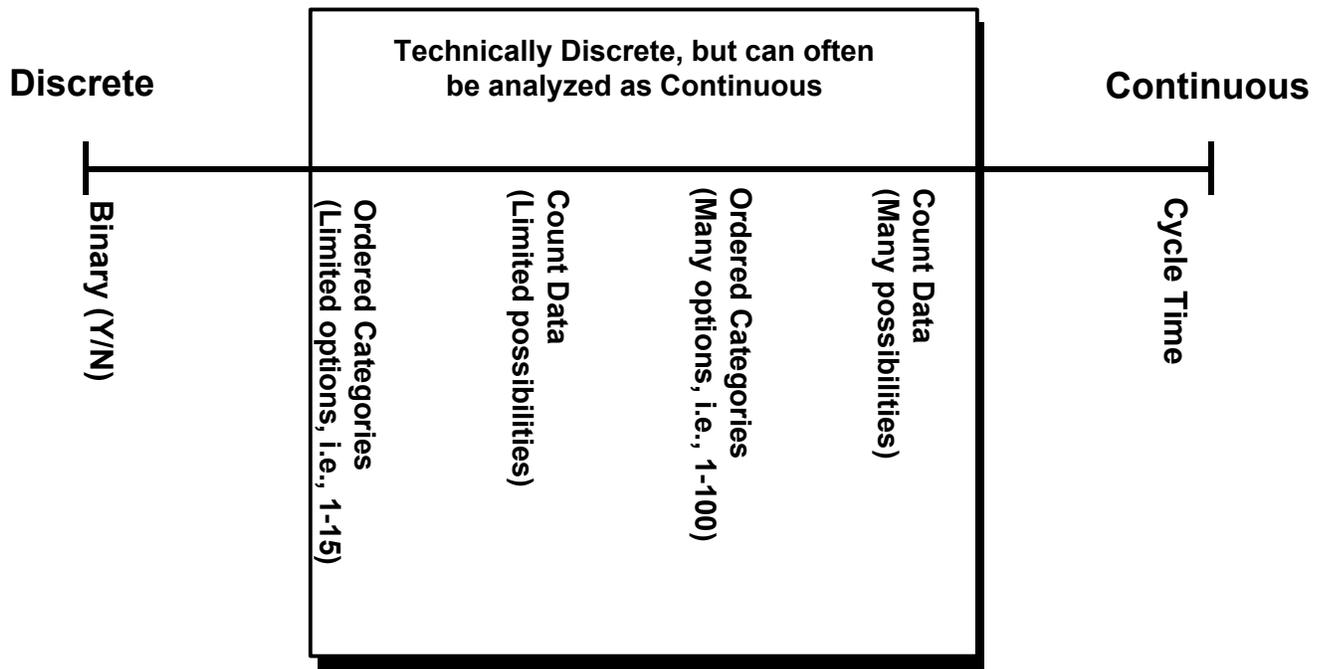
***Use Continuous Data  
Whenever Possible***



# Types Of Data

## Data Type Is An Important Consideration

<p>■ <b>Discrete Data</b></p> <ul style="list-style-type: none"> <li>- Binary (Yes/No, Defect/No Defect)</li> <li>- Ordered categories (1-5)</li> <li>- Counts</li> </ul>	<p>■ <b>Continuous Data</b></p> <ul style="list-style-type: none"> <li>- Can be broken down into increments</li> <li>- Infinite number of possible values</li> </ul>
<p>■ <b>Examples</b></p> <ul style="list-style-type: none"> <li>- Number of incomplete applications</li> <li>- Percent of responding with a "5" on survey</li> <li>- Number of Green Belts trained</li> </ul>	<p>■ <b>Examples</b></p> <ul style="list-style-type: none"> <li>- Cycle time (measured in days, hours, minutes, etc.)</li> <li>- Weight (measured in tons, pounds, etc.)</li> </ul>





## *Importance Of Data Type*

**Sometimes we have choices. When we do, we should choose continuous data**

<b>Project Y</b>	<b>Discrete Y Measure</b>	<b>Continuous Y Measure</b>
Time to process	% within specifications	Actual times for each unit
Delivery time	Number late	Actual time deviated from target
Customer satisfaction	Yes/no questions	Rating 1-100
Policies lost due to price	Number lost	Difference from competition

***The More Continuous We Can Make The Data,  
The More It Will Tell Us About Our Process***



## *GOLF Exercise*

---

Point of this exercise is to calculate Z two ways:

- 1) any score over par is a defect - use discrete DPMO
- 2) take mean & std deviation and calculate Z using continuous method

Points to make:

Two golfers have same Z using discrete method, but different using continuous. Why?

What would happen if one of the scores changed by one stroke so it changes from defect to not (or the other way around).

Z using discrete method would make quantum change - Z using continuous measures in smaller increments.



# Class Exercise

Calculate the following for your assigned golfer:

Mode

Mean

Median

Standard Deviation

$$Z_{\text{calculated}} = (\text{USL} - \text{MEAN})/s$$

$$P(\text{defect}) ((\# \text{ of scores} > \text{USL})/\# \text{ of scores})$$

$Z_{\text{table}}$  ( look up using the  $P(\text{defect})$ )

Who is the better golfer?

Note: USL=72



Scot	Serge	Bill	Bob
62	73	69	73
65	69	61	67
71	70	68	67
76	70	77	67
62	67	76	73
66	72	69	75
67	70	64	71
67	68	61	72
62	71	65	68
73	69	69	68





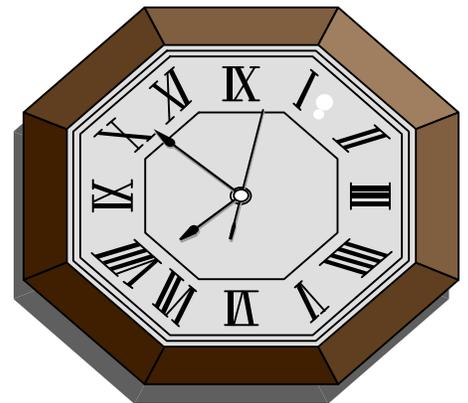
# *Defect Counting Exercise (discrete and continuous)*

Just a good exercise after teaching discrete and continuous to reinforce the concepts.



## *Another example, tying it all together:*

- **20 weeks worth of Master Black belt weekly time sheets are attached.**
- **Develop an L1 spreadsheet for the possible defects as shown on the next page. Pay close attention to opportunities.**
- **Which line item might be a good Black Belt project?**
- **Are any of these defects continuous?**
- **Use Minitab where applicable to determine  $Z_{shift}$ .**
- **Do the MBB's have a problem with control or with technology?**





# The Inspection Sheet

Define the defect, unit, and opportunity for each characteristic:

Characteristic: **FW**  
Defect: **Missing FW**  
Unit: \_\_\_\_\_  
Opportunity: \_\_\_\_\_

Characteristic: **Hours**  
Defect: **Missing Hours**  
Unit: \_\_\_\_\_  
Opportunity: \_\_\_\_\_

Characteristic: **Hours**  
Defect: **Hours out of Spec**  
Unit: \_\_\_\_\_  
Opportunity: \_\_\_\_\_

*(hours spec is 40 to 70)*

Characteristic: **Entry In Box**  
Defect: **Entry Not In Box**  
Unit: \_\_\_\_\_  
Opportunity: \_\_\_\_\_

Characteristic: **Signature**  
Defect: **Missing Signature**  
Unit: \_\_\_\_\_  
Opportunity: \_\_\_\_\_

Characteristic	D	U	OP	TOP	D/TOP	DPMO	Shift	Z.B
missing FW							1.50	
missing hours							1.50	
entry not in box							1.50	
missing signature							1.50	
hours out of spec							1.50	
other?							1.50	
Total							1.50	



# Example

<u>Master Black Belt Time Sheet</u> <b>1</b>	<u>Master Black Belt Time Sheet</u> <b>2</b>	<u>Master Black Belt Time Sheet</u> <b>3</b>
67.3 Berezowitz, Bill	57.6 Berezowitz, Bill	62.1 Berezowitz, Bill
59.4 Jeung, Kein	58.1 Jeung, Kein	72.1 Jeung, Kein
72.2 Manson, Lowry	46.3 Manson, Lowry	63.1 Manson, Lowry
57.7 Spencer, Stephanie	52.9 Spencer, Stephanie	73.4 Spencer, Stephanie
61.2 Turall, Sam	59.0 Turall, Sam	47.2 Turall, Sam
60.6 Webster, Scot	54.0 Webster, Scot	76.2 Webster, Scot
Attested	Attested	Attested
<i>Scot Webster</i>	<i>Kein Jeung</i>	<i>Lowry Manson</i>



# Example

	4
56.1	Berezowitz, Bill
61.0	Jeung, Kein
68.2	Manson, Lowry
53.7	Spencer, Stephanie
53.6	Turall, Sam
60.6	Webster, Scot
Attested	<i>Brett Farue</i>

	5
59.6	Berezowitz, Bill
72.5	Jeung, Kein
66.7	Manson, Lowry
63.2	Spencer, Stephanie
53.5	Turall, Sam
60.9	Webster, Scot
Attested	<i>Sam Turall</i>

	6
50.4	Berezowitz, Bill
54.6	Jeung, Kein
61.8	Manson, Lowry
55.8	Spencer, Stephanie
54.4	Turall, Sam
63.0	Webster, Scot
Attested	<i>Scot Webster</i>



# Example



Master Black Belt Time Sheet 7

52.3	Berezowitz, Bill
45.8	Jeung, Kein
45.3	Manson, Lowry
45.6	Spencer, Stephanie
40.0	Turall, Sam
49.7	Webster, Scot

Attested *Dr. William  
Berezowitz*



Master Black Belt Time Sheet 8

62.9	Berezowitz, Bill
60.5	Jeung, Kein
67.2	Manson, Lowry
69.8	Spencer, Stephanie
66.3	Turall, Sam
68.4	Webster, Scot

Attested *Kein Jeung*



Master Black Belt Time Sheet 9

45.5	Berezowitz, Bill
51.5	Jeung, Kein
50.4	Manson, Lowry
57.1	Spencer, Stephanie
56.9	Turall, Sam
50.7	Webster, Scot

Attested *Lowry Manson*



# Example

  
*Master Black Belt Time Sheet* 10

57.5	Berezowitz, Bill
65.5	Jeung, Kein
61.4	Manson, Lowry
60.7	Spencer, Stephanie
59.5	Turall, Sam
71.4	Webster, Scot

Attested *Stephanie Spencer*

**BM**  
*Master Black Belt Time Sheet* 11

33.5	Berezowitz, Bill
47.4	Jeung, Kein
48.6	Manson, Lowry
40.4	Spencer, Stephanie
57.0	Turall, Sam
40.5	Webster, Scot

Attested

  
*Master Black Belt Time Sheet* 12

64.0	Berezowitz, Bill
54.2	Jeung, Kein
61.2	Manson, Lowry
70.5	Spencer, Stephanie
65.7	Turall, Sam
55.2	Webster, Scot

Attested *Sam Turall*



# Example

*Master Black Belt Time Sheet* 13

53.3	Berezowitz, Bill
61.0	Jeung, Kein
63.3	Manson, Lowry
50.5	Spencer, Stephanie
53.6	Turall, Sam
56.2	Webster, Scot

Attested *Scot Webster*

*Master Black Belt Time Sheet* 14

61.1	Berezowitz, Bill
62.5	Jeung, Kein
66.3	Manson, Lowry
60.5	Spencer, Stephanie
62.9	Turall, Sam
57.8	Webster, Scot

Attested *Dr. William  
Berezowitz*

*Master Black Belt Time Sheet* 15

57.5	Berezowitz, Bill
67.4	Jeung, Kein
52.1	Manson, Lowry
58.3	Spencer, Stephanie
48.3	Turall, Sam
56.1	Webster, Scot

Attested *Lowry Manson*



# Example

*Master Black Belt Time Sheet* 17

63.7
53.8
45.4
49.7
55.1
67.8

Attested Stephanie  
Spencer

*Master Black Belt Time Sheet* 18

65.0	Berezowitz, Bill
55.3	Jeung, Kein
59.9	Manson, Lowry
54.3	Spencer, Stephanie
64.4	Turall, Sam
61.9	Webster, Scot

Attested Sam Turall

*Master Black Belt Time Sheet* 19

48.3	Berezowitz, Bill
56.7	Jeung, Kein
57.0	Manson, Lowry
57.0	Spencer, Stephanie
58.5	Turall, Sam
53.3	Webster, Scot

Attested Mikel Harry



# Example

			
<i>Master Black Belt Time Sheet</i>		<i>Master Black Belt Time Sheet</i>	
		<b>20</b>	
<input type="text" value="65.8"/>	Berezowitz, Bill	<input type="text" value="78.9"/>	Berezowitz, Bill
<input type="text" value="61.2"/>	Jeung, Kein	<input type="text" value="65.0"/>	Jeung, Kein
<input type="text" value="73.3"/>	Manson, Lowry	<input type="text" value="82.3"/>	Manson, Lowry
<input type="text" value="69.6"/>	Spencer, Stephanie	<input type="text" value="77.7"/>	Spencer, Stephanie
<input type="text" value="69.6"/>	Turall, Sam	<input type="text" value="73.6"/>	Turall, Sam
<input type="text" value="68.6"/>	Webster, Scot	<input type="text" value="73.6"/>	Webster, Scot
Attested by	<input type="text" value="Kein Jeung"/>	Attested by	<input type="text"/>



# *Step 1: Select CTQ Characteristic Tool Summary*

---

## ■ **QFD**

- *Select CTQ; narrow project focus*
- *Step: 1*

## ■ **Process Mapping**

- *Understand process steps; narrow project focus*
- *Steps: 1, 6, Improve Phase*

## ■ **FMEA**

- *Identify and prevent failures; narrow project focus*
- *Steps: 1, 6, 12*



## *Take Aways—Step 1*

---

- *A Quality Function Deployment (QFD) translates **customer needs** into detailed **process/product requirements**.*
  
- *The QFD assists in narrowing the focus of the project by **prioritizing** actions according to their impact on the customer.*
  
- *The QFD is a matrix which relates the customer **wants** (CTQs) to **how** we might satisfy those wants.*
  - *Each **how** is rated highly if it has a strong chance of satisfying a **want***
  - *We multiply the how rating times the importance rating of the want to get the **priority** of the action.*



## *Take Aways—Step 1*

---

- **A *process map*** is a graphical representation of steps, events, operations, and relationships of resources in a process.
  - *used to identify potential breakdowns, rework loops, and sources of variation in a process*
  
- **An *FMEA*** is used to identify the potential failure modes of a process or product
  - *identifies ways a product or process can fail and the effects of these failures*
  - *rates the severity of the failures*
  - *rates the ability to detect the failures*
  - *quantifies the likelihood of the failures*
  - *prioritizes activities to mitigate or prevent the failures from occurring*



## *Take Aways—Step 1*

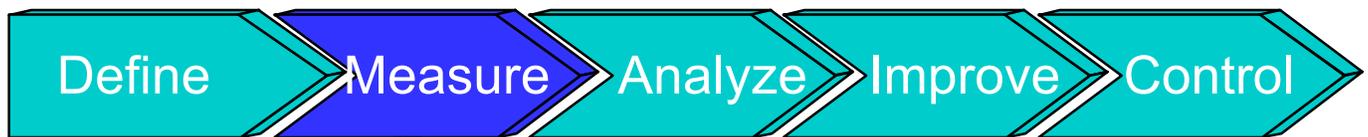
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### ■ *Review of Terms:*

- *Cause - deficiency that results in a failure mode*
- *Failure Mode - the manner in which a part or process can fail to meet specification*
- *Effect - the impact on the customer if the failure mode is not prevented or corrected*



# Measure Phase



**1. Select CTQ Characteristics**

*Deliverable: Identify Measurable CTQ That Will Be Improved*

*Tools:*

- QFD
- Process Mapping
- FMEA
- Discrete vs. Continuous Data

**2. Define Performance Standards**

*Deliverable: Determine and Confirm Specification Limits For Your Y.*

**3. Measurement System Analysis**

*Deliverable: Measurement System Adequate to Measure Y.*

*Tools:*

- Continuous Gage R&R
- Test/Retest
- Attribute Gage R&R



# *Define Performance Standards Objectives*

## **Step 2: Define Performance Standards**

*By the end of Step 2, the BB/GB will have:*

- a. *Defined a defect.*

*What are the customer's acceptance criteria for the part/product or process?*

- b. *Established how to measure the quality of the part/product or process.*

*Where are the data coming from?*

*How do you measure the process?*

*What are the units of measure?*

*Is it a discrete or continuous measure?*

- c. *Determined the Performance Standard for their project.*

- d. *Gained consensus with their team on the Performance Standard for their project.*



## *Performance Standards*

---

**A Performance Standard** is the requirement(s) or specification(s) imposed by the customer on a specific CTQ.

*It answers the questions:*

*What does the customer want?*

*What is a good product/process?*

*What is a defect?*

*Some examples:*

### *Blueprints*

- ✓ *Sizes and dimensions are provided on parts*

### *Contracts*

- ✓ *Type of turbine, fuel efficiency, time of delivery, warranty information*



## *Establishing a Performance Standard*

- *The goal of a performance standard is to translate the customer need into a measurable characteristic.*
  - *Operational Definition*
  - *Target*
  - *Specification Limits*
  - *Defect Definition*
  
- *A characteristic should be measurable.*
  
- *Translate the Voice of the Customer to the Voice of the Process.*



## *Operational Definitions*

### ■ *Definition:*

- *An operational definition is a precise description that tells how to get a value for the characteristic (CTQ) you are trying to measure. It includes “What Something Is” and “How to Measure It”*

### ■ *Purpose:*

- *To Remove Ambiguity so that Everyone has the same understanding*
- *To provide a clear way to measure the characteristic*
  - *Identifies what to measure*
  - *Identifies how to measure it*
  - *Makes sure that no matter who does the measuring, the results are essentially the same*
  - *Must be useful to both you and the customer*

***At a Minimum—  
A Clear Definition of a Defect is Required***



## *What is a Defect?*

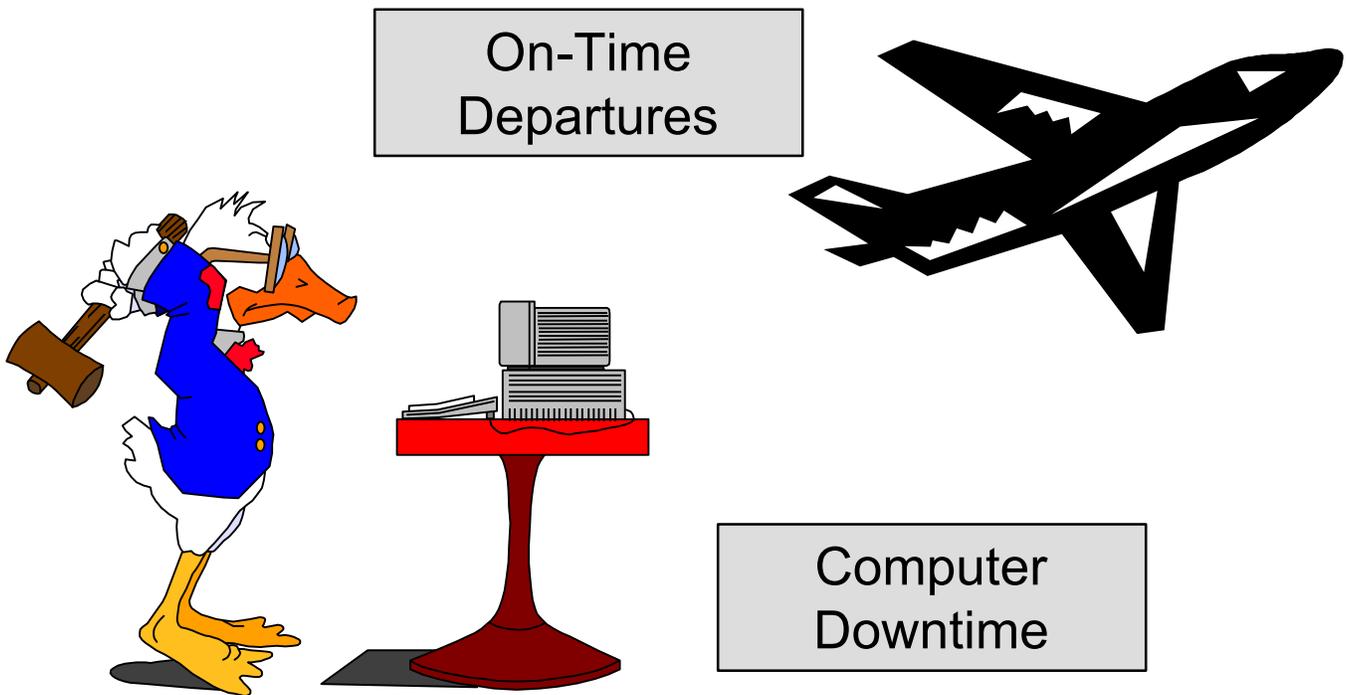
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- *Defect - Anything that results in customer dissatisfaction. Anything that results in a nonconformance.*

*Characteristic measures and defects are defined operationally.*

## *Class Exercise*

### *Examples*



- *Work as a class or individually.*
- *Your task*
  - *Write operational definitions for the examples above*
  - *Compare results if you work individually*



## *Possible Solutions to the Exercise*

### ■ *On-Time Departures*

- *An on-time departure could be one in which the door to the jetway is closed before the scheduled departure time. A passenger may think an on-time departure is one in which the plane takes off at the scheduled departure time. Both ways use a discrete measurement: on-time or late. A better way to measure departure time is to calculate the difference between the scheduled departure time and the actual departure time. An operational definition for actual departure time could be the time the door to the jetway is closed.*

### ■ *Computer Downtime*

- *Computer downtime occurs when a computer stops operating or processing during operational use. Time computer went down is based on operations log. Downtime is over when computer returns to normal operations.*



# Operational Definition – Partner Exercise (25 Minutes)-Optional Exercise

## Desired Outcomes

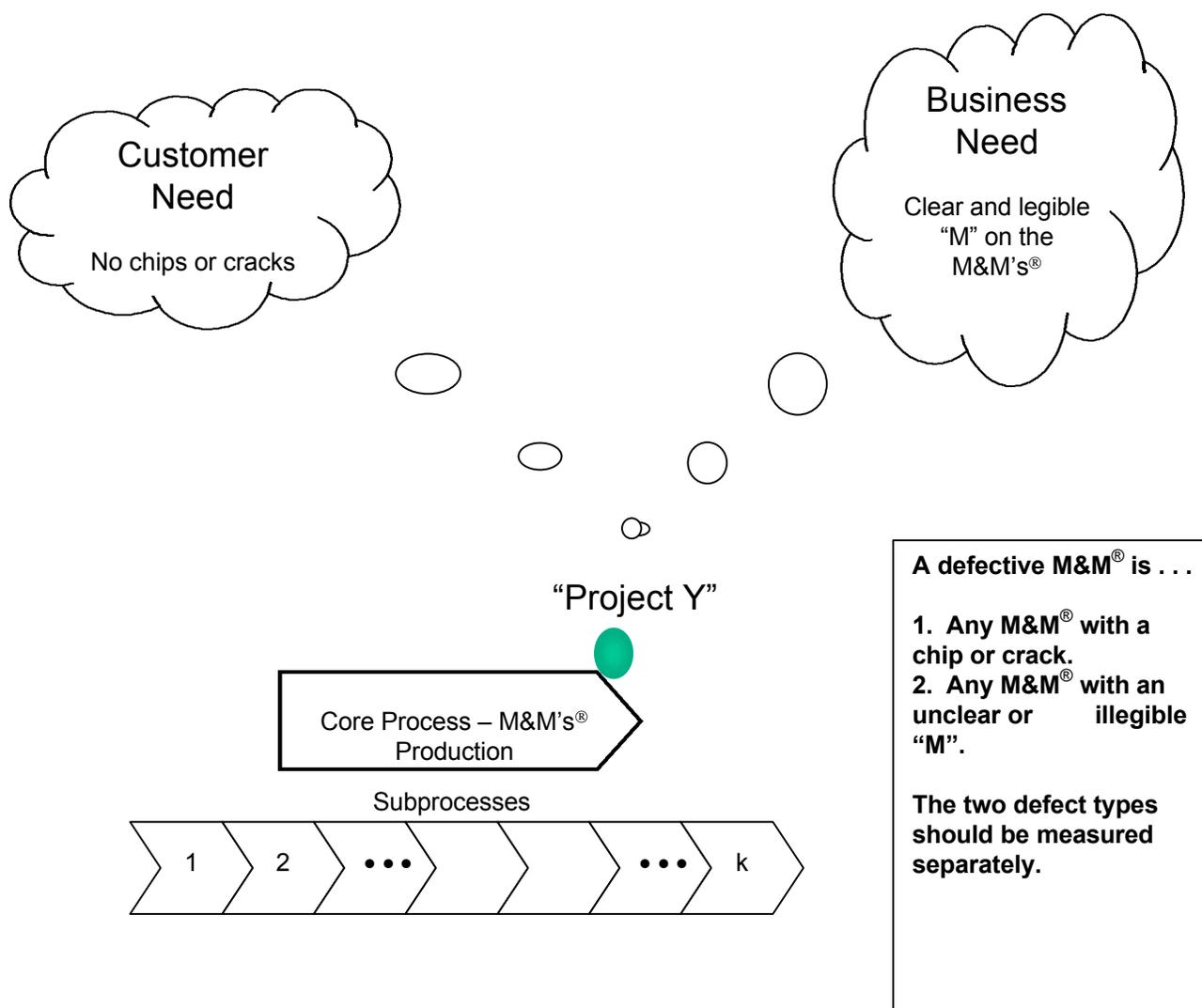
- Practice applying operational definitions
- Collect data on the number of defects in a package of M&M's®

What	How	Who	Time
<b>Partner Preparation</b>	<ul style="list-style-type: none"><li>■ Find a partner for the exercise.</li><li>■ Determine timing for each activity below.</li><li>■ Read the background information.</li></ul>	All	
<b>Develop An Operational Definition</b>	<ul style="list-style-type: none"><li>■ Develop an operational definition for one of the defect types found in an M&amp;M, either, 1) chips and cracks, or 2) unclear and illegible "M".</li><li>■ The definition should include:<ul style="list-style-type: none"><li>– What</li><li>– How</li><li>– Importance to customer</li></ul></li></ul>	Partners	
<b>Measure And Record Data</b>	<ul style="list-style-type: none"><li>■ With your partner apply your operational definition to your package of M&amp;M's®.</li><li>■ Use the form on the following page to record the total number of M&amp;M's® you inspect and the number of defective M&amp;M's®.</li><li>■ Note: If an M&amp;M has one or more chips/cracks, classify the M&amp;M's® as defective.</li></ul>	Partners	
<b>Close Exercise</b>	<ul style="list-style-type: none"><li>■ Brainstorm the challenges of developing an operational definition for this exercise, and how these challenges may impact your own project work.</li><li>■ Choose a spokesperson to report out on your operational definition, the challenges you experienced, and how these may impact your project work in the future.</li></ul>	Partners	



# Operational Definition – Partner Exercise (continued)

- Customers of M&M's<sup>o</sup> candy have various needs related to the consumption of the candy. Because the candy should "... melt in your mouth, not in your hands," one of the Project Y CTQs is for the candy to have no chips or cracks.
- Part of the internal process for making the candy is printing the letter "M" on the candy. While not a high priority for external customers it is important to internal customers for marketing and product branding.





# Operational Definition – Partner Exercise (continued)

## Data Collection Check Sheet

Date:	Location:
-------	-----------

Operational Definition: \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Data Collector's Name	# Of Pieces Inspected	# Of Pieces Chipped Or Cracked	# Of Pieces With Unclear Or Illegible "M"

## Data Summary Sheet

Data Collector's Name	# Of Pieces Inspected	% Of Pieces Chipped Or Cracked	% Of Pieces With Unclear Or Illegible "M"	% Of Pieces Defective



## *Operational Definition – Table Team (5 Minutes)*

---

### *Define Your Project Y*

- ***Write an operational definition for your Project Y***
- ***Write the definition on a flip chart***
- ***Report out***

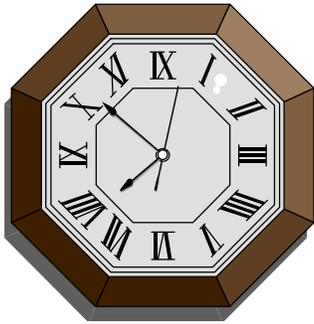


## *Reminders for Operational Definitions*

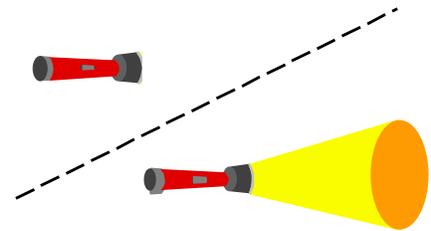
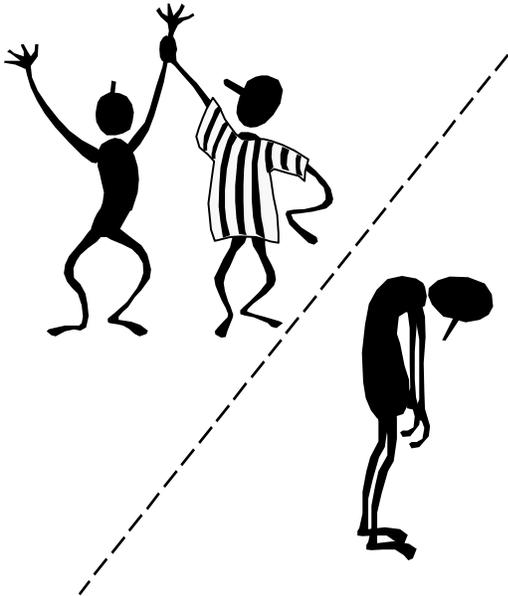
- *Remember to:*
  - *Remove ambiguity so everyone has the same understanding of words and instructions*
  - *Make sure that no matter who does the measuring, the results are consistent*
- *Check that your definition has:*
  - *Specific and concrete criteria (What)*
  - *A method to measure (How)*
  - *Usefulness to both you and the customer*



# *Continuous and Discrete Data*



**Continuous**



**Discrete**



## *The Basic Nature of Data*

*Two kinds of data can be used for measuring process capability:*

### ■ **Continuous Data**

- *Characterizes a product or process feature in terms of its size, weight, volts, time, or currency*
- *The measurement scale can be meaningfully divided into finer and finer increments of precision*
- *To apply the normal distribution, one must necessarily use continuous data*

### ■ **Discrete Data**

- *Counts the frequency of occurrence: e.g., the number of times something happens or fails to happen*
- *Is not capable of being meaningfully subdivided into more precise increments*
- *The Poisson and binomial models are used in connection with this type of data*
- *The validity of inferences made from discrete data are highly dependent upon the number of observations. The sample size required to characterize a discrete product or process feature is **much larger** than that required when continuous data is used.*



# Discrete vs. Continuous

## Discrete Data

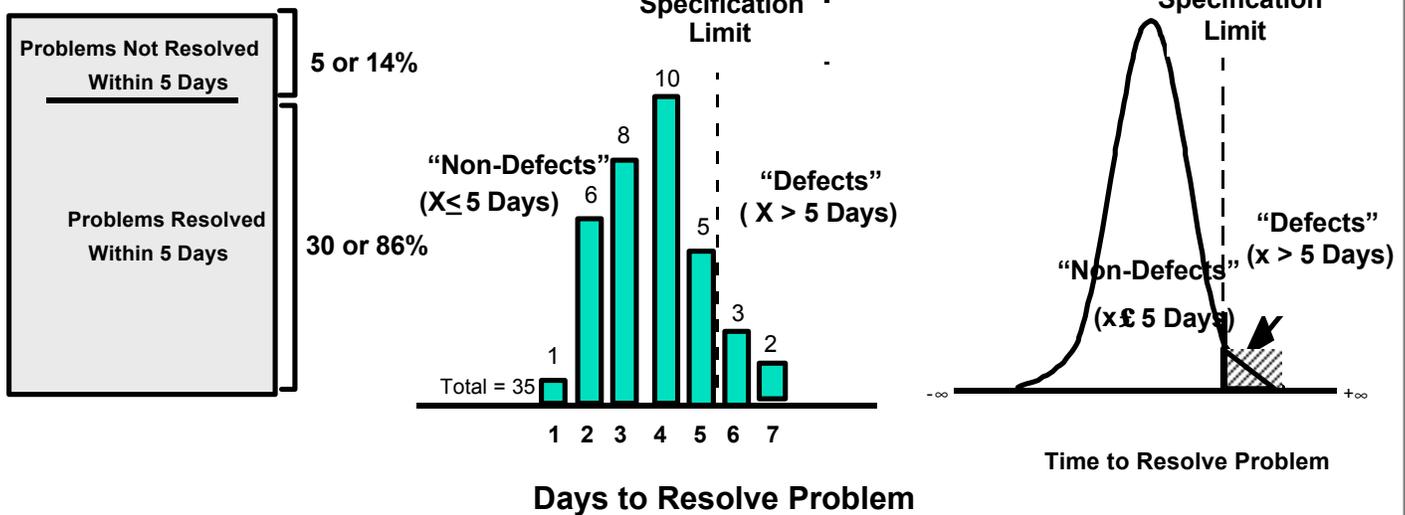
Values Can Only Be Zero or One, e.g., Good or Bad.

## Ordinal Data

Values Can Vary Only By Whole Units, e.g., count. (May be used, with caution, like continuous data.)

## Continuous Data

Values Can Be Measured To Any Degree of Precision.

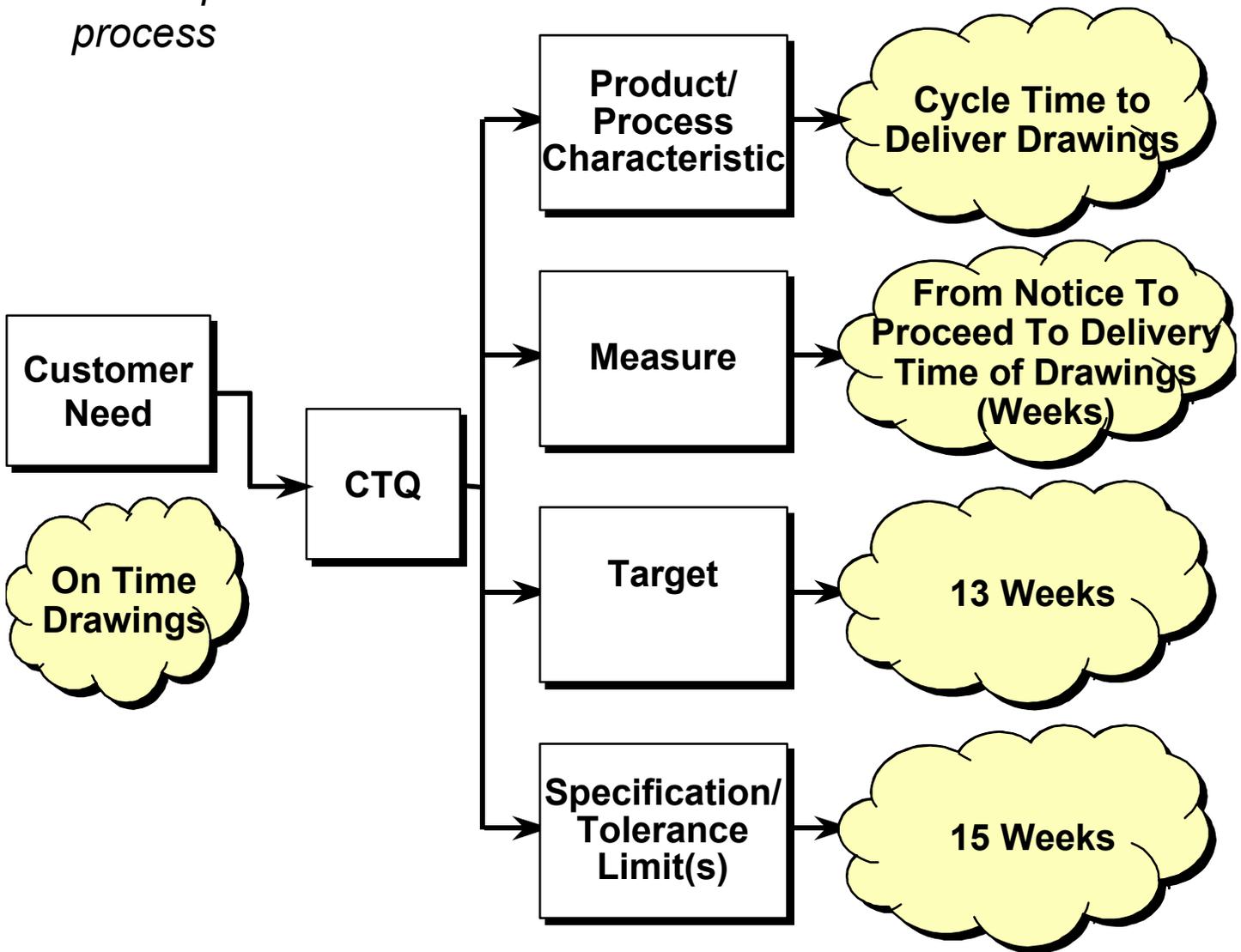


**Defect rates can be calculated using either discrete or continuous data.**



## Establishing a Performance Standard

- A performance standard translates customer needs into quantified requirements for our product or process





## Measurement Matrix

CTQ Type	Performance Standard Source	Measurement Method	
		Continuous	Discrete
<b>Dimension</b>	Drawings	Actual dimension	Good/Bad In/Out Tol. Pass/Fail
<b>Time</b>	Standards, Customers, Quotes/Bids	Actual time	Under/Over Estimate
<b>Money</b>	Quotes/Bids, Budgets	Actual cost	Under/Over Budget
<b>Completeness</b>	Process	% Complete	Present/Absent
<b>Accuracy/ Quality</b>	Standards, Process	Number of Errors	Good/Bad



## *The Road to Six Sigma*

---

### ■ **Less than 4-sigma**

- *Low hanging fruit*
- *Collect little bits of data*
- *Seven basic tools*
- *Discrete data OK*

### ■ **4-sigma to 5-sigma**

- *Process characterization and optimization*
- *Need continuous data*

### ■ **5-sigma to 6-sigma**

- *Design for Six Sigma*
- *Need continuous data*



## *Project B3095.1: Reducing Rotor Blasting Time*

**Defect:** Any time the “prep for Non-destructive Testing” takes more than 25 hours for a compressor rotor

**Opportunity:** Every compressor rotor Non-destructive Testing.

**Measure:** From the start time of “prep for blast” to the end time of “prep for blast.” Measured continuously in hours.

**Specification Limit:**  $USL = 25$  hours



## *Performance Standard Exercise: 20 mins.*

- *Working in teams, for two or more projects in your team*
  - *Identify your CTQ*
  - *Define the performance standard*
    - ✓ *define the measurable characteristic*
    - ✓ *determine whether it is continuous or discrete*
      - *If the characteristic is discrete, can you measure it differently so that it can be continuous?*
    - ✓ *determine the specification limits if applicable*
    - ✓ *define a defect*

*One or two groups will be asked to present their findings.*



## *Take Aways—Step 2*

---

- **A performance standard** answers these questions:
  - What does the customer want?
  - What is a good product/process?
  - What is a defect?
  
- The goal of a performance standard is to translate the customer need into a **measurable** characteristic.
  
- An **operational definition** is a precise description that tells how to get a value for the characteristic you are trying to measure.



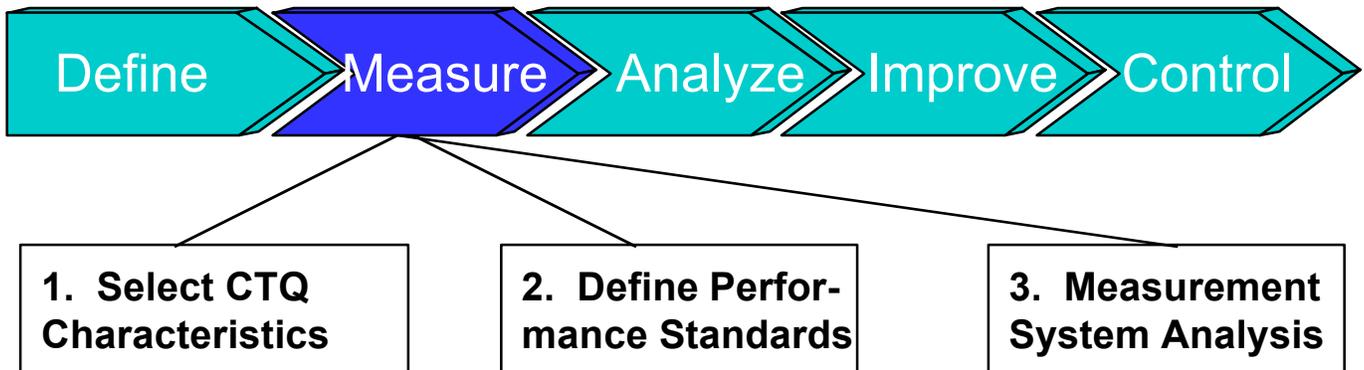
## *Take Aways—Step 2*

---

- *A defect is a **nonconformance** or anything that results in customer dissatisfaction.*
- ***Discrete Data** is categorical data such as go and no-go and is summarized as frequencies of defects. It is not capable of being subdivided into more precise increments.*
- ***Continuous Data** is based on some continuum of values a measurement can take and is summarized using measures of central tendency and dispersion.*



# Measure Phase



*Deliverable: Identify Measurable CTQ That Will Be Improved*

*Tools:*

- QFD
- Process Mapping
- FMEA
- Discrete vs. Continuous Data

*Deliverable: Determine and Confirm Specification Limits For Your Y.*

*Deliverable: Measurement System Adequate to Measure Y.*

*Tools:*

- Continuous Gage R&R
- Test/Retest
- Attribute Gage R&R
- Calibration
- Destructive Gage R & R



## *Establish Data Collection Plan Objectives*

---

### **Step 3: Measurement System Analysis**

- *To develop a written strategy for collecting the data you will use in your project.*
- *To define a clear strategy for collecting reliable data efficiently.*
- *To develop a common reference document for all team members to promote clear communication about the purpose and methods for data collection and provide the link between the data collection effort and the project goals.*



## *Data Collection Plan*

---

- *Focuses on the project Y and performance standards for Y.*
- *Uses the required performance range for your project Y to help you select a measurement tool.*
- *Helps ensure that resources are used effectively to collect only data that is critical to the success of the project.*
- *Requires the team to consider the problem to be solved, determine what data is needed to solve the problem, and then formulate a strategy for collecting the data.*
- *Considers potential Xs for the selected Y.*



## *Items to Consider*

---

- *Costs versus the potential benefits of collecting new data.*
  
- *If the cost to collect data exceeds the project benefits, you may:*
  - *Look for other ways to collect data*
  - *Consider reducing the number of samples needed by accepting a higher level of risk*
  - *Look for an alternate Y for the project CTQ*
  - *Re-examine the project charter*
  - *Challenge your team to think beyond the existing data collection mechanisms*



# *Measurement Systems Analysis Objectives*

---

## **Step 3: Measurement System Analysis**

*By the end of Step 3, the BB/GB will have:*

- a. Established the capability of the measurement system and the data.*
- b. Gained consensus with the project team on any actions needed regarding the measurement system.*



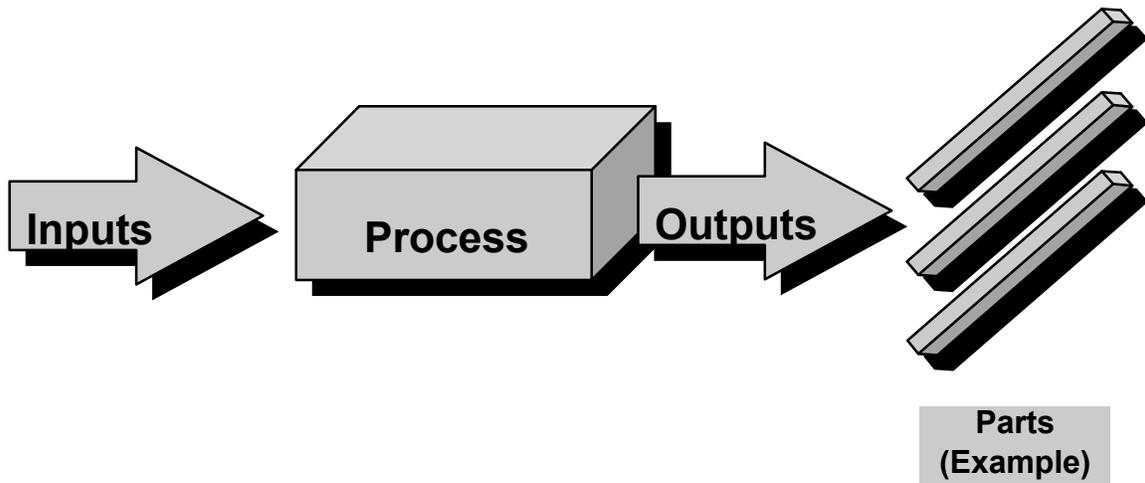
## *Measurement Systems Analysis Objectives*

*By the end of the training program, the participant will be able to:*

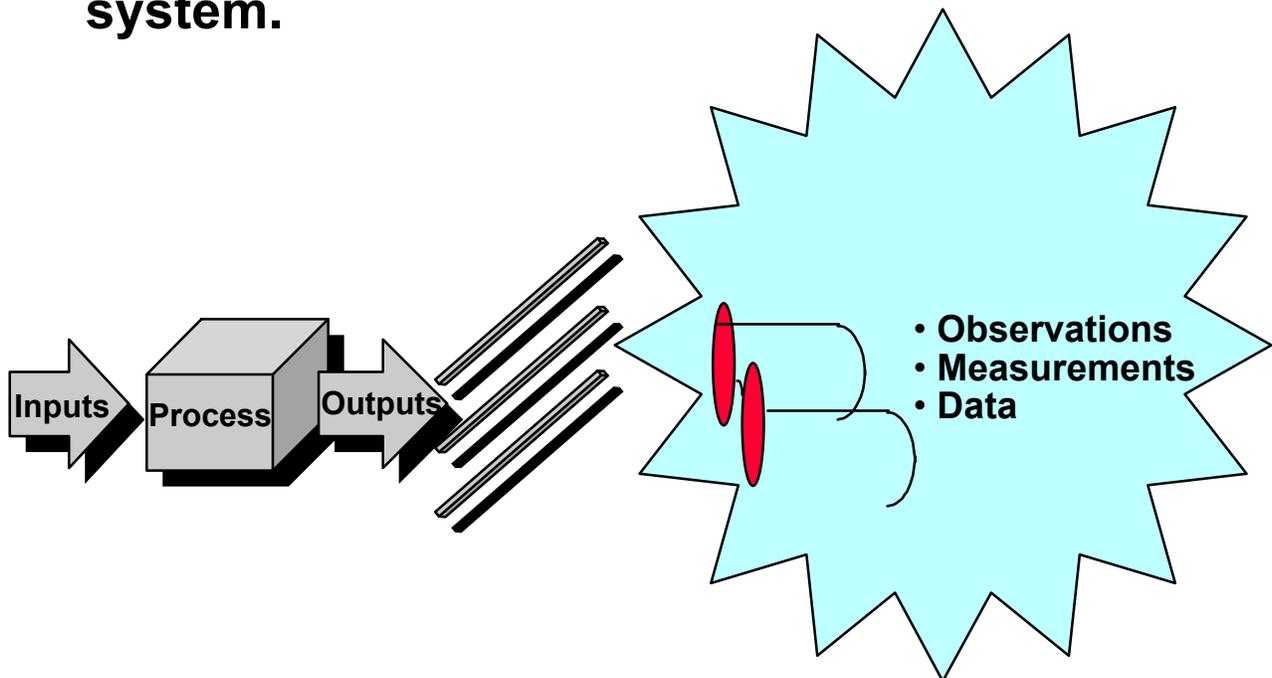
- *Understand measurement as a system which includes operators, gages and environment.*
- *Define the terms resolution, precision, accuracy bias, stability and linearity as used in Measurement Systems Analysis (MSA).*
- *Using the MSA Checklist, document the existing measurement system.*
- *Conduct a Test-Retest study and analyze the results.*
- *Conduct a Gage R&R study and analyze the results. Understand short form GR & R and attribute R & R. and the effect of tolerance on GR & R.*
- *Calculate both the appraiser variation (reproducibility) and equipment variation (repeatability).*
- *Identify the sources of variation in a measurement system.*
- *Define the concepts of calibration standards and destructive G R & R.*



## Evaluation of a Process



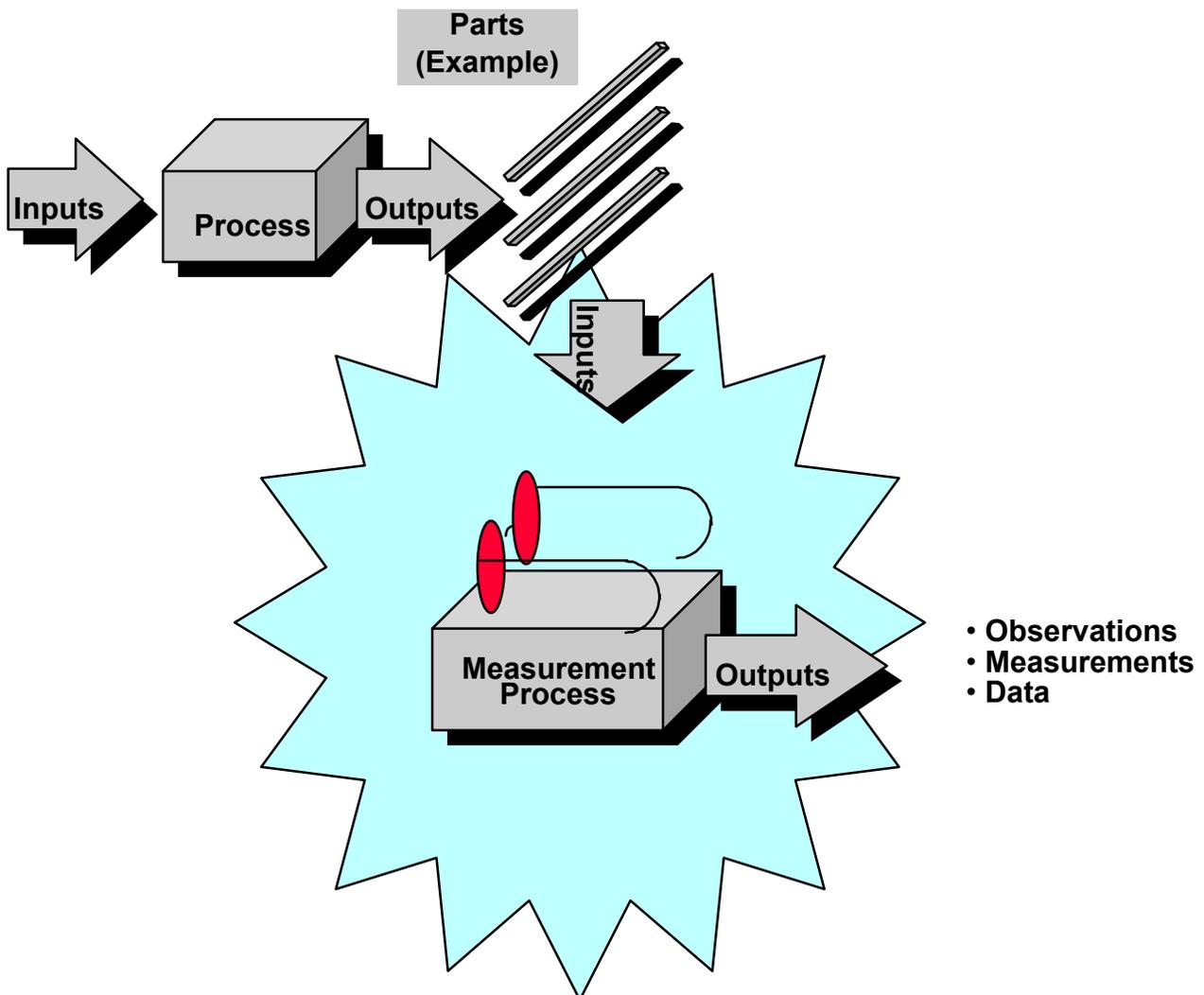
**Any information we gather about process behavior must first pass through a sensory system.**





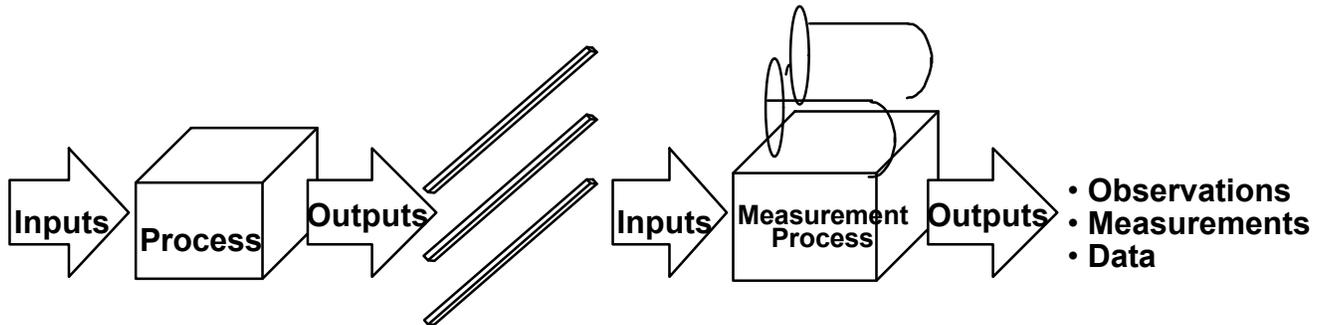
# The Measurement Process

*In other words, we must submit the output from the first process to a second process*

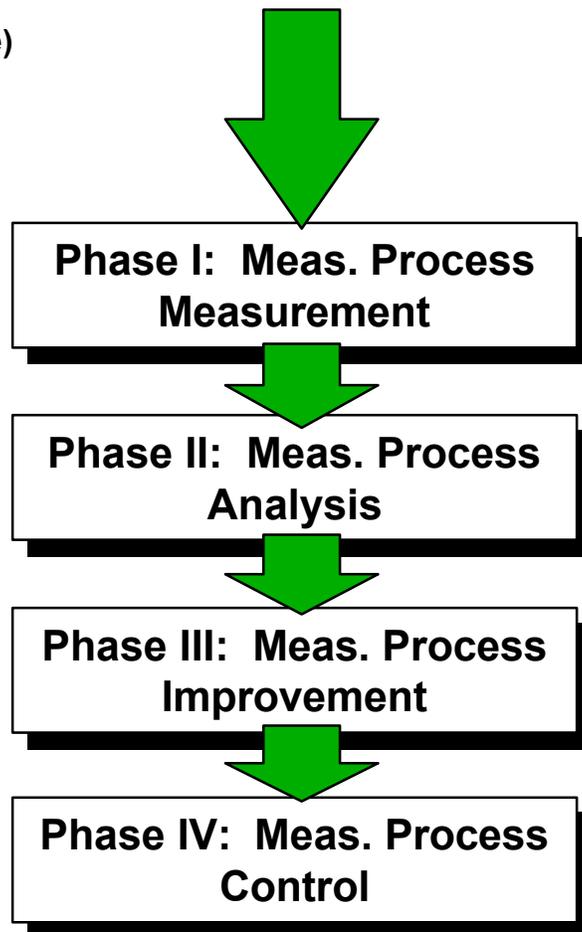




# Managing the Measurement Process

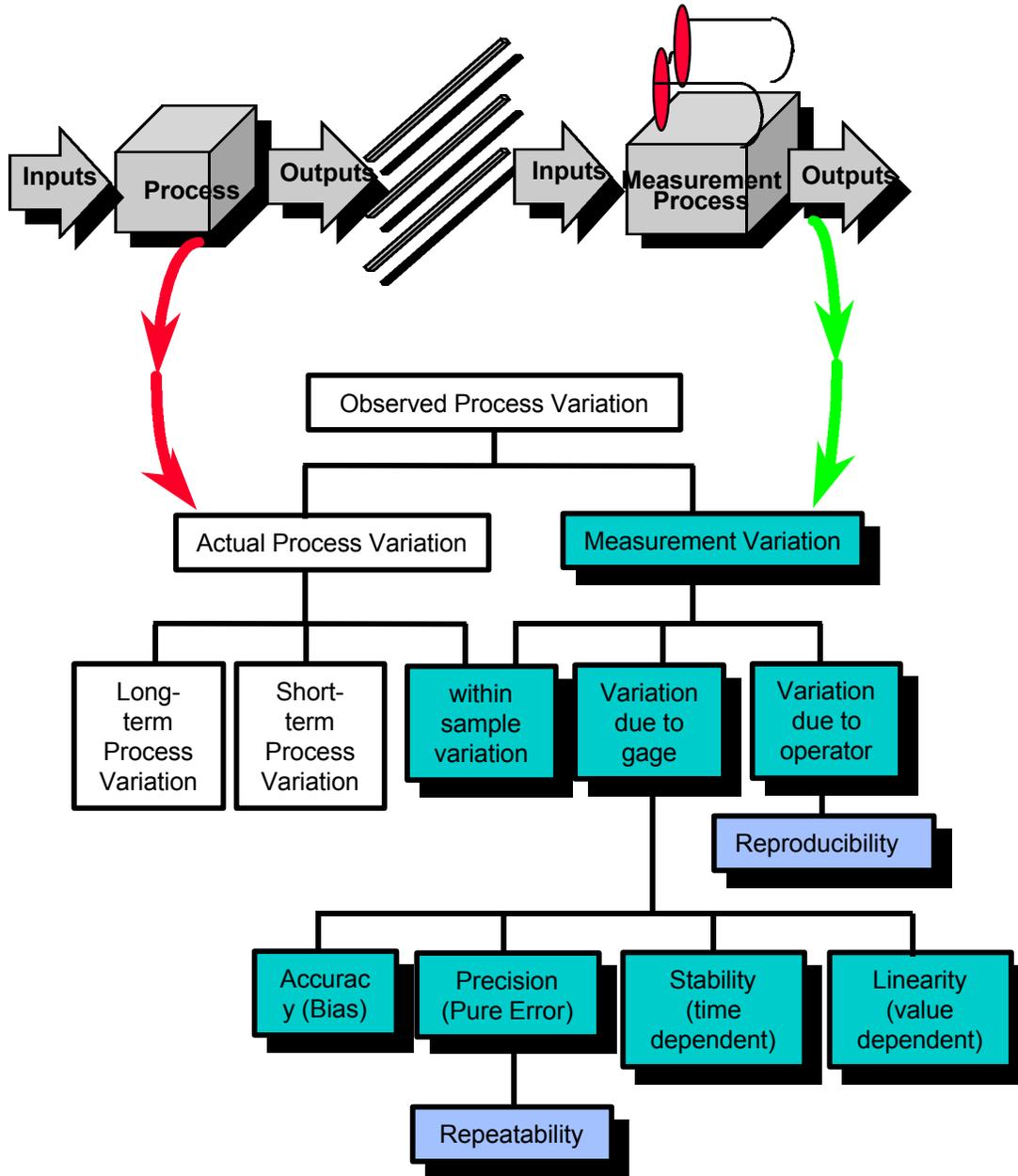


Parts  
(Example)





# Possible Sources of Variation

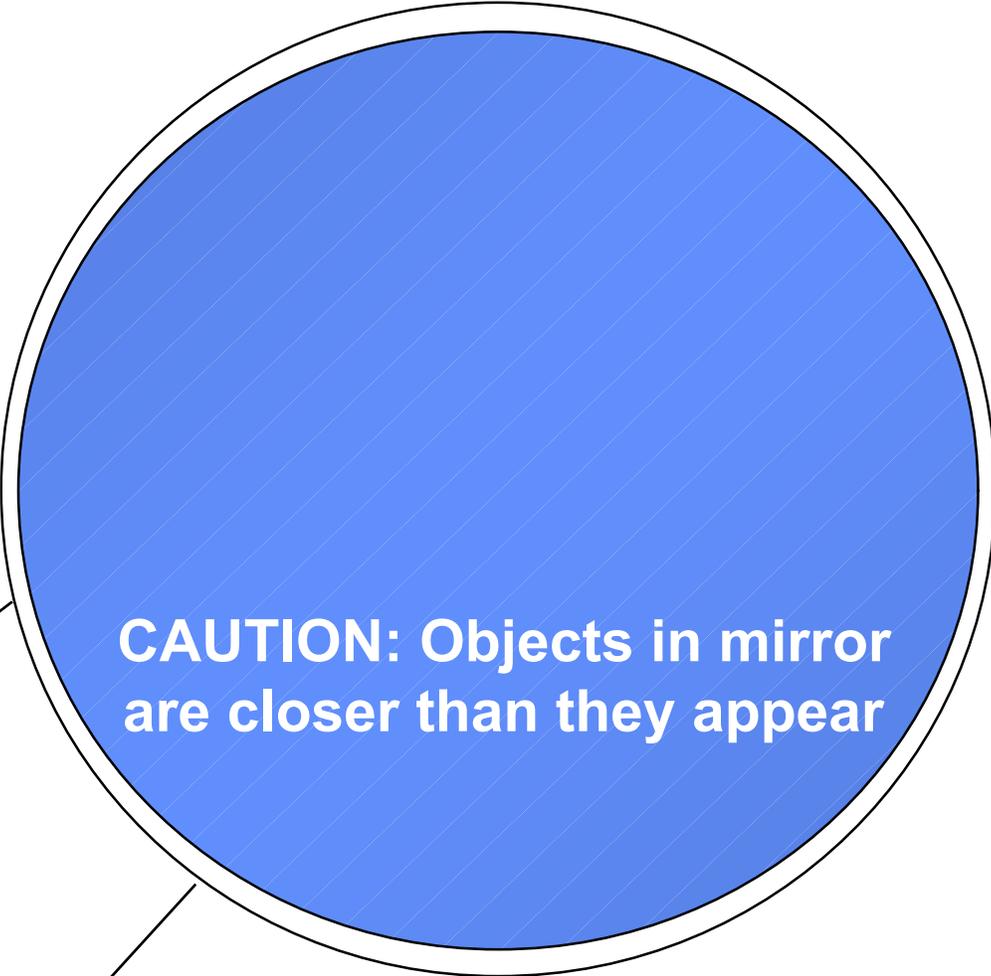


To address actual process variability, the variation due to the measurement system must first be identified and separated from that of the process



## *Possible Sources of Variation*

**A measurement system will not willingly disclose the type of distortion, inaccuracy or imprecision it is transmitting to our data. We must actively force it to reveal its hidden effects.**



**CAUTION: Objects in mirror are closer than they appear**



# Measurement System Analysis

---

**Accuracy** – the differences between **observed** average measurement and a **standard**

**Repeatability** – variation when one person repeatedly measures the same unit with the same measuring equipment

**Reproducibility** – variation when two or more people measure the same unit with the same measuring equipment

**Stability** – variation obtained when the same person measures the same unit with the same equipment over an extended period of time

**Linearity** – the consistency of the measurement system across the entire range of the measurement system



## *Measurement (Gage) Requirements*

---

***The gage should have adequate precision based on two different comparisons:***

■ ***Precision***

- ***The gage should be able to resolve the tolerance into approximately ten levels. If this condition is not met, the project team may be unable to achieve its goals.***

■ ***Accuracy***

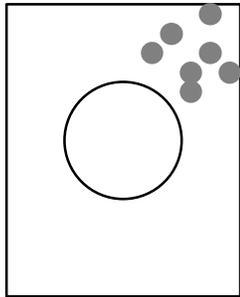
- ***The gage noise must be less than the process noise, otherwise it will be impossible to see ordinary process variation.***

***If these two conditions are not met, the gage may be inadequate and data from the gage may be of no value. In that case, the gage must be either improved (use MAIC) or replaced (use DFSS) with a device that has acceptable precision.***

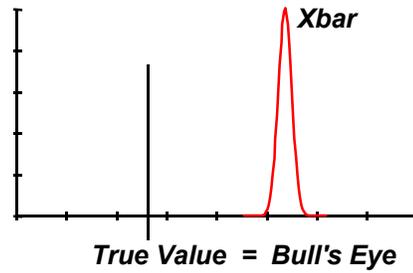


# Precision & Accuracy

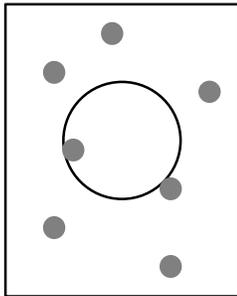
## Target Analogy



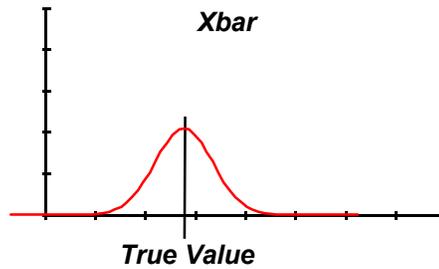
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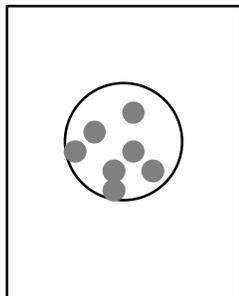
I. Precise, not accurate



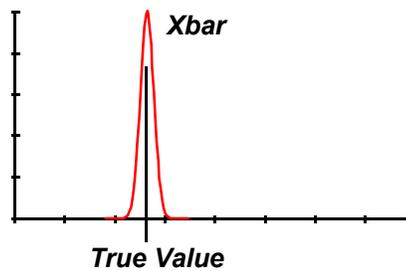
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II. Accurate, not precise



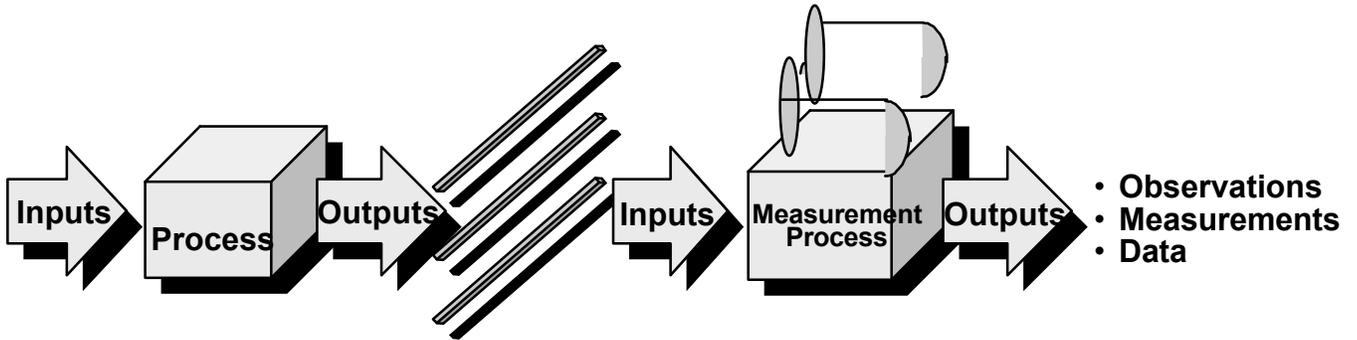
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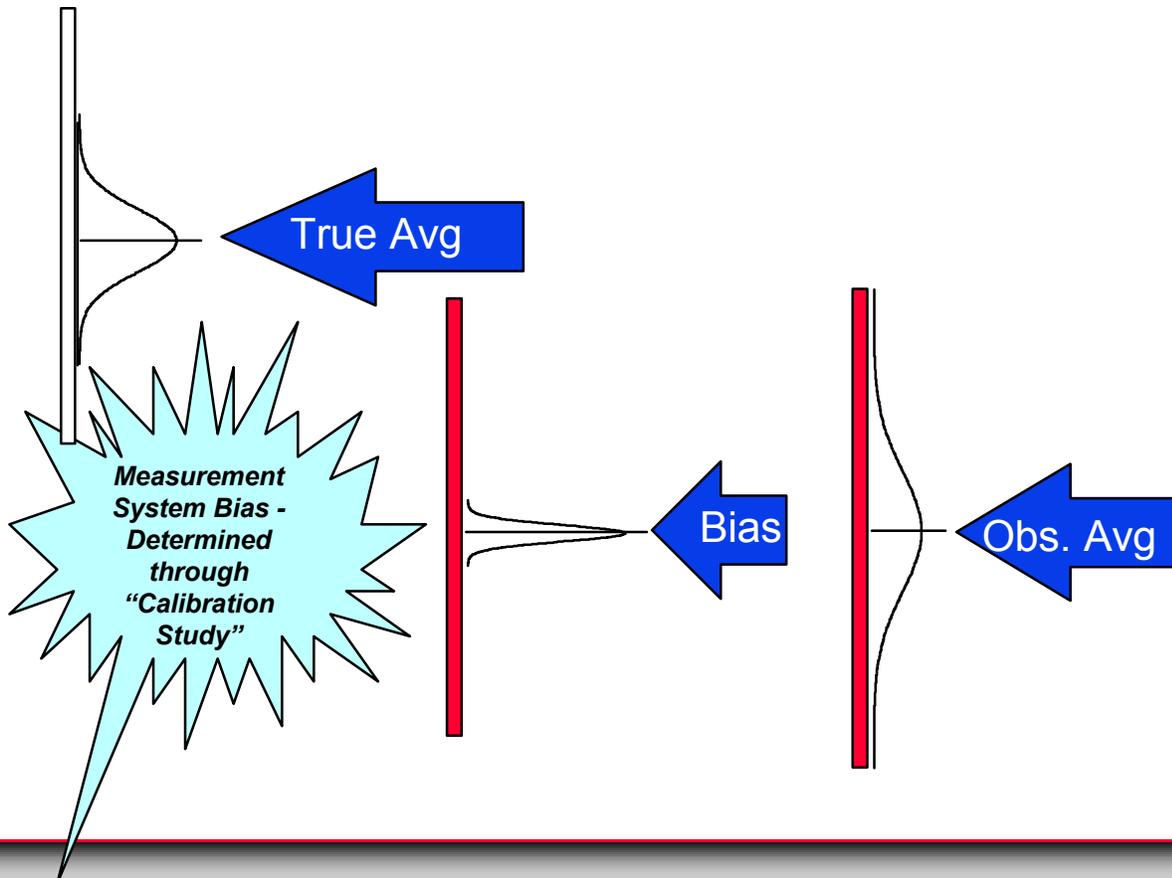
III. Precise and accurate



# Accuracy (Bias) - Shift in the Average

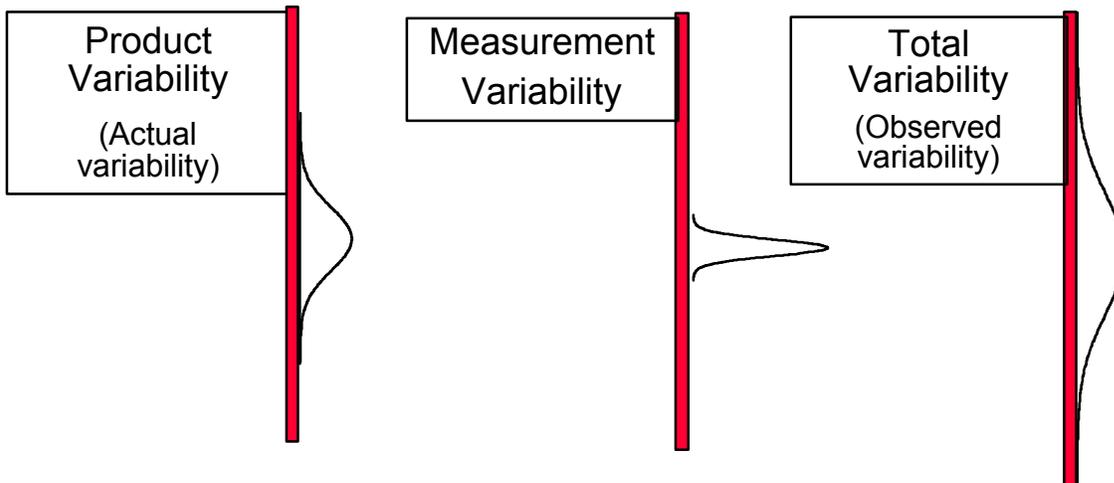
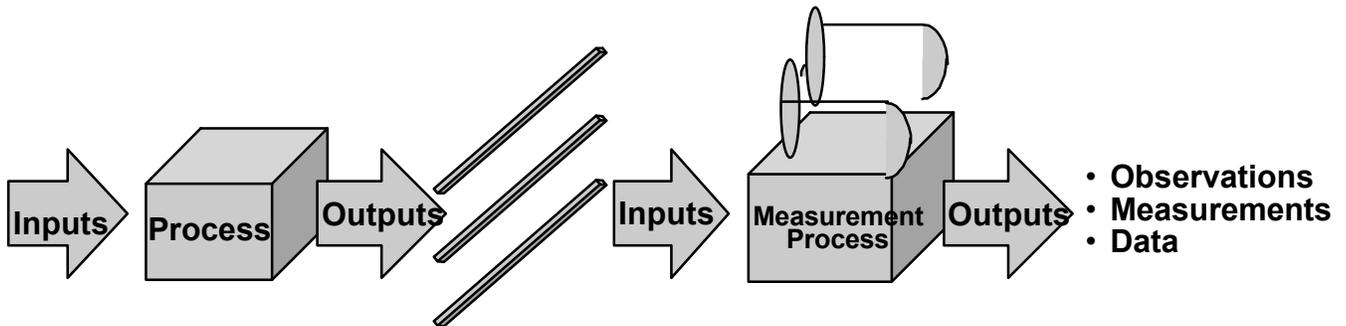


$$\mu_{\text{Actual (Part)}} + \mu_{\text{Meas. System}} = \mu_{\text{Observed (Total)}}$$

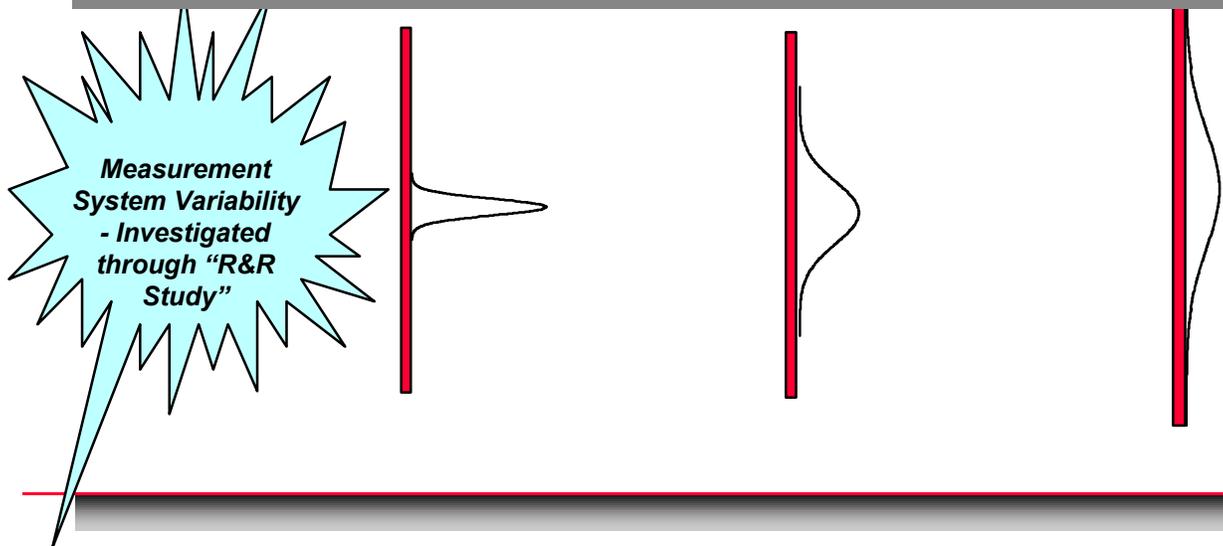




# Measurement System Variation



$$\sigma^2_{\text{Actual (Part)}} + \sigma^2_{\text{Meas. System}} = \sigma^2_{\text{Observed (Total)}}$$





## *Measurement Device (Gage) Resolution*

---

*The Gage scale should divide, or resolve, the Tolerance into at least **10 parts**.*

*If this is not true, the Gage is inadequate—it cannot even SEE the variation that may, or may not exist. Its data is of no value in determining the process results or the true capability of the process.*

*If the gage Resolution is inadequate, it must be replaced with a device that has acceptable Resolution—then data can start to be gathered and analyzed.*



## *Resolution & Significant Digits*

*Example—A steel rule marked only in whole inches:*



*Consider:*

- 1. Only measures falling exactly on the scale marks (1, 2, 3, ..., 11, 12) are exact.*
- 2. Measures falling between marks are doubtful or uncertain — i.e., 11.2 in. requires an estimate of the 0.2 figure (might it not be 0.1 or 0.3? Or perhaps 0.18 or 0.26?)*
- 3. The measured result of 11.2 in. contains three significant digits—two exact digits (the “11”) and one doubtful digit (the “0.2”).*
- 4. The measurement should NOT be reported as: 11.21 or 11.214 or 11.2143 inches. Nor as 11.20 nor 11.200 nor 11.2000. Any of these would mislead the user of the data into thinking there are 4 or 5 significant digits when, at the best, we have only three due to the resolution of the device.*



# *Elements of Measurement System Variability*

---

*Men & Women*

*Method*

*Material*

*Measurement*

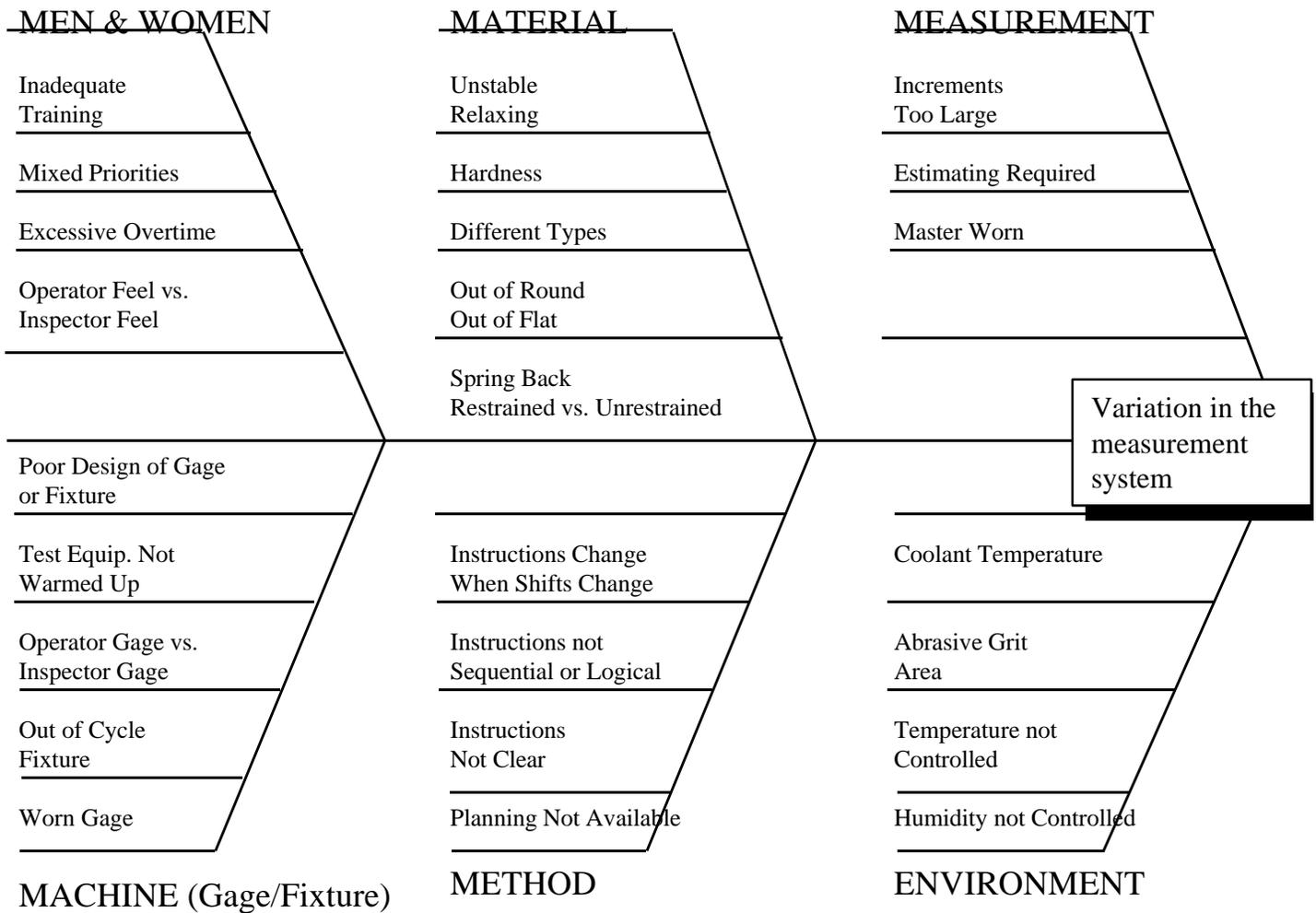
*Machine*

*Environment*

*Identify Potential Sources of Variation  
in the Measurement Process*



# Identify Sources of Variation in the Measurement System





## *Measurement System Adequacy*

- *The MSA Checklist*
- *The Test-Retest Study: a first look*
- *The Gage Reproducibility and Repeatability Study*



# Measurement Systems Analysis Checklist

Review the following Guide & Checklist BEFORE conducting an in-depth measurement system study.

## **GUIDE to Measurement Systems Analysis and validation of data sources:**

1. *What is the measurement procedure used?*
2. *Briefly describe the measurement procedure. What standards apply? Are used?*
3. *What is the "precision" (measurement error) of the system?*
4. *How has the precision been determined?*
5. *What does the gage (measurement system) Supplier state is the device's:*

= *Discrimination* (Resolution)?

= *Accuracy* (Bias)?

= *Precision* (Measurement Error)?

6. *Do you have results of a:*

= *Test-Retest Study?* (determines Measurement Error or "lack-of-precision")

= *Gage R&R Study?* (allocates the error between device and operator(s))

*If so, what are they?*

7. *Are different measurement systems (gages, scales, etc.) used to gather the same data? Identify which data comes from which device.*



## *Test-Retest Study*

*Best Practice Hint: Do Test-Retest before Gage R&R—a quick look at the situation.*

**Why:** *Determine the “Precision” of the system, instrument, device, or gage—where Precision = Measurement Error = Repeatability*

**How:**

- Repeatedly measure the same item.*
- Same conditions, operator, device, and “location” on item—same, same, same.*
- Completely mount and dismount item for each measurement—“exercise” gage through full range of normal use.*

**Data:** *Twenty (20) or more measurements. If measurements are “difficult” or “expensive,” then 10-15 may be OK. More is better. Calculate the sample Mean ( $\bar{X}$ ) and Standard Deviation ( $s$  or  $SD$ ) of the repeated measurements.*



## *Test-Retest Study*

### **Guidelines:**

1. **Device Precision**, really “lack-of-precision,” should be less than 1/10 of the Tolerance:

$$SD < 1/10 \times \text{Tolerance}$$

*If SD exceeds 1/10 X Tolerance, then the measurement system is unacceptable—the device introduces excessive “noise” into the data, it has a problem with **repeatability**. Action is required to find and remove the sources of this noise, this **error**—up to the possible replacement of the device.*

2. **Device Accuracy** may be estimated if you know the true-value of the Test Unit:

$$\text{Inaccuracy} = \text{Bias} = \bar{X} - \text{“True-Value”}$$

*If you use a “Standard Unit,” the True-Value is known. Otherwise, you do not know the True-Value and, therefore, are not able to determine the device Accuracy or Bias.*



## Test-Retest Study

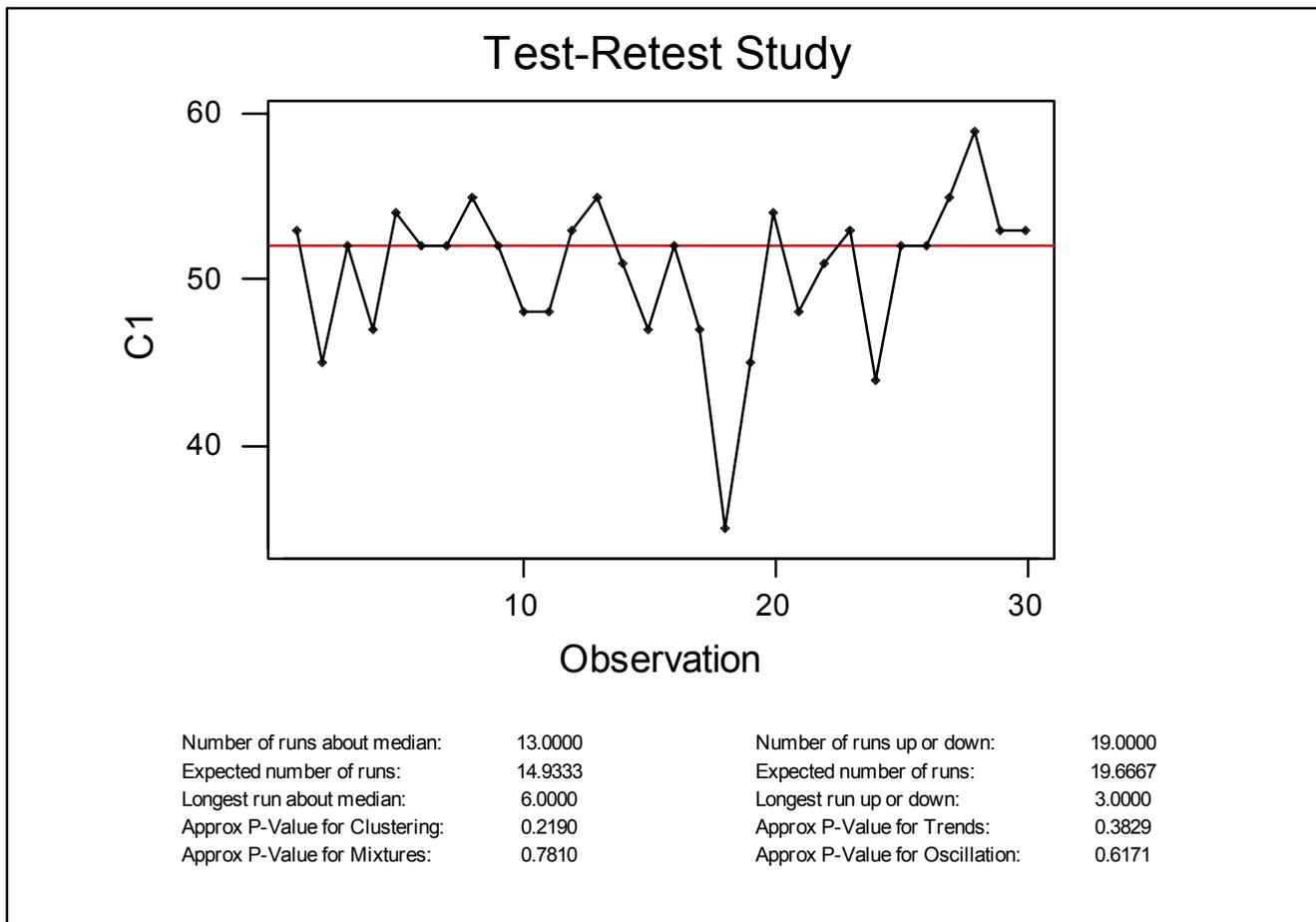
**Example:** Test-Retest study on a measuring device. Thirty (30) repeat measures were taken on a Working Standard test item of given thickness of 50 mils. The measuring device is used for a measurement where the Tolerance Width is 20 mils (+/- 10). The data, in mils, is below:

53	48	48
45	53	51
52	55	53
47	51	44
54	47	52
52	52	52
52	47	55
55	35	59
52	45	53
48	54	53



## Analysis

**Step 1:** Plot the 30 measures in the sequence taken. Look for “patterns” or trends that may indicate the device is “shifting” as the measurements are taken.



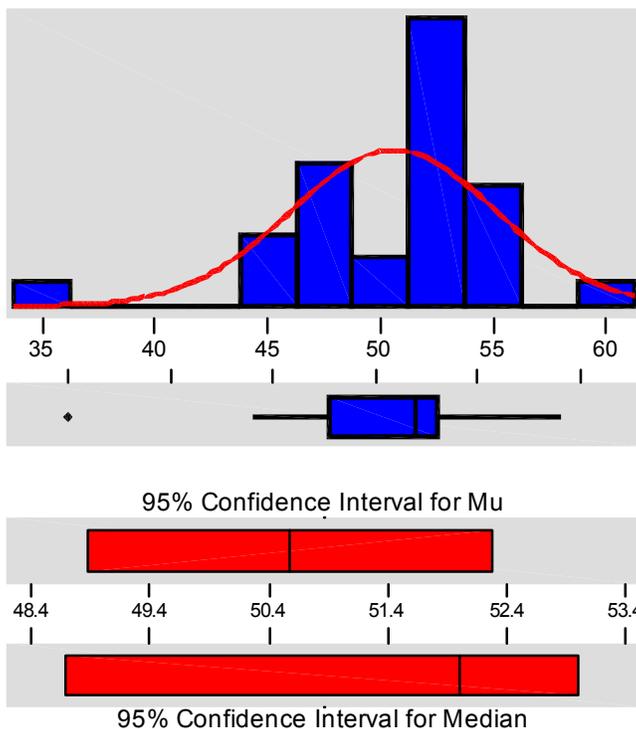


## Analysis (cont.)

Step 2: Calculate sample statistics ( $\bar{X}$  and SD) and plot a histogram of the measurements:

Note that  $SD = 4.6$  and  $\bar{X} = 50.6$ —as shown below:

### Descriptive Statistics



Variable: C1

Anderson-Darling Normality Test

A-Squared: 1.210  
P-Value: 0.003

Mean 50.5667  
StDev 4.5613  
Variance 20.8057  
Skewness -1.38288  
Kurtosis 3.50411  
N 30

Minimum 35.0000  
1st Quartile 47.7500  
Median 52.0000  
3rd Quartile 53.0000  
Maximum 59.0000

95% Confidence Interval for Mu  
48.8634 52.2699

95% Confidence Interval for Sigma  
3.6327 6.1319

95% Confidence Interval for Median  
48.6861 53.0000



## *Analysis (cont.)*

*Step 3: Conclusions = given that the Tolerance Width = 20 (+/- 10), the Test-Retest data shows an unacceptable level of device Precision:*

$$SD = 4.6 > 1/10 \times (20)$$

$$SD = 4.6 \gg 2.0$$

*Device has “problems” with Measurement Error*

*Given that the Standard test item had a known thickness of 50 mils, the estimate of Accuracy is:*

$$Inaccuracy = Bias = 50.6 - 50.0 = +0.6$$

*The acceptability of this level of Bias depends on the application. If it remains consistent from reading to reading, then all measurements could be “adjusted” by subtracting the known Bias value of +0.6 mils.*



# *How to Conduct A Gage Reproducibility & Repeatability (Gage R&R) Study*

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- 1. Collecting the Data*
- 2. Performing the Calculations*
- 3. Analyzing the Results*



## *Collecting the Data*

- *Communicate the data collection plan*
  - *data to be collected*
  - *method to use*
  - *cost of the data collection process*
  - *expected benefits of data collection to the project*
  - *expected time frame*
- *Train employees on the data collection plan*
  - *operational definitions*
  - *measurement tools*
  - *procedures and checklists*
- *Collect the data*
  - *project Y*
  - *potential project Xs*



## *Collecting the Data*

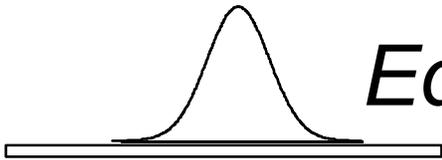
*In order to get and estimate variation in the “Real” Measurement System - follow the process*

- 1. Follow actual process*
- 2. Use the people that usually measure*
- 3. Follow the planning for the job*
- 4. Perform the study in the usual environment*
- 5. Use the gages used for the job*

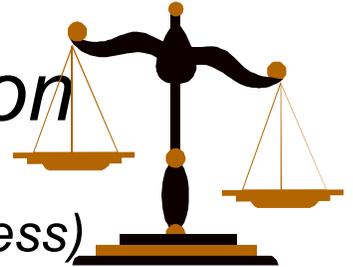
**MAKE IT BUSINESS AS USUAL  
AS MUCH AS POSSIBLE**



# *Types of Variation Estimated by the Gage R&R*



## *Equipment Variation*



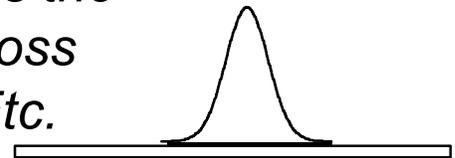
*(Sources of variation from within the process)  
Within Gage - Within Operator - Within Part - Etc.*

*The variation introduced into the measurement process from within one or more elements of the measurement process - such as: within operator variation - within gage variation - within part variation - within method variation.*



## *Appraiser Variation*

*(Source of variation from across the process) Across Gages - Across Operators - Across Parts - Etc.*

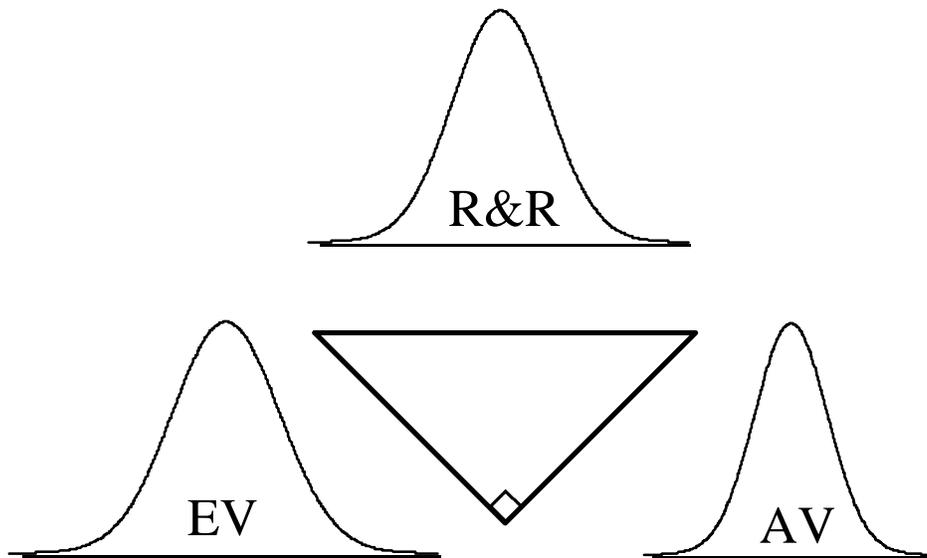


*The variation introduced into the measurement process by effects going across the measurement process - such as different appraisers - different part configurations - different checking methods.*



## *Relationship Between EV, AV and R&R*

*R&R is the Reproducibility (AV) and Repeatability (EV) of the Measurement System. It represents the total variation in the Measurement System.*



$$\sigma^2_{\text{Equipment}} + \sigma^2_{\text{Appraiser}} = \sigma^2_{\text{Total (R\&R)}}$$



## *Relating R&R to Specification Window*

*How much of the tolerance is used up by the Measurement System variability?*



*About 50% of the tolerance, in this example, is used up by the Measurement System variability. This leaves only 50% for the Process variability!*



# Analyzing the Gage R&R Results

## Rules of Thumb

1. *R&R less than 10% - Measurement System acceptable.*
2. *R&R 10% to 30% - maybe acceptable - make decision based on classification of characteristic, hardware application, customer input, etc.*
3. *R&R over 30% - not acceptable. find problem, re-visit the fishbone diagram, remove root causes.*
4. *A “signal-to-noise” Ratio =  $(\text{StdDev}_{\text{parts}} / \text{StdDev}_{\text{GR\&R}}) \times 1.41$  and rounded*  
*Guidelines:*
  - < 2—no value for process control, parts all “look” the same*
  - = 2—can see two groups—high/low, good/bad*
  - = 3—can see three groups—high/mid/low*
  - >= 4—acceptable measurement system (higher is better)*
5. *Effective resolution—50%, or more, of Xbar chart outside control limits—implies part variation “exceeds” Measurement System variation.*



## *Gage R & R Studies*

---

- *Gage R & R Studies should be performed over the range of expected observations.*
- *The primary objective of a Gage R & R study is to quantify the level of measurement variability.*
- *A secondary objective of the Gage R & R study is to separate the contributions of variability from different sources.*
- *The three methods for a Gage R & R study for continuous data are:*
  - *Short form*
  - *Long form*
  - *ANOVA*
- *Only short form and ANOVA are covered in this course. The ANOVA method is preferred over the long form method, since it gives more information than the other methods.*
- *The discrete/attribute data Gage R & R method is also presented.*



---

# *Short Form Gage Repeatability and Reproducibility*



## Short Form Gage Reproducibility Example

Part	Operator A	Operator B	Range(A-B)
1	4	2	2
2	3	4	1
3	6	7	1
4	5	7	2
5	9	8	1
(Tolerance = 20)			Sum of Ranges: 7
			Average Range: 1.4

$$\text{Average Range} = \Sigma R/5 = (\bar{R}) = 7/5 = 1.4$$

$$\text{Gage Error (Gage R\&R)} = \frac{5.15}{1.19} (\bar{R}) = \frac{5.15 (1.4)}{1.19} = 6.1$$

$$\text{Gage R\&R as a \% of Tolerance} = (\text{Gage R\&R} \times 100) / \text{Tolerance} = (6.1 \times 100/20) = 30.5\%$$

Note: In the short form method, repeatability and reproducibility cannot be separated.



## Short Form Gage R&R Example

- Gage Error is calculated by multiplying the average range by a constant (4.33 in our example). The constant value is derived from the ratio  $5.15/d^*$ , where  $d^*$  is determined from the following table. For our example,  $d^* = 1.19$ , for 5 parts and 2 operators.
- 5.15 STD represent 99% confidence level for a normal distribution.
- $d^*$  Values for the Distribution of the Average Range

Number of parts	Number of Operators		
	<u>2</u>	<u>3</u>	<u>4</u>
1	1.41	1.91	2.24
2	1.28	1.81	2.15
3	1.23	1.77	2.12
4	1.21	1.75	2.11
5	1.19	1.74	2.10
6	1.18	1.73	2.09
7	1.17	1.73	2.09
8	1.17	1.72	2.08
9	1.16	1.72	2.08
10	1.16	1.72	2.08

---

---

### Notes



## *Exercise – Use the Short Method to Determine the Gage Error*

Dimension “D” = 2.000 +/- .015

Part	Operator A	Operator B	Range
1	2.003	2.001	
2	1.998	2.003	
3	2.007	2.006	
4	2.001	1.998	
5	1.999	2.003	
Sum of Ranges:			



---

## *Gage R&R Process: Guidelines*

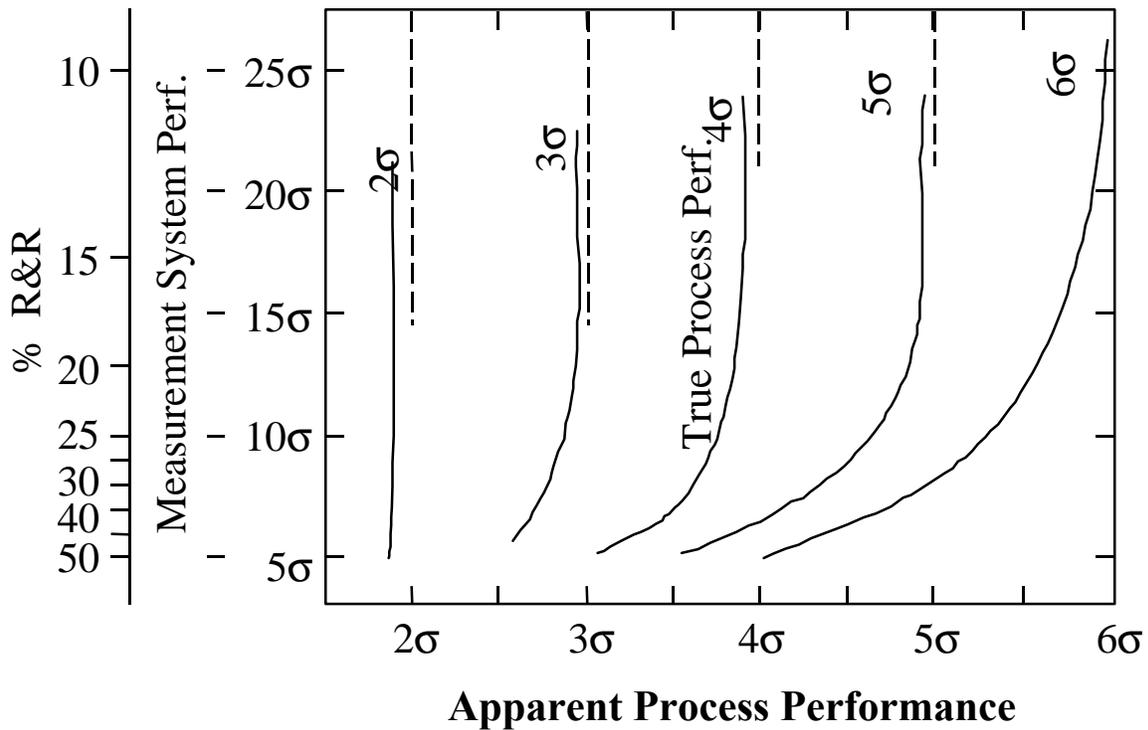
---

1. R&R less than 10% - measurement system acceptable.
2. R&R 10% to 30% - maybe acceptable - make decision based on classification of characteristic, hardware application, customer input, Sigma level of your process.
3. R&R over 30% - not acceptable. Find problem. Remove root cause.





# Sigma Conversion Chart



***The Measurement System Needs to be an Order of Magnitude Better than the Desired Process Sigma***



## *Solution to Short Form Exercise*

Use the short method to determine the gage error.

Dimension "D" = 2.000 +/- .015

Part	Operator A	Operator B	Range
1	2.003	2.001	0.002
2	1.998	2.003	0.005
3	2.007	2.006	0.001
4	2.001	1.998	0.003
5	1.999	2.003	0.004
Sum of Ranges:			0.015

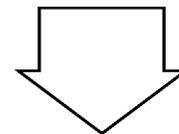
**Calculations:**

n = number of parts

Average Range,  $(\bar{R}) = \Sigma R/n = 0.015/5 = 0.003$

Gage Error (GRR) =  $(5.15/1.19) (\bar{R}) = (4.33)(0.003) = 0.013$

GRR as a % of Tolerance =  $(0.013 \times 100) / 0.030 = 43.3\%$



*Unacceptable gage*



---

## *Gage R&R Bolt Exercise - Optional*

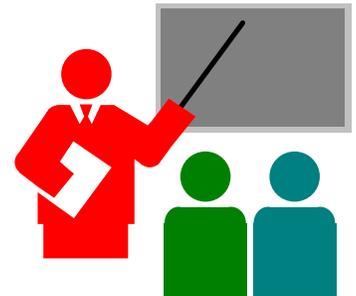


## *The Gage Reproducibility & Repeatability (GR&R) Study: Optional*

1. *Set up “data sheet”—Minitab.*
2. *Perform study—collect & enter Data.*
3. *Perform calculations & prepare Charts.*
4. *Analyze—interpret & draw conclusions.*
5. *Take action, make recommendations—keep, improve, or replace the measurement system.*

### ***BEST PRACTICE Hints:***

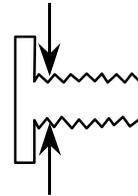
- *randomize readings*
- *use graphs and charts to analyze*
- *ANOVA is “best” study method*
- *base conclusions on more than Gage R&R% of Tolerance alone*





## *Gage R&R “Bolt” Exercise: Optional*

- *Each team performs Gage R&R study on a bolt measuring process.*



- *The feature of interest = bolt diameter.*
- *Three operators measure diameter of ten bolts, three times:  $3 \times 10 \times 3 = 90$ . The order they are measured should be random.*
- *Team may have one size of bolt, but different teams may have different sizes. [Bolt tolerance(s) on a later page P]*
- *Prepare Minitab “data sheet” for the study.*
- *Take measurements & input data.*
- *Analyze & interpret results—numeric and graphical outputs via Minitab.*





## *More Best-Practice Hints: Optional*

- *Review RESOLUTION of gage (“Scale Ticks”) = record data to one UNCERTAIN DIGIT.*
- *Establish “rule” used for setting uncertain digit:*
  - *1/10th of interval*
  - *nearest 1/4th of interval*
  - *how is “rounding” done*
- *Avoid “dumb decimal digits”—if data is in 1/1,000ths, then record 0.0052 as 5.2 'mils' [5.2 mm, etc.]; if in '10 thousands', then record 0.03465 as 346.5; etc., etc. [reduces entry error]*
- *Make notes of unusual readings.*
- *Randomize parts within operators.*
- *Think about how data is being “subgrouped” by operator, trial, part—and by sequence within operator—and overall sequence.*



***What questions can this data answer?***



## *“Bolt”- Specifications: Optional*

---

### **Two sizes:**

$3/8'' = 6/16'' \quad 0.375'' \quad +/- \quad 0.010'' \quad \text{Big}$   
 $[0.365 \quad 0.385]$

or  $LSL = 365 \quad USL = 385 \quad \text{mils}$

*“Tolerance” = Tol Width = 20 mils*

---

$5/16'' \quad 0.3125'' \quad +/- \quad 0.010'' \quad \text{Small}$   
 $[0.3025 \quad 0.3225]$

or  $LSL = 302.5 \quad USL = 322.5 \quad \text{mils}$

*“Tolerance” = Tol Width = 20 mils*

---



## Common Example:

**We will review the Bolt Exercise, but first...Greenville a Gage R&R study:**

- 3 Operators, 3 Trials, 10 Parts (pins) = 90 data points
- Data in whole mils, no decimal (scale ticks = mils)
- Range of  $X_{ijk}$  = 342 to 390;  $Xbar = 364.54$
- Spec Width = (USL - LSL) = 20 mils



**Example file = Grr02-gvl6-97.mtw**

TestSeq	OpSeqNo	Oper(i)	Trial(j)	Part(k)	Xijk
1	1	1	1	4	370
2	2	1	1	7	372
3	3	1	1	9	390
4	4	1	1	3	360
5	5	1	1	10	355
6	6	1	1	8	360
7	7	1	1	1	352
8	8	1	1	2	390
9	9	1	1	6	342
10	10	1	1	5	360
11	1	2	1	8	360
12	2	2	1	1	355
13	3	2	1	4	370
14	4	2	1	3	365
15	5	2	1	6	350
16	6	2	1	9	390
17	7	2	1	10	352
18	8	2	1	7	372
19	9	2	1	5	360
20	10	2	1	2	390
21	1	3	1	3	365
:	:	:	:	:	:
:	:	:	:	:	:



## Analyzing Gage R&R Results—Review

Use graphs and charts to analyze

- ANOVA is “best” study method
- look at more than Gage R&R% of tolerance

MINITAB FILE: Grr02-gvl6-97.mtw

The screenshot shows the Minitab software interface. The 'Graph' menu is open, displaying various plotting options. The data table below shows the following information:

	C1		C4	C5	C6	C7	C8	C9
	OpSeqNo		Part(k)	Xijk				
1	1		4	370				
2	2	1	7	372				
3	3	1	9	390				
4	4	1	3	360				
5	5	1	10	355				
6	6	1	8	360				
7	7	1	1	352				
8	8	1	2	390				



# Graphical Analysis

**Boxplot**

C1 OpSeqNo  
C2 Oper(i)  
C3 Trial(j)  
C4 Part(k)  
C5 Xijk

**Graph variables: Y (measurement) vs X (category)**

Graph	Y	X
1	Xijk	'Oper(i)'
2	Xijk	'Trial(j)'
3	Xijk	'Part(k)'

**Data display:**

Item	Display	For each	Group variable
1	IQRRange Box	Graph	
2	Outlier S>>	Graph	
3			

Annotation  Frame  Regions

**Boxplot Options**

**Transpose X and Y**

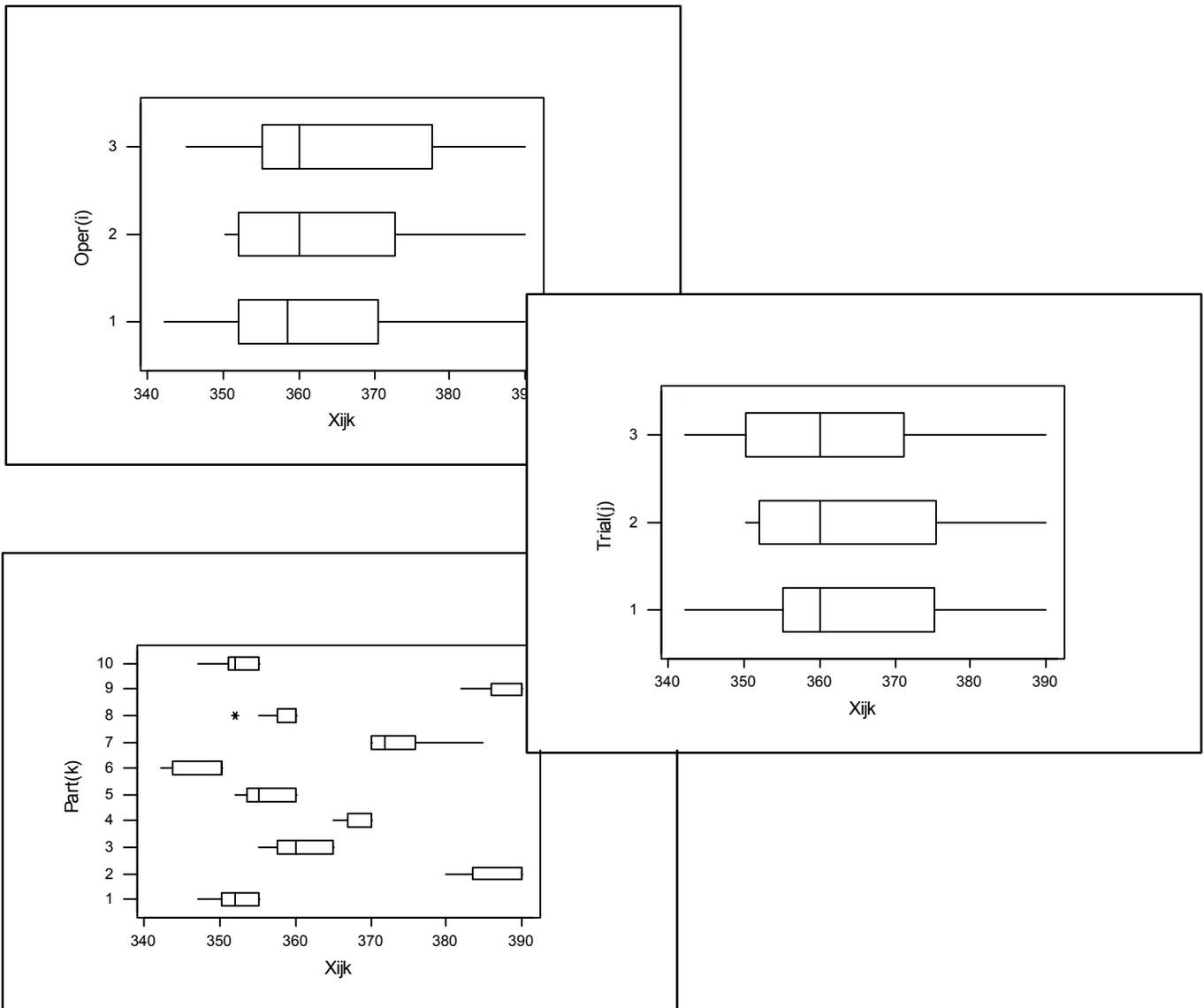


# Graphical Analysis

What “factors” affect the measurements, the  $X_{ijk}$ ?

- Operator (i)?
- Trial (j)?
- Part (k)?

What does this say about the measurement system?





# *Gage R&R Study: ANOVA Method*



# Gage R&R Study: ANOVA Method

MINITAB FILE: Grr02-gvl6-97.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the 'Quality Tools' sub-menu is selected, with 'Gage R&R Study...' highlighted. The worksheet 'GRR02-1.MTW' is open, showing a table with columns C1 through C9 and rows 1 through 8. The data in the table is as follows:

	C1	C2	C3	C4	C5	C6	C7	C8	C9
↓	OpSeqNo	Oper(i)	Trial(j)						
1	1	1							
2	2	1							
3	3	1							
4	4	1							
5	5	1	1	10	355				
6	6	1	1	8	360				
7	7	1	1	1	352				
8	8	1	1	2	390				

At the bottom of the window, a status bar reads: "Perform a Gage R&R study using the ANOVA or Xbar-R methods".



# ANOVA Method

1. Select Columns for Part, Operator, & Measurement

**Gage R&R Study**

C1	OpSeqNo
C2	Oper(i)
C3	Trial(j)
C4	Part(k)
C5	Xijk

Part numbers: 'Part(k)'  
Operators: 'Oper(i)'  
Measurement data: Xijk

Method of Analysis  
 ANOVA  
 Xbar and R

Buttons: Select, Help, Gage Info..., Options..., OK, Cancel

2. Select the ANOVA method.

3. Select Options.

**Gage R&R Study - Options**

Study variation: 5.15 (number of standard deviations)  
Process tolerance: 20  
Process variation: (6\*historical sigma)

Draw plots on separate pages, one plot per page

Title:

Buttons: Help, OK, Cancel

4. Enter tolerance width.

5. Select OK.



# Analyzing Gage R&R

## Gage R&R Study - ANOVA Method

### ANOVA Table With Operator\* Part Interaction

Source	DF	SS	MS	F	P
Parts	9	16728.1	1858.68	186.698	0.00000
Operators	2	192.4	96.18	9.661	0.00141
<b>Oper*Part</b>	<b>18</b>	<b>179.2</b>	<b>9.96</b>	<b>1.089</b>	<b>0.38509</b>
Repeatability	60	548.7	9.14		
Total	89	17648.3			

**Step 1: Is Operator x Part Interaction significant?**  
*Some Parts read differently by different Operators?*

### ANOVA Table Without Operator\* Part Interaction

Source	DF	SS	MS	F	P
Parts	9	16728.1	1858.68	199.181	0.00E+00
Operators	2	192.4	96.18	10.307	1.07E-04
Repeatability	78	727.9	9.33		
Total	89	17648.3			

**Interaction NOT significant—ANOVA rerun**

**Next Step = Analyze Gage R&R results**



# Step 2: Analyze Gage R&R Results

## Gage R&R Study - ANOVA Method Gage R&R

“Distances” = 99% of observed variation per category (in mils in this case)

**the “Components of Variation”**

Source	VarComp	StdDev	5.15*Sigma
Total Gage R&R	12.23	3.4966	18.0077
Repeatability	9.33	3.0548	15.7321
Reproducibility	2.89	1.7014	8.7624
Operator	2.89	1.7014	8.7624
Part-To-Part	205.48	14.3347	73.8236
Total Variation	217.71	14.7550	75.9881

Source	%Contribution	%Study	Var %Tolerance
Total Gage R&R	5.62	23.70	90.04
Repeatability	4.29	20.70	78.66
Reproducibility	1.33	11.53	43.81
Operator	1.33	11.53	43.81
Part-To-Part	94.38	97.15	369.12
Total Variation	100.00	100.00	379.94

“Study Variation”

Gage R&R as % of given Tolerance Width

Number of Distinct Categories = 6



How do these add??  
—  
by “Sum of Squares”

- What are “good” results?:
- Gage R&R Variation small vs. Total or Study Variation—implies Part-to-Part Variation is major source-of-variation (SOV) in measurement system.
  - Gage R&R Variation small vs. Tolerance Width.

See...  
**RULES of THUMB**  
...(next page)



## *Rules of Thumb*

### **Analyzing Gage R&R Results**

#### **A. R&R% of Tolerance**

1. *R&R less than 10% - Measurement System “acceptable”*
2. *R&R 10% to 30% - May be acceptable - make decision based on classification of Characteristic, Application, Customer Input, etc.*
3. *R&R over 30% - Not acceptable. Find problem, re-visit the Fishbone Diagram, remove Root Causes. Is there a better gage on the market, is it worth the additional cost?*

***DANGER: these rules of thumb may NOT apply when process improvement/ process control is the goal! ALL data should be analyzed statistically and graphically before drawing conclusions!—SEE FOLLOWING***



## *Rules of Thumb*

### **B. % Contribution (or Gage R&R StdDev):**

**GR&R Variance should be “small” compared to Part-to-Part Variance**—applies in cases where Tolerance Width is not meaningful, and %Tolerance is unavailable—such as one sided specs

### **C. Number of Distinct Categories**

A “Signal-to-Noise” Ratio =

$(\text{StdDev}_{\text{parts}}/\text{StdDev}_{\text{GR\&R}}) \times 1.41$  and **rounded**

#### **Guidelines:**

< 2  $\bar{P}$  no value for process control, parts all “look” the same

= 2  $\bar{P}$  can see two groups—high/low, good/bad

= 3  $\bar{P}$  can see three groups—high/mid/low

<sup>3</sup> 4  $\bar{P}$  **acceptable measurement system**  
(higher is better)

### **D. Effective Resolution**

50%, or more, of Xbar Chart outside control limits—implies part variation “exceeds” Measurement System variation



## Step 3: Apply Gage R&R Rules

### Gage R&R Study - ANOVA Method

Source	VarComp	StdDev	5.15*Sigma
Total Gage R&R	12.23	3.4966	18.0077
Repeatability	9.33	3.0548	15.7321
Reproducibility	2.89	1.7014	8.7624
Operator	2.89	1.7014	8.7624
Part-To-Part	205.48	14.3347	75.981
Total Variation	217.71	14.7550	75.981

Rule B

Rule A

Source	%Contribution	%Study Var	%Tolerance
Total Gage R&R	5.62	23.70	90.04
Repeatability	4.29	20.70	78.66
Reproducibility	1.33	11.53	43.81
Operator	1.33	11.53	43.81
Part-To-Part	94.38	97.15	369.12
Total Variation	100.00	100.00	379.94

Number of Distinct Categories = 6

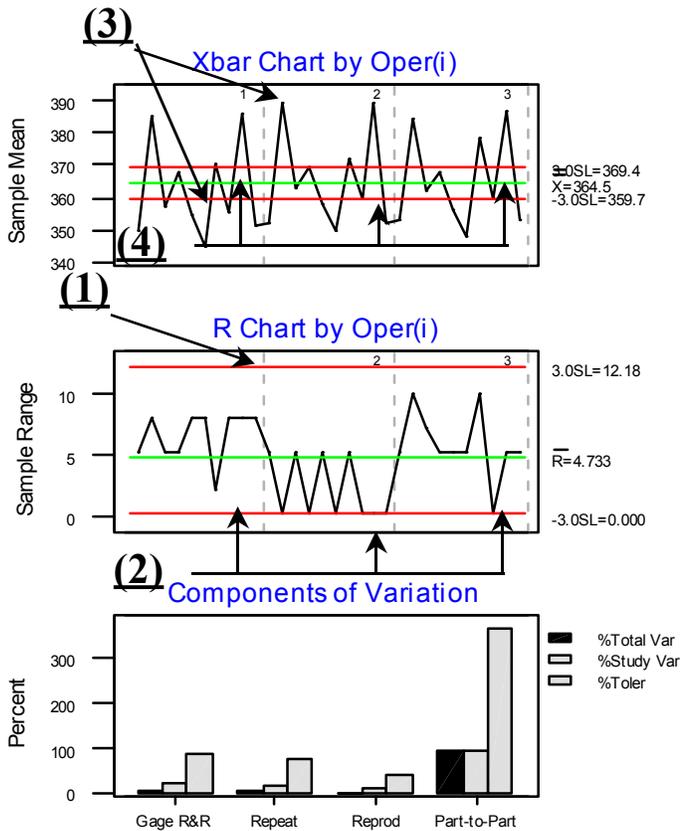
Rule C

Graphics

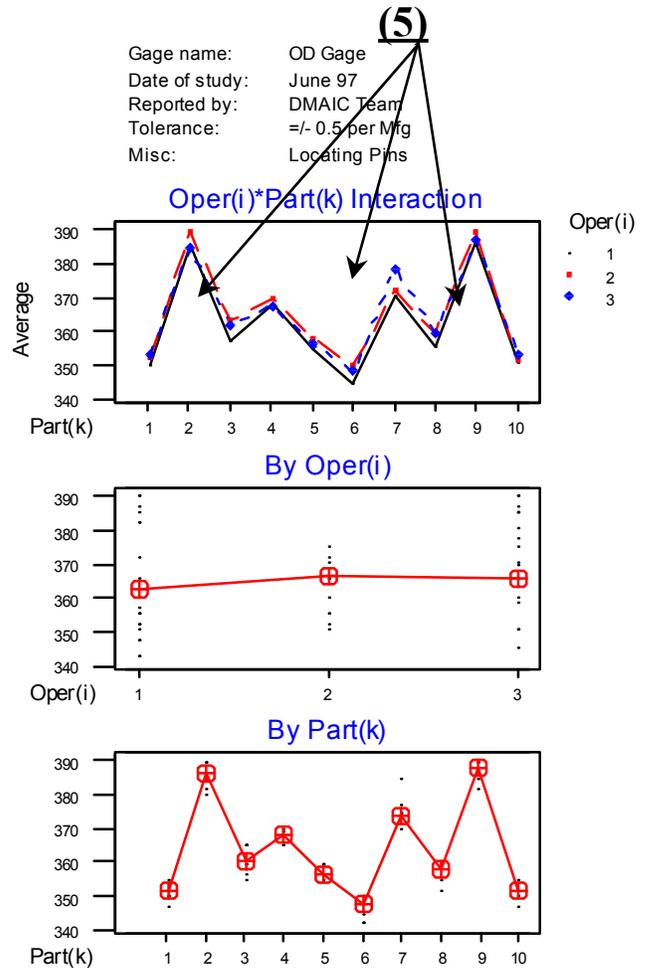


# Gage R&R Graphical Output

Gage R&R (ANOVA) for Xijk



Gage name: OD Gage  
 Date of study: June 97  
 Reported by: DMAIC Team  
 Tolerance: +/- 0.5 per Mfg  
 Misc: Locating Pins



**Conclusion: Gage not OK for given specs, but can "see" part-to-part variation.**



# “Bolt” Gage R&R

## Now let’s look at “Bolt” Gage R&R:

### Steps:

#### 1. Visual “feel” for Gage R&R data:

Box Plots by Operator, Trial, Part  
Graph >Boxplot

#### 2. Run Gage R&R ANOVA Method:

Stat >Quality Tools >Gage R&R Study

#### 3. Analyze Gage R&R Results:

• is Operator x Part Interaction significant? \_\_\_\_\_



• apply Rules of Thumb:

A. Gage R&R% of Tol. = \_\_\_\_\_ [ $<30\%$ ?]

B. %Contribution = \_\_\_\_\_ [Gage R&R vs Part StdDev?]

C. No. Distinct Cat. = \_\_\_\_\_ [ $\geq 4$ ?]

D. Effective Res. = \_\_\_\_\_ [ $\geq 50\%$ ?]

#### 4. Graphical Output:



1) Stability: R Chart \_\_\_\_\_

2) Consistency Within: R Chart pattern for appraisers \_\_\_\_\_

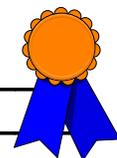
3) Effective Resolution: Xbar Chart -  $> 50\%$  points outside \_\_\_\_\_

4) Consistency Between: Xbar consistency between operators \_\_\_\_\_

5) Systematic Shift: Operator/Part Interaction Plot \_\_\_\_\_

### **CONCLUSIONS—*is Measurement System OK ?***

- for given Tolerance? \_\_\_\_\_
- to see Part-Part variation? \_\_\_\_\_
- comments? \_\_\_\_\_



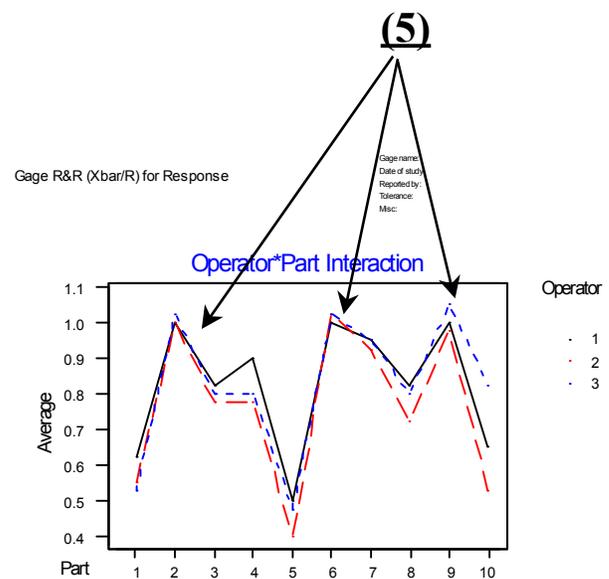
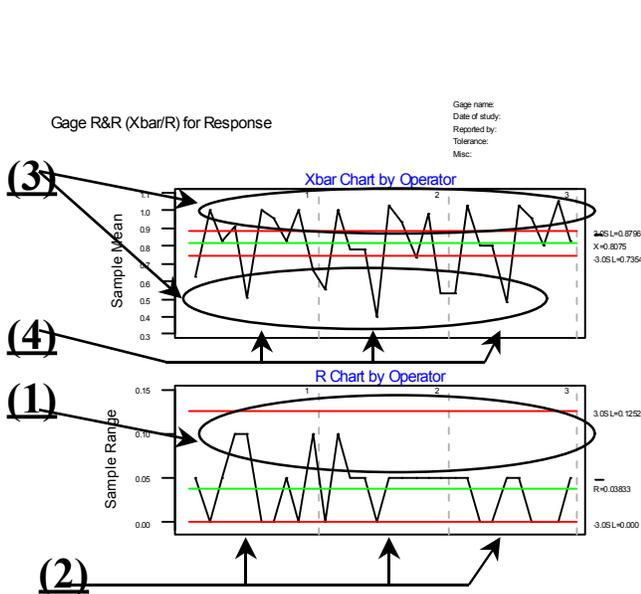
[Hint: make maximum use of graphs and charts to prep reports, support conclusions]



# Example

## Acceptable Gage R&R for both Process Improvement and Product Acceptance:

Source	%Contribution	%Study Var	%Tolerance
Total Gage R&R	6.332	25.164	23.51
Repeatability	3.509	18.733	17.50
Reproducibility	2.823	16.802	15.70
Part-to-Part	93.668	96.782	90.42
Total Variation	100.000	100.000	93.43
Number of distinct categories =	5		





## *Review*

---

### **Best Practices:**

- *Prepare data entry form = 6 columns*
- *Randomize readings*
- *Use graphs and charts to analyze*
- *ANOVA = “best” study method*
- *Base conclusions on more than Gage R&R% of Tolerance alone*





## Review

### Rules of Thumb

#### **A. R&R% of Tolerance**

1. R&R less than 10% = “acceptable”
2. R&R 10% to 30% = May be acceptable
3. R&R over 30% - **Not acceptable**

**B. % Contribution (or Gage R&R StdDev) = “small”**  
compared to Part-to-Part Variance—cases where  
Tolerance Width is not meaningful, and %Tolerance is  
unavailable—one sided specs.

#### **C. Number of Distinct Categories**

- < 2  $\bar{P}$  no value for process control, parts all “look”  
the same
- = 2  $\bar{P}$  can see two groups—high/low, good/bad
- = 3  $\bar{P}$  can see three groups—high/mid/low
- <sup>3</sup> 4  $\bar{P}$  **acceptable measurement system**  
(higher is better)

**D. Effective Resolution = Xbar Chart > 50% outside  
control limits**



# *Attribute Gage R & R*



## *Analyzing Gage R&R with Attribute Data: AR&R's*



- The long form method of analysis used Minitab and Anova to determine the adequacy of continuous data collected on the measurement system.
- If the data to be collected can only be classified as attribute, a slightly modified analysis method must be used.
- The objectives are unchanged, as it is still desirable to determine the adequacy of the measurement system, and whether to focus on equipment, appraiser, or both for improvement purposes.



# AR&R Data Sheet

AR&R Data										
Sample	'True'	Operator 1			Operator 2			Operator 3		
	Answer	Trial1	Trial2	Trial3	Trial1	Trial2	Trial3	Trial1	Trial2	Trial3
1	N	N	N	N	N	N	N	N	N	N
2	N	N	N	N	N	N	N	N	N	N
3	N	N	N	N	N	N	N	D	N	N
4	D	D	D	D	D	D	D	D	D	D
5	D	D	D	D	D	D	D	D	N	D
6	N	N	N	N	N	N	N	N	N	N
7	D	N	D	N	D	D	D	D	D	D
8	N	N	N	N	D	N	D	N	N	N
9	N	N	N	N	N	N	N	N	N	N
10	N	N	N	N	N	N	N	D	N	D
11	D	D	D	D	D	D	D	D	D	D
12	D	N	N	N	D	D	D	D	D	D
13	D	D	D	D	D	D	D	D	D	D
14	N	N	N	N	N	N	N	N	N	N
15	D	D	D	D	D	D	D	D	D	D
16	N	D	D	D	N	N	N	N	N	N
17	N	N	N	N	N	N	N	N	N	N
18	N	N	N	N	N	N	N	N	N	N
19	N	N	N	N	D	D	D	N	N	N
20	N	N	N	N	N	N	N	D	D	N
21	D	D	D	D	D	D	D	D	D	N
22	N	N	N	N	D	D	D	N	N	N
23	N	N	N	N	D	D	D	N	N	N
24	N	N	N	N	N	N	N	D	D	D
25	N	N	N	N	N	N	N	N	N	N
26	D	D	D	D	D	D	D	D	D	D
27	N	N	N	N	N	N	N	N	N	N
28	N	N	N	N	N	N	N	N	N	N
29	N	N	N	N	N	N	N	N	N	N
30	D	N	D	N	D	D	D	D	D	D
31	D	D	D	D	D	D	D	D	D	D
32	N	N	N	N	N	N	N	N	N	N
33	N	D	N	N	D	D	N	N	N	N
34	N	N	N	N	N	N	N	N	N	N
35	N	N	N	N	N	N	N	N	D	N
36	D	D	D	D	D	D	D	D	D	D
37	N	N	N	N	N	N	N	D	N	N
38	N	N	N	N	N	N	N	N	N	D
39	N	N	N	N	N	N	N	N	N	N
40	N	N	N	N	D	D	D	N	N	N



# AR&R Example



Sample	Number of Defects									Total
	Trial1	Trial2	Trial3	Trial1	Trial2	Trial3	Trial1	Trial2	Trial3	
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	1	0	0	1
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	1	0	1
6	0	0	0	0	0	0	0	0	0	0
7	1	0	1	0	0	0	0	0	0	2
8	0	0	0	1	0	1	0	0	0	2
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	1	0	1	2
11	0	0	0	0	0	0	0	0	0	0
12	1	1	1	0	0	0	0	0	0	3
13	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0
16	1	1	1	0	0	0	0	0	0	3
17	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0
19	0	0	0	1	1	1	0	0	0	3
20	0	0	0	0	0	0	1	1	0	2
21	0	0	0	0	0	0	0	0	1	1
22	0	0	0	1	1	1	0	0	0	3
23	0	0	0	1	1	1	0	0	0	3
24	0	0	0	0	0	0	1	1	1	3
25	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0
30	1	0	1	0	0	0	0	0	0	2
31	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0
33	1	0	0	1	1	0	0	0	0	3
34	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	1	0	1
36	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	1	0	0	1
38	0	0	0	0	0	0	0	0	1	1
39	0	0	0	0	0	0	0	0	0	0
40	0	0	0	1	1	1	0	0	0	3
# of N	5	2	4	6	5	5	5	4	4	40



# AR&R Analysis Example



AR&R Measurement System				
Input Section				
Operators	Discrepancies	No. of Samples	No. of Repeats	Total_Opps
Oper 1	11	40	3	120
Oper 2	16	40	3	120
Oper 3	13	40	3	120
Total	40	120	n/a	360

			Confidence Limits	
			on Zmeasure	
Output Section			alpha= 0.05	
ppm	Yield	Zmeas	lower	upper
91667	90.83%	1.33	1.00	1.68
133333	86.67%	1.11	0.82	1.42
108333	89.17%	1.24	0.92	1.56
111111	88.89%	1.22	1.04	1.40

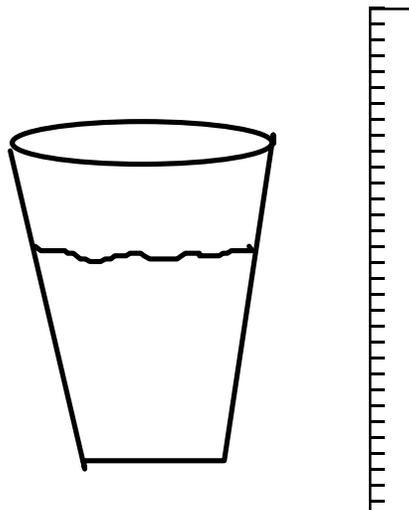


## *GR&R Break Out-Optional Exercise*

Your team will complete a measurement study using MINITAB gage R&R. You have five cups of water (parts), a ruler (the gage) and three operators.

Decide the role of each team member. Conduct the measurement study and analyze the results.

Report out gage R&R, AV, EV values and interpretations.

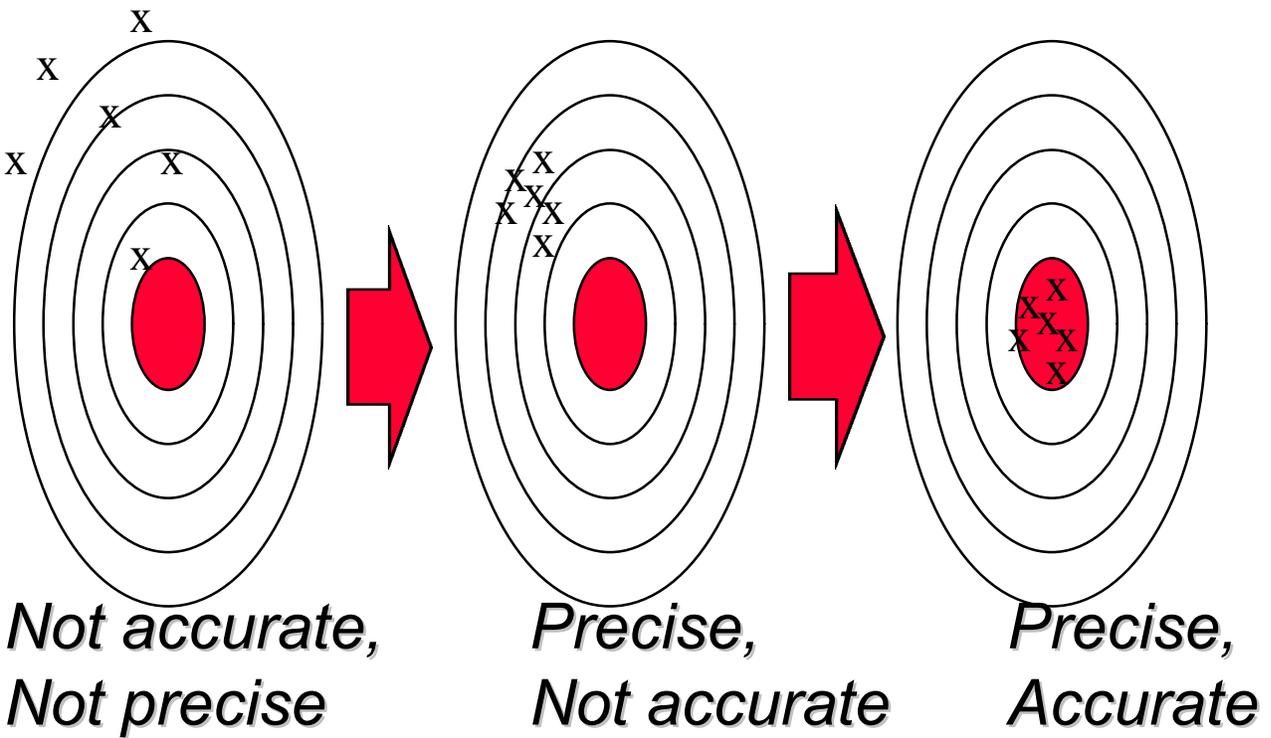




# *Calibration Standards*



# Gage Improvement Roadmap



Decrease  
Variation  
through:  
•6-Sigma  
DMAIC  
•Replication

Re-center  
through:  
•Calibration



# *Destructive Gage R&R*



# *Temporal Measurement System Analysis*

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## **Objective:**

*Understand the limitations that destructive and temporal elements place on continuous data Gage R&R analyses*



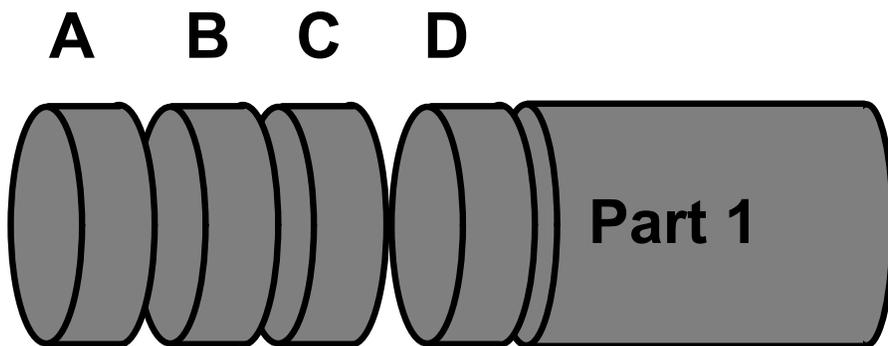
## *Destructive Testing*

- Some tests (torque, yield strength, tensile, modulus, elongation, hardness, rheometer, etc.) are destructive tests.
- The part cannot be measured by more than one operator because it is destroyed.
- Parts should be selected to minimize within “part” variability by taking one, homogeneous part and dividing it into sections.
- The main part is what the ANOVA table will call a “part” and the subsections are available for multiple observations.



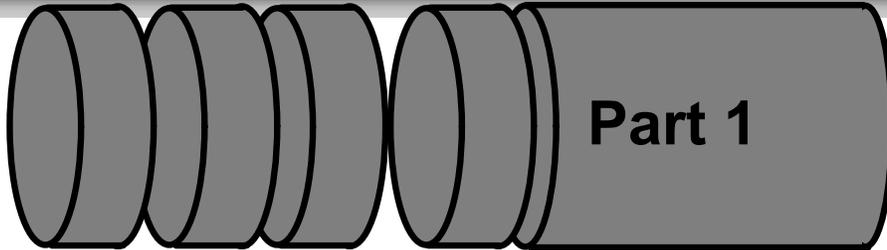
## Exercise

- Each operator measures subsections of the the same overall part
- Since each operator measures a *portion* of the same part, the parts must be as homogeneous as possible.
- ANOVA may be is used to analyze data from a destructive test GR&R Study.
- It is helpful to label parts with numbers and subsections with letters to help avoid confusion.

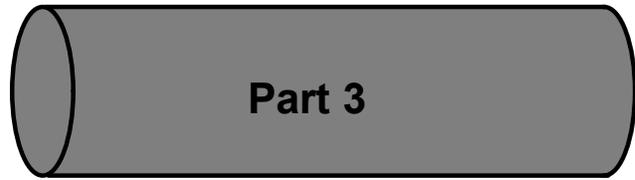
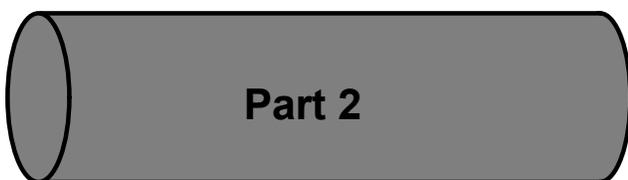




## *Exercise*



Within part variation



Between part variation



## *Temporal Effects*



- Gage R&R depends upon our ability to observe the same part or event multiple times.
- If the item measured is an *event*, it may not happen the same way twice. For this reason, Gage R&R may be impossible.
- If the event is recorded via video or audio tape, it may be possible to conduct a Gage R&R study.
- If the event cannot be recorded, but multiple judges can observe at once, reproducibility can be estimated, but repeatability cannot be.



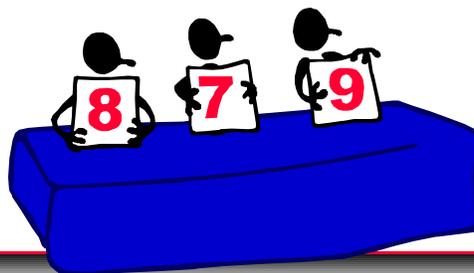
## *Temporal Effects*



### **Example**

Many Olympic sports are judged based on a numerical score sheet filled out by a judge. The difference among different judge's readings of one, live event is a measure of reproducibility.

For this same case, repeatability can be estimated only if the competition is taped. A random sample of 10 performances, shown to five judges two times each would allow estimates of both repeatability and reproducibility





## *Temporal Effects*



### **Example**

Call centers field service calls from around the world at centralized locations. Call duration and call quality are both recorded and tracked very closely.

Call quality is calculated based on a scoring sheet filled out by a quality monitor. Calls are scored on a continuous scale from 1 - 100 and can be assumed to be continuous for the purposes of a Gage R&R calculation.



## *Temporal Effects*

### **Example, continued**

In one monitoring scenario, monitors listen in live and score the calls. For a live listen it is impossible to calculate repeatability, but if two monitors listen at the same time, it is possible to calculate reproducibility.

In another monitoring scenario, calls are recorded at random and scored at a later date. For this recorded scenario, both repeatability and reproducibility may be calculated.





## *Summary: Important Questions To Ask*

- *Are we capturing the correct data? Does the data reflect what is happening in the process?*
- *How big is the measurement error?*
- *Can we detect process improvement if and when it happens?*
- *What are the sources of measurement error?*
- *Are the measurements being made with measurement units which are small enough to properly reflect the variation present?*
- *Is the Measurement System stable over time?*
- *Is the Measurement System “capable” for this study?*
- *How much uncertainty should be attached to a measurement when interpreting it?*
- *How do we improve the measurement system?*
- *Is the data attribute or not?*
- *Do we need to complete calibration standards ?*
- *Do we need to complete a destructive G R & R ?*



## *Take Aways—Step 3*

- *Variation in the measurement system will contribute to the observed variation in a process.*
  
- *Sources of variation in a measurement system are:*
  - *gage*
  - *operator*
  - *environment*
  
- *The resolution is the ability of the gage to see the variation in the process.*
  - *The gage scale should divide the tolerance into at least ten parts*
  - *The gage should be accurate: mean close to the true mean of the process, and precise: small variation*



## *Take Aways—Step 3*

- *The MSA Checklist, Test-Retest Study, and Gage Reproducibility and Repeatability study are used to validate a measurement system.*
  
- *The key word in the Test-Retest Study is “same.”*
  - *The **same** operator should use the **same** gage to measure the **same** specimen repeatedly*
  - *The calculated standard deviation from the repeated measurements should not be greater than ten percent of the tolerance width*
  
- *The Gage Reproducibility and Repeatability Study is a more detailed study of the measurement system.*
  - *repeatability (equipment variation or EV)*
  - *reproducibility (appraiser variation or AV)*



## *Take Aways - Step 3*

---

- *Short form Gage R and R provides a quick way of determining acceptability of gage variations.*
- *However, short form Gage R & R does not provide a way of separating gage repeatability and reproducibility.*
- *The ANOVA method of Gage R & R is the better method to determine this since, it separates interaction effects, better determines causality, and aids in the variability reduction of continuous data.*
- *Attribute/ Discrete data requires the use of the attribute R & R tool.*
- *Destructive Gage R & R - Avoid it! Is there a better way to turn a destructive test into a non-destructive gage R&R study?*
  - *Can the part be divided up so that each piece is identical and so allow you to capture a true replicate?*



## *Take Aways-Step 3*

- Can you test a different, but similar part that won't be destroyed?*
- Is there a different gage that won't destroy the part yet gives similar analysis?*
- If you must conduct Destructive Gage R&R...remember that sample variability gets confounded (combined) with operator and the gage. Therefore minimize sample variability as much as possible!*



## *Measure Deliverables*

---

- *Identify the Measurable Customer CTQ*
- *Define and Confirm Specifications for the Y*
- *Ensure Measurement System is Adequate to Measure Y*



---

# *Introduction to the Analyze Phase*

- *Using Statistics to Solve Problems*
- *The 12 Step Process*



# The 12 Step Process

Step	Description	Focus	Tools	SSQC Deliverables
<b>Define</b>				
A	Identify Project CTQs			Project CTQs (1)
B	Develop Team Charter			Approved Charter (2)
C	Define Process Map			High Level Process Map (3)
<b>Measure</b>				
1	Select CTQ Characteristics	Y	Customer, QFD, FMEA	Project Y (4)
2	Define Performance Standards	Y	Customer, Blueprints	Performance Standard for Project Y (5)
3	Measurement System Analysis	Y	Continuous Gage R&R, Test/Retest, Attribute R&R	Data Collection Plan & MSA (6), Data for Project Y (7)
<b>Analyze</b>				
4	Establish Process Capability	Y	Capability Indices	Process Capability for Project Y (8)
5	Define Performance Objectives	Y	Team, Benchmarking	Improvement Goal for Project Y (9)
6	Identify Variation Sources	X	Process Analysis, Graphical Analysis, Hypothesis Tests	Prioritized List of all Xs (10)
<b>Improve</b>				
7	Screen Potential Causes	X	DOE-Screening	List of Vital Few Xs (11)
8	Discover Variable Relationships	X	Factorial Designs	Proposed Solution (13)
9	Establish Operating Tolerances	Y, X	Simulation	Piloted Solution (14)
<b>Control</b>				
10	Define & Validate Measurement System on X's in Actual Application	Y, X	Continuous Gage R&R, Test/Retest, Attribute R&R	MSA
11	Determine Process Capability	Y, X	Capability Indices	Process Capability Y, X (15)
12	Implement Process Control	X	Control Charts, Mistake Proof, FMEA	Sustained Solution (15), Documentation (16),



# Analyze Phase



**4. Establish Process Capability**

**5. Define Performance Objectives**

**6. Identify Variation Sources**

*Deliverable: Baseline Current Process*

*Tools:*

- *Basic statistics*
- *Graphical Analysis*
- *Sampling*
- *Continuous Zst, ZIt*
- *Normality*
- *Discrete Zst, ZIt*

*Deliverable: Statistically Define the Goal of the Project*

*Tools:*

- *Benchmarking*

*Deliverable: List of Statistically Significant X's, Chosen Based on Analysis of Historical Data*

*Tools:*

- *Process Analysis*
- *Graphical Analysis*
- *Hypothesis Testing*
- *Regression Analysis*
- *GLM*



## *Objectives*

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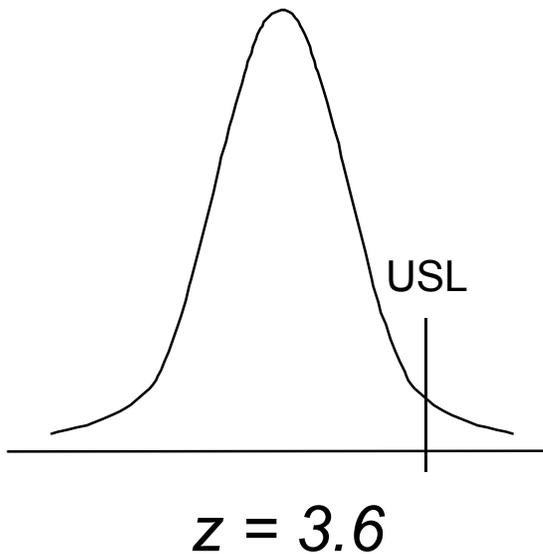
- *At the end of this section the student will be able to:*
  - *establish the capability of the process*
  - *establish an improvement goal—the performance objective*
  - *study the stability, shape, center, and spread of the process*
  - *determine the vital Xs that impact the project Y*
  - *make recommendations for the Improve phase*



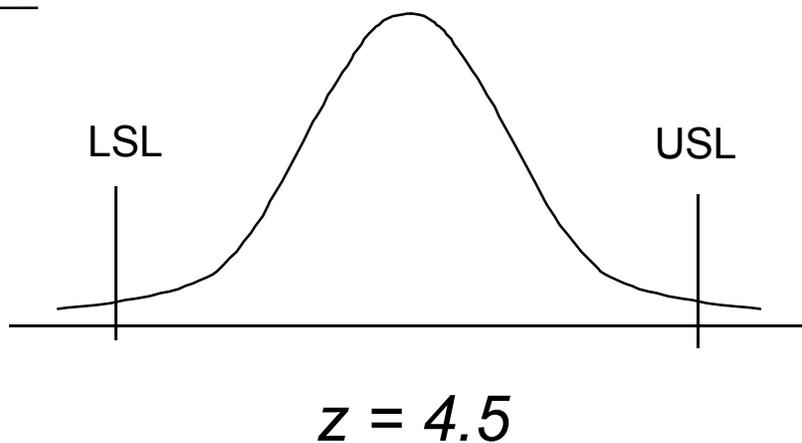
## Capability

- Express the capability of a process in terms of a standard measure.

Process A



Process B

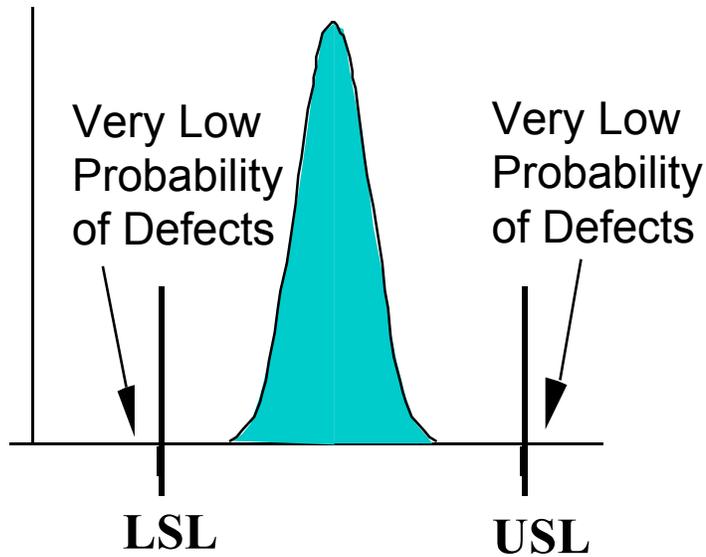


**Is process B more capable than Process A?**

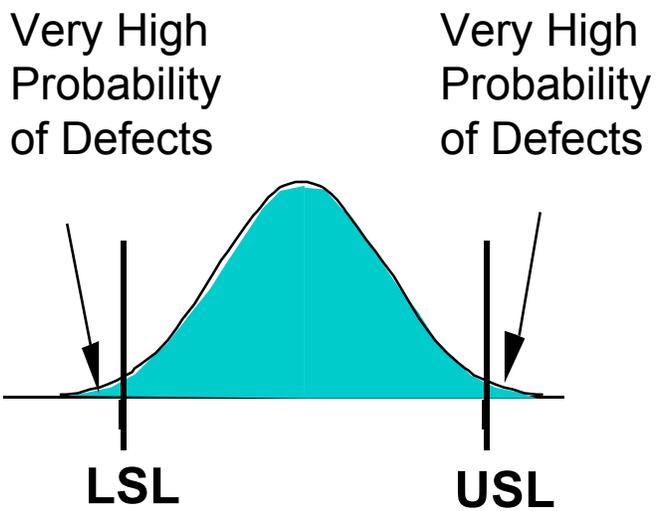


# Process Capability

**Excellent Process Capability**



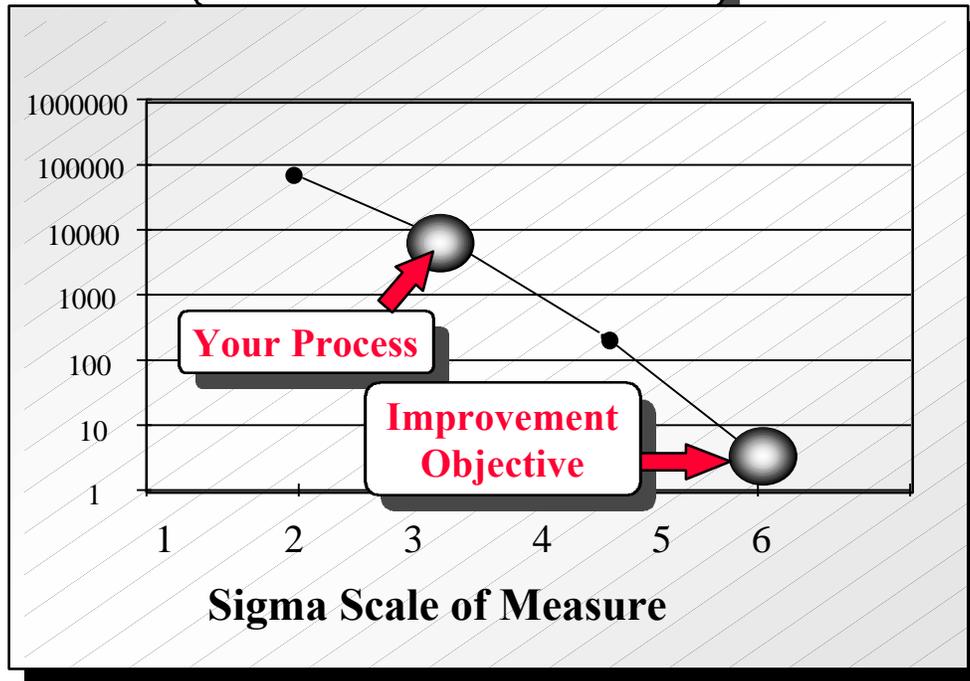
**Poor Process Capability**



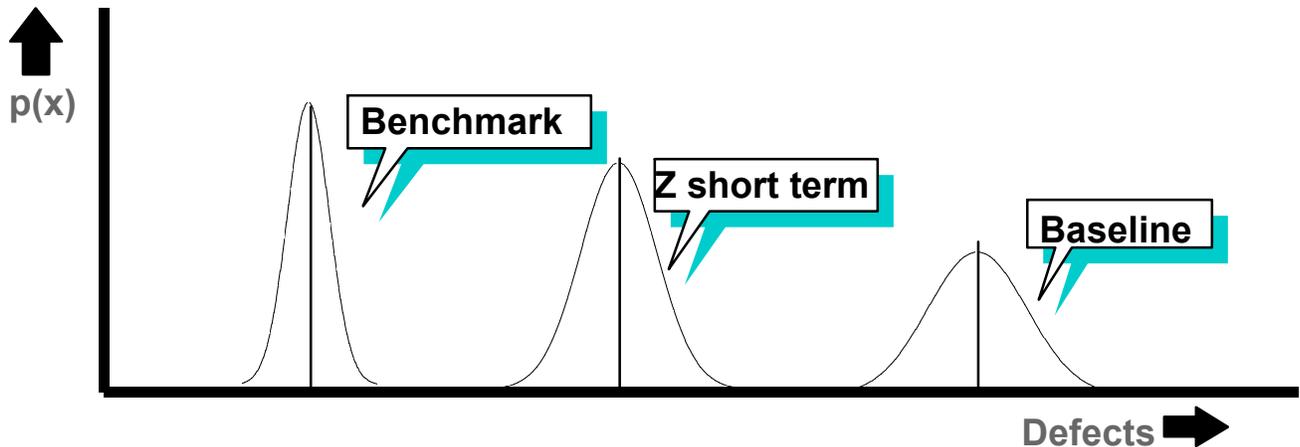


# The Performance Objective

## The Basic Objective



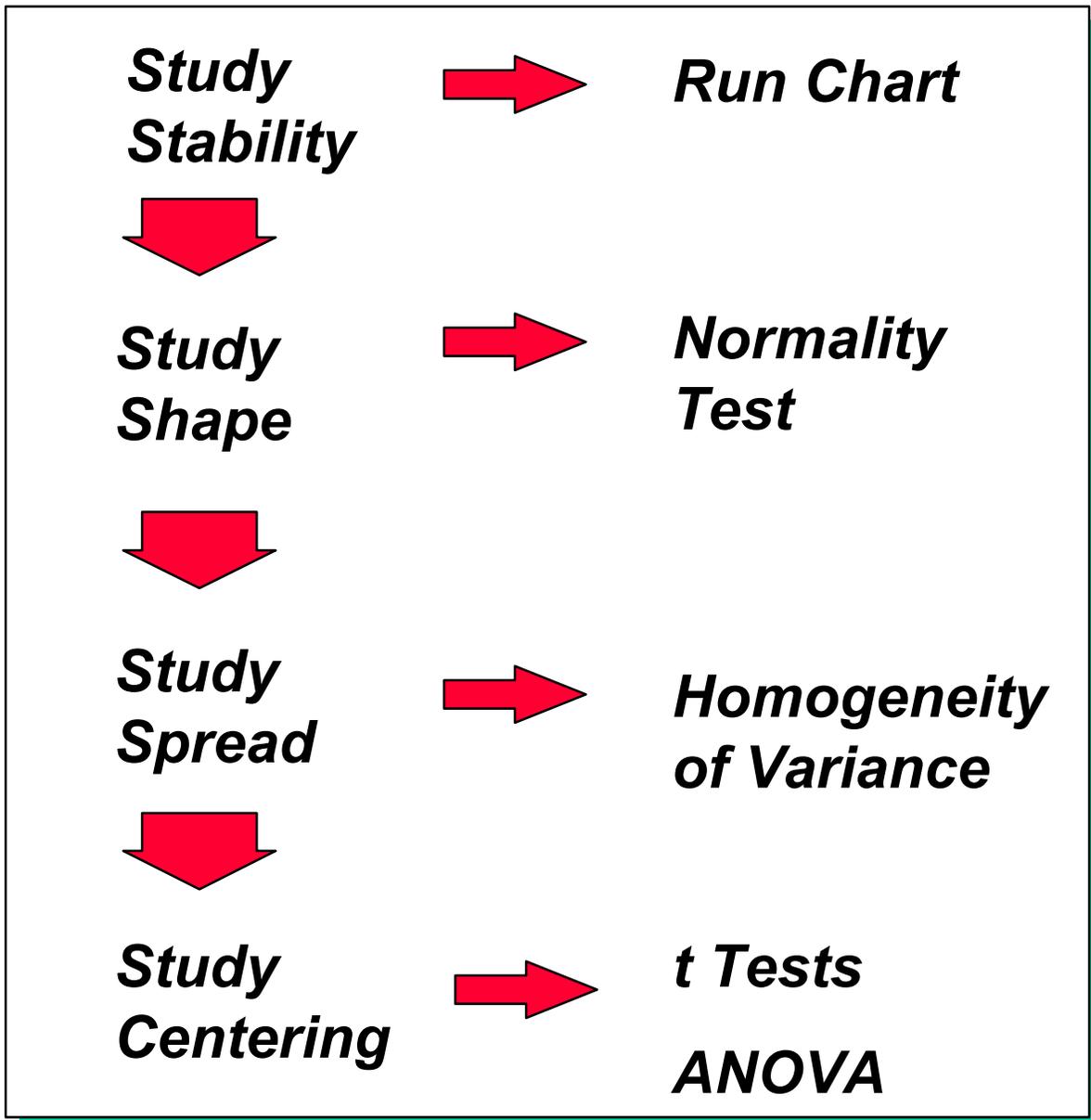
- Depending on your current baseline and process entitlement, set realistic and achievable improvement goals.





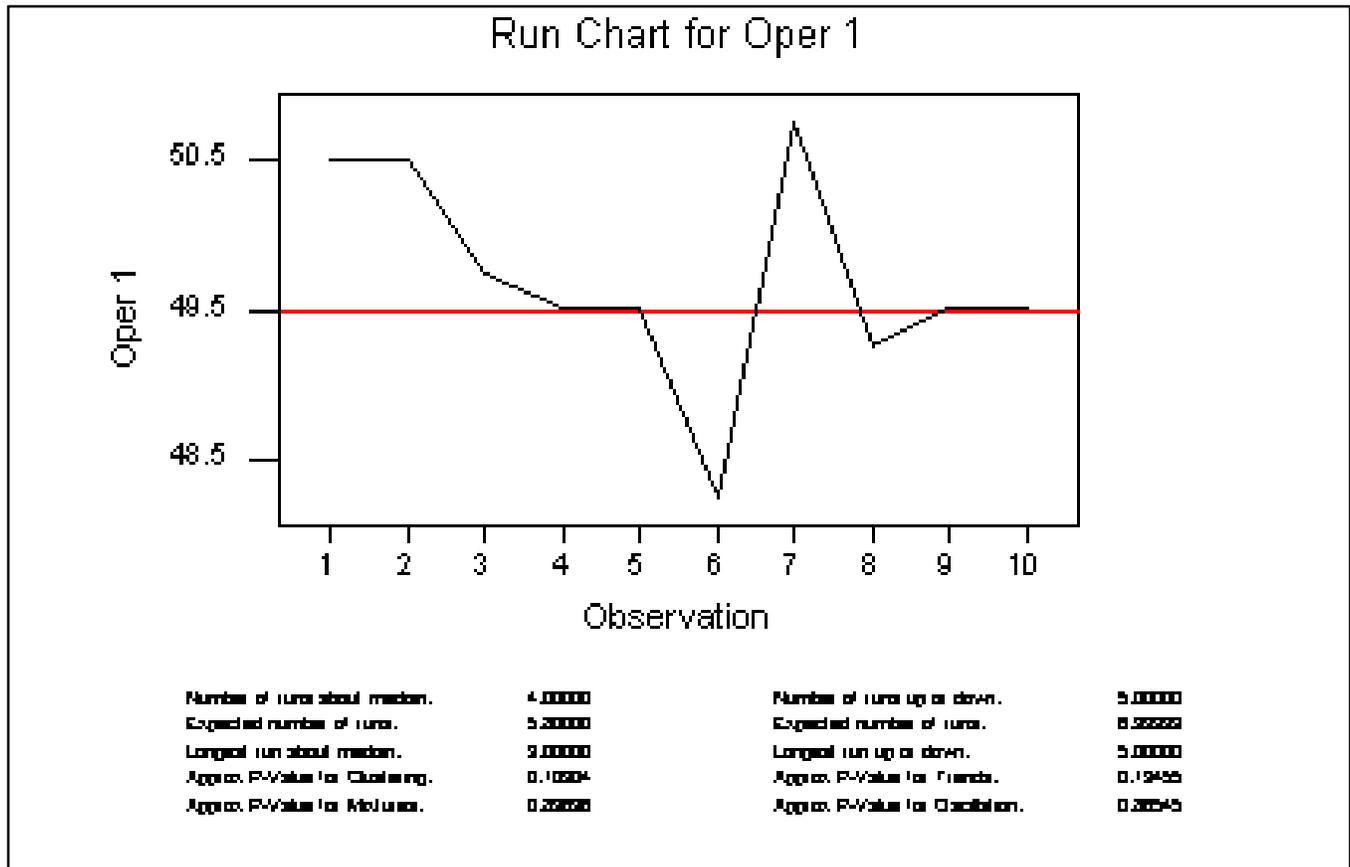
## *Understanding our Process*

- *To understand our current process, analyze the data.*





## Study Stability

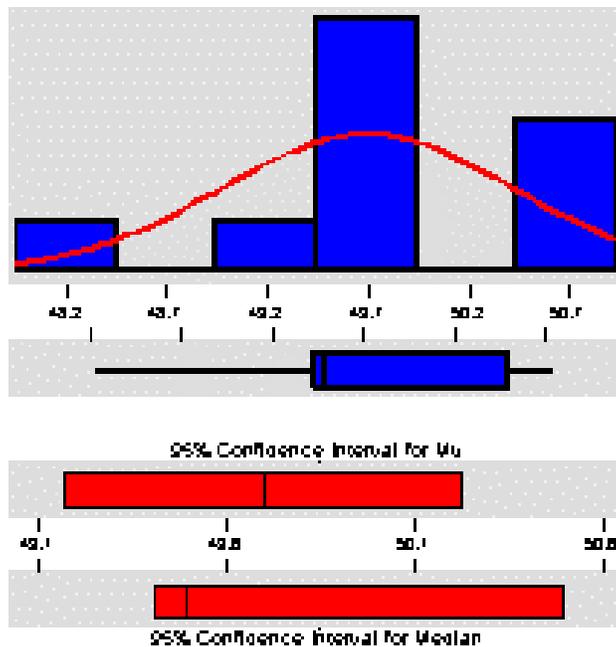


- *Understand your current process*
- *Try to find special causes of variation*
- *Look at the data over time: run chart*
- *How stable is your process?*



# Study Shape

## Descriptive Statistics



Variable: Oper 1

### Anderson-Darling Normality Test

A-Squared: 0.566  
P-Value: 0.113

Mean 49.7000  
StDev 0.7341  
Variance 0.53889  
Skewness -1.5E-01  
Kurtosis 0.547827  
n 10

Minimum 48.2500  
1st Quartile 49.4375  
Median 49.5000  
3rd Quartile 50.5000  
Maximum 50.7500

95% Confidence Interval for  $\mu$   
49.1749 50.2251

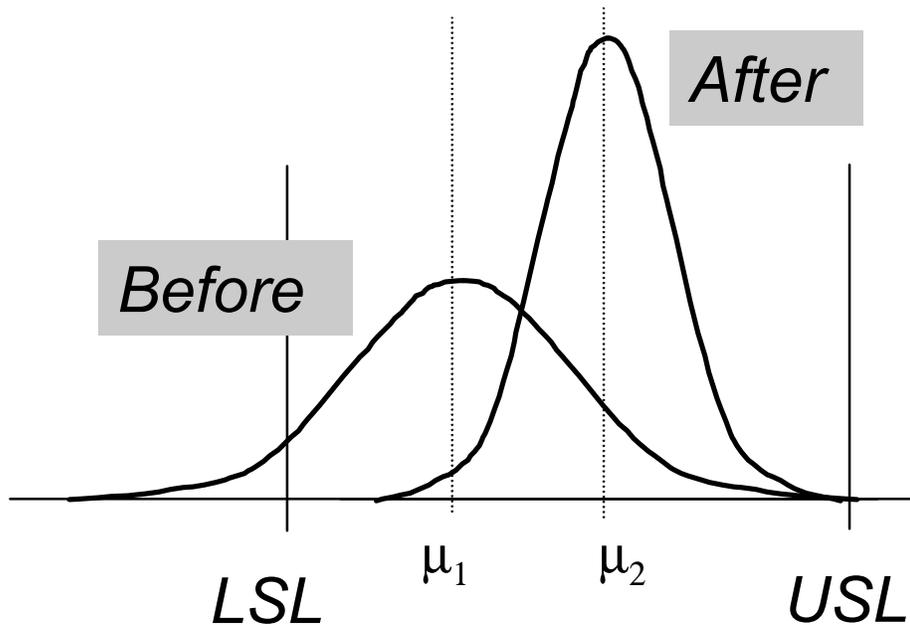
95% Confidence Interval for  $\sigma$   
0.5049 1.3402

95% Confidence Interval for Median  
49.4144 50.5000

- *Determine the mean and variation of your process.*
  - *what special causes may be leading to these values?*
- *Determine if the data is normally distributed*
  - *if no, why not?*



## *Study Center & Spread*

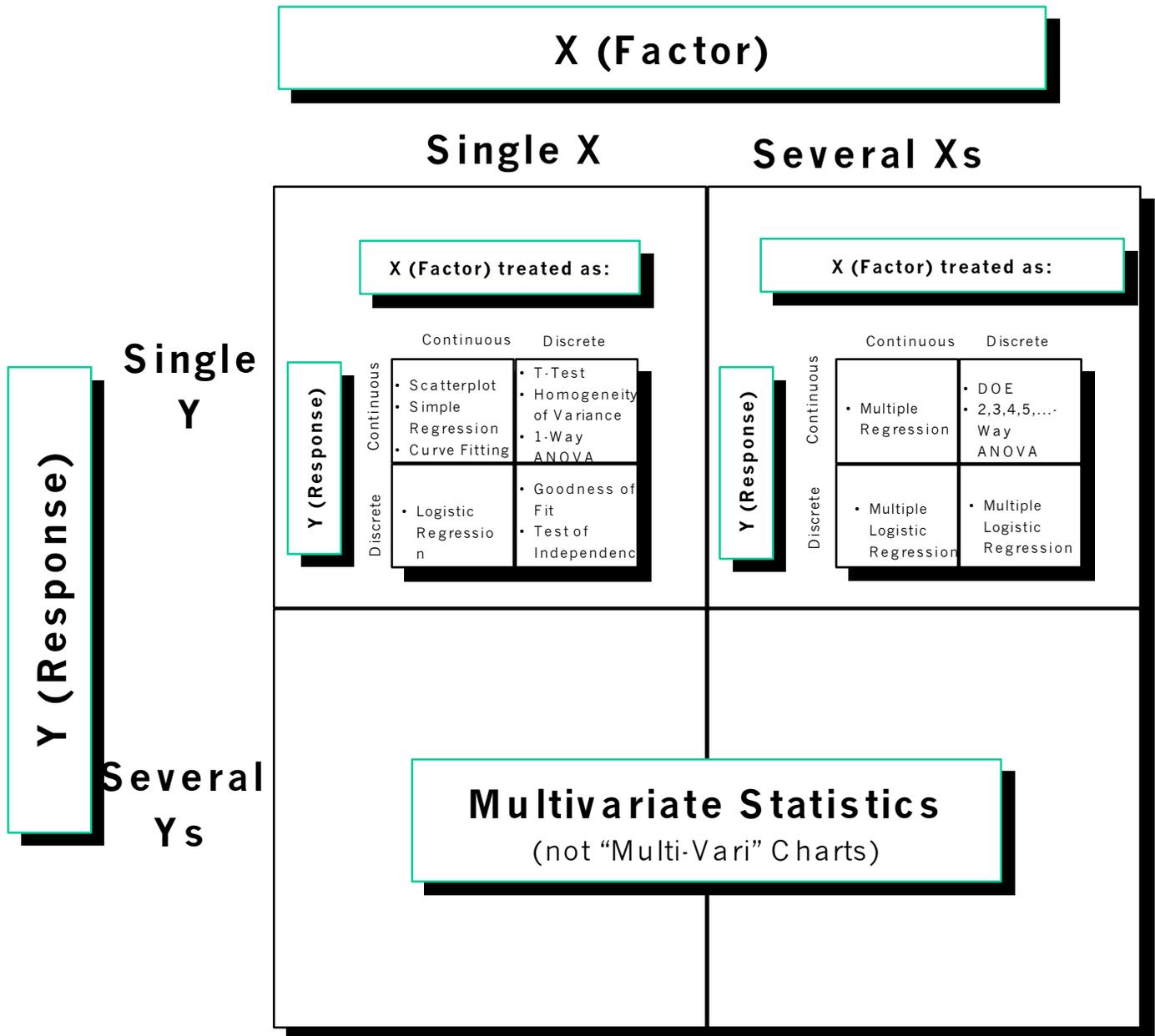


- *Changing one or more vital Xs in the process may affect the mean and the variation of the process Y.*
- *We can statistically show that the mean and the variation of the process has shifted by using hypothesis tests.*

*Goal: To reduce variation and get process on target*



# Analyzing the Data



*The correct tool depends upon the data*



## *Analyze Deliverables*

- *Review defect definition, target, and LSL & USL*
- *Establish Baseline Process Capability*
- *Is there a statistically significant difference between the Xs?*
- *Review results with team members*
- *Establish Financial Baseline*

### ***Discrete***

- *L1 Spreadsheet or Minitab Product Report*
- *Chi-square Test*

### ***Continuous***

- *Minitab Process Capability*
- *Normality Test*
- *2 sample t-test or ANOVA*
- *Homogeneity of Variance Tests*
- *Regression Analysis*



# Analyze Phase



**4. Establish Process Capability**

*Deliverable: Baseline Current Process*

*Tools:*

- Basic statistics
- Graphical Analysis
- Sampling
- Continuous Zst, Zlt
- Normality
- Discrete Zst, Zlt

**5. Define Performance Objectives**

*Deliverable: Statistically Define the Goal of the Project*

*Tools:*

- Benchmarking

**6. Identify Variation Sources**

*Deliverable: List of Statistically Significant X's, Chosen Based on Analysis of Historical Data*

*Tools:*

- Process Analysis
- Graphical Analysis
- Hypothesis Testing
- Regression Analysis
- GLM



# *Establish Process Capability Objectives*

---

## **Step 4: Establish Process Capability**

*By the end of Step 4, the BB/GB will have:*

- a. Calculated the baseline capability of the process using either continuous or discrete data.*
- b. Conducted a normality test to verify the data is normally distributed.*



## *Continuous Data Objectives*

---

- *By the end of the training program, the participant will be able to:*
  - *Use continuous data to describe their process by its average, standard deviation and normal curve*
  - *Understand the relevance of specifications created by product/process designers (target values, upper and lower specification limits)*
  - *Apply statistical principles of the Standard Normal Probability Distribution to predict the probability of a defect and process capability*
  - *Perform basic functions of Minitab: navigate within the menus, calculate 6s statistics, produce 6s reports, and load data files*



## *Statistics*

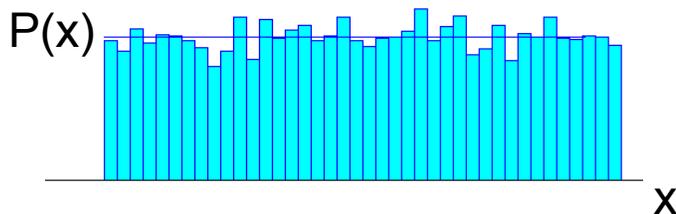
- *Statistics is concerned with making inferences about general populations and about characteristics of general populations.*
- *We study outcomes of random experiments.*
- *If a particular outcome is not known in advance, then we do not know the exact value assigned to the variable of that outcome.*
  - *the number of invoices received weekly*
  - *the cost in dollars of reworking each part*
  - *the number of surfaces that are rough on a cast part*
  - *the number of calls received every Monday between the hours of 8-9 a.m.*
- *We call such a value a **random variable**.*



## *Some Distributions*

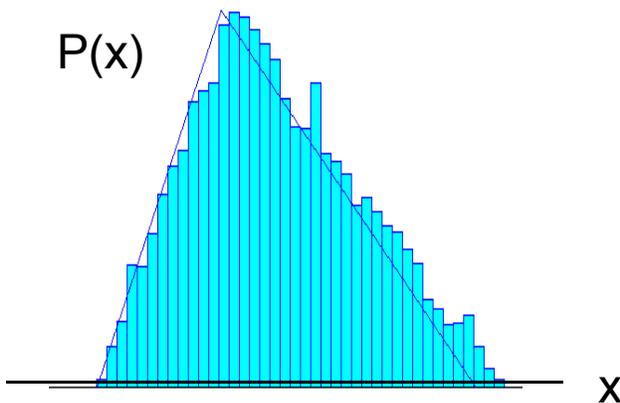
- *A random variable can be expressed in terms of a distribution.*

### *Uniform Distribution*



*All permissible values are equally likely*

### *Triangular Distribution*

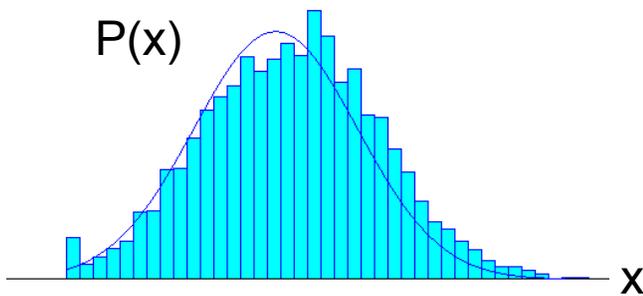


*Approximates many processes and may take on shape of many distributions*



# Distributions

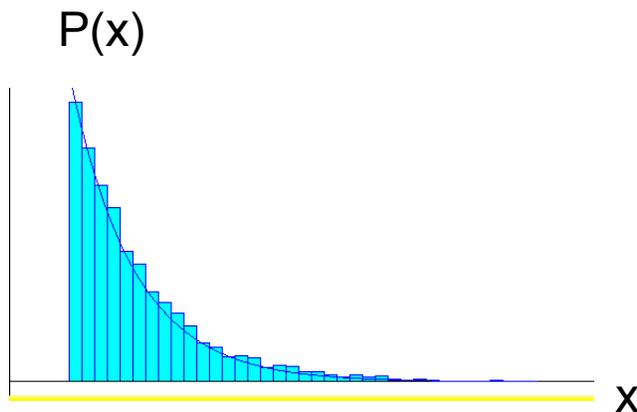
## Normal Distribution



*Process/repair times*

*Error fluctuations about an operating point*

## Exponential Distribution



*Time between Arrivals*

*Time between random (unrelated) failures*

*Events with no memory from one to the next*



## *Generating Distributions*

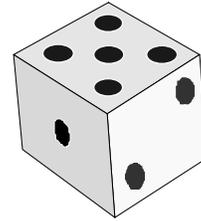
- *Distributions are the result of random events.*
- *Examine probabilities*
- *Probability that an event will occur:*

$$\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$



## *Probability - A Fair Die*

### *Example 1*



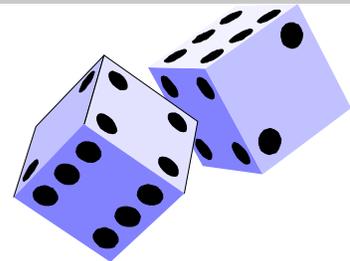
*Rolling one Fair DIE:*

- *What's the probability of rolling a 3 ...  
 $P(X=3)$ ?*
- *What's the probability of rolling an even  
number ...  $P(X \text{ is even})$ ?*
- *What's the probability of rolling a number  $> 4$   
...  $P(X > 4)$ ?*
- *What's the probability of rolling a number  
 $\geq 4$  ...  $P(X \geq 4)$ ?*



## *Probability - Two Fair Dice*

### *Example 2*



*Rolling TWO Fair DICE:*

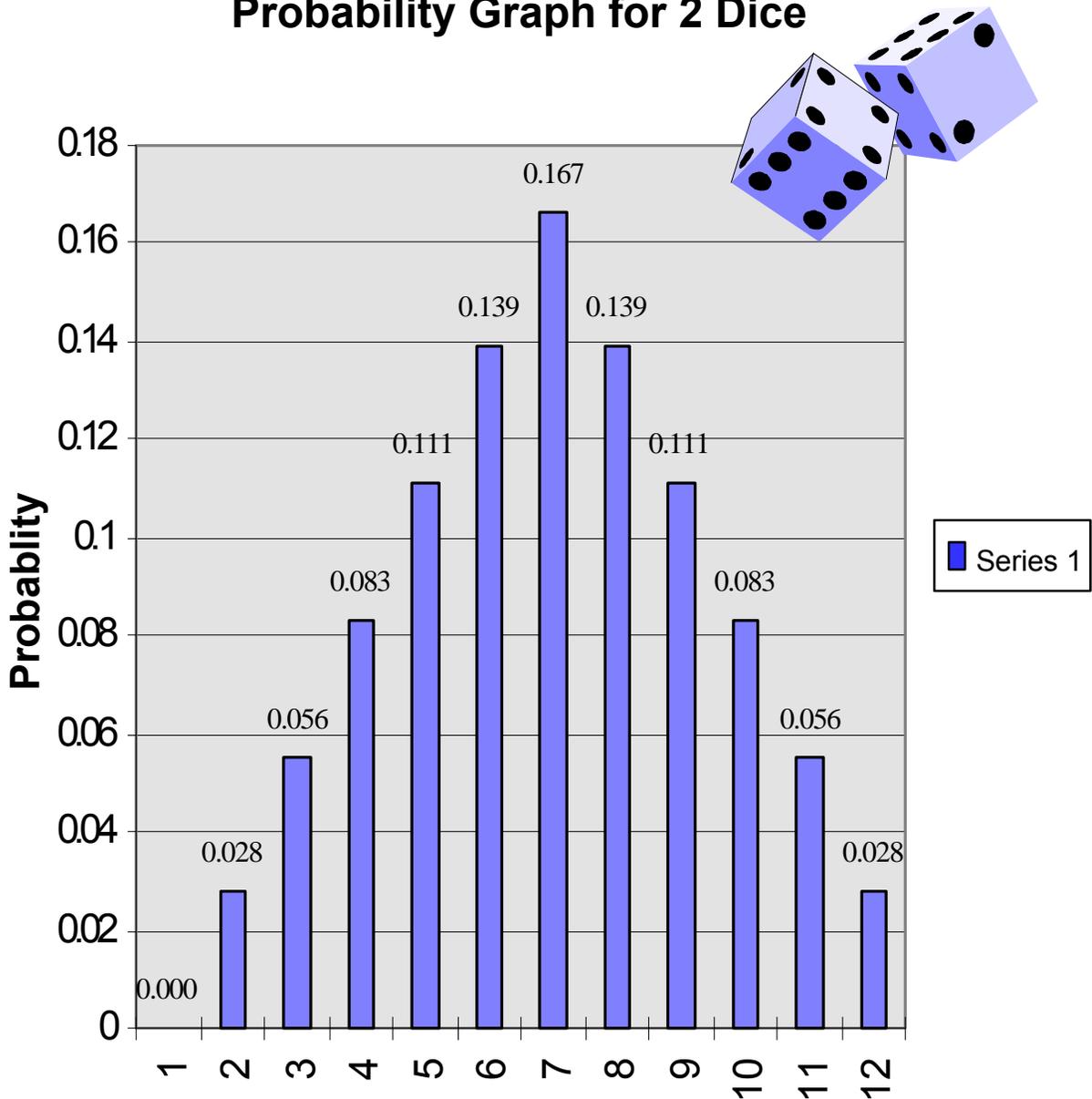
- *What's the probability of rolling a 3 ...  $P(x=3)$ ?*
- *What's the probability of rolling an even number ...  $P(x \text{ is even})$ ?*
- *What's the probability of rolling a number  $>4$ ...  $P(X>4)$ ?*
- *What's the probability of rolling a number  $\geq 4$  ...  $P(X\geq 4)$ ?*





# Probability Curve - Two Fair Dice

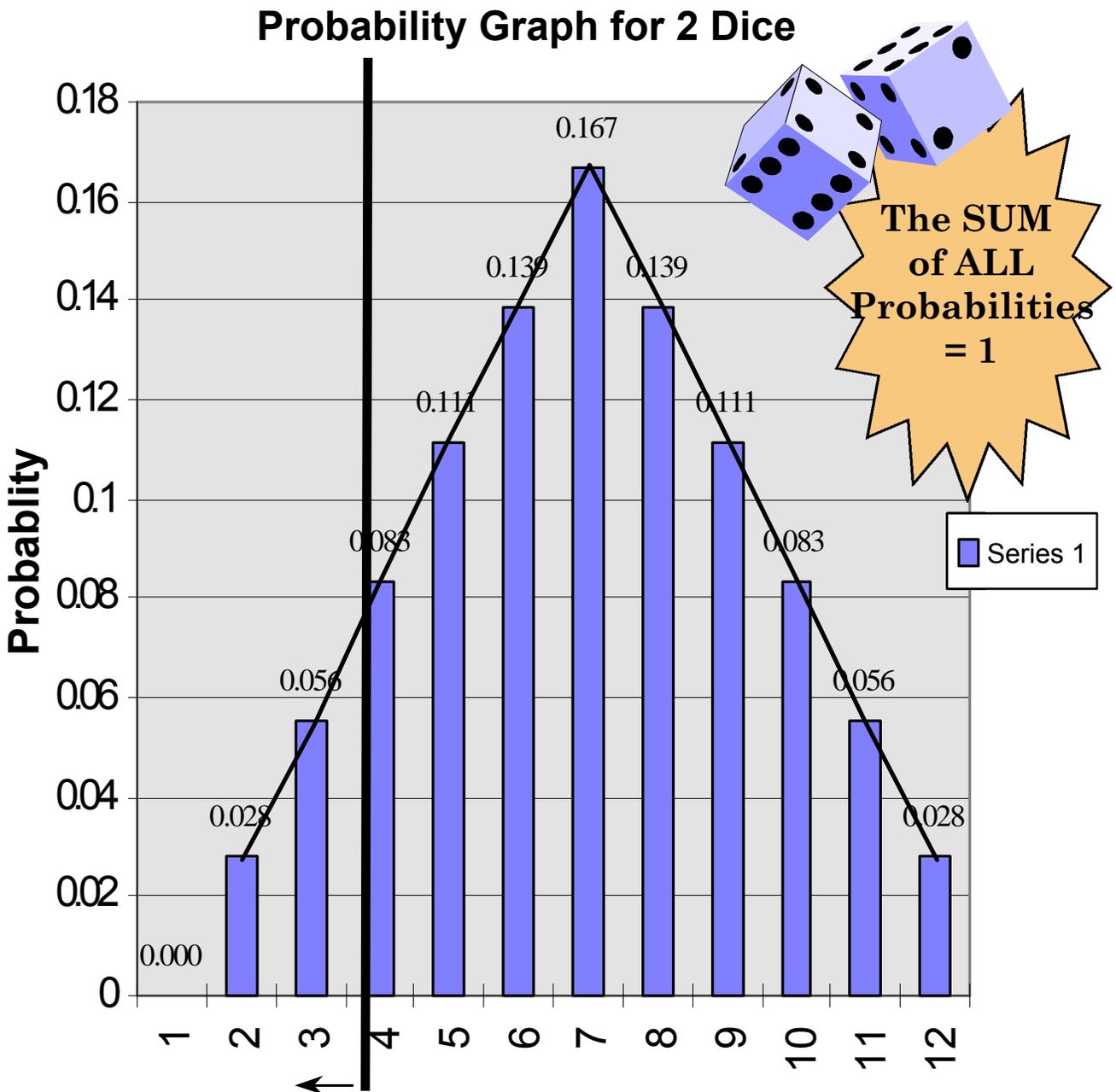
## Probability Graph for 2 Dice



**Probability Curve for Dice  $\frac{1}{4}$  SUM of Probabilities = 1**



# Probability Curve - Two Fair Dice

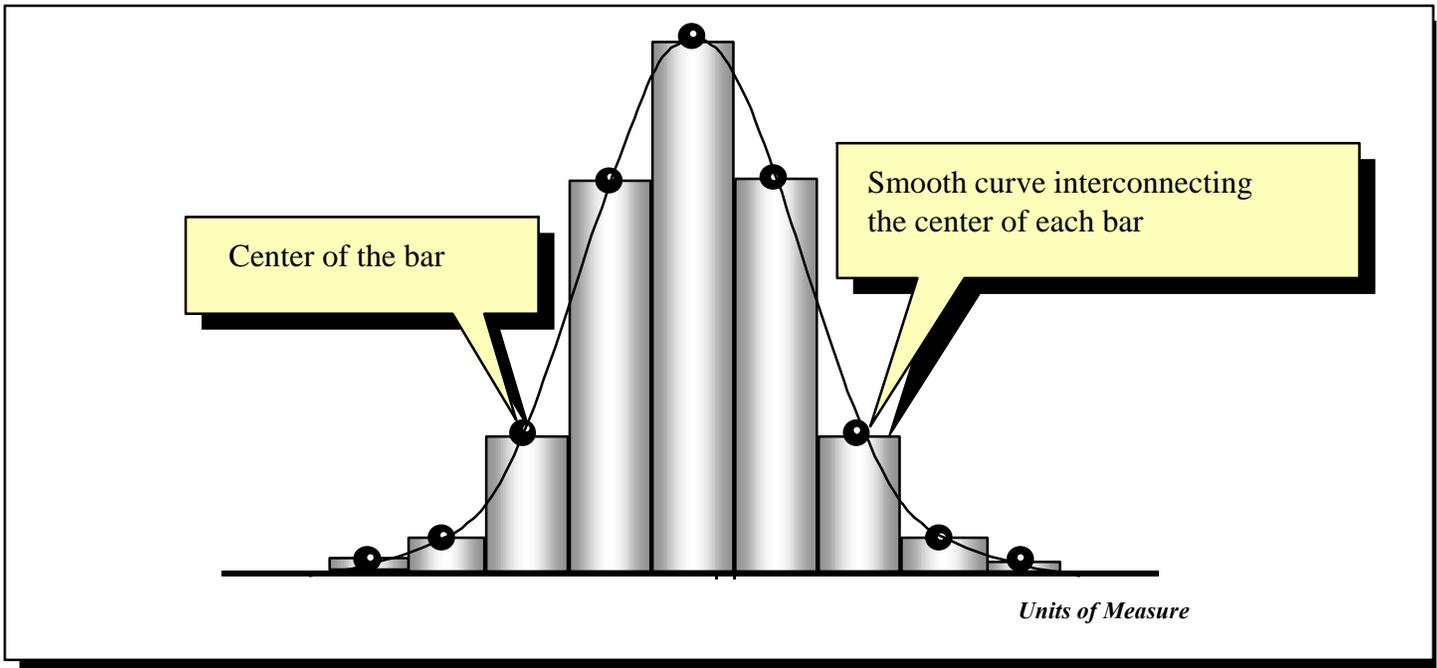


$P(x \leq 4)$  is Sum of Prob  $\leq 4 \dots = 0.166$



# Forming the Normal Curve

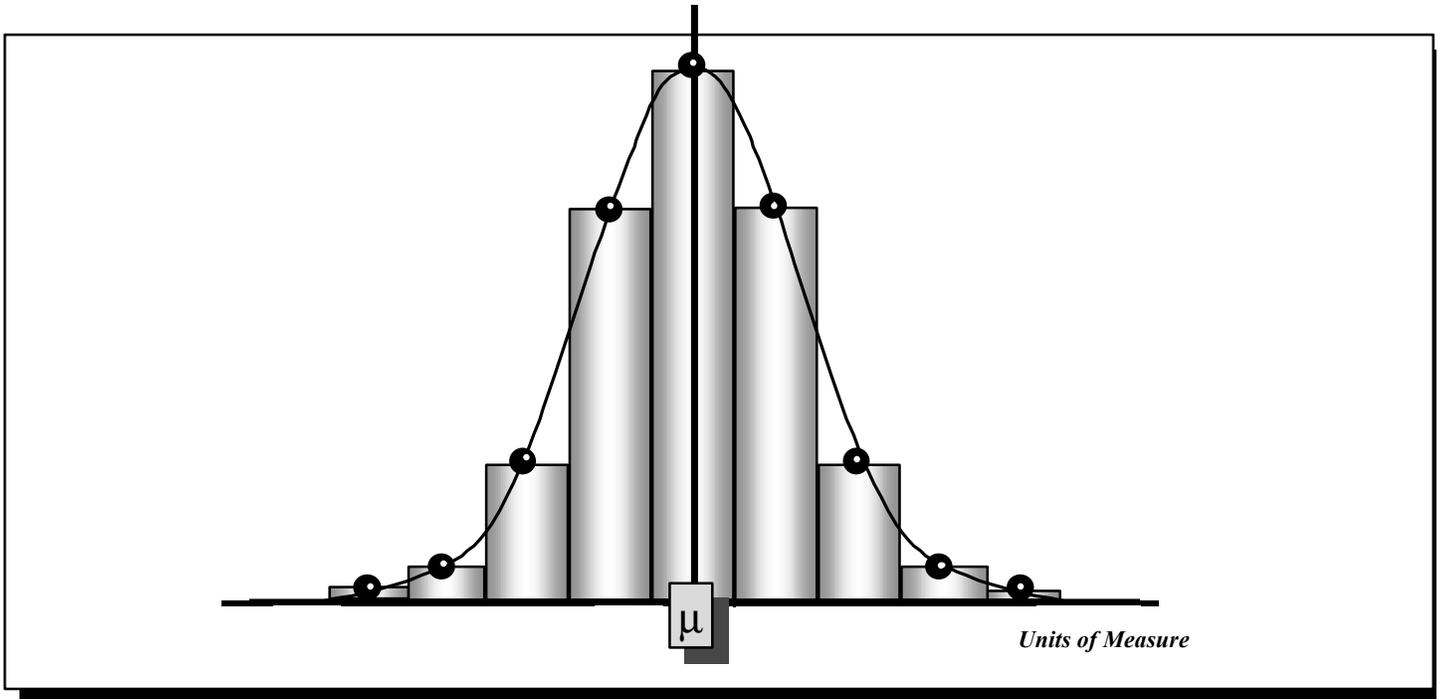
## Our Focus



*The Normal Curve is a graphical representation of the mathematical expression used to describe a Normal Distribution. This distribution is the result of a process experiencing only random variation.*



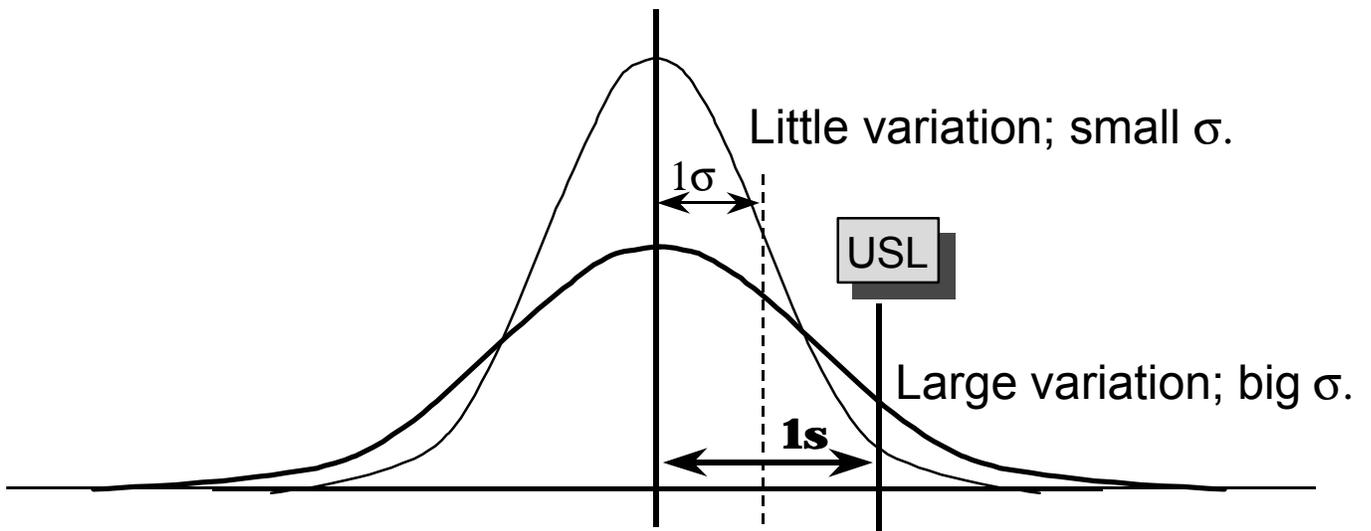
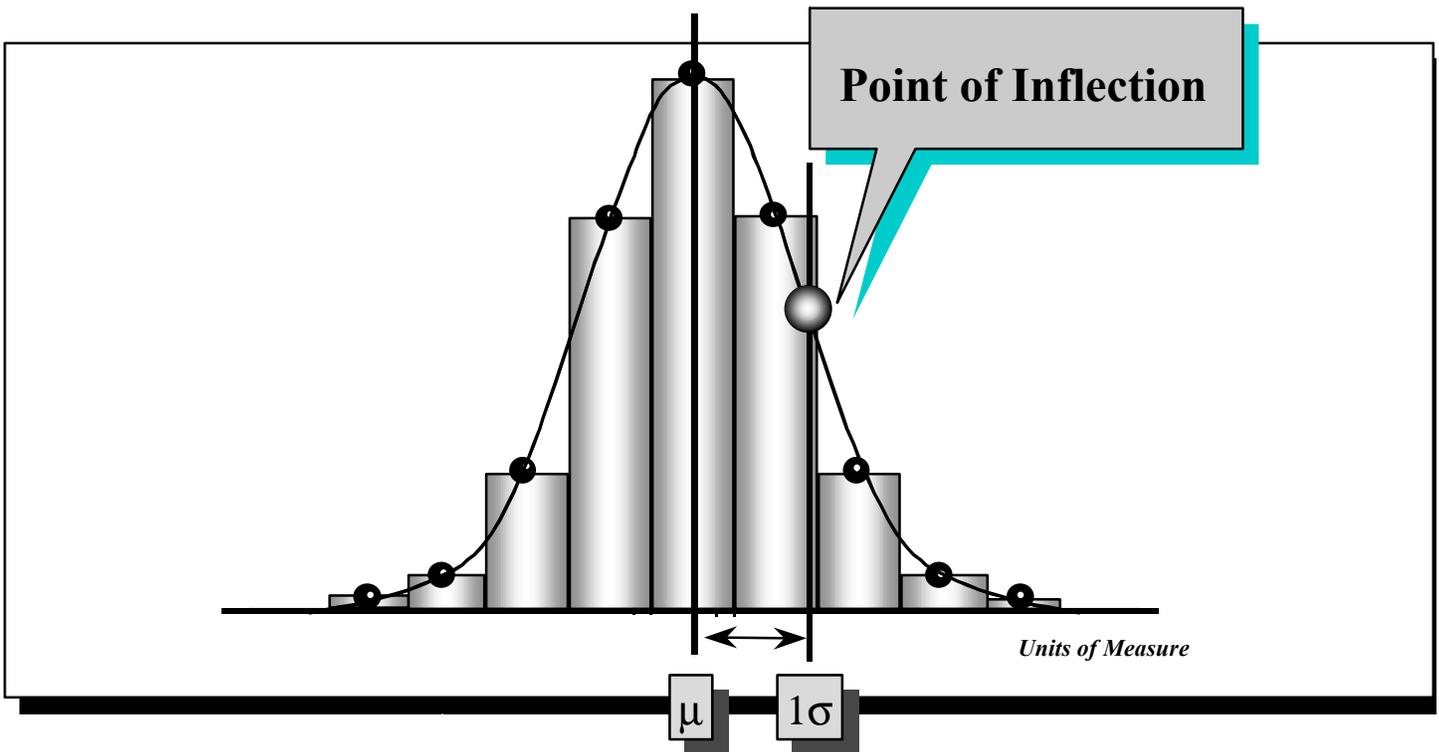
## *Process Mean ( $\bar{m}$ )*



*The Mean is the average of the data points. The Normal Curve is symmetrical about the Mean.*



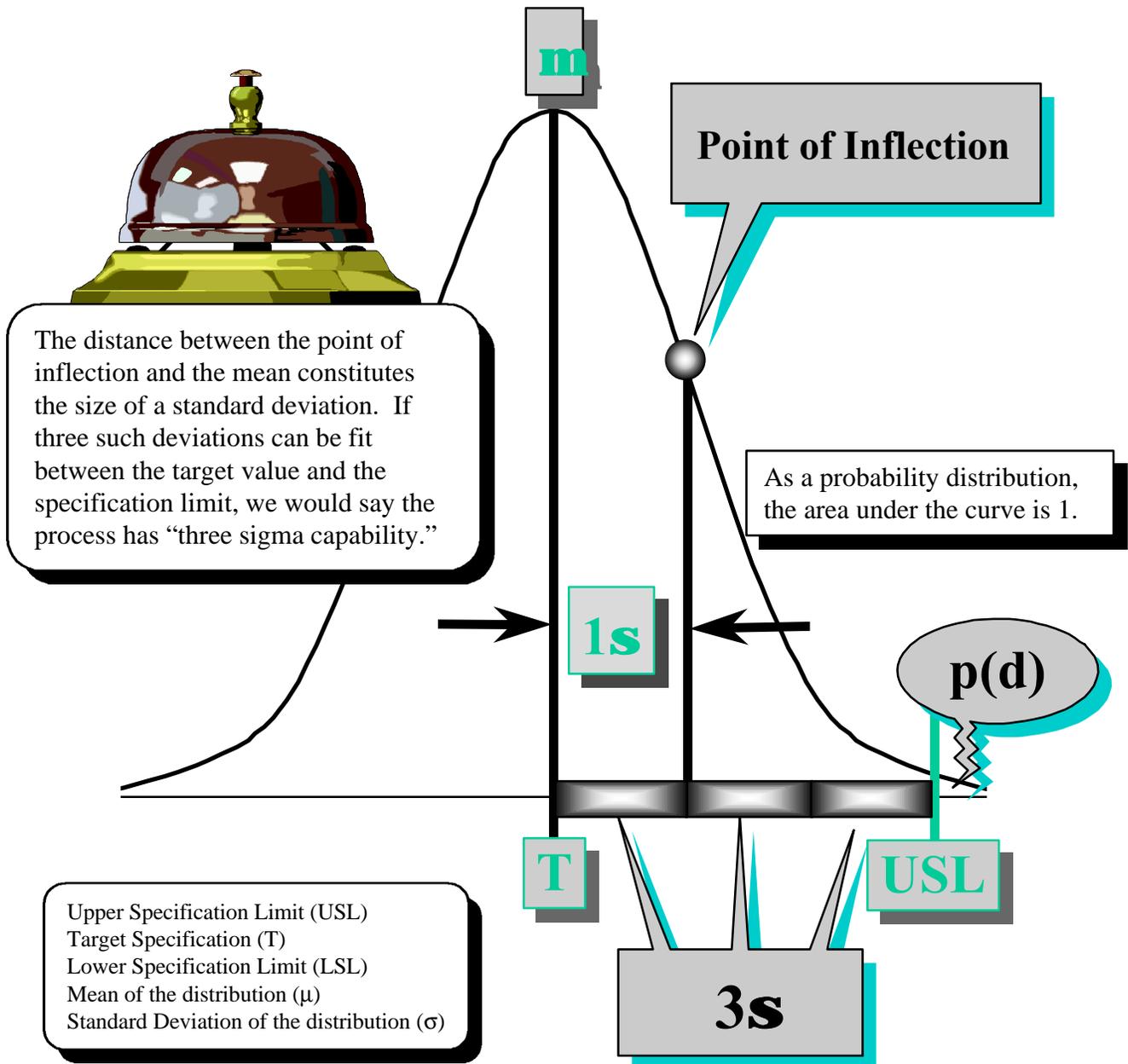
# Standard Deviation ( $s$ )



**The Standard Deviation is a measure of variability.**



# The Normal Distribution





## *The Normal Probability Distribution*

**$\mu$**  (mu), a measure of **central tendency**, is the mean or average of all values in the population. When only a sample of the population is described, mean is more properly denoted by  $\bar{x}$  (x bar).

**$\sigma$**  (sigma) is a measure of **dispersion or variability**. With smaller values of  $\sigma$ , all values in the population lie closer to the mean. When only a sample of the population is described, standard deviation is more properly denoted by  $s$ .

Both  $\mu$  and  $\sigma$  are specific values for any given population, and they change as the members of the population vary.



## The Computational Equations

**Population  
Mean**

$$m = \frac{\sum_{i=1}^N X_i}{N}$$

**Population  
Variance**

$$s^2 = \frac{\sum_{i=1}^N (X_i - m)^2}{N}$$

**Population  
Standard  
Deviation**

$$s = \sqrt{\frac{\sum_{i=1}^N (X_i - m)^2}{N}}$$

**Sample  
Mean**

$$\hat{m} = \bar{X} = \frac{\sum_{i=1}^n X_i}{n}$$

**Sample  
Variance**

$$\hat{s}^2 = s^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$$

**Sample  
Standard  
Deviation**

$$\hat{s} = s = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}}$$



# Exercise: Standard Deviation & Measures of Variation

	X	$\bar{X}$	Dev=(X - $\bar{X}$ )	$(X - \bar{X})^2$
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
$\Sigma$				
$\Sigma X$				
$\Sigma X^2$				
$\sigma^2$				



$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n} \quad \hat{\sigma} = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}}$$



## Deviation = Measures of Variation

Random data ("1" to "99", n = 10 samples):

n	Xi	Xbar	Dev =(Xi-Xbar)	(Xi-Xbar)^2
1	20	35.7	-15.70	246.49
2	1	35.7	-34.70	1204.09
3	6	35.7	-29.70	882.09
4	35	35.7	-0.70	0.49
5	54	35.7	18.30	334.89
6	67	35.7	31.30	979.69
7	43	35.7	7.30	53.29
8	99	35.7	63.30	4006.89
9	5	35.7	-30.70	942.49
10	27	35.7	-8.70	75.69
	-----	-----	-----	-----
	357		0.0	8726.10

### Results & Take-Aways:

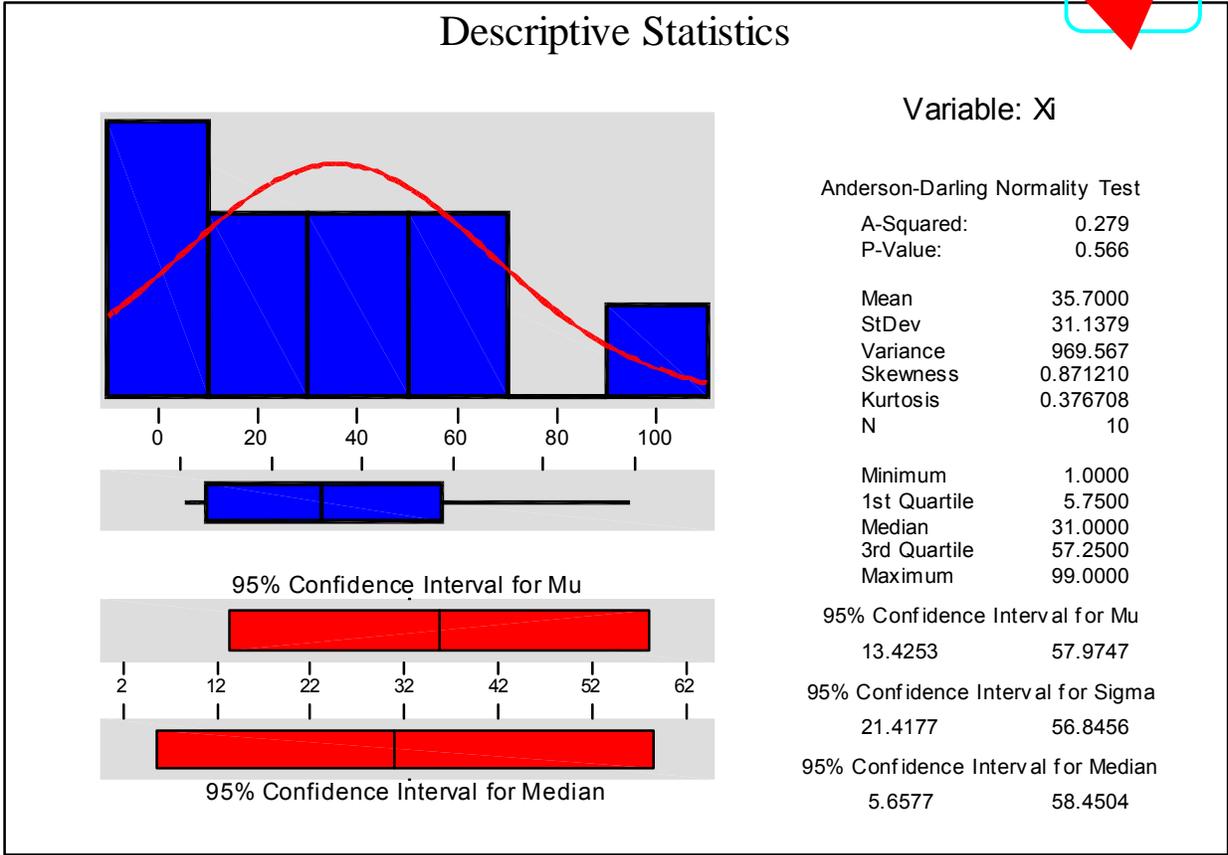
- Deviation = measure of *Variation*, measure of *Spread*
- SST = 8726.1 ... large Deviations = large SST
- Variance = "Average" SST = SST/df [df = deg. of freedom = n-1]  
 $s^2$  = Sample Variance = SST/9 = 969.6  
 $\sigma^2$  = Population Variance = SST/10 = 872.6
- Standard Deviation = Square Root of Variance:  
Sample StDev =  $s = \sqrt{969.6} = 31.1$   
Population SD =  $\sigma = \sqrt{872.6} = 29.5$

*It's all Variation ... Deviation, SST, Variance,  
& Standard Deviation = Measures of Spread*



# Check: via Descriptive Statistics Tool

Descriptive Statistics						
Variable	N	Mean	Median	TrMean	StDev	SE Mean
Xi	10	<b>35.70</b>	31.00	32.13	<b>31.14</b>	9.85
Variable	Minimum	Maximum	Q1	Q3		
Xi	1.00	99.00	5.75	57.25		



*Minitab Descriptive Statistics give sample Variance & Standard Deviation*



## Summary—Measures of Variation:

- **Range** = (Max - Min)
- **Deviation** =  $(X_i - \bar{X})$  [ $\bar{X}$  = mean or  $\mu$ ]
- **Sum-of-Squares (of the squared Deviations)** = **SST**
- **Variance** = **Average SST** =  $SST/df$  ( $df$  = degrees of freedom)
- **Std Deviation** = **SqrRoot (Variance)**  
*and*
- **Coeff. of Variation** = **CV** =  $(StDev/\bar{X}) * 100$   
*i.e., ratio of StDev to Mean—expressed as %*

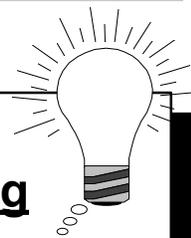
*SST is key concept in 'measuring' variation.*

*The greater SST, the greater the variation.*

*SST is **always conserved**: given a set of numbers (data), it is fixed & unchanging.*

*SST can be allocated to different **COMPONENTS OF VARIATION** -- i.e., **Within and Between Group**.*

**COMPONENTS OF VARIATION ...**  
**the KEY IDEA for Process Centering**  
**and ANOVA**





# Components of Variation ... how can we subgroup the data?

n	X(i)	SubGrp
1	20	0
2	1	1
3	6	0
4	35	1
5	54	0
6	67	1
7	43	1
8	99	1
9	5	1
10	27	1

By Even vs. Odd  
or  
... by Hi vs. Low  
or ...

Sorted by  
SubGrp

What is  $SS_0 = SS_{\text{even}}$  ?

$SS_0 =$  \_\_\_\_\_

$SS_1 = SS_{\text{odd}}$  ?

$SS_1 =$  \_\_\_\_\_

**SSW = Within Groups Variation**

$SSW = SS_0 + SS_1 =$  \_\_\_\_\_

Does  $SSW = SST = 8726.1??$

If not, where is the difference?

n	X(i)	SubGrp
1	20	0
3	6	0
5	54	0
2	1	1
4	35	1
6	67	1
7	43	1
8	99	1
9	5	1
10	27	1

... the difference ... is the **Between Group Variation:**  
 $SST = SSW + SSB$   
 In this case,  $SSB =$  \_\_\_\_\_

**SST is conserved and may be allocated to COMPONENTS OF VARIATION**



# Components of Variation ...and ANOVA <sup>3/4</sup>a different view

Here is SSB and SSW

## One-way Analysis of Variance Analysis of Variance for X(i)

Source	DF	SS	MS	F	P
SubGrp	1	350	350	0.33	0.579
Error	8	8376	1047		
Total	9	8726			

Individual 95% CIs For Mean  
Based on Pooled StDev

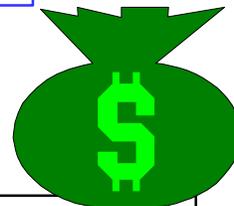
Level	N	Mean	StDev	-----+-----+-----+-----
0	3	26.67	24.68	(-----*-----)
1	7	39.57	34.54	(-----*-----)

Pooled StDev = 32.36      0    25    50

... and  $s_{ST}$  = Pooled StDev, based on SSW —  
it is the square-root of  $MS_{error}$

$MS_{error}$  is the “Within-Groups-Variance”

Finally, the P-value of 0.579 shows the  
Between-Group-Variation (SSB) is not  
significant



Measure the *Total Variation* of your  
product/process ... then analyze its *Components*.

A key issue is *subgrouping*.



## *The Normal Distribution*

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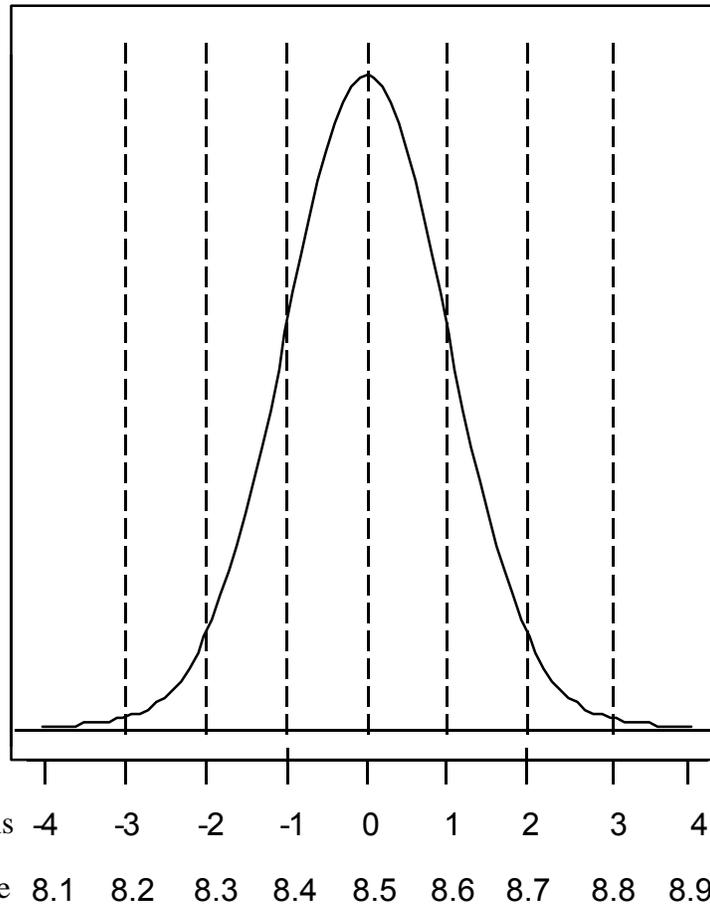
- *Generated as a result of a process experiencing random variation.*
- *Characterized by mean,  $\mu$ , and standard deviation,  $\sigma$ .*
- *All normal distributions can be related to the standard normal distribution with mean 0 and standard deviation 1.*



## Mean & Standard Deviation Example

$$\bar{x} = 8.5$$

$$s = 0.1$$



### Data

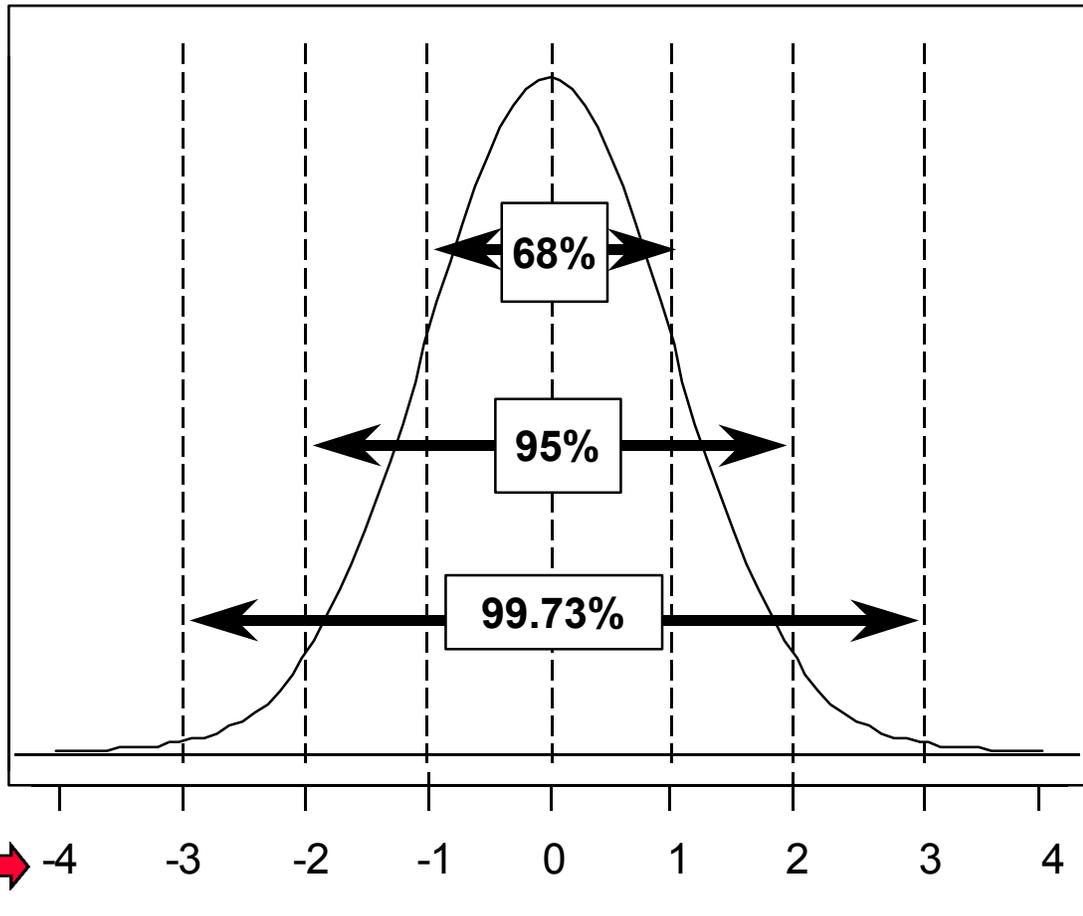
8.30935  
8.61892  
8.62944  
8.48459  
8.52606  
8.54337  
8.53467  
8.48189  
8.54814  
8.39409

***The x-axis can be represented by units of standard deviations or units of measure.***



# Normal Curve & Probability Areas

Area Under the Curve = 1



Output

***A randomly selected item has a 99.73% chance of being between -3 and 3 standard deviations from the mean.***

What's the probability of being: greater than (less than) the mean?

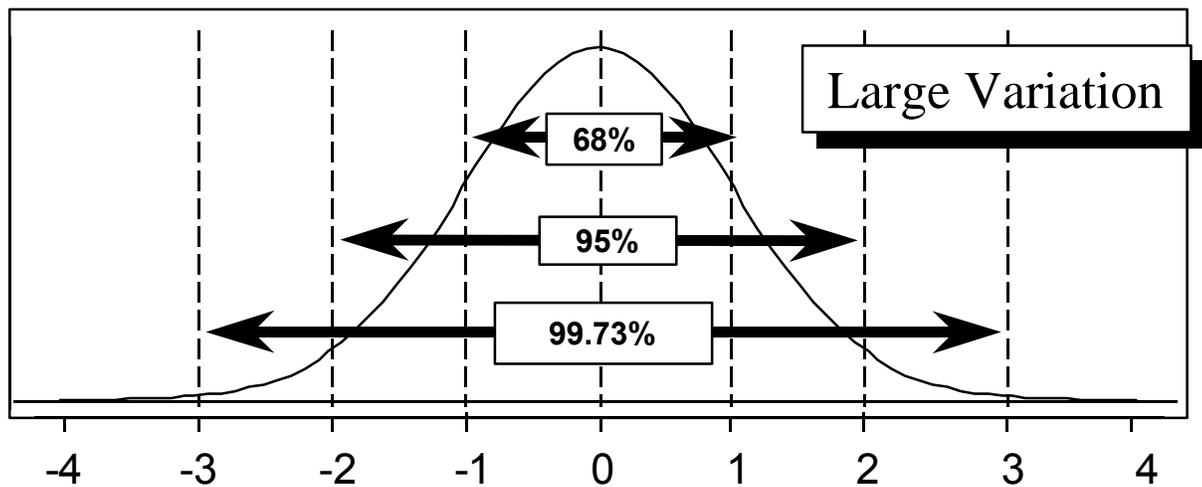
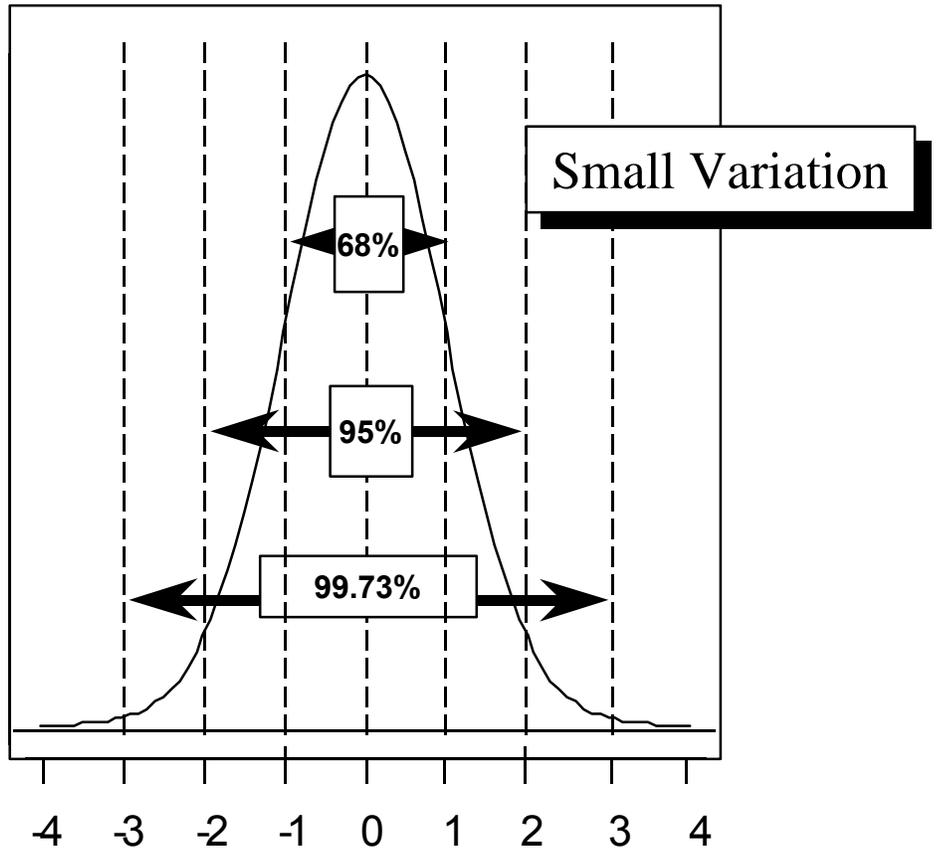
greater than (less than) 3 standard deviations?

greater than (less than) -1 standard deviation?



## Variation & Probability Areas

*The probability areas remain the same for any given region under the normal curve regardless of differences in variation.*

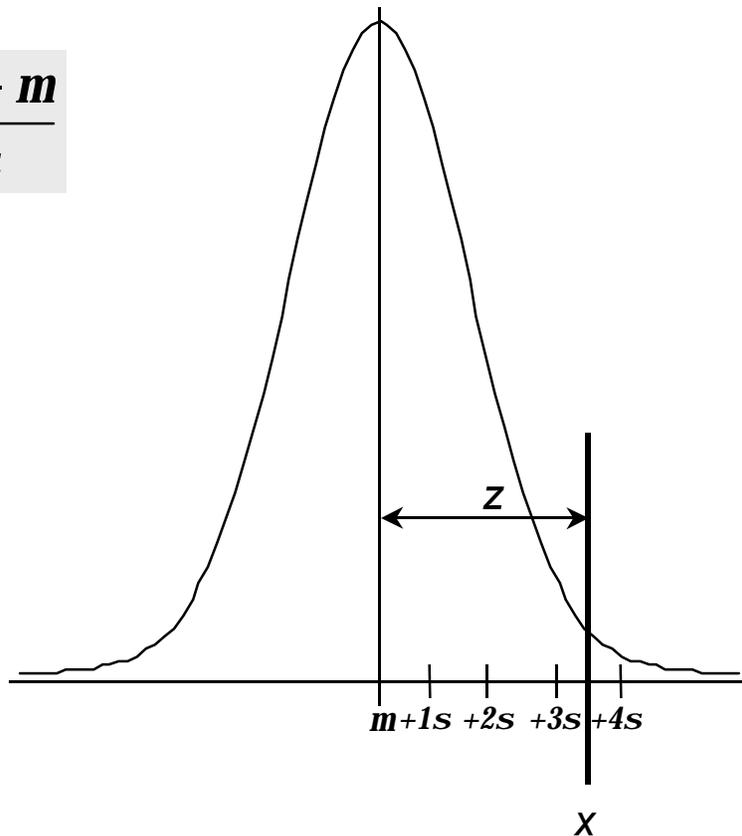




## Calculating Z

You can calculate a Z value for any given value of  $x$ . Z is the number of standard deviations which will fit between the mean and the value of  $x$ .

$$Z = \frac{x - m}{s}$$

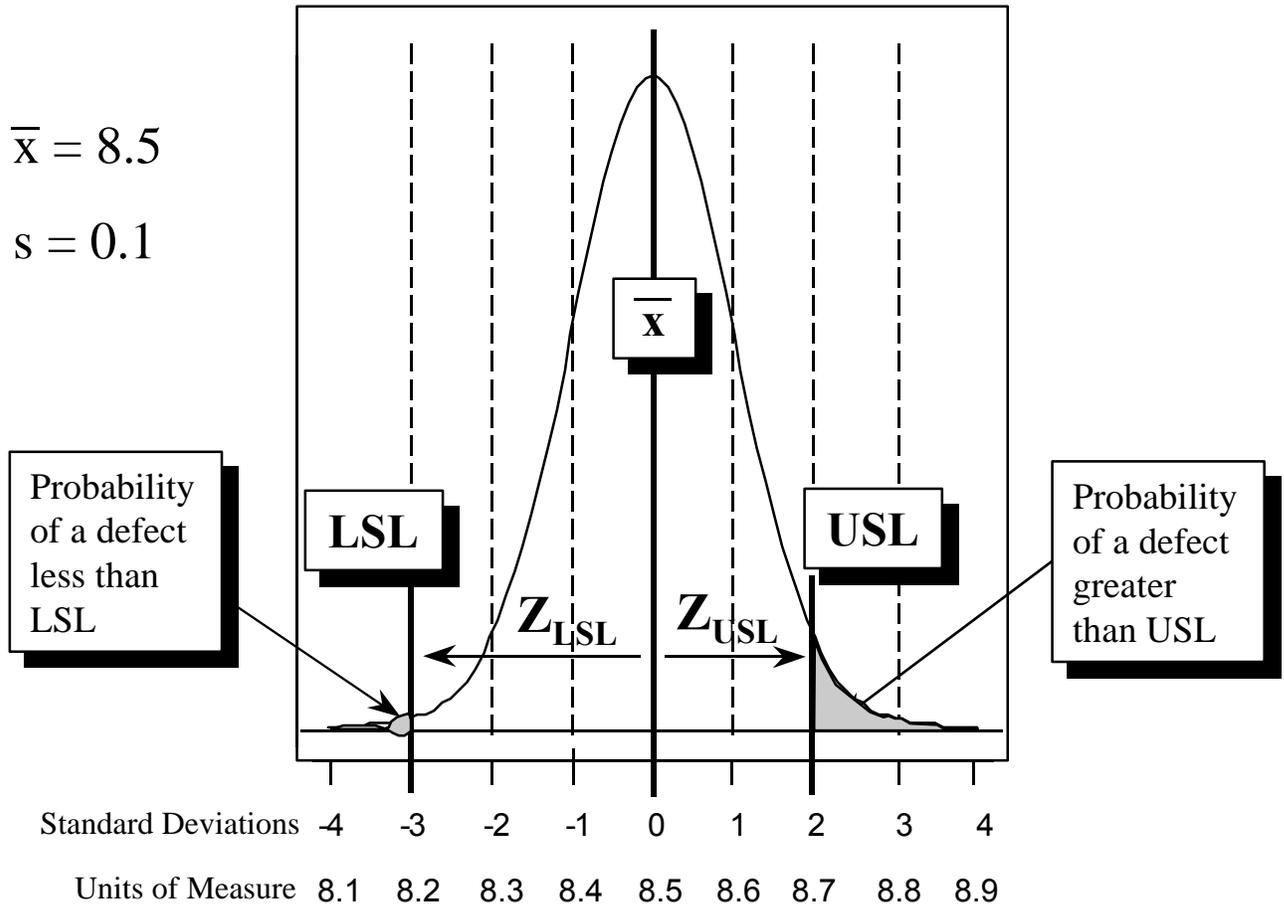




# Calculating Capability

$$\bar{x} = 8.5$$

$$s = 0.1$$

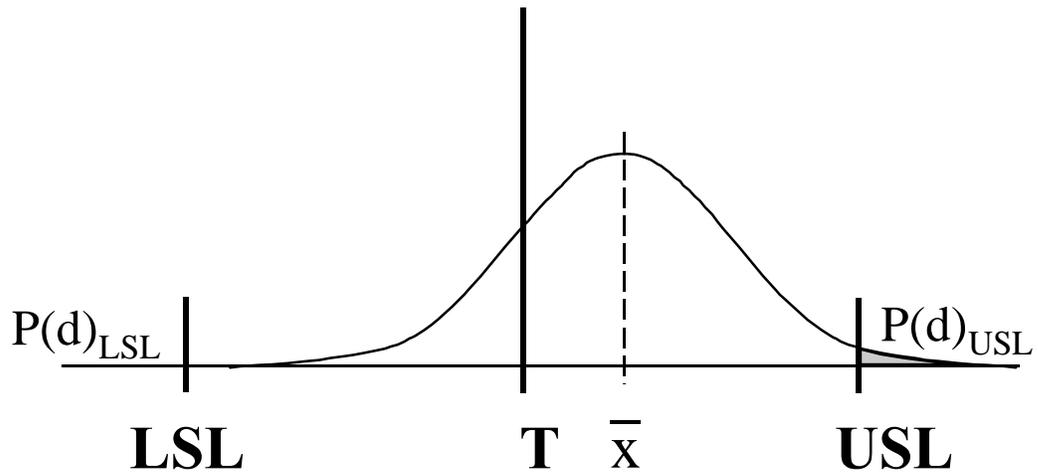


$$Z_{USL} = \frac{USL - \bar{x}}{s} = \frac{8.7 - 8.5}{0.1} = \frac{0.2}{0.1} = 2$$

$$Z_{LSL} = \frac{\bar{x} - LSL}{s} = \frac{8.5 - 8.2}{0.1} = \frac{0.3}{0.1} = 3$$



## Z-Bench



$$Z_{LSL} = \frac{\bar{x} - LSL}{S}$$



$P(d)_{LSL}$  = from Z table

$$Z_{USL} = \frac{USL - \bar{x}}{S}$$



$P(d)_{USL}$  = from Z table

$$P(d)_{Total} = P(d)_{LSL} + P(d)_{USL}$$

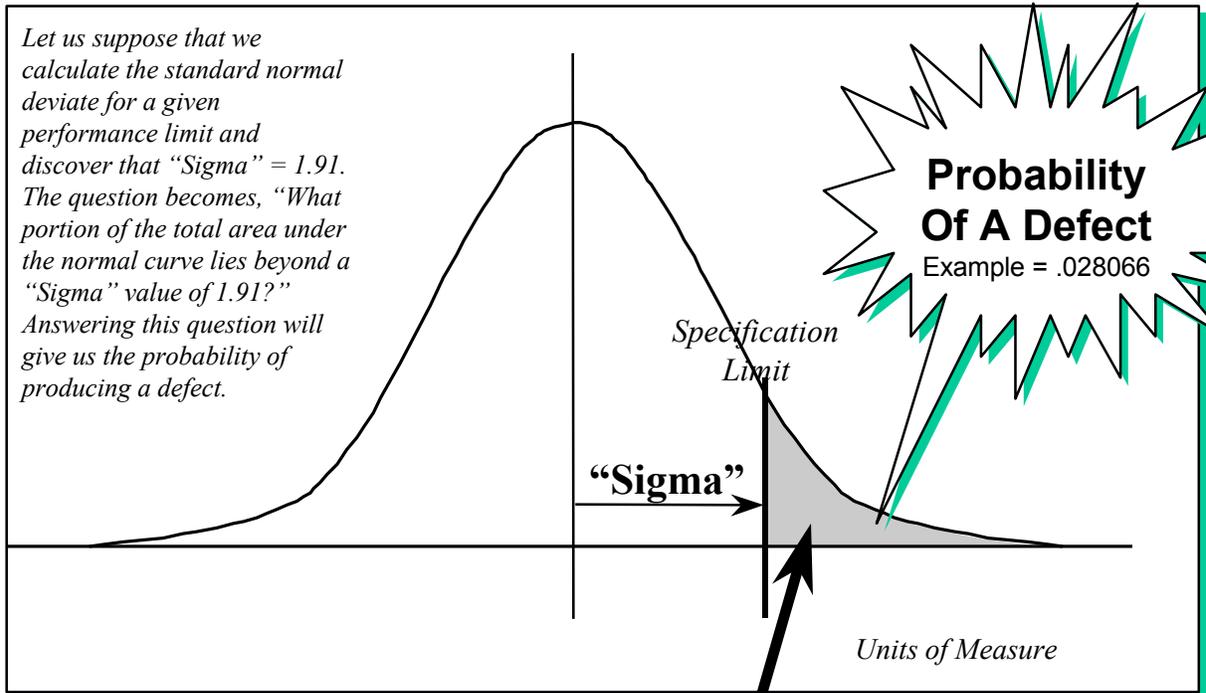


$Z_{Bench}$  = from Z table



# Probability of a Defect

Let us suppose that we calculate the standard normal deviate for a given performance limit and discover that "Sigma" = 1.91. The question becomes, "What portion of the total area under the normal curve lies beyond a "Sigma" value of 1.91?" Answering this question will give us the probability of producing a defect.



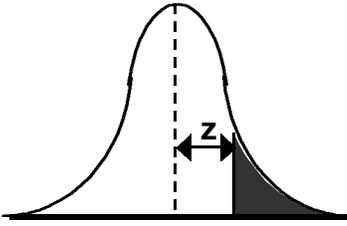
"Sigma" = Z

1.20	.420740315	1.71	.043632958	3.22	.000640954	4.73	.000001153
1.21	.401293694	1.76	.039203955	3.27	.000537758	4.78	.000000903
1.22	.382088115	1.81	.035147973	3.32	.000450127	4.83	.000000705
1.23	.363143837	1.86	.031442861	3.37	.000375899	4.88	.000000550
1.24	.344430316	1.91	.028066724	3.42	.000313179	4.93	.000000428
1.25	.325939421	1.96	.024998022	3.47	.000260317	4.98	.000000332
1.26	.307663041	2.01	.022215724	3.52	.000215873	5.03	.000000258
1.27	.289591261	2.06	.019699396	3.57	.000178601	5.08	.000000199
1.28	.271724078	2.11	.017429293	3.62	.000147419	5.13	.000000154
1.29	.254059672	2.16	.015386434	3.67	.000121399	5.18	.000000118
1.30	.236585924	2.21	.013552660	3.72	.000099739	5.23	.000000091
1.31	.219291735	2.26	.011910681	3.77	.000081753	5.28	.000000070
1.32	.202167115	2.31	.010444106	3.82	.000066855	5.33	.000000053
1.33	.185201072	2.36	.009137469	3.87	.000054545	5.38	.000000041
1.34	.168382616	2.41	.007976235	3.92	.000044399	5.43	.000000031
1.35	.151700751	2.46	.006946800	3.97	.000036057	5.48	.000000024
1.36	.135154474	2.51	.006036485	4.02	.000029215	5.53	.000000018
1.37	.118732786	2.56	.005233515	4.07	.000023617	5.58	.000000014
1.38	.102424678	2.61	.004527002	4.12	.000019047	5.63	.000000010
1.39	.864333937	2.66	.003906912	4.17			
1.40	.822466031	2.71	.003364033	4.22			
1.41	.776870614	2.76	.002889938	4.27			
1.42	.727535260	2.81	.002476947	4.32			
1.43	.675463900	2.86	.002118083	4.37			
1.44	.620764287	2.91	.001807032	4.42			
1.45	.563552914	2.96	.001538097	4.47			
1.46	.503942910	3.01	.001306156	4.52			

"Sigma" = 1.91

**Table Of Area Under The Normal Curve**

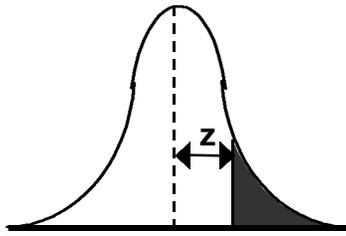
To the right of the "Sigma" value, you will note the tail area



# Single-Tail Z Table

(Values of Z from 0.00 to 4.99)

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
<b>0.00</b>	5.00e-001	4.96e-001	4.92e-001	4.88e-001	4.84e-001	4.80e-001	4.76e-001	4.72e-001	4.68e-001	4.64e-001
<b>0.10</b>	4.60e-001	4.56e-001	4.52e-001	4.48e-001	4.44e-001	4.40e-001	4.36e-001	4.33e-001	4.29e-001	4.25e-001
<b>0.20</b>	4.21e-001	4.17e-001	4.13e-001	4.09e-001	4.05e-001	4.01e-001	3.97e-001	3.94e-001	3.90e-001	3.86e-001
<b>0.30</b>	3.82e-001	3.78e-001	3.74e-001	3.71e-001	3.67e-001	3.63e-001	3.59e-001	3.56e-001	3.52e-001	3.48e-001
<b>0.40</b>	3.45e-001	3.41e-001	3.37e-001	3.34e-001	3.30e-001	3.26e-001	3.23e-001	3.19e-001	3.16e-001	3.12e-001
<b>0.50</b>	3.09e-001	3.05e-001	3.02e-001	2.98e-001	2.95e-001	2.91e-001	2.88e-001	2.84e-001	2.81e-001	2.78e-001
<b>0.60</b>	2.74e-001	2.71e-001	2.68e-001	2.64e-001	2.61e-001	2.58e-001	2.55e-001	2.51e-001	2.48e-001	2.45e-001
<b>0.70</b>	2.42e-001	2.39e-001	2.36e-001	2.33e-001	2.30e-001	2.27e-001	2.24e-001	2.21e-001	2.18e-001	2.15e-001
<b>0.80</b>	2.12e-001	2.09e-001	2.06e-001	2.03e-001	2.00e-001	1.98e-001	1.95e-001	1.92e-001	1.89e-001	1.87e-001
<b>0.90</b>	1.84e-001	1.81e-001	1.79e-001	1.76e-001	1.74e-001	1.71e-001	1.69e-001	1.66e-001	1.64e-001	1.61e-001
<b>1.00</b>	1.59e-001	1.56e-001	1.54e-001	1.52e-001	1.49e-001	1.47e-001	1.45e-001	1.42e-001	1.40e-001	1.38e-001
<b>1.10</b>	1.36e-001	1.33e-001	1.31e-001	1.29e-001	1.27e-001	1.25e-001	1.23e-001	1.21e-001	1.19e-001	1.17e-001
<b>1.20</b>	1.15e-001	1.13e-001	1.11e-001	1.09e-001	1.07e-001	1.06e-001	1.04e-001	1.02e-001	1.00e-001	9.85e-002
<b>1.30</b>	9.68e-002	9.51e-002	9.34e-002	9.18e-002	9.01e-002	8.85e-002	8.69e-002	8.53e-002	8.38e-002	8.23e-002
<b>1.40</b>	8.08e-002	7.93e-002	7.78e-002	7.64e-002	7.49e-002	7.35e-002	7.21e-002	7.08e-002	6.94e-002	6.81e-002
<b>1.50</b>	6.68e-002	6.55e-002	6.43e-002	6.30e-002	6.18e-002	6.06e-002	5.94e-002	5.82e-002	5.71e-002	5.59e-002
<b>1.60</b>	5.48e-002	5.37e-002	5.26e-002	5.16e-002	5.05e-002	4.95e-002	4.85e-002	4.75e-002	4.65e-002	4.55e-002
<b>1.70</b>	4.46e-002	4.36e-002	4.27e-002	4.18e-002	4.09e-002	4.01e-002	3.92e-002	3.84e-002	3.75e-002	3.67e-002
<b>1.80</b>	3.59e-002	3.51e-002	3.44e-002	3.36e-002	3.29e-002	3.22e-002	3.14e-002	3.07e-002	3.01e-002	2.94e-002
<b>1.90</b>	2.87e-002	2.81e-002	2.74e-002	2.68e-002	2.62e-002	2.56e-002	2.50e-002	2.44e-002	2.39e-002	2.33e-002
<b>2.00</b>	2.28e-002	2.22e-002	2.17e-002	2.12e-002	2.07e-002	2.02e-002	1.97e-002	1.92e-002	1.88e-002	1.83e-002
<b>2.10</b>	1.79e-002	1.74e-002	1.70e-002	1.66e-002	1.62e-002	1.58e-002	1.54e-002	1.50e-002	1.46e-002	1.43e-002
<b>2.20</b>	1.39e-002	1.36e-002	1.32e-002	1.29e-002	1.25e-002	1.22e-002	1.19e-002	1.16e-002	1.13e-002	1.10e-002
<b>2.30</b>	1.07e-002	1.04e-002	1.02e-002	9.90e-003	9.64e-003	9.39e-003	9.14e-003	8.89e-003	8.66e-003	8.42e-003
<b>2.40</b>	8.20e-003	7.98e-003	7.76e-003	7.55e-003	7.34e-003	7.14e-003	6.95e-003	6.76e-003	6.57e-003	6.39e-003
<b>2.50</b>	6.21e-003	6.04e-003	5.87e-003	5.70e-003	5.54e-003	5.39e-003	5.23e-003	5.08e-003	4.94e-003	4.80e-003
<b>2.60</b>	4.66e-003	4.53e-003	4.40e-003	4.27e-003	4.15e-003	4.02e-003	3.91e-003	3.79e-003	3.68e-003	3.57e-003
<b>2.70</b>	3.47e-003	3.36e-003	3.26e-003	3.17e-003	3.07e-003	2.98e-003	2.89e-003	2.80e-003	2.72e-003	2.64e-003
<b>2.80</b>	2.56e-003	2.48e-003	2.40e-003	2.33e-003	2.26e-003	2.19e-003	2.12e-003	2.05e-003	1.99e-003	1.93e-003
<b>2.90</b>	1.87e-003	1.81e-003	1.75e-003	1.69e-003	1.64e-003	1.59e-003	1.54e-003	1.49e-003	1.44e-003	1.39e-003
<b>3.00</b>	1.35e-003	1.31e-003	1.26e-003	1.22e-003	1.18e-003	1.14e-003	1.11e-003	1.07e-003	1.04e-003	1.00e-003
<b>3.10</b>	9.68e-004	9.35e-004	9.04e-004	8.74e-004	8.45e-004	8.16e-004	7.89e-004	7.62e-004	7.36e-004	7.11e-004
<b>3.20</b>	6.87e-004	6.64e-004	6.41e-004	6.19e-004	5.98e-004	5.77e-004	5.57e-004	5.38e-004	5.19e-004	5.01e-004
<b>3.30</b>	4.83e-004	4.66e-004	4.50e-004	4.34e-004	4.19e-004	4.04e-004	3.90e-004	3.76e-004	3.62e-004	3.49e-004
<b>3.40</b>	3.37e-004	3.25e-004	3.13e-004	3.02e-004	2.91e-004	2.80e-004	2.70e-004	2.60e-004	2.51e-004	2.42e-004
<b>3.50</b>	2.33e-004	2.24e-004	2.16e-004	2.08e-004	2.00e-004	1.93e-004	1.85e-004	1.78e-004	1.72e-004	1.65e-004
<b>3.60</b>	1.59e-004	1.53e-004	1.47e-004	1.42e-004	1.36e-004	1.31e-004	1.26e-004	1.21e-004	1.17e-004	1.12e-004
<b>3.70</b>	1.08e-004	1.04e-004	9.96e-005	9.57e-005	9.20e-005	8.84e-005	8.50e-005	8.16e-005	7.84e-005	7.53e-005
<b>3.80</b>	7.23e-005	6.95e-005	6.67e-005	6.41e-005	6.15e-005	5.91e-005	5.67e-005	5.44e-005	5.22e-005	5.01e-005
<b>3.90</b>	4.81e-005	4.61e-005	4.43e-005	4.25e-005	4.07e-005	3.91e-005	3.75e-005	3.59e-005	3.45e-005	3.30e-005
<b>4.00</b>	3.17e-005	3.04e-005	2.91e-005	2.79e-005	2.67e-005	2.56e-005	2.45e-005	2.35e-005	2.25e-005	2.16e-005
<b>4.10</b>	2.07e-005	1.98e-005	1.89e-005	1.81e-005	1.74e-005	1.66e-005	1.59e-005	1.52e-005	1.46e-005	1.39e-005
<b>4.20</b>	1.33e-005	1.28e-005	1.22e-005	1.17e-005	1.12e-005	1.07e-005	1.02e-005	9.77e-006	9.34e-006	8.93e-006
<b>4.30</b>	8.54e-006	8.16e-006	7.80e-006	7.46e-006	7.12e-006	6.81e-006	6.50e-006	6.21e-006	5.93e-006	5.67e-006
<b>4.40</b>	5.41e-006	5.17e-006	4.94e-006	4.71e-006	4.50e-006	4.29e-006	4.10e-006	3.91e-006	3.73e-006	3.56e-006
<b>4.50</b>	3.40e-006	3.24e-006	3.09e-006	2.95e-006	2.81e-006	2.68e-006	2.56e-006	2.44e-006	2.32e-006	2.22e-006
<b>4.60</b>	2.11e-006	2.01e-006	1.92e-006	1.83e-006	1.74e-006	1.66e-006	1.58e-006	1.51e-006	1.43e-006	1.37e-006
<b>4.70</b>	1.30e-006	1.24e-006	1.18e-006	1.12e-006	1.07e-006	1.02e-006	9.68e-007	9.21e-007	8.76e-007	8.34e-007
<b>4.80</b>	7.93e-007	7.55e-007	7.18e-007	6.83e-007	6.49e-007	6.17e-007	5.87e-007	5.58e-007	5.30e-007	5.04e-007
<b>4.90</b>	4.79e-007	4.55e-007	4.33e-007	4.11e-007	3.91e-007	3.71e-007	3.52e-007	3.35e-007	3.18e-007	3.02e-007



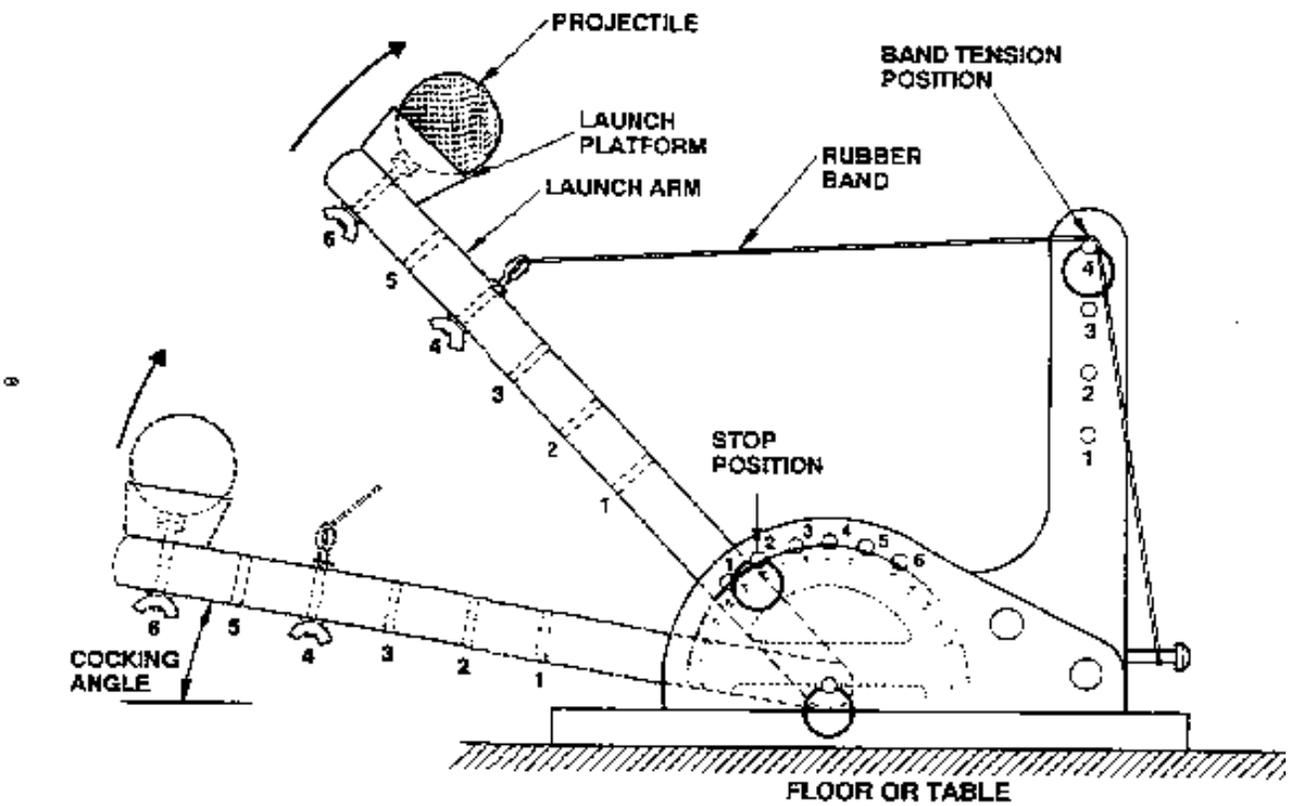
# Single-Tail Z Table

(Values of Z from 5.00 to 9.99)

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
5.00	2.87e-007	2.72e-007	2.58e-007	2.45e-007	2.33e-007	2.21e-007	2.10e-007	1.99e-007	1.89e-007	1.79e-007
5.10	1.70e-007	1.61e-007	1.53e-007	1.45e-007	1.37e-007	1.30e-007	1.23e-007	1.17e-007	1.11e-007	1.05e-007
5.20	9.96e-008	9.44e-008	8.95e-008	8.48e-008	8.03e-008	7.60e-008	7.20e-008	6.82e-008	6.46e-008	6.12e-008
5.30	5.79e-008	5.48e-008	5.19e-008	4.91e-008	4.65e-008	4.40e-008	4.16e-008	3.94e-008	3.72e-008	3.52e-008
5.40	3.33e-008	3.15e-008	2.98e-008	2.82e-008	2.66e-008	2.52e-008	2.38e-008	2.25e-008	2.13e-008	2.01e-008
5.50	1.90e-008	1.79e-008	1.69e-008	1.60e-008	1.51e-008	1.43e-008	1.35e-008	1.27e-008	1.20e-008	1.14e-008
5.60	1.07e-008	1.01e-008	9.55e-009	9.01e-009	8.50e-009	8.02e-009	7.57e-009	7.14e-009	6.73e-009	6.35e-009
5.70	5.99e-009	5.65e-009	5.33e-009	5.02e-009	4.73e-009	4.46e-009	4.21e-009	3.96e-009	3.74e-009	3.52e-009
5.80	3.32e-009	3.12e-009	2.94e-009	2.77e-009	2.61e-009	2.46e-009	2.31e-009	2.18e-009	2.05e-009	1.93e-009
5.90	1.82e-009	1.71e-009	1.61e-009	1.51e-009	1.43e-009	1.34e-009	1.26e-009	1.19e-009	1.12e-009	1.05e-009
6.00	9.87e-010	9.28e-010	8.72e-010	8.20e-010	7.71e-010	7.24e-010	6.81e-010	6.40e-010	6.01e-010	5.65e-010
6.10	5.30e-010	4.98e-010	4.68e-010	4.39e-010	4.13e-010	3.87e-010	3.64e-010	3.41e-010	3.21e-010	3.01e-010
6.20	2.82e-010	2.65e-010	2.49e-010	2.33e-010	2.19e-010	2.05e-010	1.92e-010	1.81e-010	1.69e-010	1.59e-010
6.30	1.49e-010	1.40e-010	1.31e-010	1.23e-010	1.15e-010	1.08e-010	1.01e-010	9.45e-011	8.85e-011	8.29e-011
6.40	7.77e-011	7.28e-011	6.81e-011	6.38e-011	5.97e-011	5.59e-011	5.24e-011	4.90e-011	4.59e-011	4.29e-011
6.50	4.02e-011	3.76e-011	3.52e-011	3.29e-011	3.08e-011	2.88e-011	2.69e-011	2.52e-011	2.35e-011	2.20e-011
6.60	2.06e-011	1.92e-011	1.80e-011	1.68e-011	1.57e-011	1.47e-011	1.37e-011	1.28e-011	1.19e-011	1.12e-011
6.70	1.04e-011	9.73e-012	9.09e-012	8.48e-012	7.92e-012	7.39e-012	6.90e-012	6.44e-012	6.01e-012	5.61e-012
6.80	5.23e-012	4.88e-012	4.55e-012	4.25e-012	3.96e-012	3.69e-012	3.44e-012	3.21e-012	2.99e-012	2.79e-012
6.90	2.60e-012	2.42e-012	2.26e-012	2.10e-012	1.96e-012	1.83e-012	1.70e-012	1.58e-012	1.48e-012	1.37e-012
7.00	1.28e-012	1.19e-012	1.11e-012	1.03e-012	9.61e-013	8.95e-013	8.33e-013	7.75e-013	7.21e-013	6.71e-013
7.10	6.24e-013	5.80e-013	5.40e-013	5.02e-013	4.67e-013	4.34e-013	4.03e-013	3.75e-013	3.49e-013	3.24e-013
7.20	3.01e-013	2.80e-013	2.60e-013	2.41e-013	2.24e-013	2.08e-013	1.94e-013	1.80e-013	1.67e-013	1.55e-013
7.30	1.44e-013	1.34e-013	1.24e-013	1.15e-013	1.07e-013	9.91e-014	9.20e-014	8.53e-014	7.91e-014	7.34e-014
7.40	6.81e-014	6.31e-014	5.86e-014	5.43e-014	5.03e-014	4.67e-014	4.33e-014	4.01e-014	3.72e-014	3.44e-014
7.50	3.19e-014	2.96e-014	2.74e-014	2.54e-014	2.35e-014	2.18e-014	2.02e-014	1.87e-014	1.73e-014	1.60e-014
7.60	1.48e-014	1.37e-014	1.27e-014	1.17e-014	1.09e-014	1.00e-014	9.30e-015	8.60e-015	7.95e-015	7.36e-015
7.70	6.80e-015	6.29e-015	5.82e-015	5.38e-015	4.97e-015	4.59e-015	4.25e-015	3.92e-015	3.63e-015	3.35e-015
7.80	3.10e-015	2.86e-015	2.64e-015	2.44e-015	2.25e-015	2.08e-015	1.92e-015	1.77e-015	1.64e-015	1.51e-015
7.90	1.39e-015	1.29e-015	1.19e-015	1.10e-015	1.01e-015	9.33e-016	8.60e-016	7.93e-016	7.32e-016	6.75e-016
8.00	6.22e-016	5.74e-016	5.29e-016	4.87e-016	4.49e-016	4.14e-016	3.81e-016	3.51e-016	3.24e-016	2.98e-016
8.10	2.75e-016	2.53e-016	2.33e-016	2.15e-016	1.98e-016	1.82e-016	1.68e-016	1.54e-016	1.42e-016	1.31e-016
8.20	1.20e-016	1.11e-016	1.02e-016	9.36e-017	8.61e-017	7.92e-017	7.28e-017	6.70e-017	6.16e-017	5.66e-017
8.30	5.21e-017	4.79e-017	4.40e-017	4.04e-017	3.71e-017	3.41e-017	3.14e-017	2.88e-017	2.65e-017	2.43e-017
8.40	2.23e-017	2.05e-017	1.88e-017	1.73e-017	1.59e-017	1.46e-017	1.34e-017	1.23e-017	1.13e-017	1.03e-017
8.50	9.48e-018	8.70e-018	7.98e-018	7.32e-018	6.71e-018	6.15e-018	5.64e-018	5.17e-018	4.74e-018	4.35e-018
8.60	3.99e-018	3.65e-018	3.35e-018	3.07e-018	2.81e-018	2.57e-018	2.36e-018	2.16e-018	1.98e-018	1.81e-018
8.70	1.66e-018	1.52e-018	1.39e-018	1.27e-018	1.17e-018	1.07e-018	9.76e-019	8.93e-019	8.17e-019	7.48e-019
8.80	6.84e-019	6.26e-019	5.72e-019	5.23e-019	4.79e-019	4.38e-019	4.00e-019	3.66e-019	3.34e-019	3.06e-019
8.90	2.79e-019	2.55e-019	2.33e-019	2.13e-019	1.95e-019	1.78e-019	1.62e-019	1.48e-019	1.35e-019	1.24e-019
9.00	1.13e-019	1.03e-019	9.40e-020	8.58e-020	7.83e-020	7.15e-020	6.52e-020	5.95e-020	5.43e-020	4.95e-020
9.10	4.52e-020	4.12e-020	3.76e-020	3.42e-020	3.12e-020	2.85e-020	2.59e-020	2.37e-020	2.16e-020	1.96e-020
9.20	1.79e-020	1.63e-020	1.49e-020	1.35e-020	1.23e-020	1.12e-020	1.02e-020	9.31e-021	8.47e-021	7.71e-021
9.30	7.02e-021	6.39e-021	5.82e-021	5.29e-021	4.82e-021	4.38e-021	3.99e-021	3.63e-021	3.30e-021	3.00e-021
9.40	2.73e-021	2.48e-021	2.26e-021	2.05e-021	1.86e-021	1.69e-021	1.54e-021	1.40e-021	1.27e-021	1.16e-021
9.50	1.05e-021	9.53e-022	8.66e-022	7.86e-022	7.14e-022	6.48e-022	5.89e-022	5.35e-022	4.85e-022	4.40e-022
9.60	4.00e-022	3.63e-022	3.29e-022	2.99e-022	2.71e-022	2.46e-022	2.23e-022	2.02e-022	1.83e-022	1.66e-022
9.70	1.51e-022	1.37e-022	1.24e-022	1.12e-022	1.02e-022	9.22e-023	8.36e-023	7.57e-023	6.86e-023	6.21e-023
9.80	5.63e-023	5.10e-023	4.62e-023	4.18e-023	3.79e-023	3.43e-023	3.10e-023	2.81e-023	2.54e-023	2.30e-023
9.90	2.08e-023	1.88e-023	1.70e-023	1.54e-023	1.39e-023	1.26e-023	1.14e-023	1.03e-023	9.32e-024	8.43e-024



# Catapult Exercise





## *Catapult Setup*

---

- *Secure catapult to desk using a C-clamp.*
- *Position tape measure to measure from the back of the catapult forward.*
- *Catapult Settings:*
  - *Band Tension Position 4*
  - *Stop Position 2*



## *Catapult Experiment*

- *A team consists of six members.*
- *Each team selects two manufacturing engineers. The other members are operators.*
- *The two manufacturing engineers determine the angle and procedure to consistently hit the distance of 50 inches.*
- *The manufacturing engineers will instruct the operators about the angle and method to hit the desired distance.*
- *One manufacturing engineer and four operators will launch the projectile ten (10) times.*
- *The other manufacturing engineer takes the role of inspector.*
- *The inspector measures the distances with 1/2 inch resolution without informing the others of the results.*
- *The inspector enters the results in chronological order into an empty Minitab worksheet.*
- *The data will be used for statistical analysis.*



# Basic Statistics Calculations - Using Minitab

*Basic Statistic Calculations for Catapult data.*

MINITAB FILE: Catapult.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and 'Basic Statistics' is selected. The 'Display Descriptive Statistics...' option is highlighted. Below the menu, a data table for 'CATAPULT.MTW' is visible. The table has columns C14 through C22. The data is as follows:

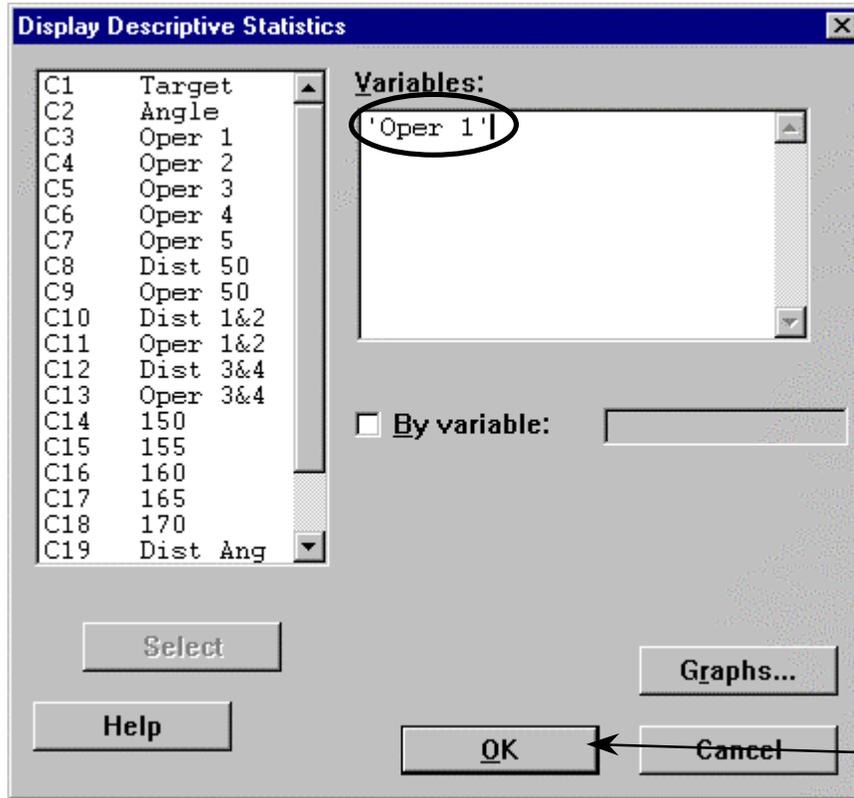
	C14	C15	C16	C17	C18	C19	C20	C21	C22
↓	150	155	160	165	170	Dist Ang	C Angle	D150&170	150&1
1	26.00	36.25	44.00	53.00	62.25	26.00	150	26.00	
2	26.00	36.25	44.00	52.50	62.50	26.00	150	26.00	
3	26.75	35.75	44.00	53.00	62.00	26.75	150	26.75	
4	27.50	36.25	44.25	53.75	63.25	27.50	150	27.50	
5	27.00	36.50	43.50	53.25	63.00	27.00	150	27.00	
6	26.75	35.00	44.00	53.50	62.75	26.75	150	26.75	
7	27.50	36.00	44.75	54.00	62.50	27.50	150	27.50	
8	26.50	36.50	44.50	54.25	61.00	26.50	150	26.50	

Calculate descriptive statistics and display in the Session window



# Tabular Output of Basic Statistical Calculations

1. Double click on "C3"



2. Click "OK"

Session

Descriptive Statistics

Variable	N	Mean	Median	Tr Mean	StDev	SE Mean
Oper 1	10	49.700	49.500	49.750	0.734	0.232

Variable	Min	Max	Q1	Q3
Oper 1	48.250	50.750	49.438	50.500



# Graphical Output of Basic Statistical Calculations

1. Double click on "C3"

Display Descriptive Statistics

Variables: 'Oper 1'

By variable:

Display Descriptive Statistics - Graphs

Histogram of data  
 Histogram of data, with normal curve  
 Dotplot of data  
 Boxplot of data  
 Graphical summary

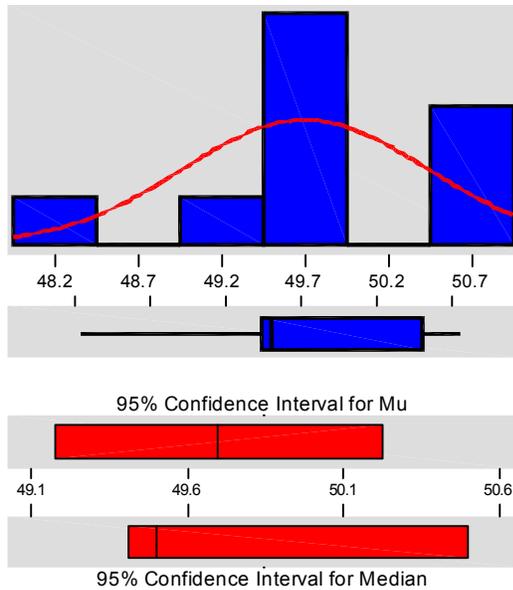
Confidence level: 95.0

Help OK Cancel

Select Help OK Cancel Graphs...

2. Click on Graphs Button to bring up the Graph Dialogue Box (See above)

## Descriptive Statistics



Variable: Oper 1

### Anderson-Darling Normality Test

A-Squared:	0.556
P-Value:	0.113
Mean	49.7000
StDev	0.7341
Variance	0.538889
Skewness	-3.5E-01
Kurtosis	0.547827
N	10
Minimum	48.2500
1st Quartile	49.4375
Median	49.5000
3rd Quartile	50.5000
Maximum	50.7500
95% Confidence Interval for Mu	49.1749 50.2251
95% Confidence Interval for Sigma	0.5049 1.3402
95% Confidence Interval for Median	49.4144 50.5000

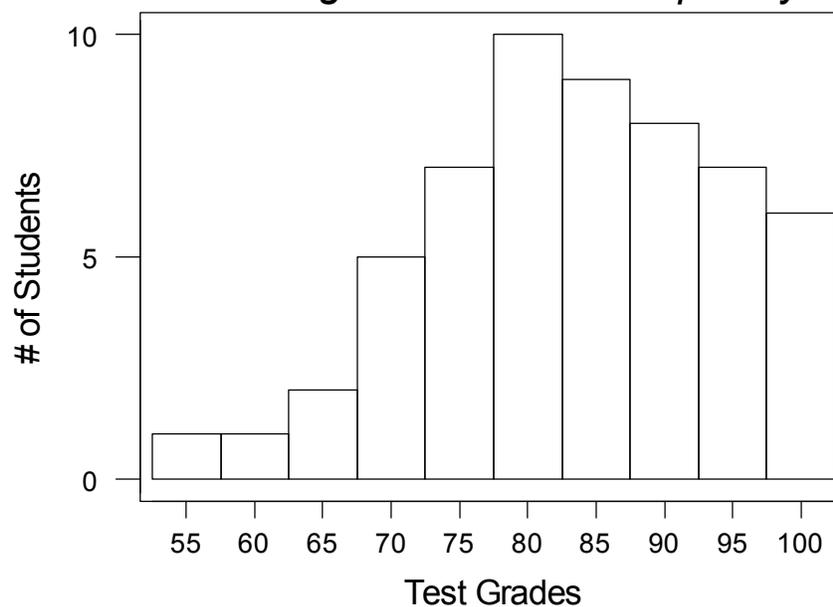


# Histogram

**Purpose:** To display variation in a process. Converts an unorganized set of data or group of measurements into a coherent picture.

**When:** To determine if process is on target meeting customer requirements. To determine if variation in process is normal or if something has caused it to vary in an unusual way.

**How:** Count the number of data points  
Determine the range (R) for entire set  
Divide range value into classes (K)  
Determine the class width (H) where  $H = R/K$   
Determine the end points  
Construct a frequency table based on values computed in previous step  
Construct a Histogram based on frequency table





# Histogram Example

MINITAB FILE: Catapult.mtw

The screenshot shows the Minitab software interface. The 'Graph' menu is open, and 'Histogram...' is selected. The background window displays a data table with the following content:

	C17	C20	C21	C22	C23
↓	165	C Angle	D150&170	150&170	
1	53.00	150	26.00	1	
2	52.50	150	26.00	1	
3	53.00	150	26.75	1	
4	53.75	150	27.50	1	
5	53.25	63.00	27.00	150	27.00

At the top of the data table, there are summary statistics: 49.500, 49.750, 0.734, and 0.2. Below these are Q1 (49.437) and Q3 (50.500). The status bar at the bottom reads 'Create histograms'.



# Histogram Output

1. Double click anywhere on the C3 line to select Oper 1 as a variable

**Graph variables:**

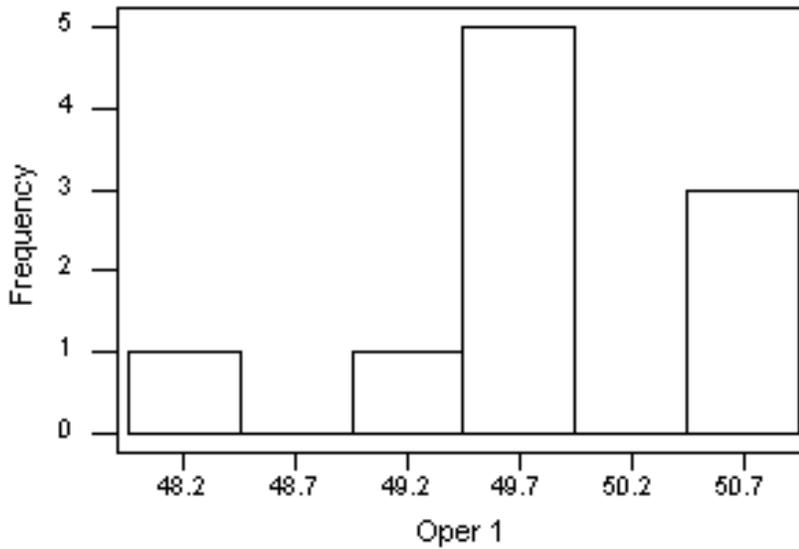
Graph	Variable
1	'Oper 1'
2	
3	

**Data display:**

Item	Display	For each	Group variables
1	Bar	Graph	
2			
3			

Buttons: Select, Annotation, Frame, Regions, Help, Options..., **OK**, Cancel

2. Click OK





## *Histogram Exercise*

*Enter your catapult data into columns C1 through C5. Label them “Oper 1” through “Oper 5.”*

- 1. Make a histogram of Oper 1.*
  - What are your observations?*
  
- 2. Make a histogram of Oper 2.*
  - Does it match the histogram for Oper 1?*
  - The LSL is 46, the target is 50, and the USL is 54 inches.*
  - Is the process on target?*
  - Is the process within the specification limits?*

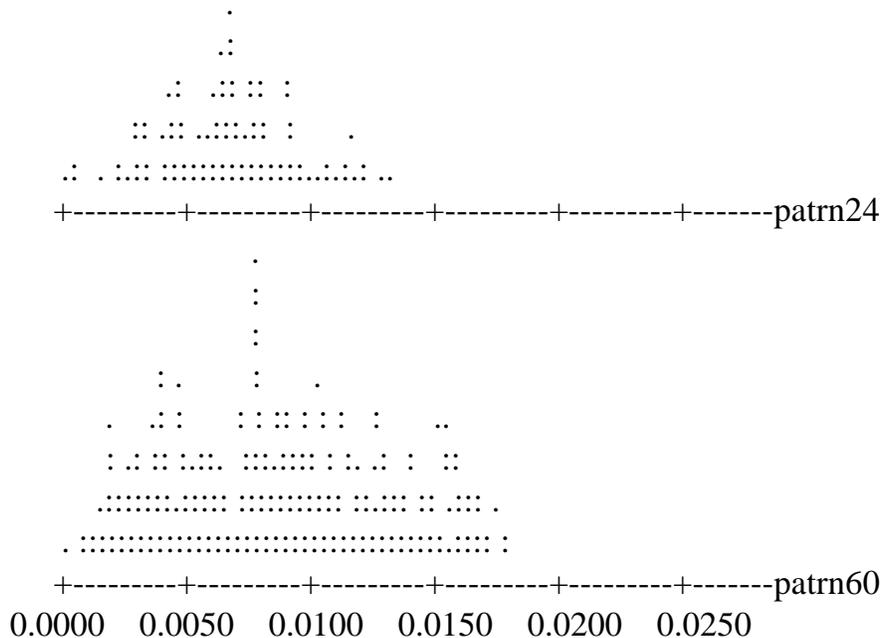


# Dot Plot

**Purpose:** *To display variation in a process.  
Quick graphical comparison of two or more processes.*

**When:** *First stages of data analysis.*

**How:** *Create an X axis.  
Scale the axis per the range in the data.  
Place a dot for each value along the X axis.  
Stack repeat dots.*





# Dot Plot Example

MINITAB FILE: Catapult.mtw

The screenshot shows the Minitab software interface with the 'Graph' menu open. The 'Character Graphs' option is selected, which has opened a sub-menu where 'Dotplot...' is highlighted. The Session window on the left contains the following commands:

```
MTB > %Describe 'C
SUBC> Confidence 9
Executing from file
MTB > Histogram 'Op
SUBC> MidPoint;
SUBC> Bar;
SUBC> ScFrame;
SUBC> ScAnnotatic
MTB >
```

The data table below shows the values for columns C17, C22, and C23:

	C17	C22	C23
↓	165	150&170	
1	53.00	1	
2	52.50	1	
3	53.00	1	
4	53.75	1	
5	53.25	1	

The status bar at the bottom of the window reads: "Create a character-style dotplot that prints in the Session window"



# Dot Plot Output

1. Double click here and here

2. Check "Same scale for all variables"

Dotplot

C1	Target
C2	Angle
C3	Oper 1
C4	Oper 2
C5	Oper 3
C6	Oper 4
C7	Oper 5
C8	Dist 50
C9	Oper 50
C10	Dist 1&2
C11	Oper 1&2
C12	Dist 3&4
C13	Oper 3&4
C14	150
C15	155
C16	160
C17	165

Variables:  
'Oper 1' 'Oper 2'

By variable:

Same scale for all variables

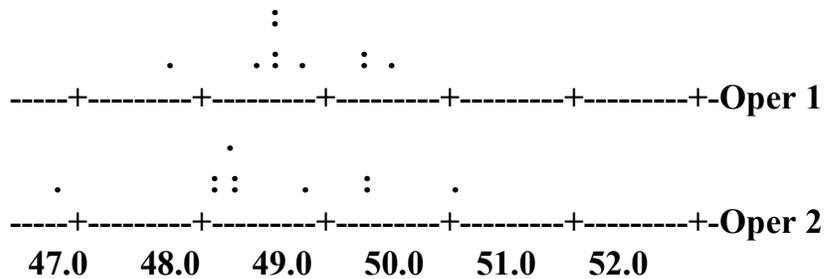
First midpoint:

Last midpoint:

Tick increment:

Select Help OK Cancel

3. Click "OK"





## *Dot Plot Exercise*

---

- *Using your catapult data, make a dot plot for each operator.*
- *What are your observations?*



## Stacking Data

Depending on what you want Minitab to do, you may need to organize your data in different ways. To create a box plot (the next tool we will demonstrate) you need stacked data.

To take your five catapult operator columns of data and stack them on top of each other, use the Stack command.

### MINITAB FILE: Your file

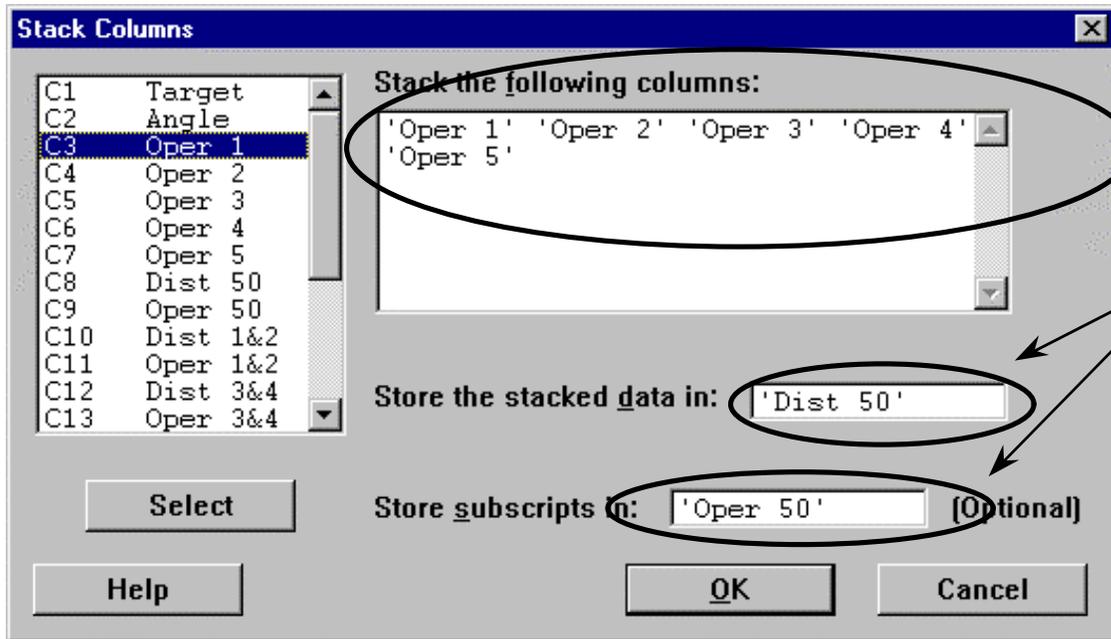
The screenshot shows the Minitab software interface. The 'Manip' menu is open, and the 'Stack/Unstack' option is selected, which has opened a sub-menu with 'Stack Columns...' highlighted. Below the menu, a spreadsheet is visible with columns C17 through C23. The data in the spreadsheet is as follows:

	C17	C18	C19	C20	C21	C22	C23
↓	165	170	Dist Ang	C Angle	D150&170	150&170	
1	53.00	62.25	26.00	150	26.00	1	
2	52.50	62.50	26.00	150	26.00	1	
3	53.00	62.00	26.75	150	26.75	1	
4	53.75	63.25	27.50	150	27.50	1	
5	53.25	63.00	27.00	150	27.00	1	



# Stacking - Input

1. Double click on each column label you want to stack



2. In these boxes enter the column numbers of the next available column in your data window. Example: "C8" or "C9"

3. Click on "OK"



# Stacking - Output

Target	Angle	Oper 1	Oper 2	Oper 3	Oper 4	Oper 5	Dist 50	Oper 50
50	162	50.50	50.50	46.50	49.00	50.00	50.50	1
50	162	50.50	49.00	50.00	50.25	49.75	50.50	1
50	162	49.75	51.50	49.25	50.50	49.75	49.75	1
50	162	49.50	50.50	48.75	49.75	50.00	49.50	1
50	162	49.50	47.00	49.00	50.00	48.75	49.50	1
50	162	48.25	48.75	49.75	50.25	50.00	48.25	1
50	162	50.75	49.00	50.00	50.00	49.75	50.75	1
50	162	49.25	49.00	49.75	50.50	50.25	49.25	1
50	162	49.50	48.75	48.00	50.00	50.75	49.50	1
50	162	49.50	49.75	50.25	49.75	50.25	49.50	1
							50.50	2
							49.00	2
							51.50	2
							50.50	2
							47.00	2
							48.75	2
							49.00	2
							49.00	2
							48.75	2
							49.75	2
							46.50	3
							50.00	3
							49.25	3
							48.75	3
							49.00	3
							49.75	3
							50.00	3
							49.75	3
							48.00	3
							50.25	3
							49.00	4
							50.25	4
							50.50	4
							49.75	4
							50.00	4
							50.25	4
							50.00	4
							50.50	4
							50.00	4
							49.75	4
							50.00	5
							49.75	5
							49.75	5
							50.00	5
							48.75	5
							50.00	5
							49.75	5
							50.25	5
							50.75	5
							50.25	5



## *Box Plot*

---

- **Purpose:**

- *To begin an understanding of the distribution of the data*
- *To get a quick, graphical comparison of two or more processes*

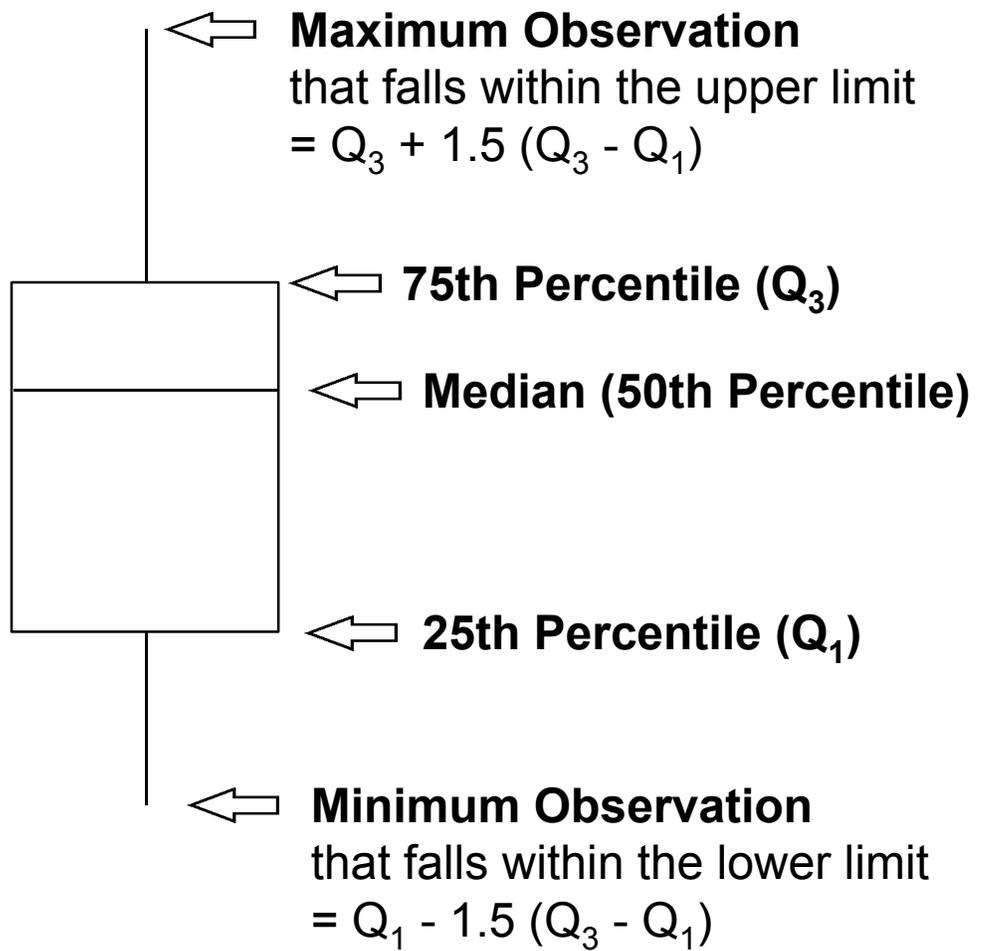
- **When:** *First stages of data analysis.*

- **How:** *Let Minitab do it.*



## Box and Whisker Plot

\* ← **Outlier**  
any point outside the lower or upper limit.





# Box and Whisker Plot Example

## MINITAB FILE: Catapult.mtw

The screenshot shows the Minitab software interface. The 'Graph' menu is open, and 'Boxplot...' is selected. The background shows a worksheet with the following data:

	C17	C20	C21	C22	C23
↓	165	C Angle	D150&170	150&170	
1	53.00	150	26.00	1	
2	52.50	150	26.00	1	
3	53.00	150	26.75	1	
4	53.75	150	27.50	1	
5	53.25	63.00	27.00	150	27.00

The 'Boxplot' dialog box is shown. The 'Graph variables' section is set to 'Y (measurement) vs X (category)'. The 'Data display' section is set to 'Item 1: IQRRange Box Graph' and 'Item 2: Outlier S>> Graph'. The 'C8 Dist 50' variable is selected in the list on the left.

1. Double clicking chooses the variables to graph

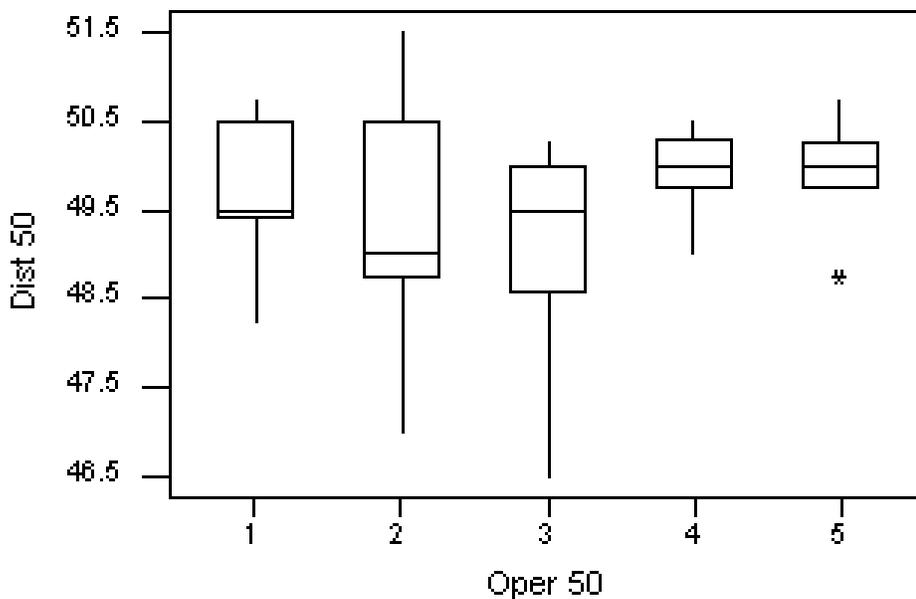


2. Click on OK



## *Box and Whisker Plot Output*

*For your catapult data, make a Box and Whisker Plot by operator.*



*What are your observations?*



## *Run Chart*

---

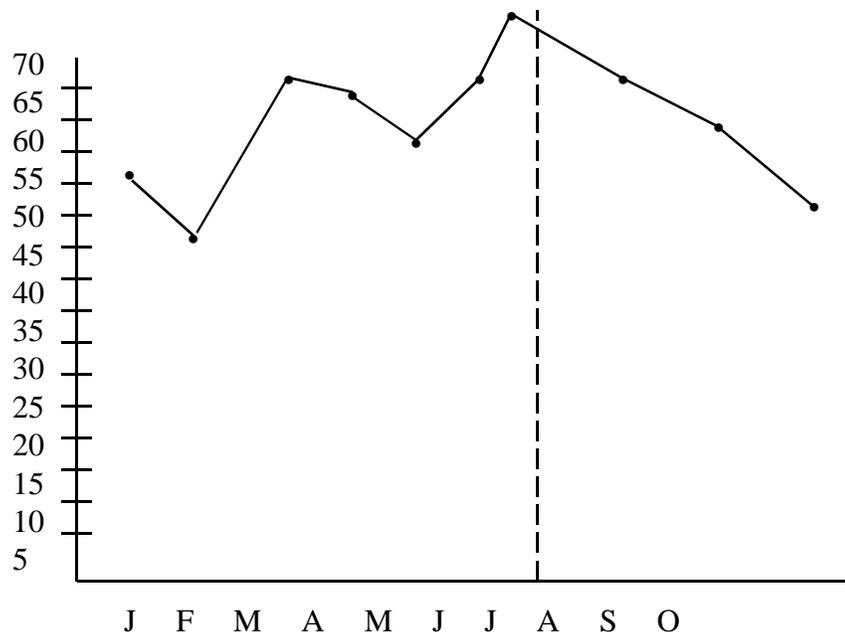
**Purpose:** *To track process over time in order to display trends and focus attention on changes in the process*

**When:** *To establish a baseline of performance for improvement*  
*To uncover changes in your process*  
*To brainstorm possible causes for trends*  
*To compare the historical performance of a process with the improved process*



## Run Chart

- How:*
- Determine what you want to measure*
  - Determine period of time to measure and in what time increments*
  - Create a graph (vertical axis = occurrences, horizontal axis = time)*
  - Collect data and plot*
  - Connect data points with solid line*
  - Calculate average of measurements, draw solid horizontal line on run chart*
  - Analyze results*
  - Indicate with a dashed vertical line when a change was introduced to the process*





# Run Chart Example

## MINITAB FILE: Catapult.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the 'Quality Tools' submenu is selected, with 'Run Chart...' highlighted. The 'Session' window on the left displays the following data:

	C17	C18	C19
↓	165	170	Dist Ang
1	53.00	62.25	26.00
2	52.50	62.50	26.00
3	53.00	62.00	26.75
4	53.75	63.25	27.50
5	53.25	63.00	27.00

The 'Catapult.mtw' data table at the bottom shows the following data:

	C22	C23
50&170		
1		
1		
1		
1		
1		

At the bottom of the Minitab window, a status bar reads: "Draw a run chart with tests for randomness".



# Run Chart Output - Subgroup Size 1

- 1. Double Click on "C8"
- 2. Enter a "1" in subgroup size
- 3. Click "OK"

**Run Chart**

Data are arranged as

Single column: 'Dist 50'

Subgroup size: 1  
(use a constant or an ID column)

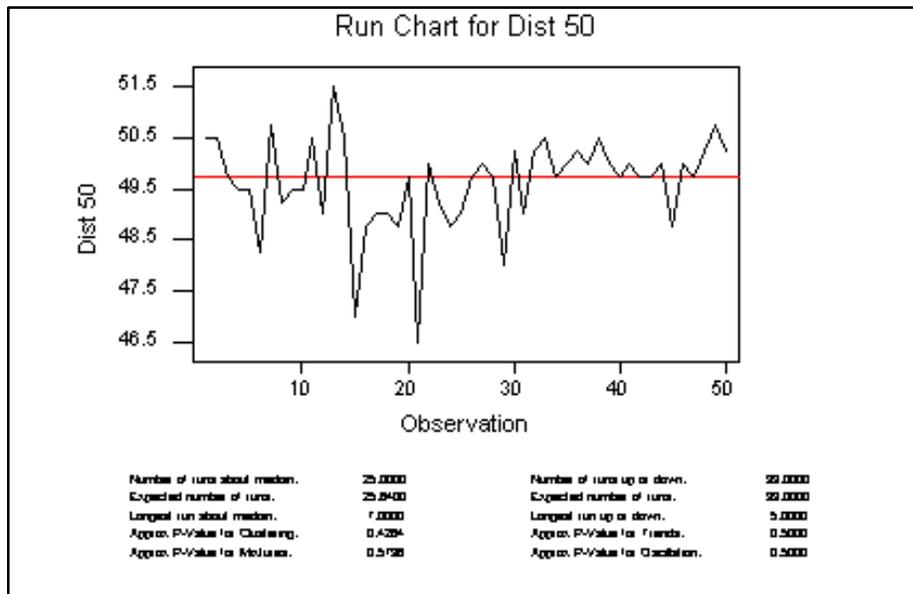
Subgroups across rows of:

For data in subgroups

Plot subgroup means

Plot subgroup medians

Select OK Cancel





# Run Chart Output - Subgroup Size 10

1. Double click on "Distance 50."
2. Enter a "10" in "Subgroup size"
3. Click "OK"

**Run Chart**

C1	Target
C2	Angle
C3	Oper 1
C4	Oper 2
C5	Oper 3
C6	Oper 4
C7	Oper 5
<b>C8</b>	<b>Dist 50</b>
C9	Oper 50
C10	Dist 1&2
C11	Oper 1&2
C12	Dist 3&4
C13	Oper 3&4
C14	150
C15	155
C16	160
C17	165
C18	170
C19	Dist Ang

Data are arranged as

Single column: 'Dist 50'

Subgroup size: 10  
(use a constant or an ID column)

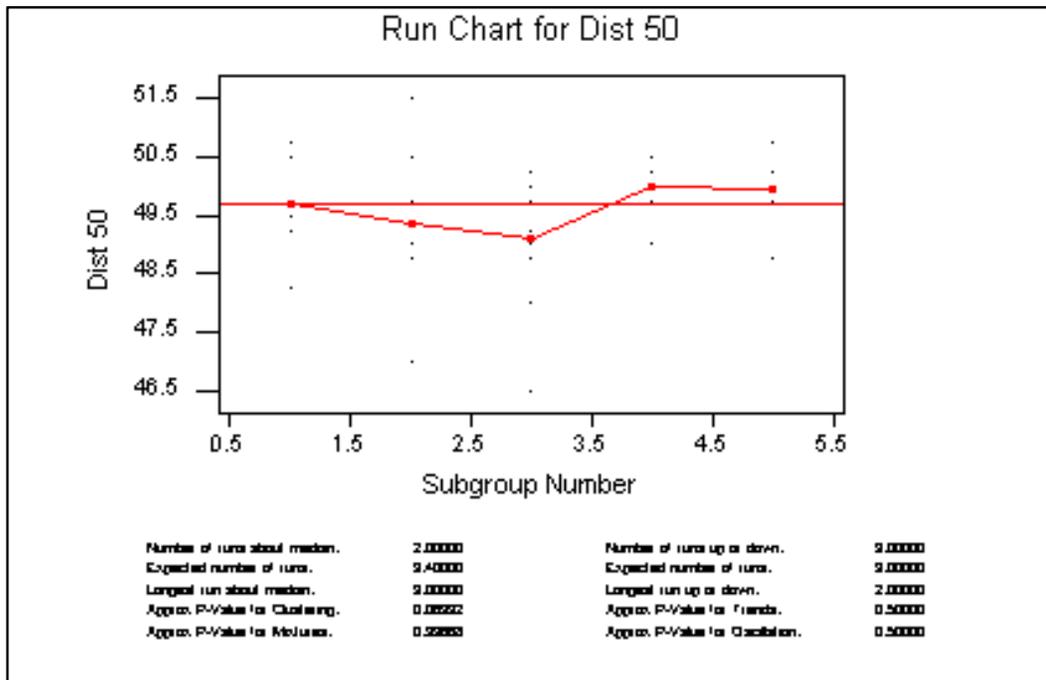
Subgroups across rows of:

For data in subgroups

Plot subgroup means

Plot subgroup medians

Buttons: Select, Help, Options..., OK, Cancel





## *Run Chart Exercise*

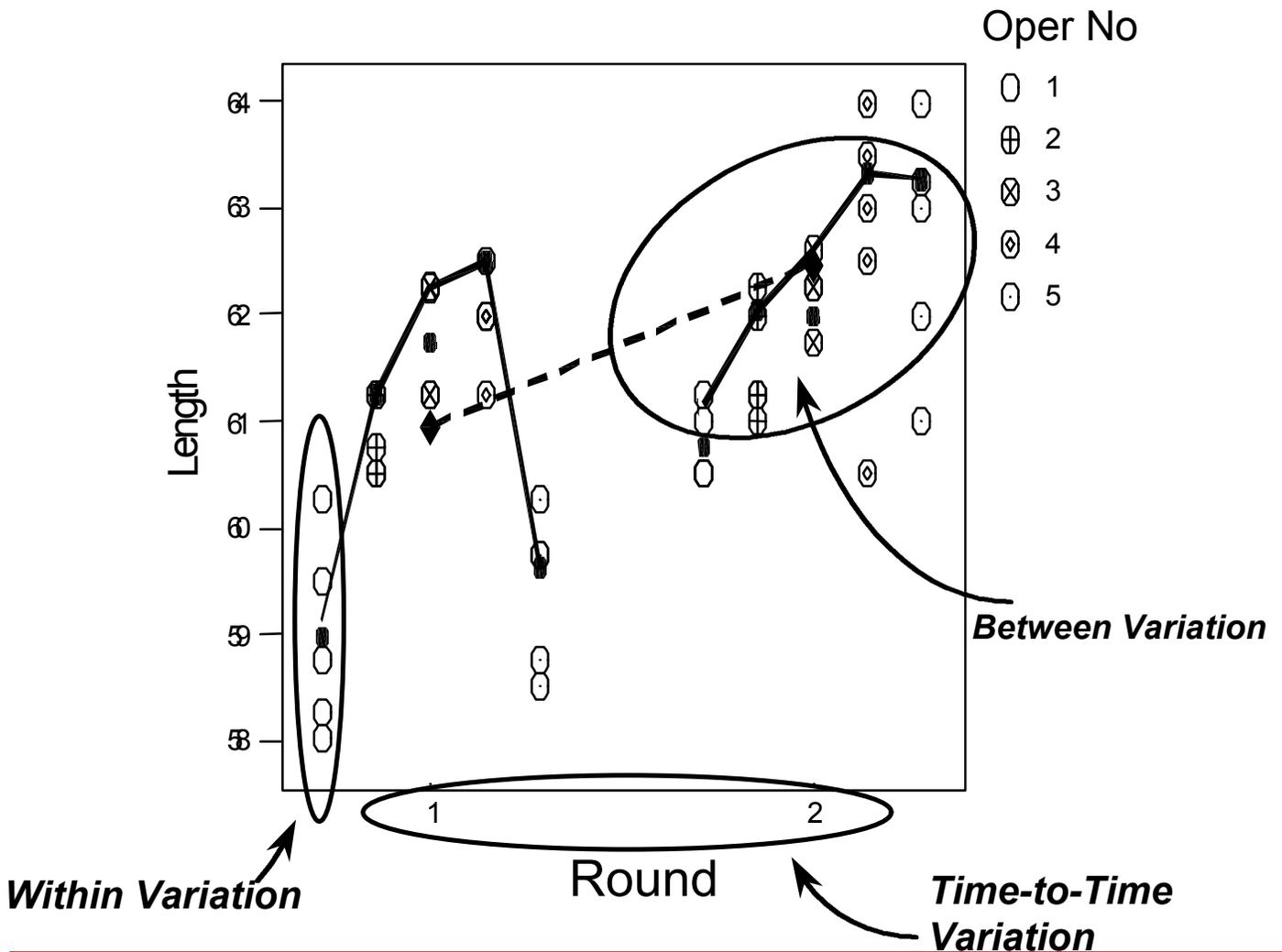
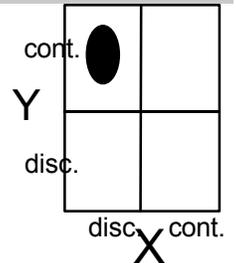
---

- *Make a run chart of your team data. Use a subgroup size of 1 and 10.*
- *What are your observations?*



# Multi-Vari Chart

Purpose: To identify the most important types or families of variation  
 To make an initial screen of process output for potential Xs





# Multi-Vari Chart - Minitab Commands

1

**Multi-Vari Chart**

Stat Graph Editor Window

Basic Statistics  
Regression  
ANOVA  
DOE  
Control Charts  
**Quality Tools**  
Reliability/Survival  
Multivariate  
Time Series  
Tables  
Nonparametrics  
EDA  
Power and Sample Size

Run Pareto Causality Capability Analysis (Weibull)... Capability Sixpack (Normal)... Capability Sixpack (Weibull)... Capability Analysis (Binomial)... Capability Analysis (Poisson)... Gage Run Chart... Gage Linearity Study... Gage R&R Study... **Multi-Vari Chart...** Symmetry Plot...

Response: Length  
Factor 1: OperNo  
Factor 2: Rounds  
Factor 3:

Options...  
OK  
Cancel

**Multi-Vari Chart - Options**

Display options  
 Display individual data points  
 Connect means for Factor 1  
 Connect means for Factor 2

Title:  
Help

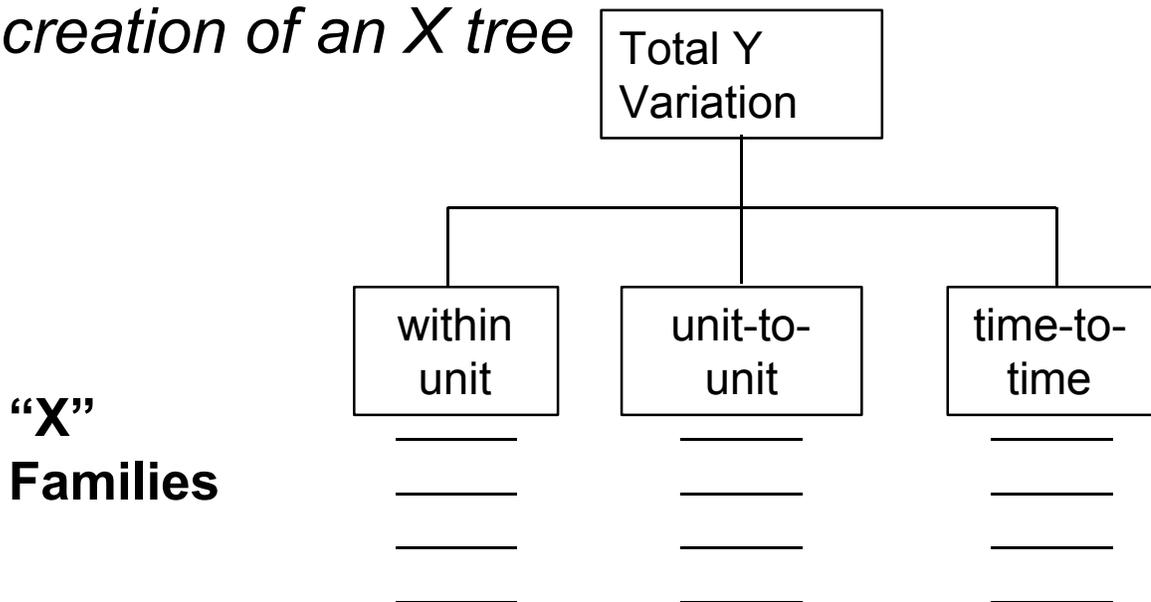
Check **Display individual...** and **Connect means...boxes**

Annotations:  
2: Enter Length for Response  
3: Enter OperNo for Factor 1  
4: Enter Rounds for Factor 2  
5: Options...  
6: Check Display individual... and Connect means...boxes



## Notes on the Multi-Vari Chart

- *Data must be in time order, preferably in the order of production (not necessarily in order of measurement)*
- *Data must be subgrouped or tagged by family of variation—the most common families of variation are: within unit, unit-to-unit (between units), and time-to-time*
- *The tagging may be guided by the creation of an X tree*



- *If a particular family of variation is unimportant, cross off that family—this narrows your search for the vital few Xs*

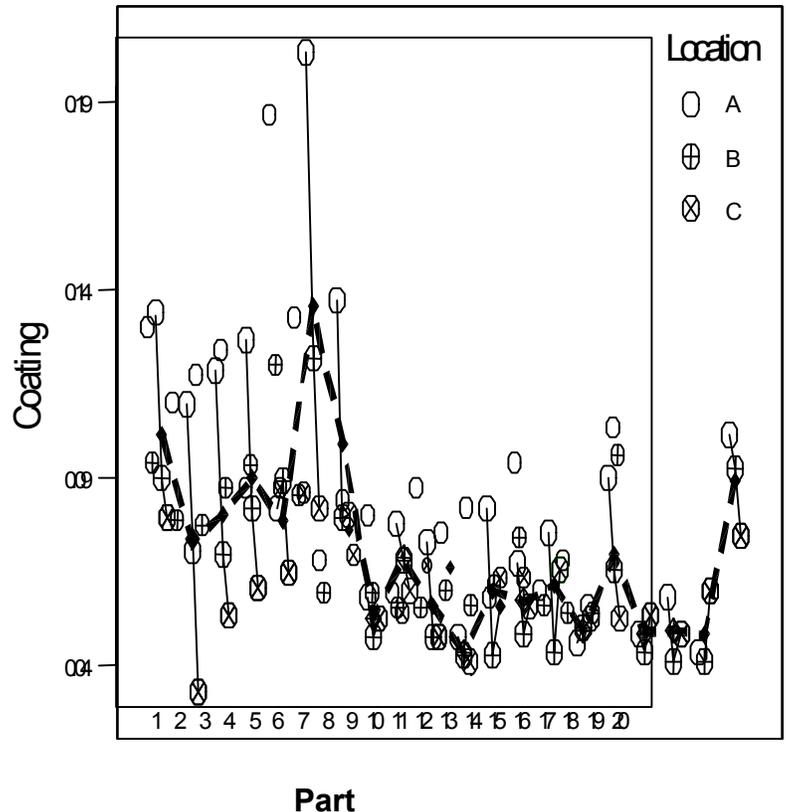
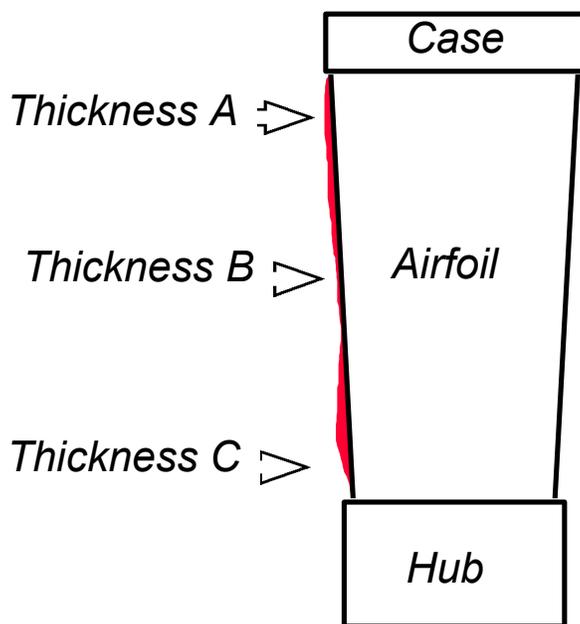


## Multi-Vari Chart Exercise

Coating thickness is a concern. The thickness of a coating applied to airfoils is being measured in three places: A, B, and C; as shown in the diagram below. Three parts produced each hour.

Which families of variation are dominant?

Multi-Vari Chart for Coating By Location - Part





## *Process Capability Catapult Example*

*What is the capability of the Catapult Process?*

*Target = 50"*

*$\bar{x} = 50.5"$  (historic data)*

*USL = 54"*

*$s = 1.1"$  (historic data)*

*LSL = 46"*

$$Z_{USL} = \frac{USL - \bar{x}}{s} = \frac{54 - 50.5}{1.1} = 3.18$$

$$Z_{LSL} = \frac{\bar{x} - LSL}{s} = \frac{50.5 - 46}{1.1} = 4.09$$

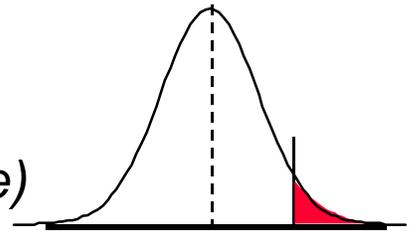


## Probability of a Defect Catapult Example

What's the probability of shooting the catapult too far?  
(Probability of a defect  $> USL$ )

$$Z_{USL} = 3.18$$

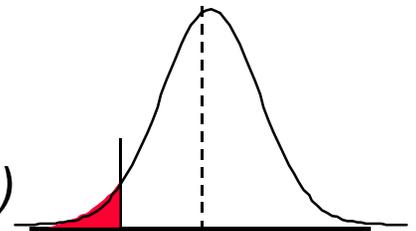
$$P(\text{defect}_{USL}) = 0.000736 \text{ (from Z table)}$$



What's the probability of shooting the catapult too short?  
(Probability of a defect  $< LSL$ )

$$Z_{LSL} = 4.09$$

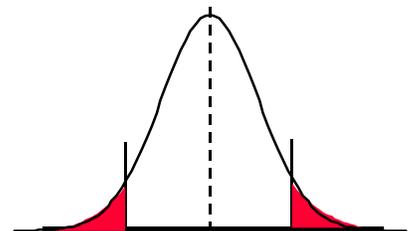
$$P(\text{defect}_{LSL}) = 0.0000216 \text{ (from Z table)}$$



What's the probability of shooting the catapult too far  
or too short? (Probability of a defect  $> USL + < LSL$ )

$$P(\text{defect}_{USL + LSL}) = 0.000736 + 0.0000216 = 0.0007576$$

$$Z_{Bench} = 3.17 \text{ (from Z table)}$$





## Process Capability Catapult Exercise

Using your data for one operator:

The LSL is 46, the target is 50, and the USL is 54 inches.

Is the process on target?

What is the Process Capability?

$$\bar{x} = \underline{\hspace{2cm}}$$

$$s = \underline{\hspace{2cm}}$$

$$Z_{USL} = \frac{USL - \bar{x}}{s} = \underline{\hspace{2cm}} \quad P(\text{defect}_{USL}) = \underline{\hspace{2cm}}$$

$$Z_{LSL} = \frac{\bar{x} - LSL}{s} = \underline{\hspace{2cm}} \quad P(\text{defect}_{LSL}) = \underline{\hspace{2cm}}$$

$$P(\text{defect}_{TOTAL}) = \underline{\hspace{2cm}}$$

$$Z_{Bench} = \underline{\hspace{2cm}} \quad (\text{from } Z \text{ table})$$

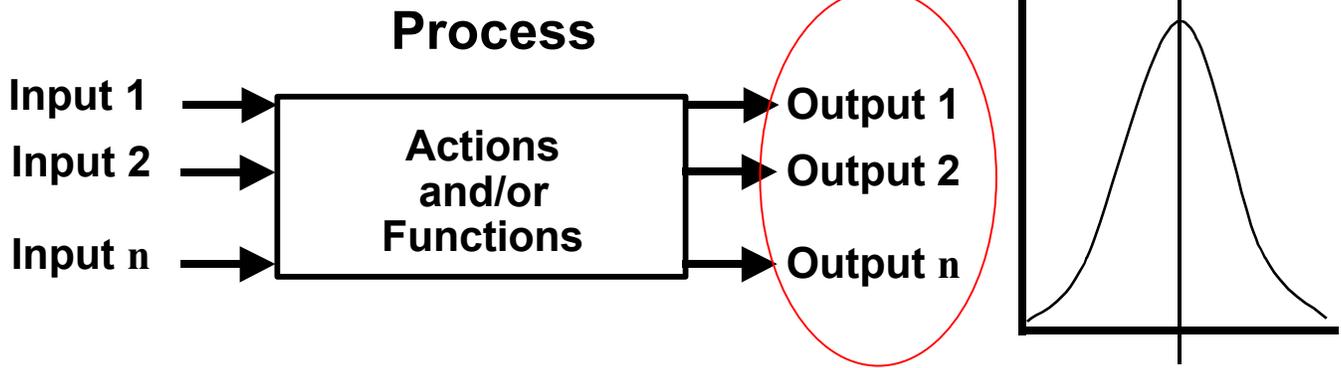


---

# ***Normality Testing***

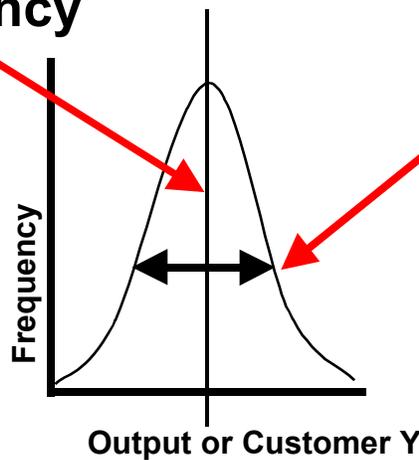


# Normality Testing



## Central Tendency

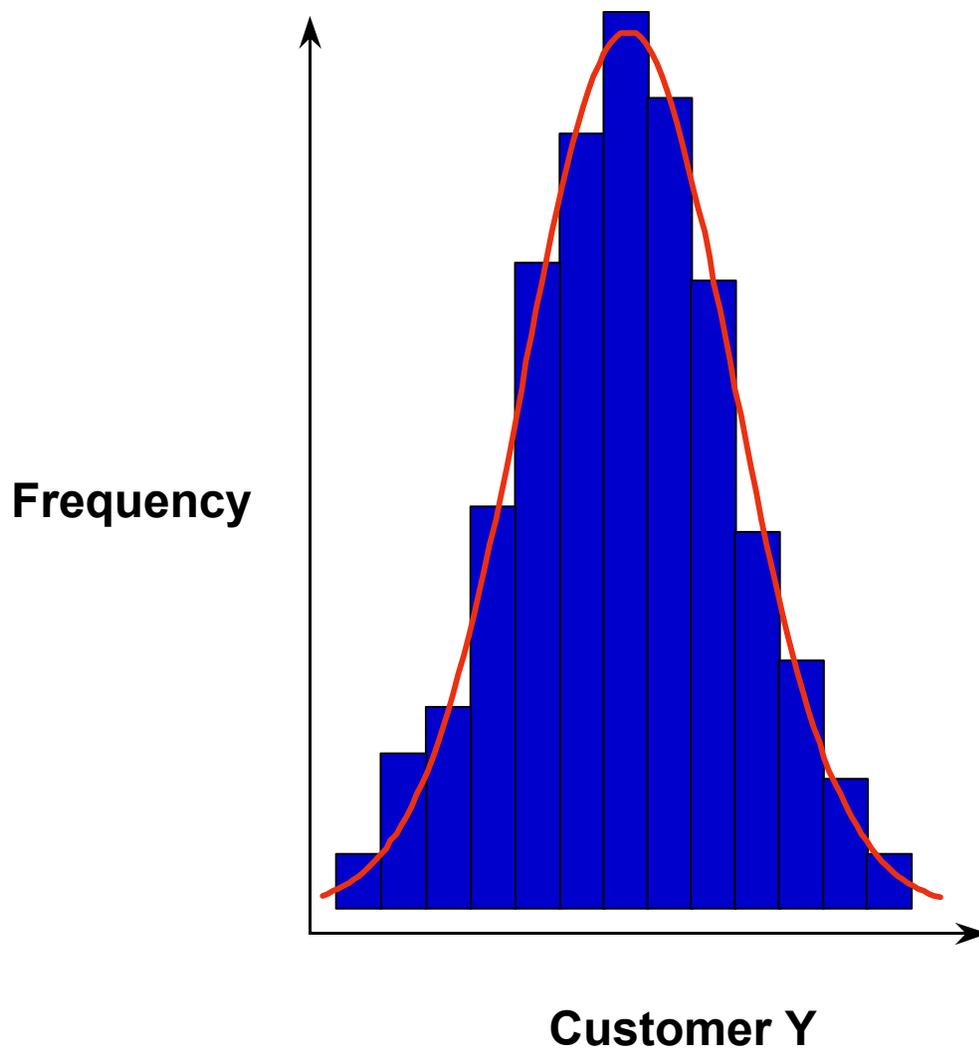
(The “Middle” of the distribution or “Nominal Performance)



**Spread or Variance**

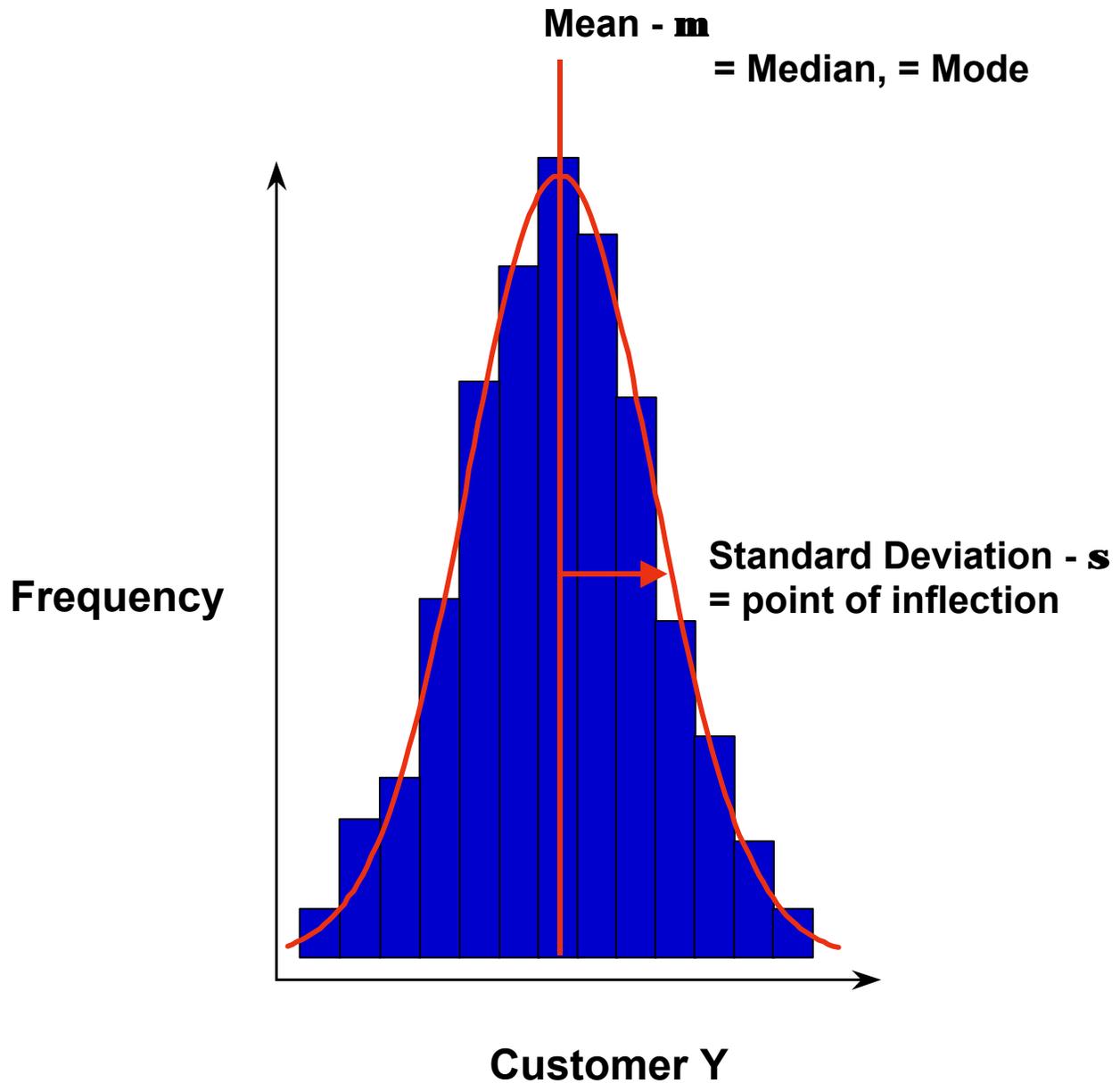


# *Normality Testing*





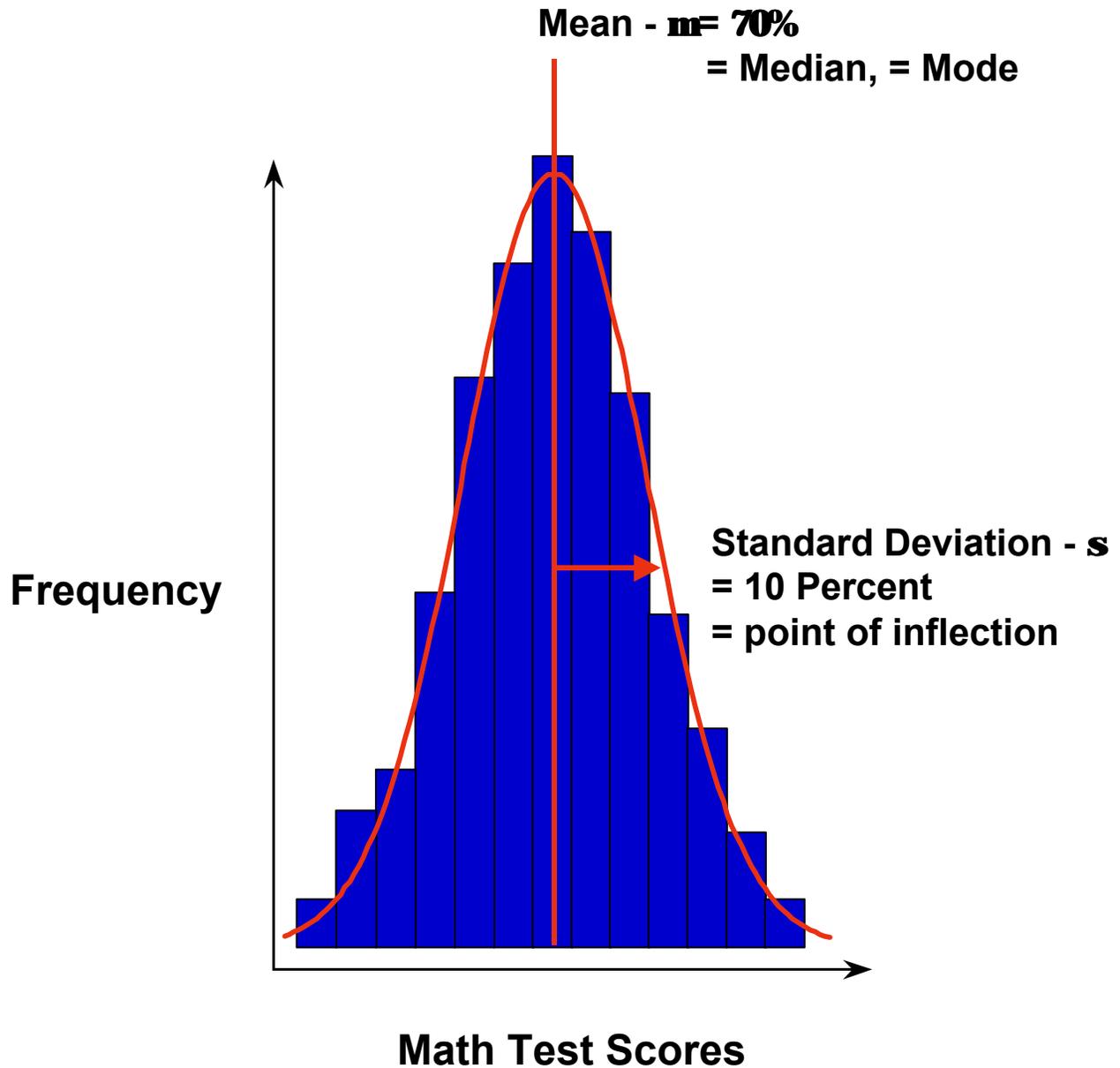
# Normality Testing





# Normality Testing

*Add values on both axes, for reference?*

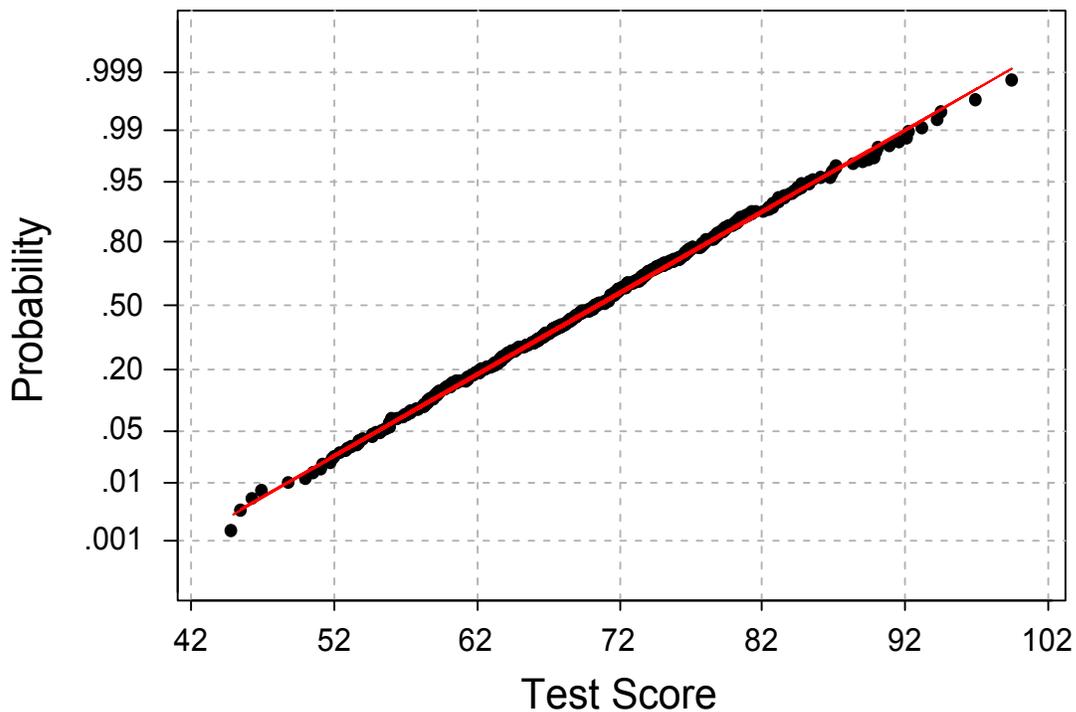




# Normality Testing

In Minitab, you can perform a Normality Test (Stat - Basic Statistics - Normality Test)

### Normal Probability Plot



Average: 70.4015  
StDev: 9.26060  
N: 500

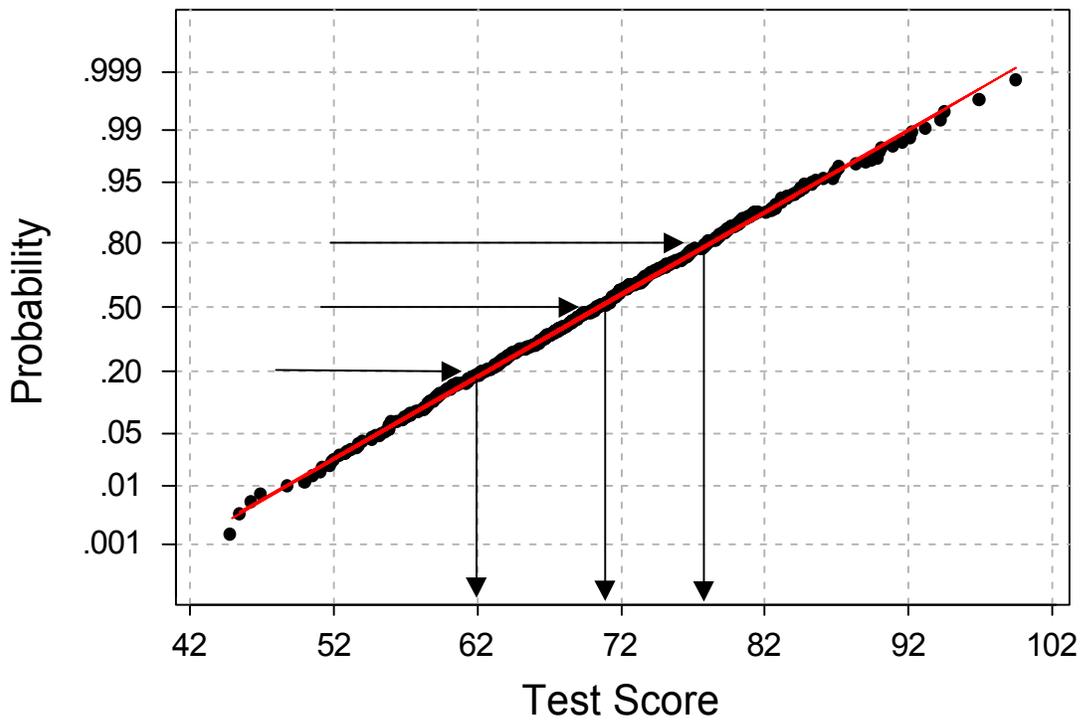
Anderson-Darling Normality Test  
A-Squared: 0.141  
P-Value: 0.973

**This data is Normal.**



# Normality Testing

## Normal Probability Plot



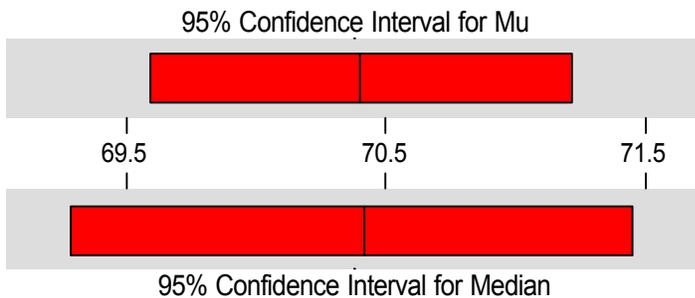
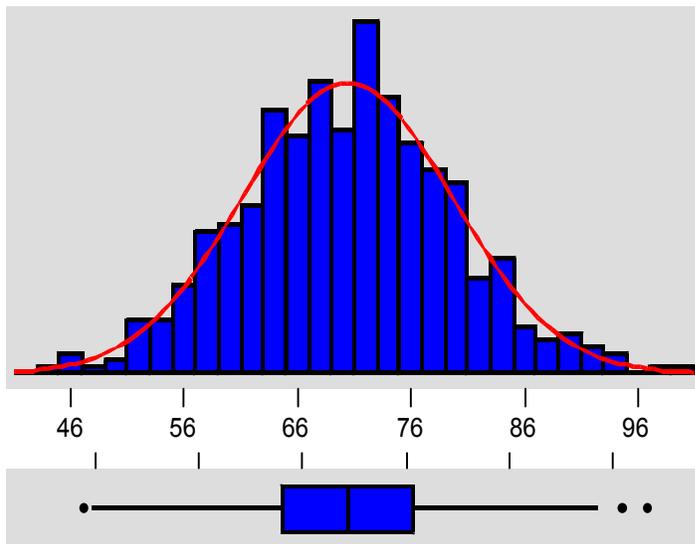
Average: 70.4015  
StDev: 9.26060  
N: 500

Anderson-Darling Normality Test  
A-Squared: 0.141  
P-Value: 0.973



# Normality Testing

## Descriptive Statistics



Variable: Test Score

**This data is Normal**

Anderson-Darling Normality Test

A-Squared:	0.141
P-Value:	0.973

Mean	70.4015
StDev	9.2606
Variance	85.7587
Skewness	7.39E-02
Kurtosis	1.23E-02
N	500

**Note that the Mean = Median**

Minimum	44.8481
1st Quartile	64.0464
Median	70.4147
3rd Quartile	76.6338
Maximum	99.5130

95% Confidence Interval for Mu	
69.5878	71.2152

95% Confidence Interval for Sigma	
8.7200	9.8732

95% Confidence Interval for Median	
69.2813	71.4524



# Normality Testing

The output of many processes are normally distributed.

But not all data is normal. There are several reasons why the data might be non-normal (such as Failure data, etc) but we won't go into detail on that at this time.

In practice, we find that one of the most common reasons for non-normal data is that we are measuring the process at the "Whole Business" level (as opposed to a more narrowed down scope). We then often encounter non-normal data.

This could be because the particular process we are measuring simply does not produce an output which is normally distributed, or it could also be that we have more than one process aggregated into our data set.

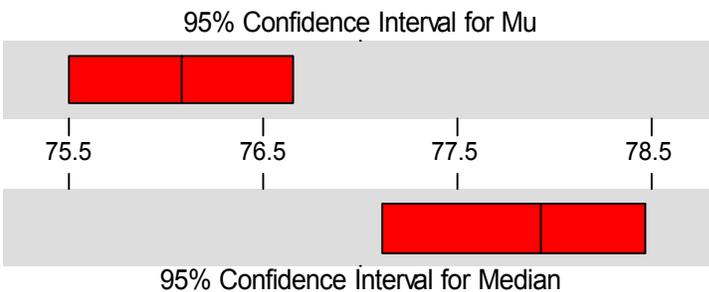
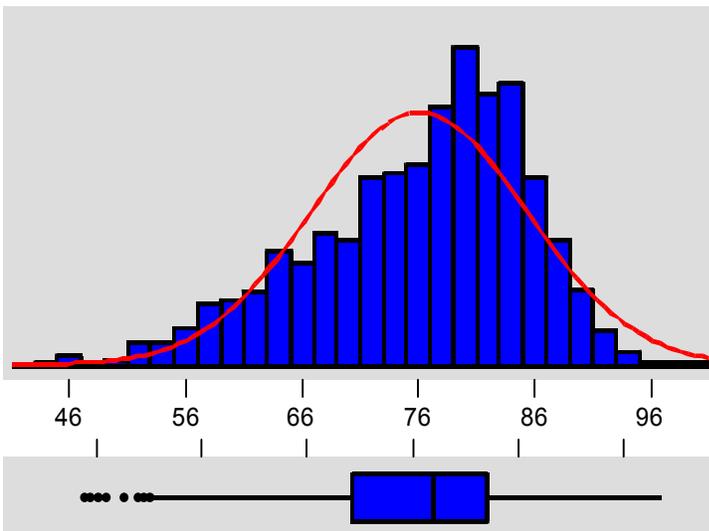
To illustrate this concept, let's take two different sets of test scores. Last year's scores had an average of 70 with a standard deviation of 10 points (as in the previous example) and, after making changes to the curriculum, this year's scores have an average of 82 and a standard deviation of 5. We have 500 test scores from each year, so the total data set has 1000 data points.

Let's see what the distribution and normality plot look like.



# Normality Testing

## Descriptive Statistics



Variable: Test Score **This data is not Normal.**

Anderson-Darling Normality Test

A-Squared: 10.513  
P-Value: 0.000

Mean 76.0732  
StDev 9.3036  
Variance 86.5561  
Skewness -6.4E-01  
Kurtosis 5.07E-03  
N 1000

Minimum 44.8481  
1st Quartile 70.2410  
Median 77.9322  
3rd Quartile 82.9794  
Maximum 99.5130

95% Confidence Interval for Mu  
75.4959 76.6506

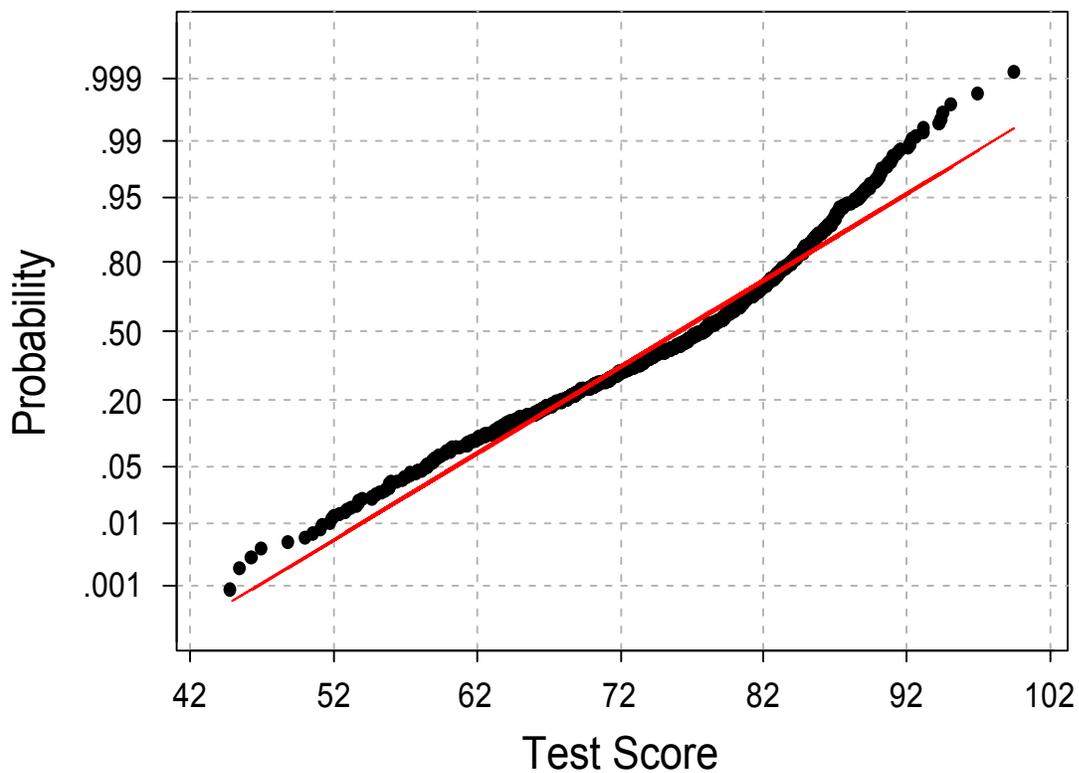
95% Confidence Interval for Sigma  
8.9129 9.7303

95% Confidence Interval for Median  
77.1074 78.4734



# Normality Testing

## Normal Probability Plot



Average: 76.0732  
StDev: 9.30356  
N: 1000

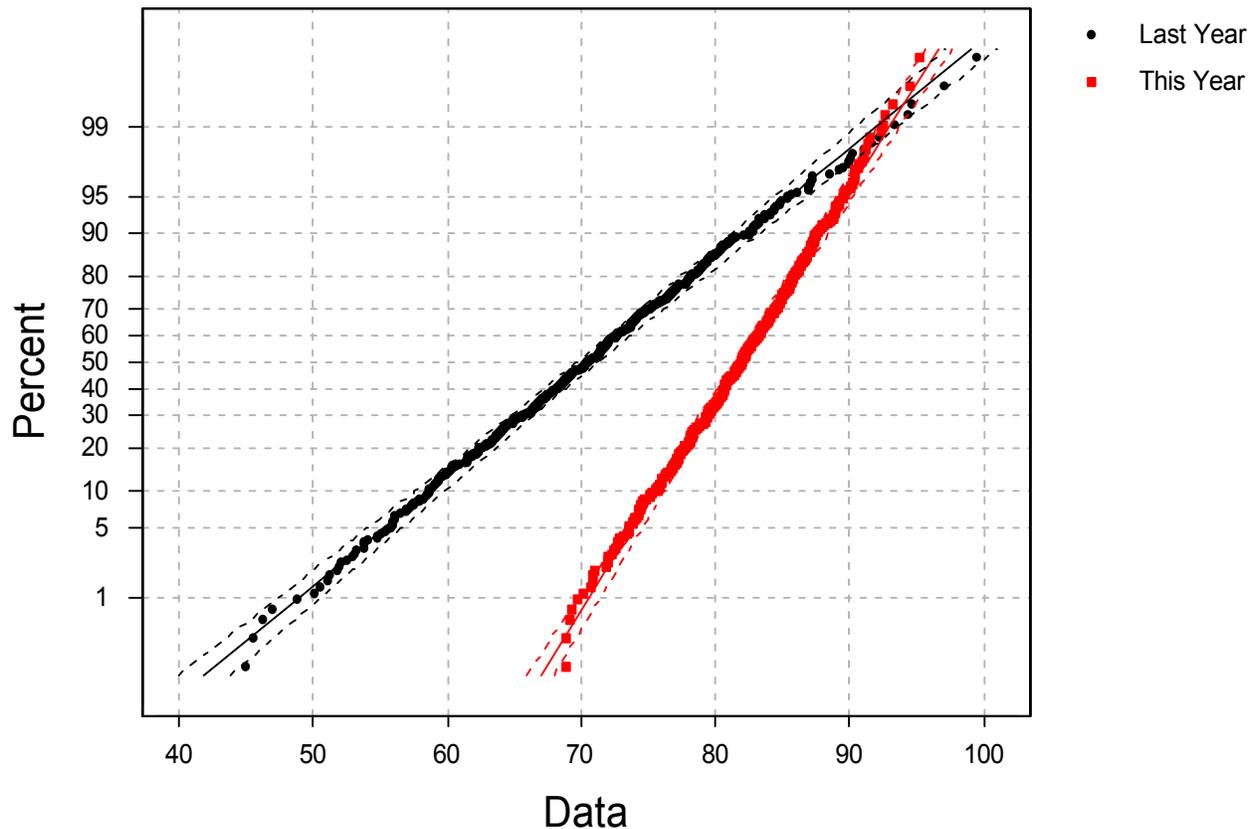
Anderson-Darling Normality Test  
A-Squared: 10.513  
P-Value: 0.000



# Normality Testing

**Minitab (Graph - Probability Plot)** You may graph more than one column or choose one column as the output separate it on a label in another column.

Normal Probability Plot for Last Year...This Year





## *Normality Testing*

---

**The first step in dealing with non-normal data is to evaluate whether or not you have more than one process output aggregated into the output distribution.**

**Sometimes this is very difficult to do. If your project data is non-normal, you should consult with your MBB for help in evaluating the data.**

**Sometimes, the output distribution is not an aggregate of multiple processes and still results in non-normal data. In that case, the mean and standard distribution can be misleading. Other statistical indicators of the central tendency and variance should be used.**

**Let's look at an example of an output distribution where the measurement is relative to a target. This example is Plastics' On time delivery data from 1999. The target date for delivery is established by the customer. We measure the number of days early or late based on the target. So, if the customer requested delivery on March 22 and it was delivered on March 26, the data point is 4 days late.....etc.**



## *Normality Testing*

---

***Let's Look At An Example ...***

***GE Plastics Lexan Q1 1999***

**Customer Expectation:** Each Order will be delivered on the day requested

**Unit:** An Order

**Measure:** Days Early/Late to the Customer Request

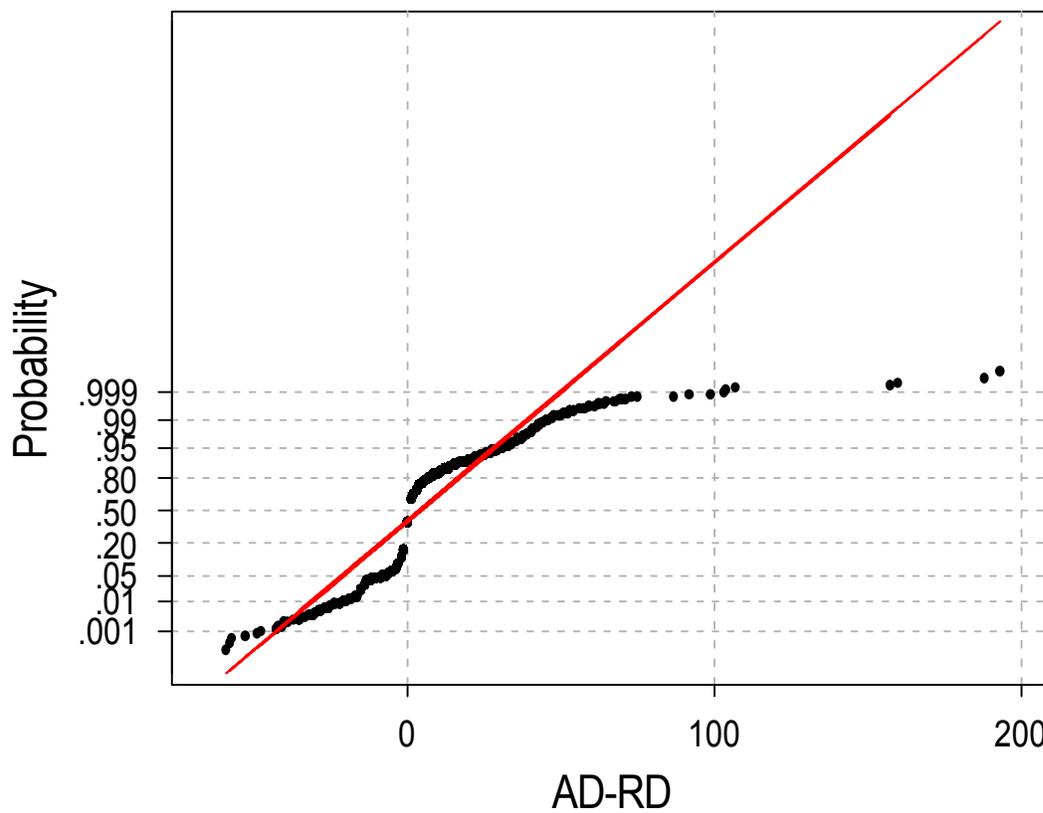
**8461 Deliveries in the 1st Quarter of 1999**



# Normality Testing

## Plastics Deliveries 1st Q '99

### Normal Probability Plot



Average: 3.90178  
StDev: 12.3470  
N: 8461

Anderson-Darling Normality Test  
A-Squared: 975.177  
P-Value: 0.000

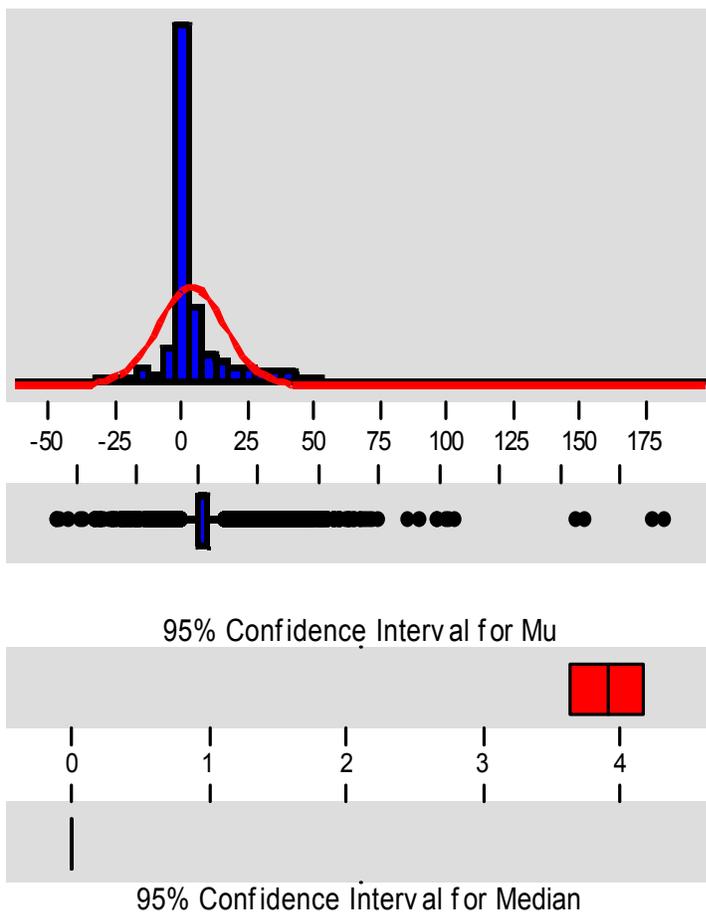
**The Data is not normal.**



# Normality Testing

## Plastics Deliveries 1st Q '99

### Descriptive Statistics



### Variable: Early/Late

#### Anderson-Darling Normality Test

A-Squared: 975.177  
P-Value: 0.000

Mean 3.9018  
StDev 12.3470  
Variance 152.448  
Skewness 2.92315  
Kurtosis 24.5556  
N 8461

Minimum -59.000  
1st Quartile 0.000  
Median 0.000  
3rd Quartile 4.000  
Maximum 193.000

95% Confidence Interval for Mu  
3.639 4.165

95% Confidence Interval for Sigma  
12.164 12.536

95% Confidence Interval for Median  
0.000 0.000

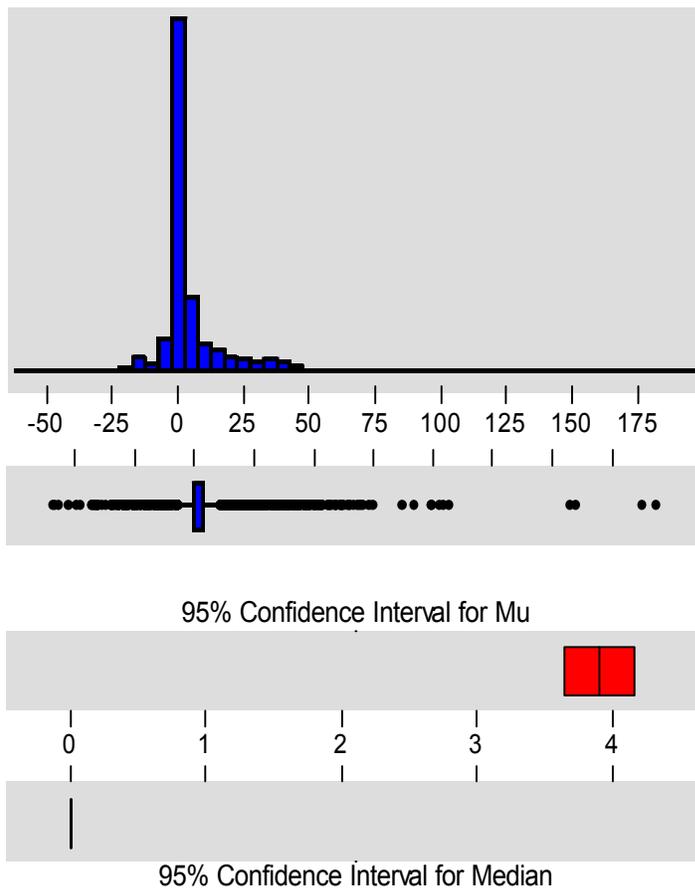
**You can see that the normal curve Minitab draws does not represent the data.**



# Normality Testing

## Plastics Deliveries 1st Q '99

### Descriptive Statistics



Variable: Early/Late

Anderson-Darling Normality Test

A-Squared: 975.177  
P-Value: 0.000

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0.000 0.000

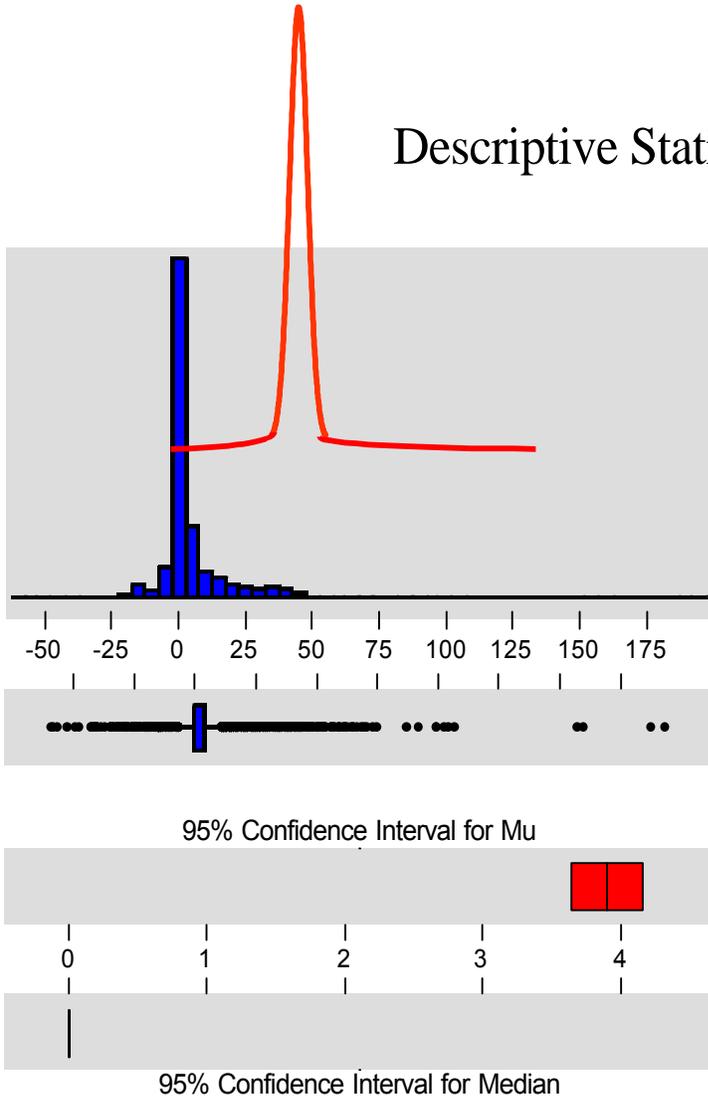
**Here is the Histogram without the attempt at fitting a normal curve.**



# Normality Testing

## Plastics Deliveries 1st Q '99

### Descriptive Statistics



Variable: Early/Late

Anderson-Darling Normality Test

A-Squared: 975.177  
P-Value: 0.000

Mean 3.9018  
StDev 12.3470  
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0.000 0.000

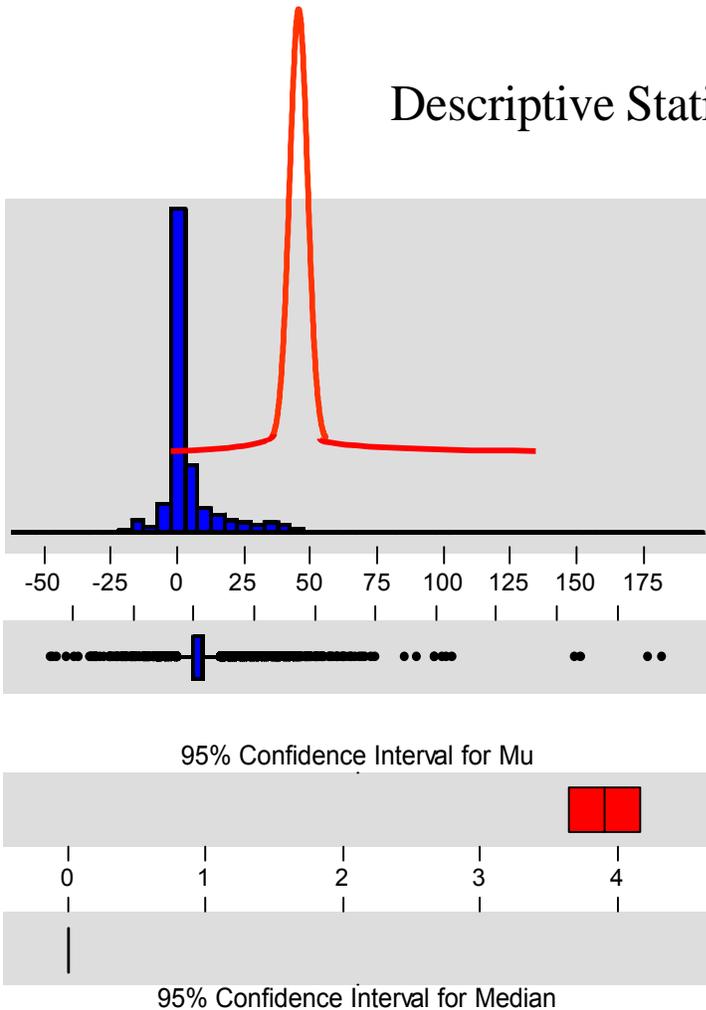
**Here is a better attempt to “fit” a curve to the data.**



# Normality Testing

## Plastics Deliveries 1st Q '99

### Descriptive Statistics



Variable: Early/Late

#### Anderson-Darling Normality Test

A-Squared:	975.177
P-Value:	0.000

Mean	3.9018
StDev	12.3470
Variance	152.448
Skewness	2.92315
Kurtosis	24.5556
N	8461

Minimum	-59.000
1st Quartile	0.000
Median	0.000
3rd Quartile	4.000
Maximum	193.000

95% Confidence Interval for Mu	
3.639	4.165

95% Confidence Interval for Sigma	
12.164	12.536

95% Confidence Interval for Median	
0.000	0.000

**Mean = 4 days late      Std. Dev. = 12 days**

**Let's see why the mean and standard deviation will not work in this case.**

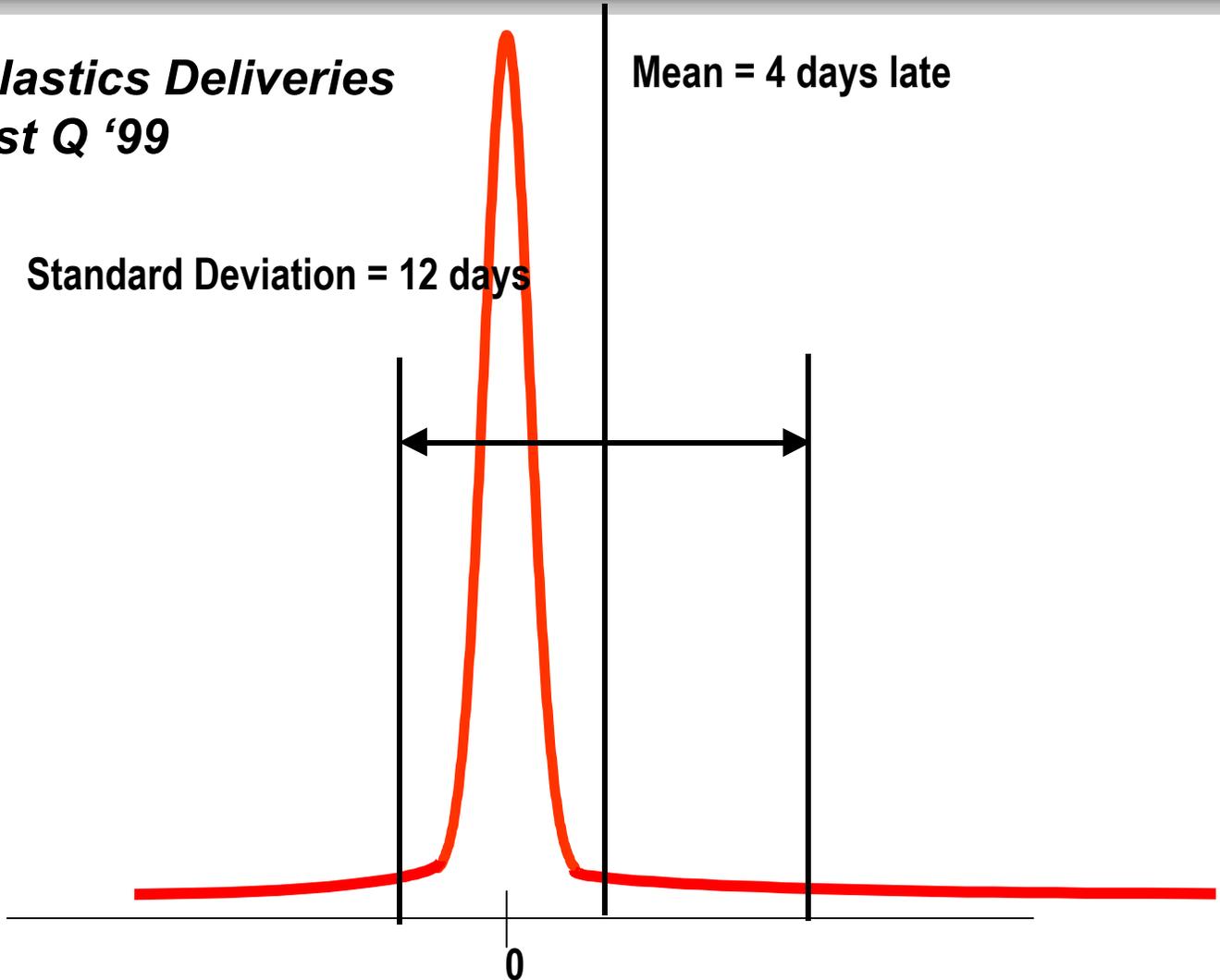


## Normality Testing

**Plastics Deliveries  
1st Q '99**

Standard Deviation = 12 days

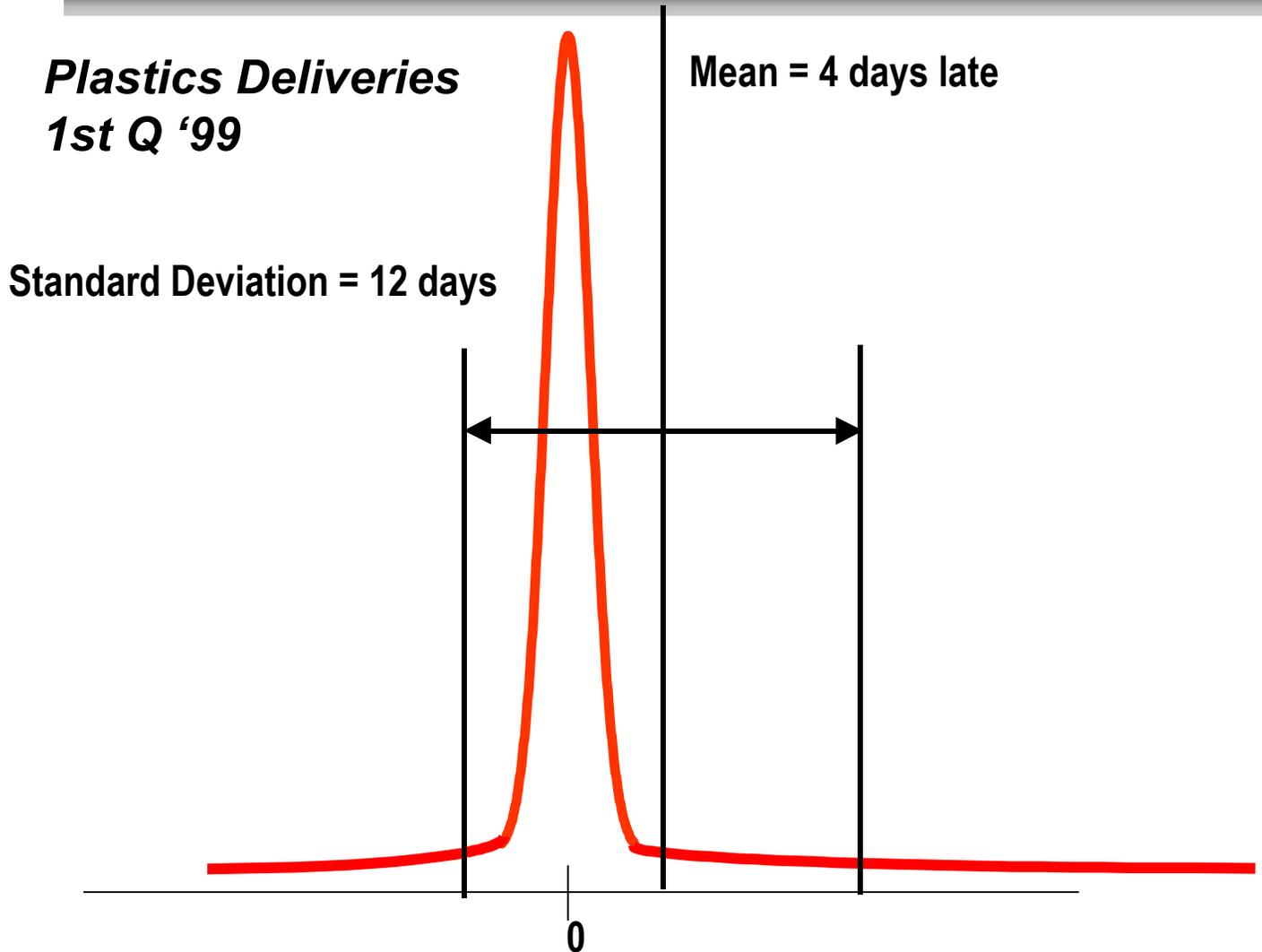
Mean = 4 days late



The red curve is the one that we “fit” to the data. The central tendency of the actual data is not well represented by the mean of 4 days late. 4 days late would indicate that we have a structural problem with our central tendency of delivering on time. This is not true if we use the median as the central tendency indicator.



## Normality Testing

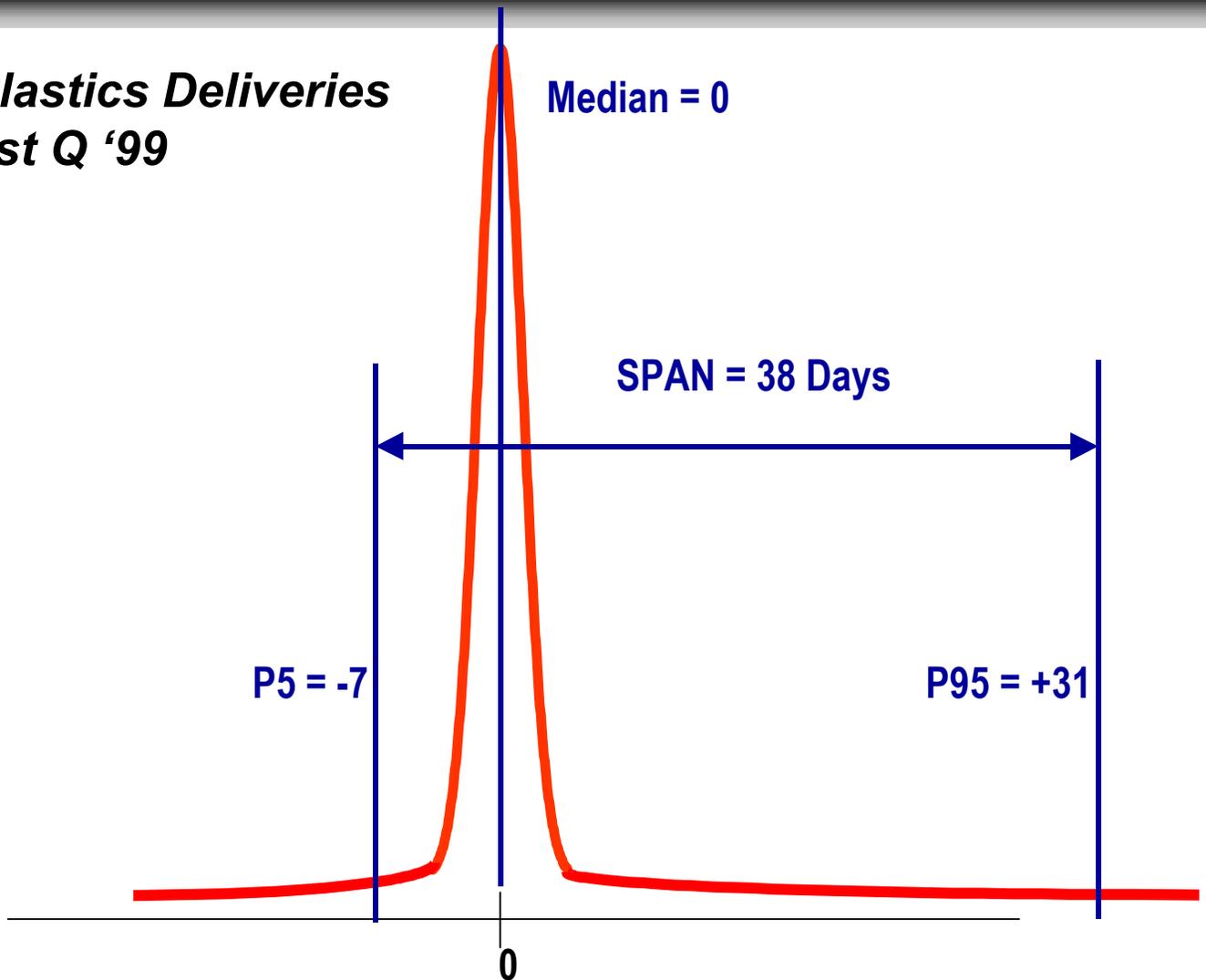


Also, the standard deviation assumes an equal probability of variation on each side of the mean. This is not the case with this data. The variation on each side of the central tendency is quite different. There is a much longer “tail” on the late side than the early side. In other words, the business is late 50% of the time, but when it’s late, it’s very late.



## Normality Testing

**Plastics Deliveries  
1st Q '99**



The median is a much better central tendency indicator in this case. The Span (distance from the 5% probability to the 95% probability) is a much better indicator of the variance in this case.

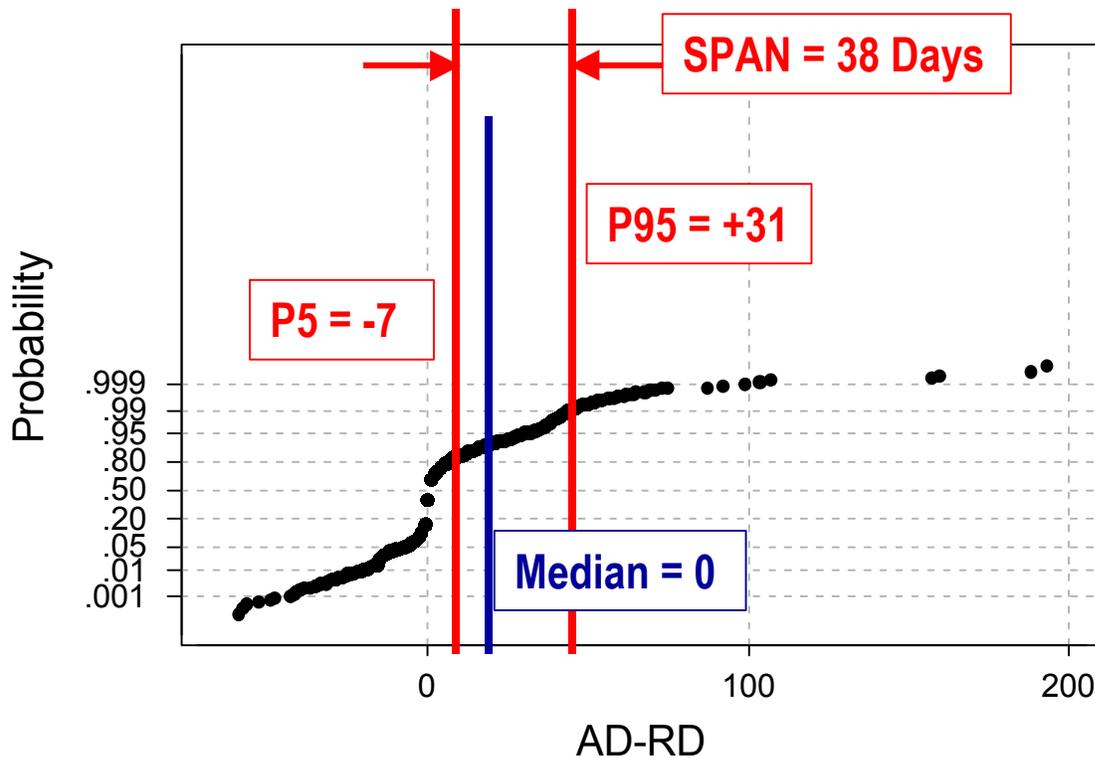
**In this case, to use mean and standard deviation would be very deceiving. The median and span are much less sensitive to the long “tails” of the Distribution.**



# Normality Testing

## Plastics Deliveries 1st Q '99

Normal Probability Plot



Average: 3.90178  
StDev: 12.3470  
N: 8461

Anderson-Darling Normality Test  
A-Squared: 975.177  
P-Value: 0.000

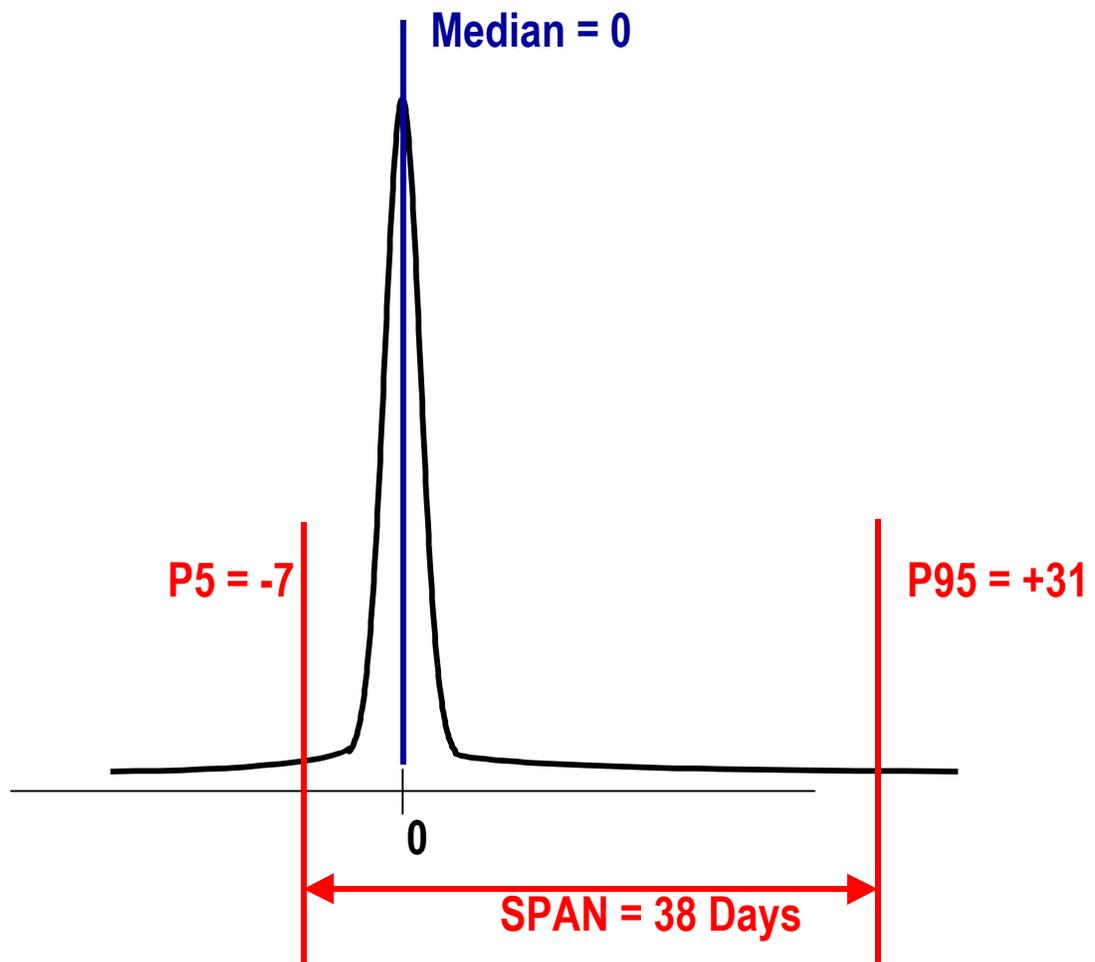
Here are the Statistical Indicators superimposed on the Normality Plot



# Normality Testing

## Two Statistical Indicators:

### Central Tendency - Characterizes The Process Location



### Variance - Characterizes the Consistency



# *Normality Testing*

---

## **Data Distribution Categories:**

Although the distribution of non-normal data may take almost any shape, in practice, most of the distributions will fall into a small set of categories:

- Normal
- Stable Operations (or Skewed Data)
- Targeted Data (as in the previous example)
- Bi-Modal (usually when you have multiple processes in the data set)

So, what are the right statistical indicators to use for the central tendency and variance in each case?



# Normality Testing

## Most Common Data Distributions and their Normality Plots

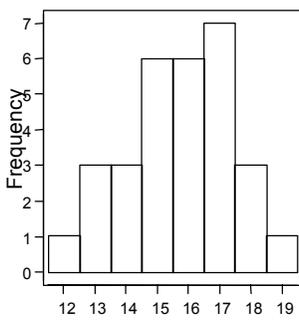
**Normal Data**

**Bimodal curve**

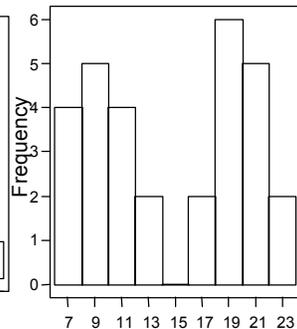
**Stable Operations Curve**

**Data measured vs. a Target**

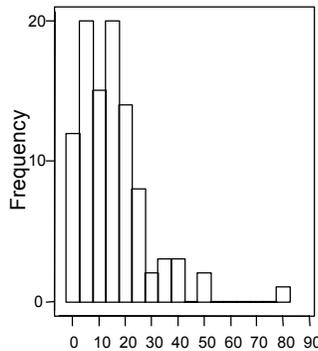
Roughly Normal Distribution



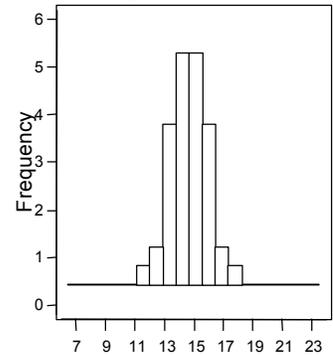
Bimodal Distribution



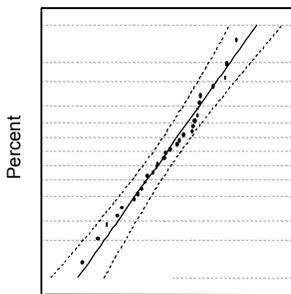
Exponential Distribution



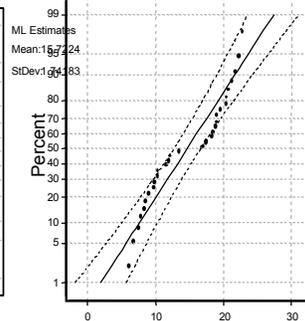
Long Tailed Distribution



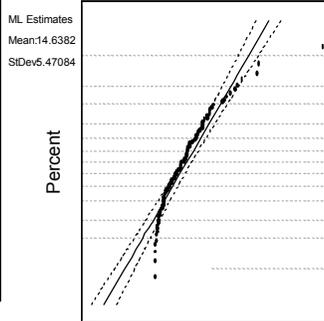
Normal Probability Plot for a Normal Distribution



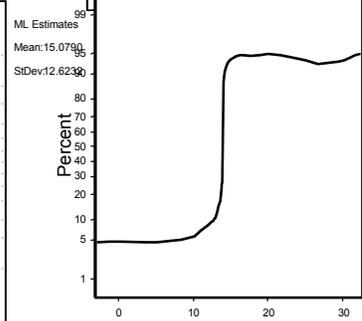
Normal Probability Plot for a Bimodal Distribution



Normal Probability Plot for an Exponential Distribution



Normal Probability Plot For Long-Tailed



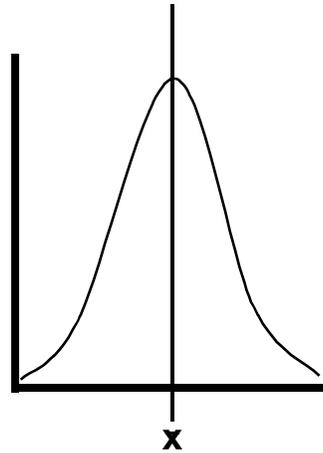


# Normality Testing

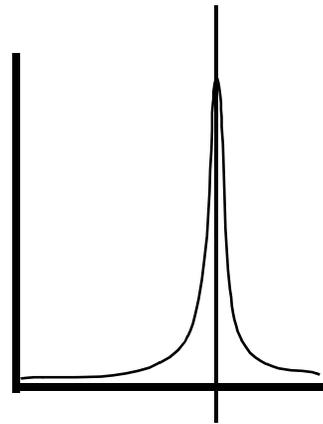
## Central Tendency - Statistical Indicators of the Distribution

Represents the nominal value of the process.

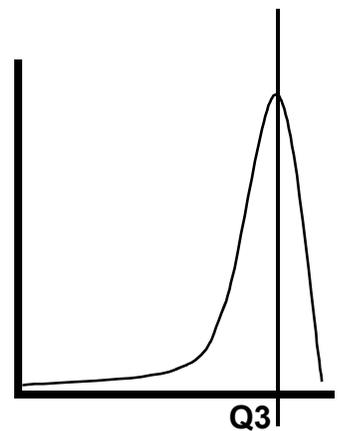
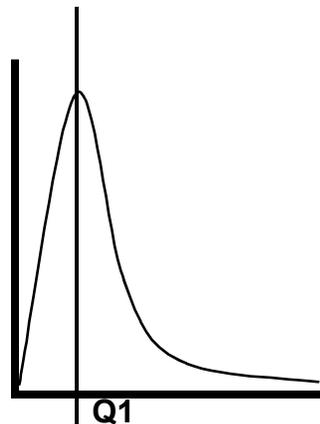
- Mean ( $\bar{x}$ )



- Median (“middle” data point)



- Quartile Values (Q1, Q3)



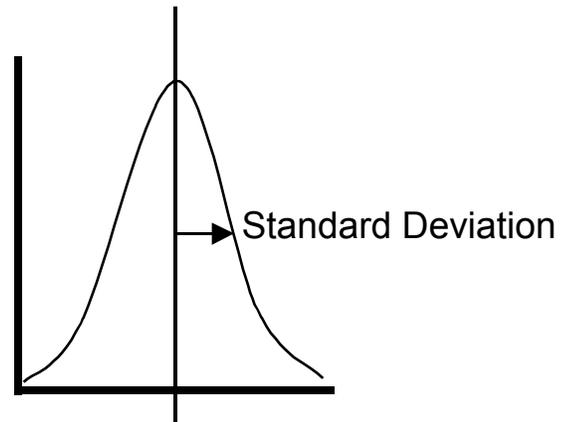


# Normality Testing

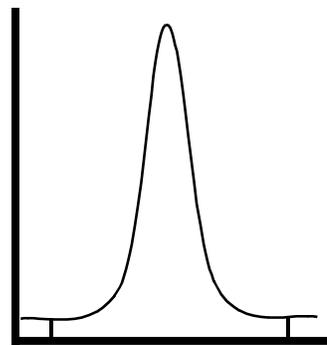
## Variance - Statistical Indicators of the Distribution

Represents the variation in the process.

- Standard Deviation ( $s$ )



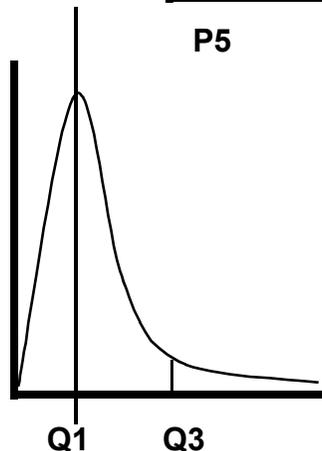
- Span ( $P95 - P5$ )



P5

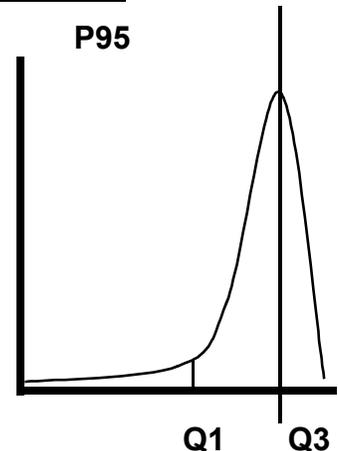
P95

- Stability Factor (SF) =  $Q1/Q3$



Q1

Q3

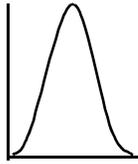
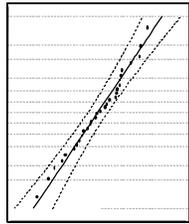
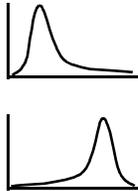
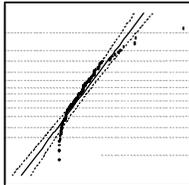
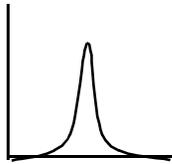
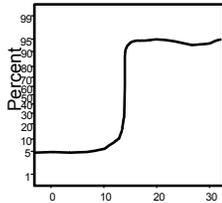
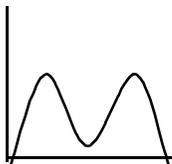
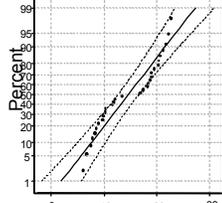


Q1

Q3



# Statistical Indicators Summary

Shape	Normality Plot	Central Tendency	Variance
 <p>normal</p>	<p>Normal Probability Plot for a Normal Distribution</p>  <p>ML Estimates Mean: 5.7224 StDev: 74.183</p>	$\text{mean} \bar{x}$	standard deviation (S)
 <p>skewed</p>	<p>Normal Probability Plot for an Exponential Distribution</p>  <p>ML Estimates Mean: 5.0790 StDev: 2.6232</p>	Quartile Q1 or Q3	stability factor (SF)
 <p>long-tailed</p>	<p>Normal Probability Plot for a Long-Tailed Distribution</p>  <p>ML Estimates Mean: 4.6382 StDev: 47084</p>	$\text{Media} \tilde{n}(x)$	span or range
 <p>bimodal</p>	<p>Normal Probability Plot for a Bimodal Distribution</p>  <p>ML Estimates Mean: 4.6382 StDev: 47084</p>	The different processes must be separated before descriptive statistics can be calculated	



# *Normality Testing*

## **Summary**

- Data is often but not always normal
- Normality Testing helps you determine if it is normal and gives you insight into how your process is behaving
- When it's not normal, it may be because of more than one underlying process
- Caution is needed in proceeding if it's not - consult your MBB.

## **Notes:**

Once you have baselined your output distribution, you must do a normality test to see if the data is normal. The results of a normality test are a key deliverable for your project. Also, learning to read normality plots can give you valuable insight into how your process behaves.

When you encounter non-normal data, you should involve your MBB in deciding how to proceed. The first consideration should be to try to evaluate if you have more than one process (central tendency) in the data set.

If you are sure you do not have more than one process in the data set, you must choose the appropriate set of statistical indicators based on the shape of your distribution. This set of statistical indicators must be used throughout your project. The goals you establish in Step 5 and the results you achieve and evaluate in Step 11 will be based on these statistical indicators.



---

# *Baseline Using Discrete Data*



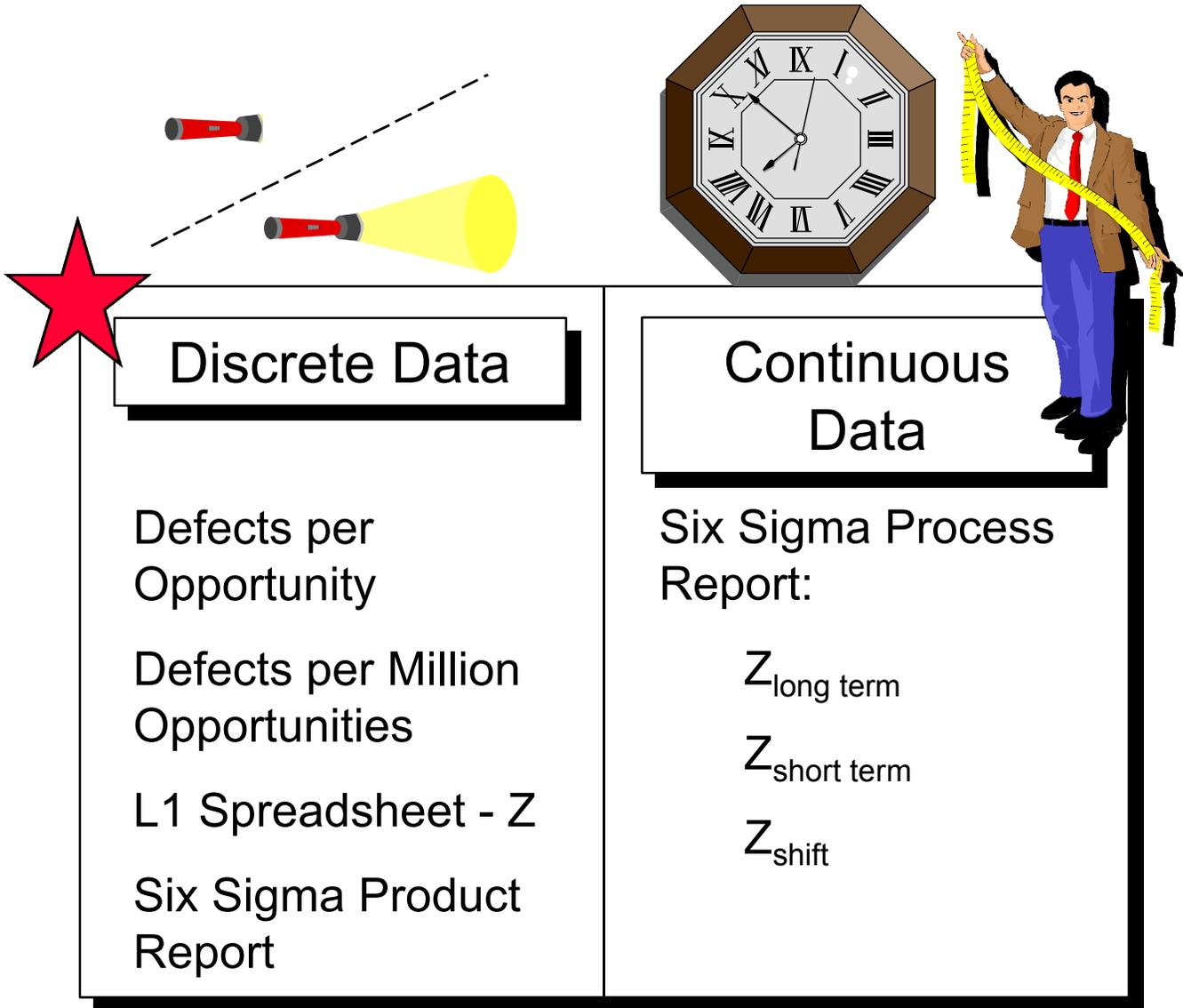
## *Discrete Data Objectives*

*By the end of the training program, the participant will be able to:*

- *Characterize their process using discrete data.*
- *Calculate the distribution of defects for a given DPU.*
- *Determine how DPU controls Throughput Yield ( $Y_{TP}$ ).*
- *Utilize Z tables to convert DPMO to “Z.”*
- *Understand the differences between Classical Yield ( $Y_C$ ), First Time Yield ( $Y_{FT}$ ), Throughput Yield ( $Y_{TP}$ ) and Rolled Yield ( $Y_{RT}$ ).*
- *Calculate submitted, observed, and escaping defect levels.*
- *Understand how complexity impacts quality.*



# Data Analysis Roadmap





# Definitions

## Unit (U)

The number of parts, sub-assemblies, assemblies, or systems **inspected or tested**.

- Squares: 4 units

## Opportunity (OP)

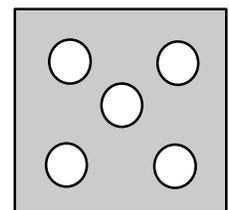
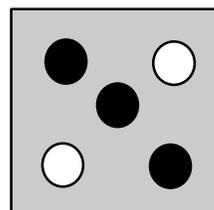
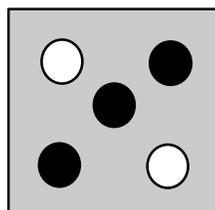
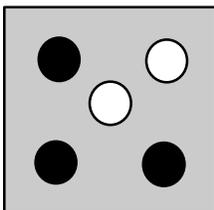
A characteristic you inspect or test.

- Circles: 5 opportunities per unit

## Defect (D)

Anything that results in customer dissatisfaction. Anything that results in a non-conformance.

- Black circles: 9 defects





## Formulas

### Defects per Unit

$$DPU = D/U$$

$$9/4 = 2.25$$

### Total Opportunities

$$TOP = U*OP$$

$$4*5 = 20$$

### Defects per Opportunity (Probability of a Defect)

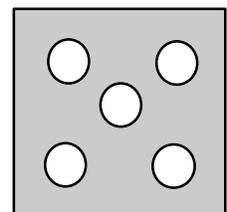
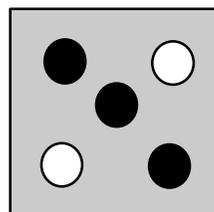
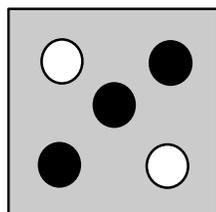
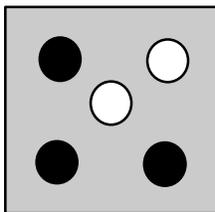
$$DPO = D/TOP$$

$$9/20 = .45$$

### Defects per Million Opportunities

$$DPMO = DPO*1,000,000$$

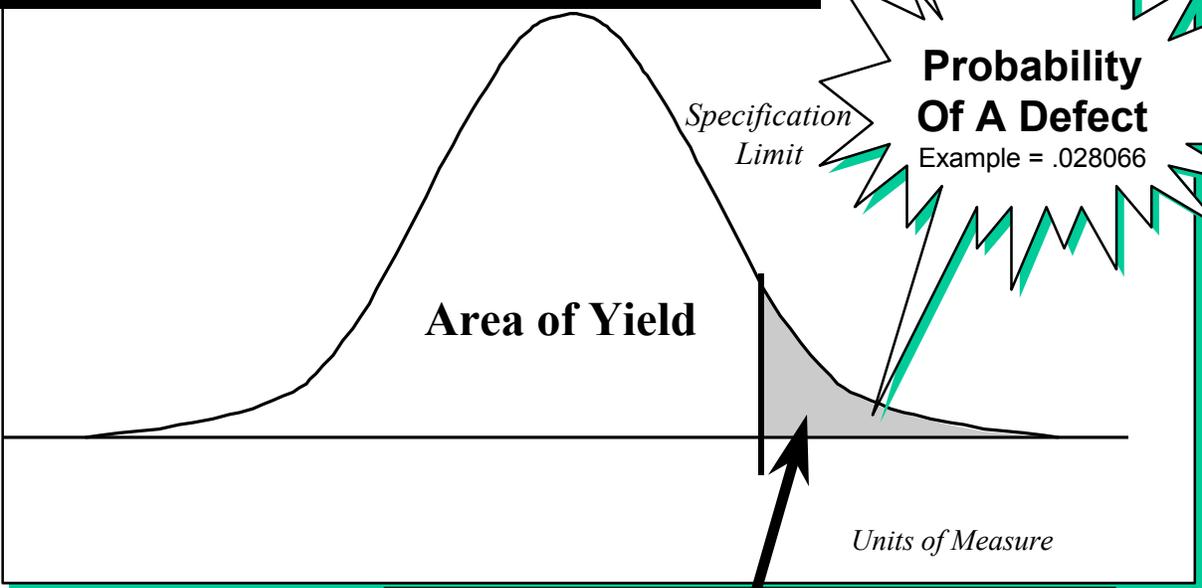
$$.45*1,000,000 = 450,000$$





# Linking DPO to Probability of a Defect

**DPO = Probability of a Defect**



A cartoon illustration of a person in a blue suit and red tie, wearing glasses and holding a magnifying glass, is looking at the table below. The table is a standard normal distribution table showing the area under the curve to the right of a given z-score.

0.20	.420740315	1.71	.043632958	3.22	.000640954	4.73	.000001153
0.25	.401293694	1.76	.039203955	3.27	.000537758	4.78	.000000903
0.30	.382088199	1.81	.035147973	3.32	.000450127	4.83	.000000705
0.35	.363143387	1.86	.031442864	3.37	.000375899	4.88	.000000550
0.40	.344631735	1.91	.028066724	3.42	.000313179	4.93	.000000428
0.45	.327434818	1.96	.024998022	3.47	.000260317	4.98	.000000332
0.50	.311468439	2.01	.022215724	3.52	.000215873	5.03	.000000258
0.55	.296532964	2.06	.019699396	3.57	.000178601	5.08	.000000199
0.60	.282539517	2.11	.017429293	3.62	.000147419	5.13	.000000154
0.65	.269451501	2.16	.015386434	3.67	.000121399	5.18	.000000118
0.70	.257142728	2.21	.013552660	3.72	.000099739	5.23	.000000091
0.75	.245507319	2.26	.011910681	3.77	.000081753	5.28	.000000070
0.80	.234432818	2.31	.010444106	3.82	.000066855	5.33	.000000053
0.85	.223809172	2.36	.009137469	3.87	.000054545	5.38	.000000041
0.90	.213605676	2.41	.007976235	3.92	.000044399	5.43	.000000031
0.95	.203793161	2.46	.006946800	3.97	.000036057	5.48	.000000024
1.00	.194369910	2.51	.006036485	4.02	.000029215	5.53	.000000018
1.05	.185308641	2.56	.005233515	4.07	.000023617	5.58	.000000014
1.10	.176596764	2.61	.004527002	4.12	.000019047	5.63	.000000010
1.15	.168224200	2.66	.003906912	4.17			
1.20	.160181154	2.71	.003364033	4.22			
1.25	.152457817	2.76	.002889938	4.27			
1.30	.145044326	2.81	.002476947	4.32			
1.35	.137930734	2.86	.002118083	4.37			
1.40	.131107142	2.91	.001807032	4.42			
1.45	.124563631	2.96	.001538097	4.47			
1.50	.118291020	3.01	.001306156	4.52			

**"Sigma" = 1.91**

**Table Of Area Under The Normal Curve**



# Converting DPMO to Z

## Sigma table

Long term DPMO	Actual Sigma (long term)	Reported Sigma (short term)
500,000	0	1.5
460,172	0.1	1.6
420,740	0.2	1.7
382,089	0.3	1.8
344,578	0.4	1.9
<b>308,538</b>	<b>0.5</b>	<b>2</b>
274,253	0.6	2.1
241,964	0.7	2.2
211,855	0.8	2.3
184,060	0.9	2.4
158,655	1	2.5
135,666	1.1	2.6
115,070	1.2	2.7
96,801	1.3	2.8
80,757	1.4	2.9
<b>66,807</b>	<b>1.5</b>	<b>3</b>
54,799	1.6	3.1
44,565	1.7	3.2
35,930	1.8	3.3
28,716	1.9	3.4
22,750	2	3.5
17,864	2.1	3.6
13,903	2.2	3.7
10,724	2.3	3.8
8,198	2.4	3.9
<b>6,210</b>	<b>2.5</b>	<b>4</b>
4,661	2.6	4.1
3,467	2.7	4.2
2,555	2.8	4.3
1,866	2.9	4.4
1,350	3	4.5
968	3.1	4.6
687	3.2	4.7
483	3.3	4.8
337	3.4	4.9
<b>233</b>	<b>3.5</b>	<b>5</b>
159	3.6	5.1
108	3.7	5.2
72	3.8	5.3
48	3.9	5.4
32	4	5.5
21	4.1	5.6
13	4.2	5.7
9	4.3	5.8
5	4.4	5.9
<b>3.4</b>	<b>4.5</b>	<b>6</b>

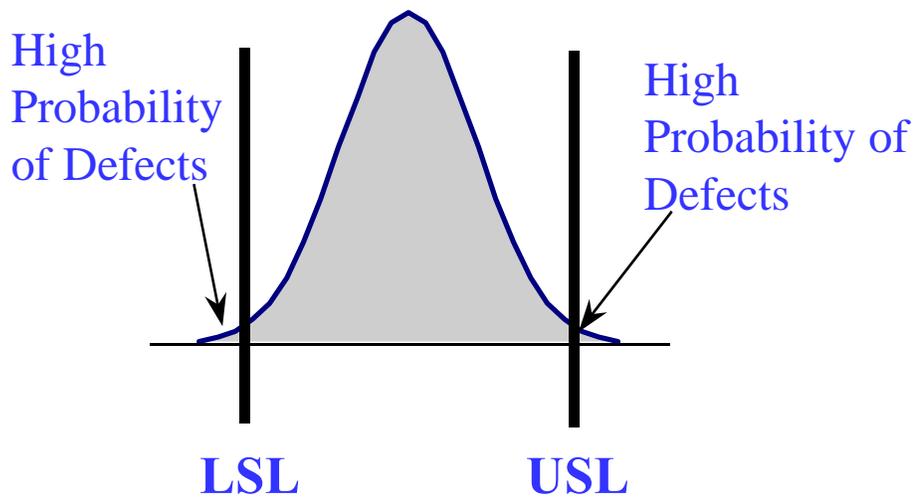
Z = Sigma  
Capability

Z	DPMO
2	308,537
3	66,807
4	6,210
5	233
6	3.4

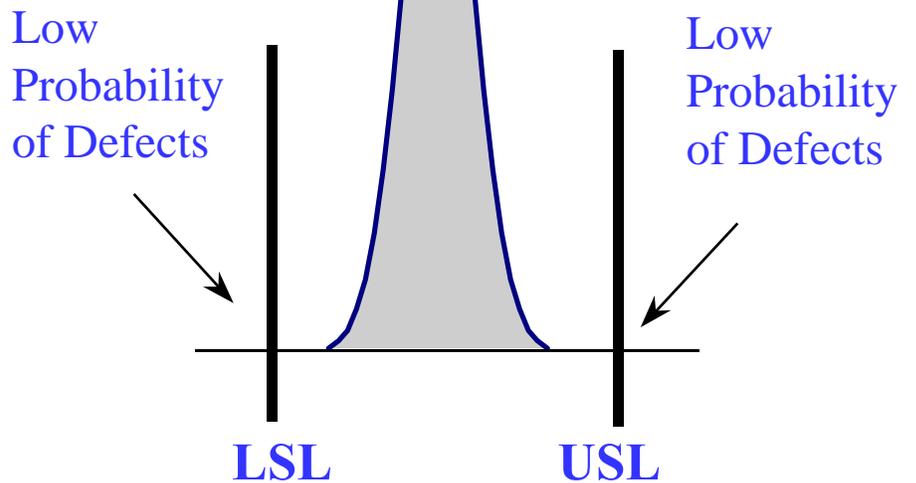


# The Normal Curve and Capability

## Poor Design Capability

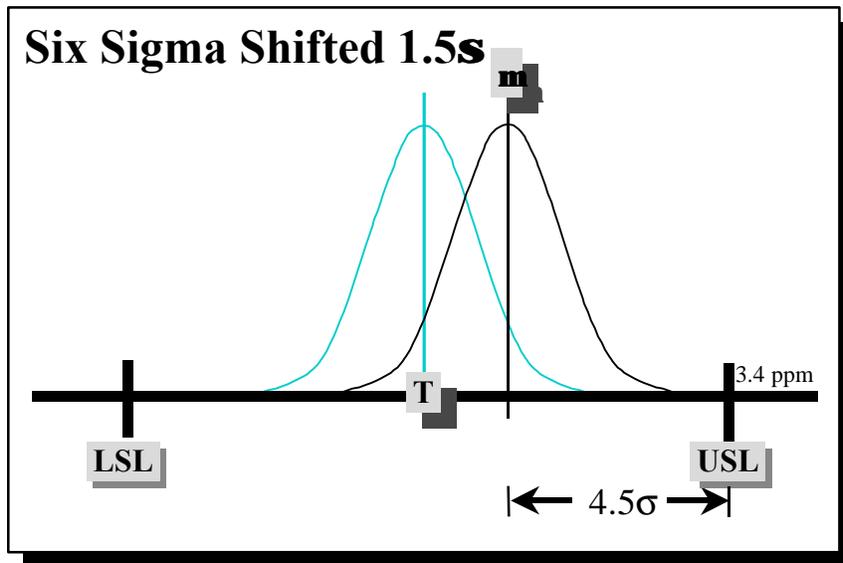
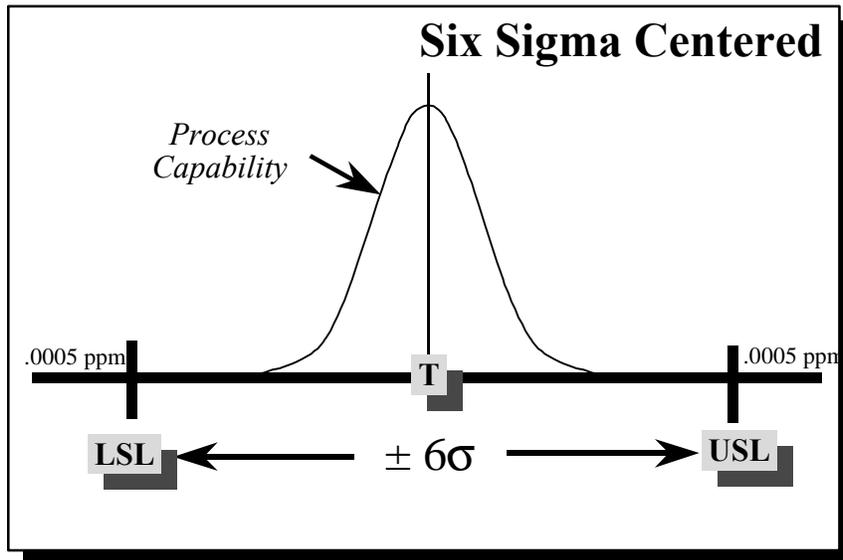


## Good Design Capability





# Generalizing the Correction



*The 1.5s shift is used as a compensatory off-set in the mean to generally account for dynamic nonrandom variations in process centering. It represents the average amount of change in a typical process over many cycles of that process.*



## Form - L1: Input

Product/CEO/ Process	Defects	Unit	Opt	Total Opt	DPU	DPO	DPMO	Shift	Long Term Capability	Sigma
	<u>D</u>	<u>U</u>	<u>OP</u>	<u>TOP</u>	<u>DPU</u>	<u>DPO</u>	<u>DPMO</u>	<u>Shift</u>	<u>Sigma-L</u>	<u>ZB</u>
Product A	2	1	659	659	2.0000	0.003035	3035	1.5	2.74	4.24
Product B	24	340	48	16320	0.0706	0.001471	1471	1.5	2.97	4.47
Product C										
Product D										
Product E										
Product F										
Product G										
Product H										
Product I										
Product J										
Product K										
Product L										
<b>Grand Total</b>	26			16979		0.001531	1531	1.5	2.96	4.46

**Opportunity** - Anything you measure or test

**Defect** - Any non-conformity in a product

**Units** - The number of units (parts, subassemblies, assemblies, or systems) inspected or tested

**Define Opportunities**

**Accumulate Defects for All Defined Opportunities**



# L1 Form Roll-Up

Business	<u>D</u>	<u>U</u>	<u>OP</u>	<u>TOP</u>	<u>DPU</u>	<u>DPO</u>	<u>DPMO</u>	<u>Shift</u>	<u>Sigma-L</u>	<u>Z.B</u>
Product-A								1.5		
Product-B								1.5		
Product-C										
<b>Grand Total</b>				0				1.5		

Product-C	<u>D</u>	<u>U</u>	<u>OP</u>	<u>TOP</u>	<u>DPU</u>	<u>DPO</u>	<u>DPMO</u>	<u>Shift</u>	<u>Sigma-L</u>	<u>Z.B</u>
Product-B										
Product-A										
Sub-Assembly-A								1.5		
Sub-Assembly-B								1.5		
Sub-Assembly-C										
<b>Grand Total</b>	0			0				1.5		

Sub-Assembly C	<u>D</u>	<u>U</u>	<u>OP</u>	<u>TOP</u>	<u>DPU</u>	<u>DPO</u>	<u>DPMO</u>	<u>Shift</u>	<u>Sigma-L</u>	<u>Z.B</u>
Sub-Assembly B										
Sub-Assembly-A										
Process-1								1.5		
Process-2								1.5		
Process-3										
<b>Grand Total</b>	0			0				1.5		

Process 3	<u>D</u>	<u>U</u>	<u>OP</u>	<u>TOP</u>	<u>DPU</u>	<u>DPO</u>	<u>DPMO</u>	<u>Shift</u>	<u>Sigma-L</u>	<u>Z.B</u>
Process 2										
Process-1										
Characteristic-1								1.5		
Characteristic-2								1.5		
Characteristic-3										
<b>Grand Total</b>	0			0				1.5		



# Six Sigma Product Report

In Minitab, create the following table:

	C1	C2	C3	C4
↓	Defects	Units	Opps	
1	2	1	659	
2	24	340	48	
3				
4				

Run the Six Sigma Product Report:

**Rollup Statistics**

Charact	Defts
1	2
2	24
Total	26

**Worksheet 1 \*\*\***

	C1	C2	C3	C6	C7	C8	C9
↓	Defects	Units	Opps				
1	2	1	659				
2	24	340	48				
3							
4							
5							
6							
7							
8							

Generate various benchmark statistics for a product



# Discrete Data Examples

1. Select Defects, Units, & Opportunities

**Six Sigma Product Report**

C1	Defects
C2	Units
C3	Opps

**Defects:**

**Units:**

**Opportunities:**

**Characteristics:**  [optional]

**Shift factors:**  [optional]

2. Select OK.

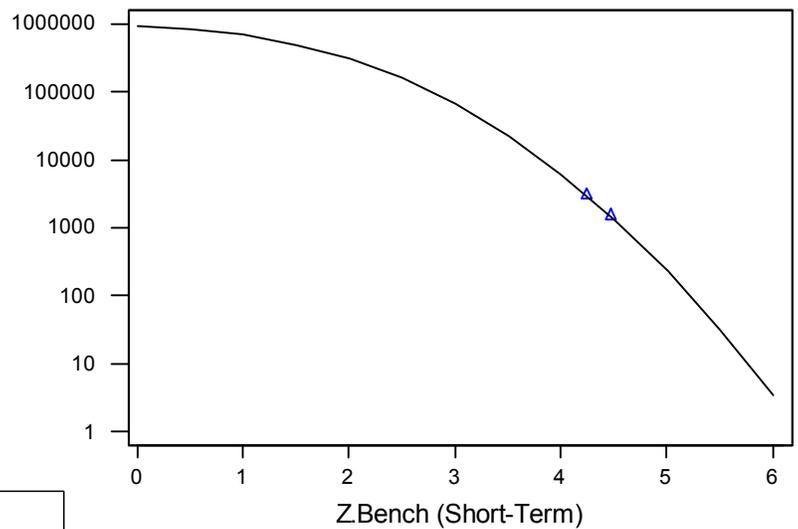


# Six Sigma Product Report Output

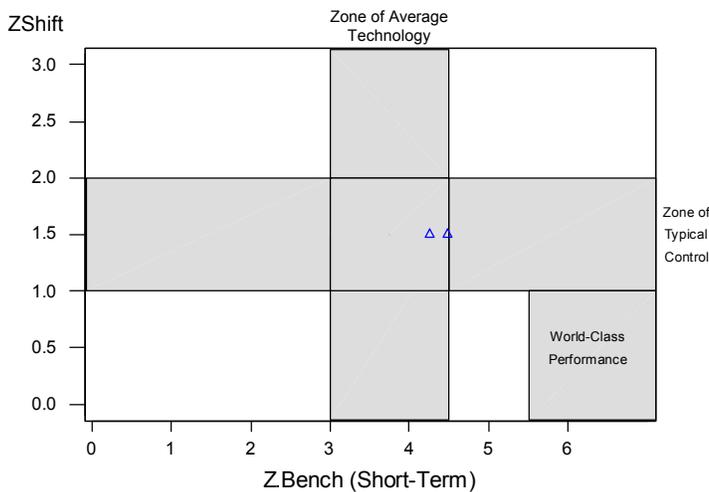
### Report 7: Product Performance

Characteristic	Defs	Units	Opps	TotOpps	DPU	DPO	PPM	ZShift	ZBench
1	2	1	659	659	2.000	0.003035	3035	1.500	4.244
2	24	340	48	16320	0.071	0.001471	1471	1.500	4.474
<b>Total</b>	<b>26</b>			<b>16979</b>		<b>0.001531</b>	<b>1531</b>	<b>1.500</b>	<b>4.461</b>

### Report 8A: Product Benchmarks



### Report 8B: Product Benchmarks

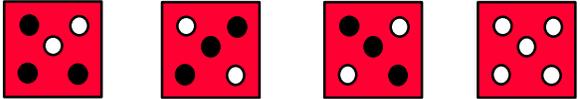




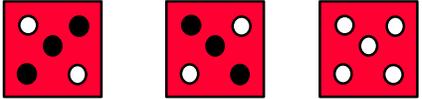
# *Yield*



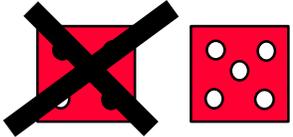
# Yield

**Four parts are made** 

1. *After first inspection: 1 passed, 3 failed*  
*Rework 3 parts*



2. *After second inspection: 1 passed, 2 failed*  
*Rework 2 parts*



3. *After third inspection: 1 passed, 1 scrapped*

**What is the yield of this process?**



## Types of Yield

### **Classical Yield = $Y_C = 3/4 = 75\%$ ?**

Classical yield is the number of defect-free parts for the whole process divided by the total number of parts inspected. If we say the yield is 3/4 or 75%, we lose valuable data on the true performance of the process. This loss of insight becomes a barrier to process improvement.

### **First Time Yield = $Y_{FT} = 1/4 = 25\%$ ?**

First time yield is the number of defect-free parts divided by the total number of parts inspected for the first time. If we say the yield is 1/4 or 25%, we are really talking about the First Time Yield (FTY). This is a better yield estimate to drive improvement.

### **Throughput Yield =**

$$Y_{TP} = P(0) = e^{-DPU} = e^{-2.25} = .1054 = 10.54\%$$

$Y_{TP}$  is the percentage of units that pass through an operation without any defects. This is the best yield estimate to drive improvement.

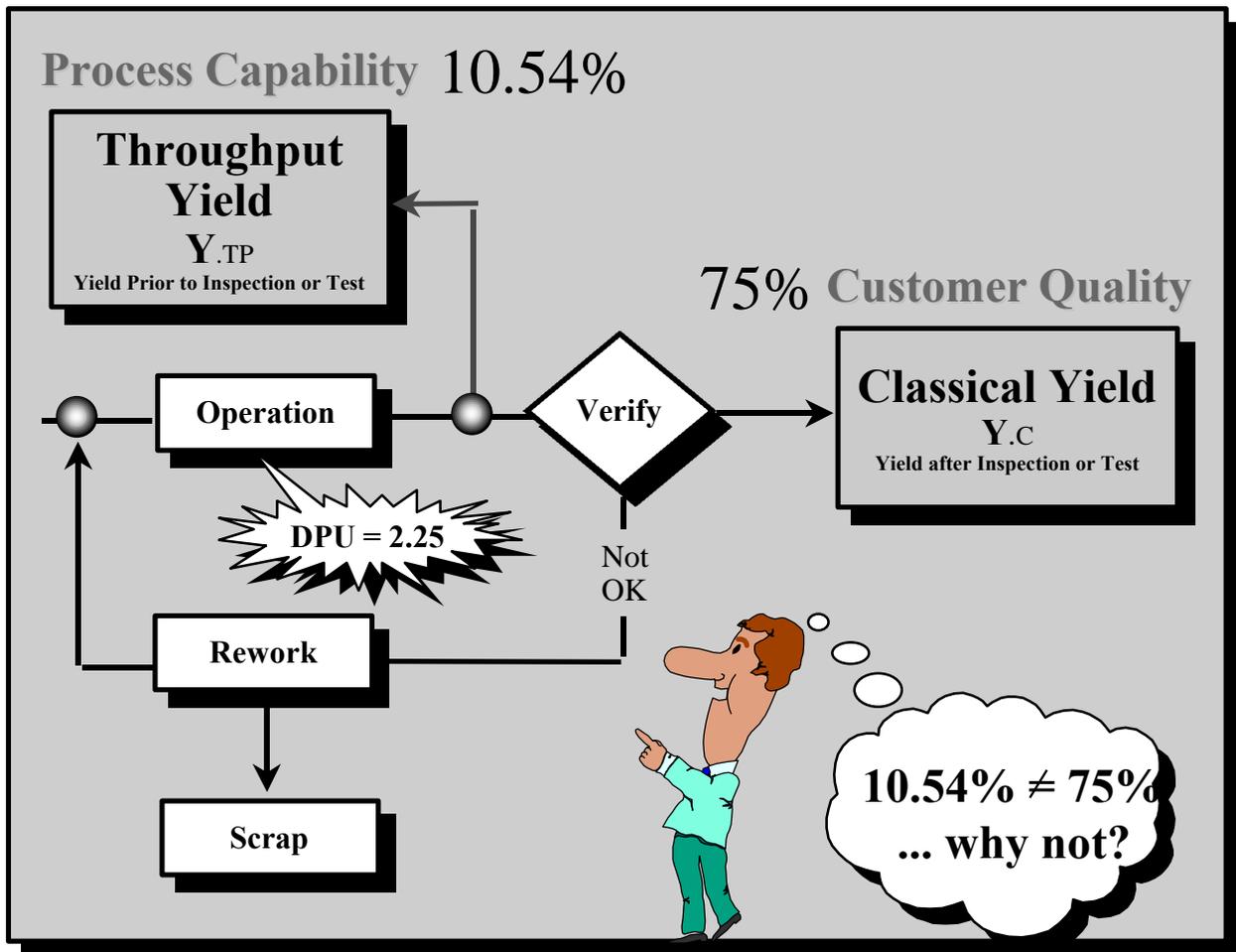


## Comparison of the Yield Models

For Example:

$$Y_C = \frac{3}{4} = .75, \text{ or } 75\%$$

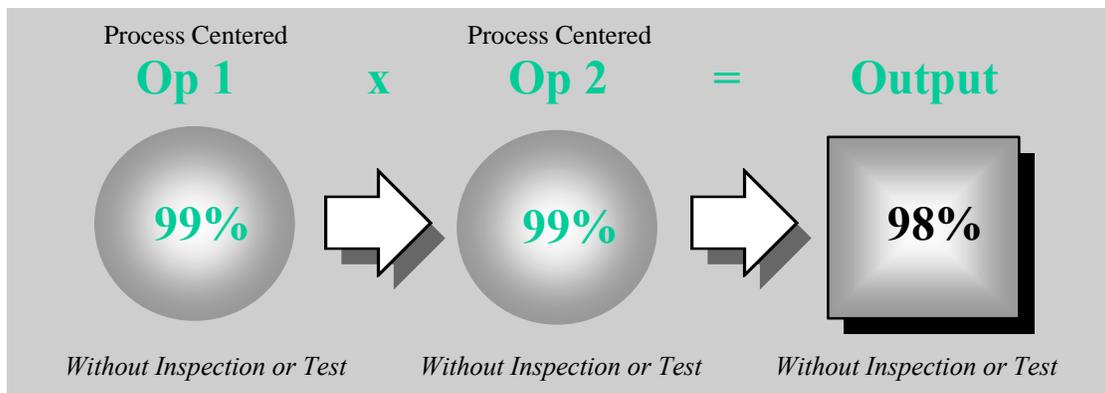
$$Y_{TP} = e^{-DPU} = e^{-2.25} = .1054, \text{ or } 10.54\%$$





## Extending the Concept

**A given process has two operations.  
Each operation has a throughput yield  
of 99 %. The rolled yield equals:**



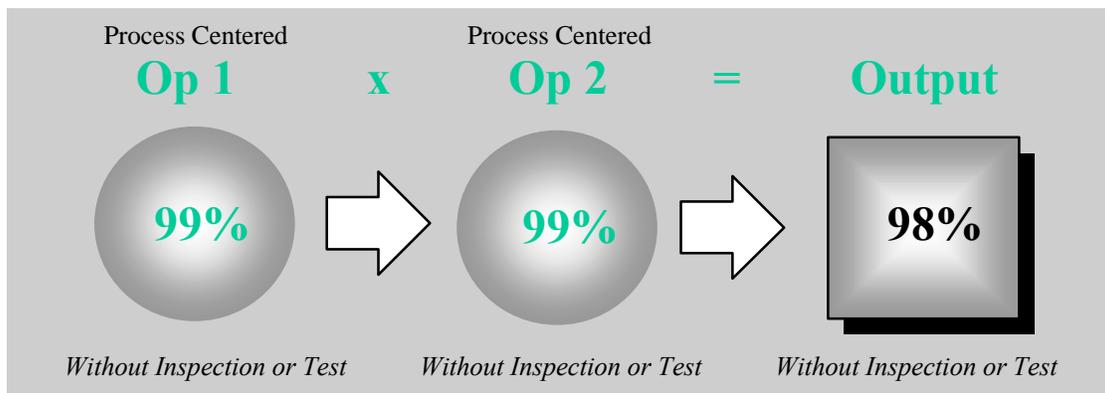
*. . . There is an 98% probability that any given unit of product could pass through both operations defect free.*

**If  $Y_{TP} = e^{-DPU} = \text{Throughput Yield}$ ,**

**Does  $Y_{RT} = e^{-TDPU} = \text{Rolled Yield}$ ?**



## Let's try it on our earlier example:



**OR**

$$Y_{RT} = e^{-TDPU}$$

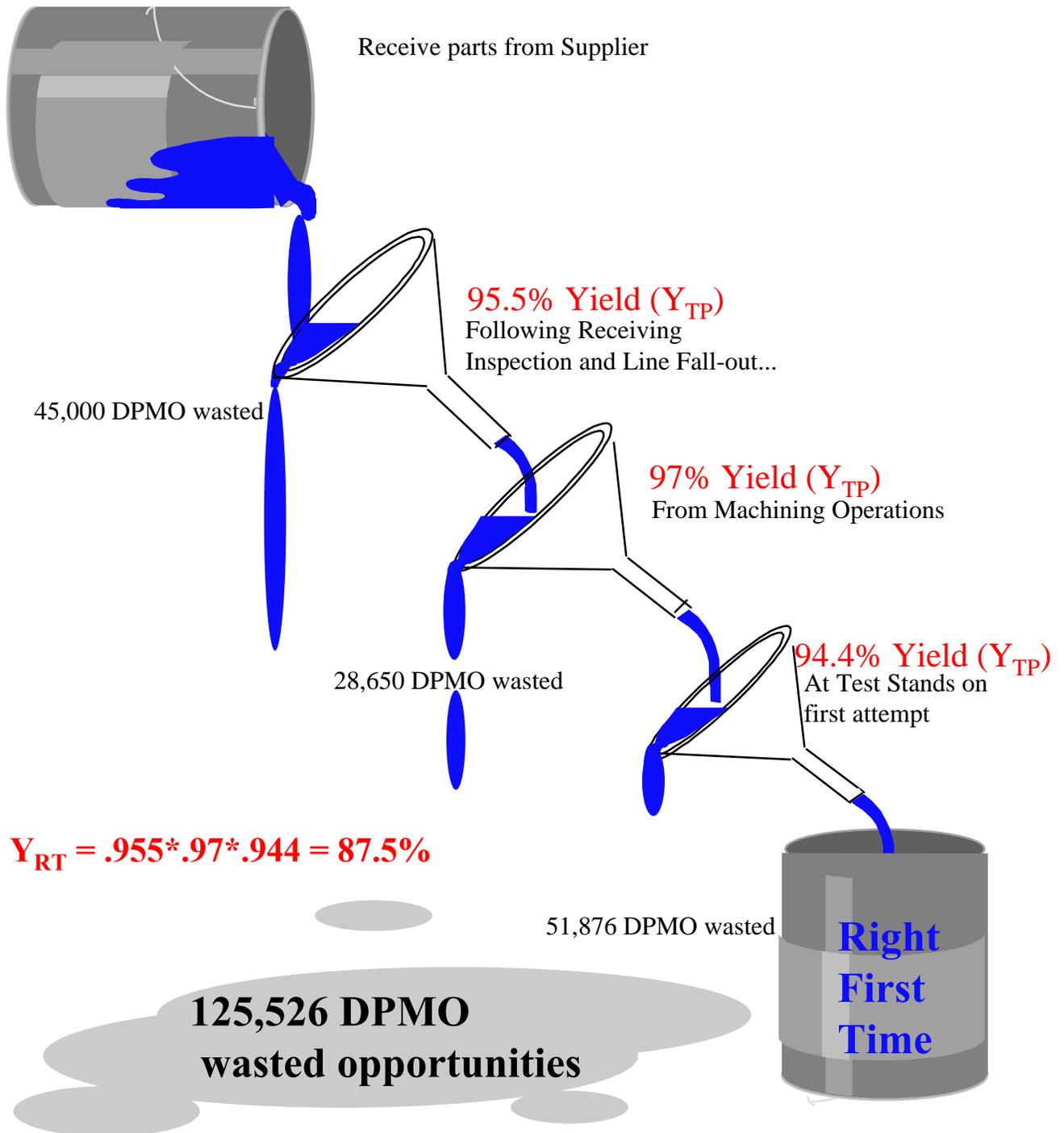
*Each Operation has a 0.01 probability of a defect. Therefore:*

$$\begin{aligned} TDPU &= .01 + .01 \text{ defect per unit} \\ &= .02 \text{ defects per unit} \end{aligned}$$

$$\begin{aligned} Y_{RT} &= e^{-.02} = .98019 \\ &\text{or } 98\% \end{aligned}$$



# Rolled Throughput Yield ( $Y_{RT}$ )



G. Reimer 12/21/94 - Charlotte NC



## Why Z = 3 Is Not Good Enough

### Rolled Yield (%) Vs Z (distribution centered)

Complexity	Capability	→			
	Number of characteristics	Z = 3.0	Z = 4.5	Z = 5.5	Z = 6.0
	1	93.32	99.865	99.997	99.99966
	20	25.09	97.334	99.941	99.9932
	60	1.58	92.214	99.820	99.9796
	100	---	87.363	99.700	99.9660
	200	---	76.324	99.402	99.9320
	500	---	50.892	98.511	99.8301
	1000	---	6.696	93.857	99.3223



## Getting to Six Sigma

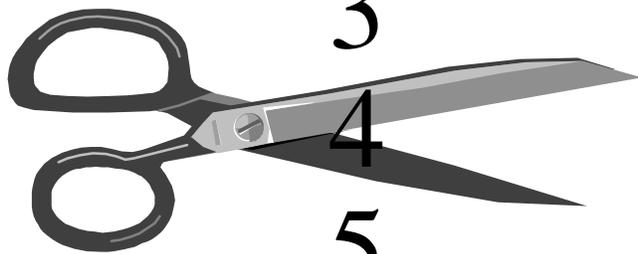
How far can **inspection** get us ?



**S**

**PPM**

2	308,537
3	66,807
4	6,210
5	233
6	3.4



(Distribution Shifted  $\pm 1.5\sigma$ )



## *The Inspection Exercise*

*Task: Count the number of times the 6th letter of the alphabet appears in the following text.*

The Necessity of Training Farm Hands for First  
Class Farms in the Fatherly Handling of Farm Live  
Stock is Foremost in the Eyes of Farm Owners.  
Since the Forefathers of the Farm Owners Trained  
the Farm Hands for First Class Farms in the  
Fatherly Handling of Farm Live Stock, the Farm  
Owners Feel they should carry on with the Family  
Tradition of Training Farm Hands of First Class  
Farmers in the Fatherly Handling of Farm Live  
Stock Because they Believe it is the Basis of Good  
Fundamental Farm Management.



# Results of the Exercise

**Observed Number of "Fs"**

↑ People	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
10																
9																
8																
7																
6																
5																
4																
3																
2																
1																

	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
Freq =																
Obs =																

Inspectors =  x  =  = Total Expected

= Total Observed

Average Efficiency per Inspector =

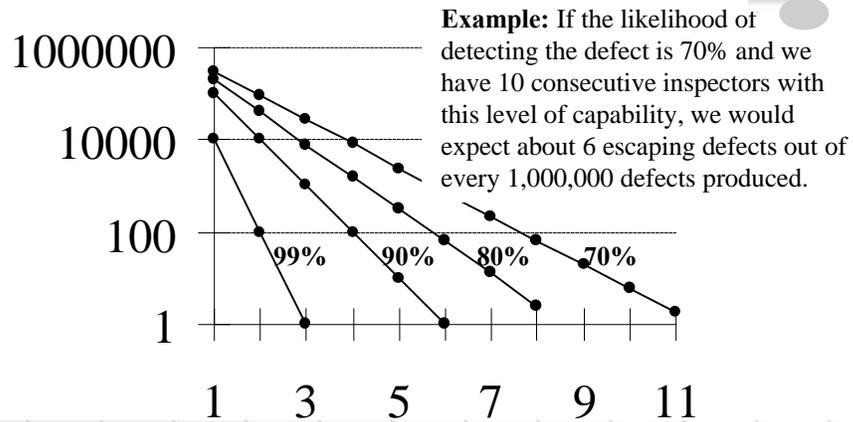


# The Impact of Added Inspection

## Escaping PPM

Note: All sigma values reflect a 1.5σ shift

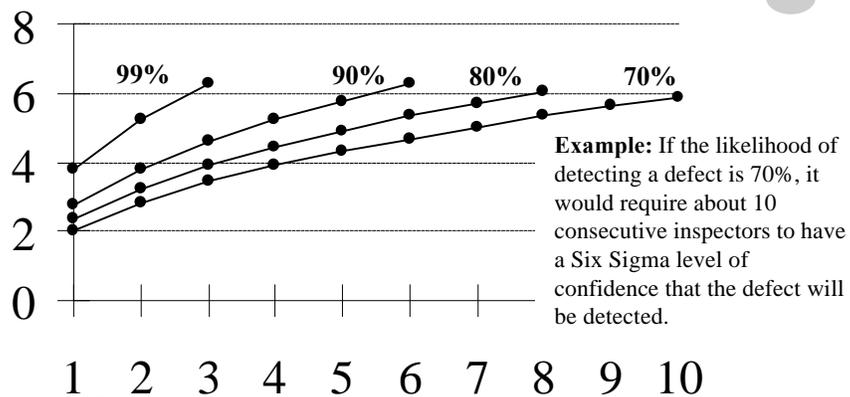
The Y axis represents the undetected defects-per-million defects. Each curve represents the inspection efficiency per inspector.



## Number of Consecutive Inspectors

## Sigma

The Y axis represents the inspection efficiency "sigma." Each curve represents the inspection efficiency per inspector.



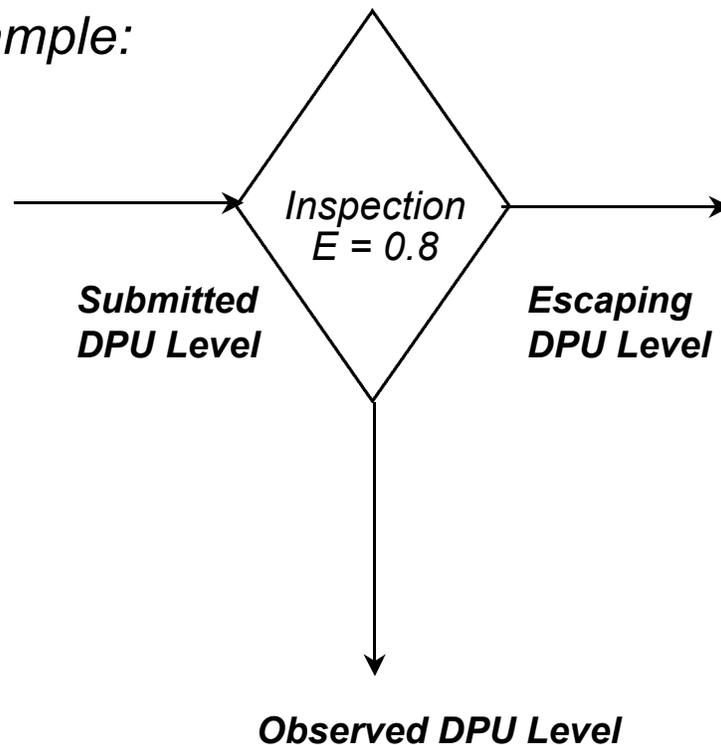
## Number of Consecutive Inspectors



## Quantification of Defects

Knowing the observed defect level and the test effectiveness, we can estimate submitted defects and escaping defects.

For example:



$$DPU_S = DPU_O + DPU_E$$

$$DPU_O = DPU_S \times E$$

$$DPU_E = DPU_S \times (1-E)$$



## Quantification of Defects

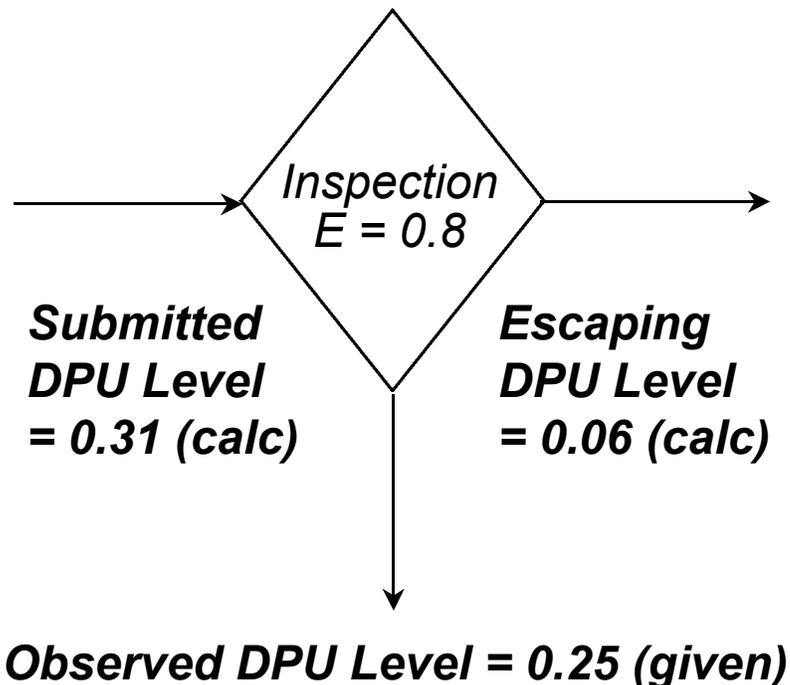
### Given:

Observed Defects = DPU of 0.25 and  $E = .8$

### Then:

Submitted Defects =  $DPU_o / E = (0.25 / 0.8)$   
= DPU of 0.31

Escaping Defects =  $DPU_s - DPU_o = (0.31 - 0.25)$   
= DPU of 0.06





## *Exercise 5:*

### **Calculating Submitted, Observed and Escaping Defect Levels**

#### **Given:**

- *The requirement is to ship the product with no more than one defect in 500 units shipped.*
- *The final test is to be performed using automatic test equipment having an effectiveness of 0.95.*

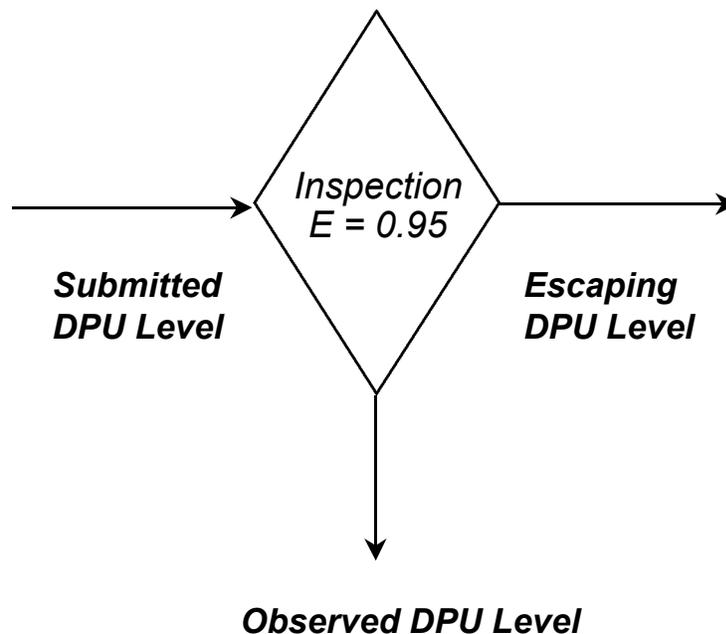
#### **Knowing the maximum level of escaping defects, we can estimate the maximum defect/unit level:**

- *Observed in the final test. (This sets a minimum throughput yield limit.)*
- *Submitted to the final test. (This sets a minimum on the combination of defects created in model assembly and from the escaping defect levels from prior tests.)*



## Exercise 5:

### Calculating Submitted, Observed and Escaping Defect Levels



1. Expressed as a decimal, what is the given acceptable maximum escaping defect level? \_\_\_\_\_
2. What then is the acceptable maximum defect level for:
  - a. Units submitted to final test? \_\_\_\_\_
  - b. Defects observed at final test? \_\_\_\_\_
3. What is the observed throughput yield figure?  
\_\_\_\_\_



## *Answers to Exercise 5:*

### **Calculating Submitted, Observed, and Escaping Defect Levels**

1.  $DPU_E = \frac{1}{500} = 0.002$

2. *Maximum acceptable submitted defects =*

$$DPU_S = \frac{\text{defects escaping}}{1 - \text{effectiveness}} = \frac{DPU_E}{1 - E} = \frac{0.002}{0.05} = 0.040 \text{ DPU}$$

2b. *Maximum acceptable observed defects =*

$$DPU_O = DPU_S - DPU_E = 0.040 - 0.002 = 0.038 \text{ DPU}$$

3.  $F_{TP} \text{ minimum} = e^{-DPU} = e^{-0.038}$

$$\frac{1}{e^{0.038}} = \frac{1}{1.0387} = 0.9627 \text{ or } 96.3\%$$



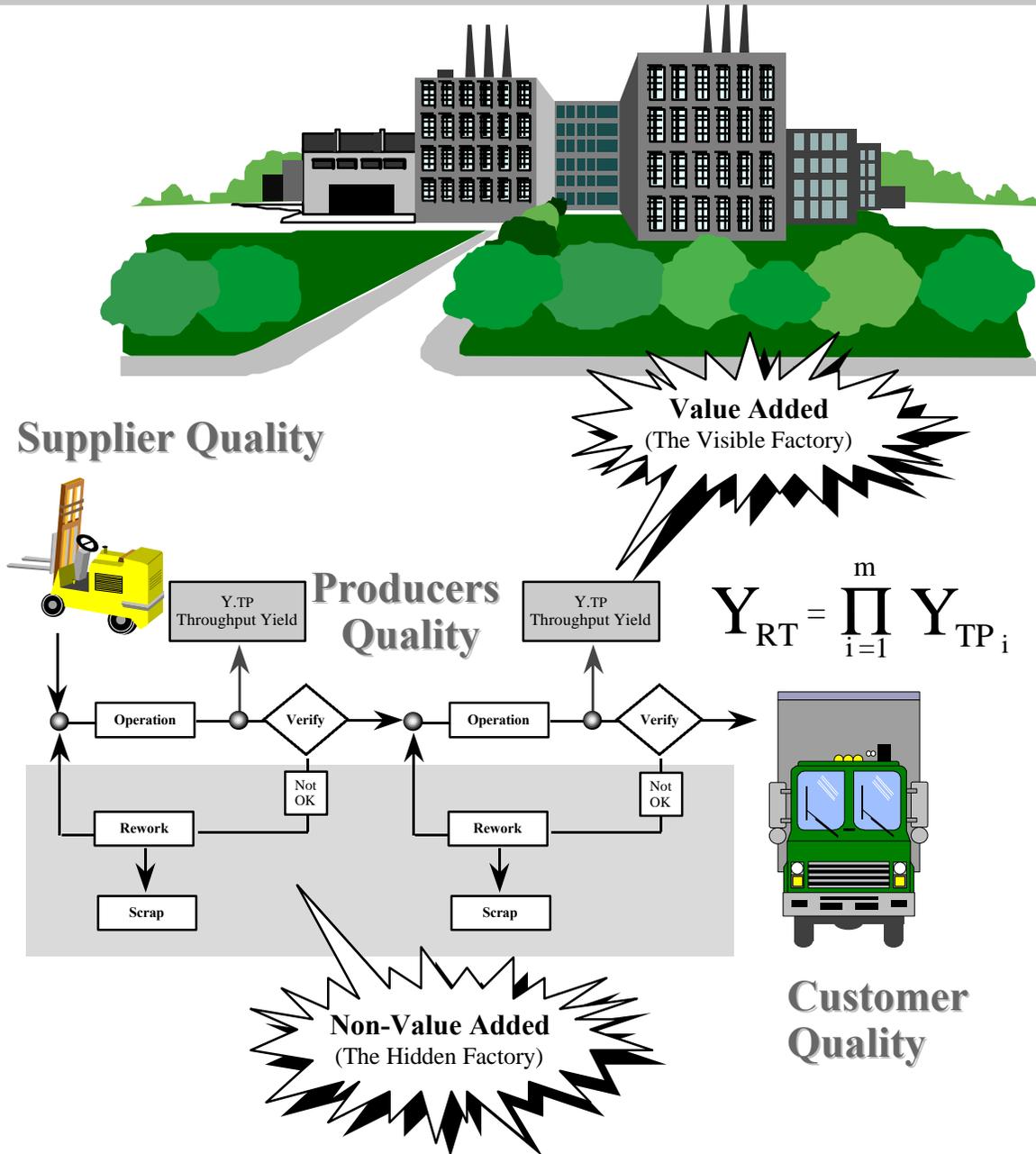
## *DPU Application Rules*

---

- 1. Track defects over all products and operations over time.*
- 2. There is no such thing as an initial good or bad DPU.*
- 3. Set aggressive defect reduction goals.*
- 4. Do not use defects for judging operator performance.*
- 5. Focus on those defects flowing to the customer.*



# A New Perspective of the Factory



*To decrease defects-per-opportunity means to increase rolled through-put yield which, in turn, improves product reliability and customer satisfaction.*



## Process Capability Objectives

- *By the end of this section the participant will be able to:*
  - *choose rational subgroups for proper sampling and analysis*
  - *calculate the within, between and total sum of squares in order to analyze and characterize the **components of variation***
  - *calculate the long and short term standard deviation and Z-values*
  - *differentiate between short term process capability and long term process capability*
  - *explain the general long term 1.5s shift*
  - *use Minitab Six Sigma Process Report to obtain short term and long term process capability measures<sup>3/4</sup>  $s_{ST}$ ,  $s_{LT}$ ,  $Z_{benchST}$ ,  $Z_{benchLT}$ ,  $Z_{shift}$ ,  $DPMO_{ST}$ ,  $DPMO_{LT}$*
  - *determine if there is a **control** problem or a **technology** problem<sup>3/4</sup> using the capability measures from above and a 2x2 matrix*



## *Process Capability Goals*

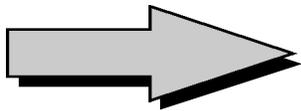
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### ■ **Goals**

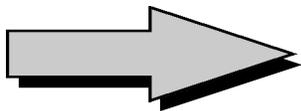
- *Familiarize participants with the key Six Sigma concepts of Control, and Technology and how they guide process improvement efforts.*
- *Enable participants to use Minitab tools to determine both long term and short term process capability measures<sup>¾</sup> including how to graphically display (via Histograms, Box Plots, Run Charts, and ANOVA results) both **special cause variation** and **common cause variation** in process data.*
- *Review and apply concepts of **Components of Variation** introduced in earlier material on **measures of variation**<sup>¾</sup> Range, Difference ( $X-Xbar$ ), Variance, Standard Deviation, and Sum of Squares Total (SST).*



## *What Is The Best Your Process Can Be?*



*A “snapshot” view of the process, free of non-random influences*



*Short-Term process capability ( $Z_{st}$ )*



## *Special Versus Common Cause Variation*

---

- *What is special (assignable) cause variation?*
  - *It is non-random variation which can be assigned to specific causes*
  - *It is controllable variation*
- *What is common (random) cause variation?*
  - *It is an inherent, natural source of variation of the process*
  - *It is not controllable variation*

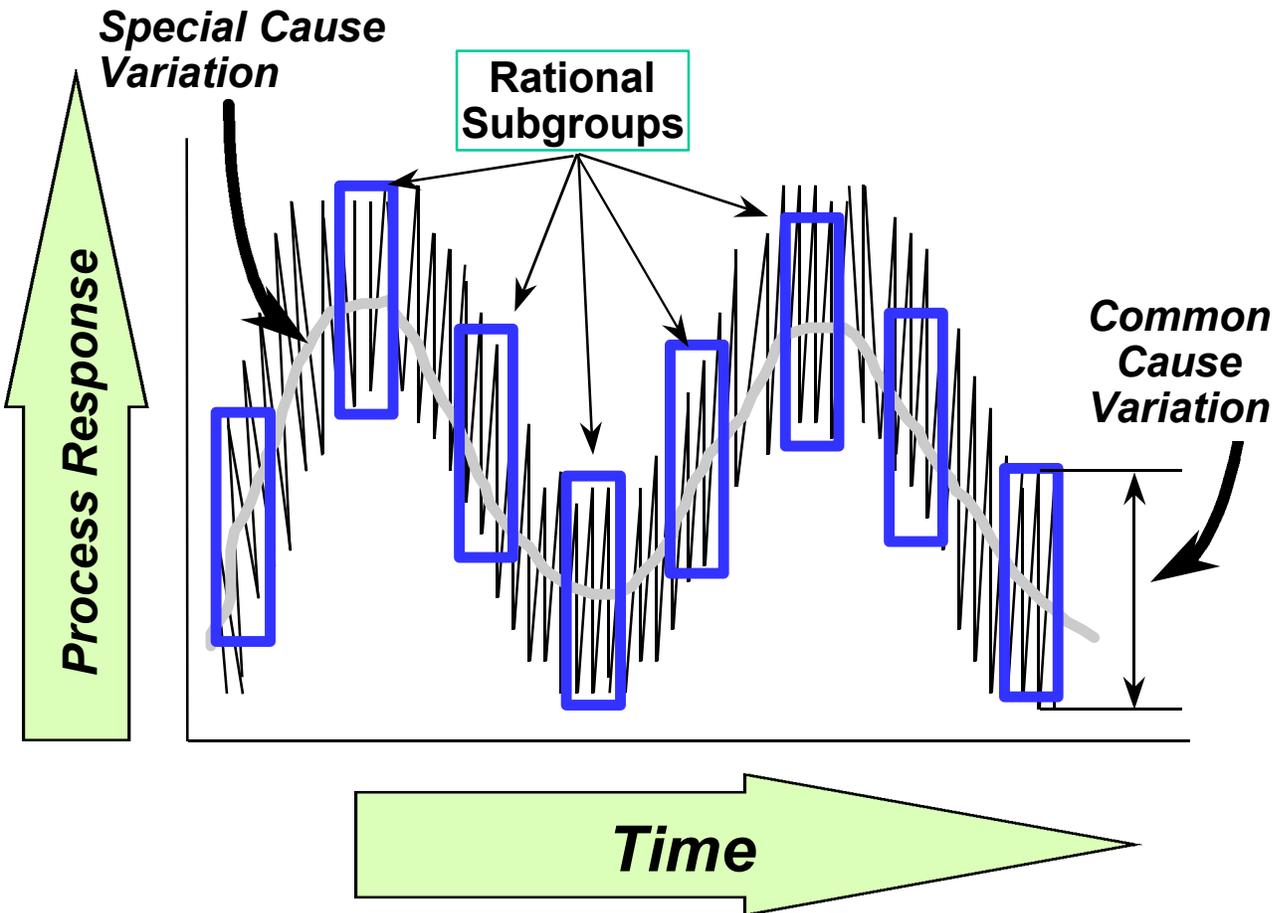


## *What is the Source of Variation?*

<b>Process</b>	<b>Special Cause</b>	<b>Common Cause</b>
NC Machines	<ul style="list-style-type: none"><li>• Operator Setup</li><li>• Variation between the machines</li><li>• Tool Wear</li><li>• Speed</li><li>• Feed Rate</li><li>• Tool Material</li><li>• Tool Angle</li></ul>	<ul style="list-style-type: none"><li>• Precision of each machine</li><li>• Material hardness variability</li></ul>
Catapult		
Your Project		



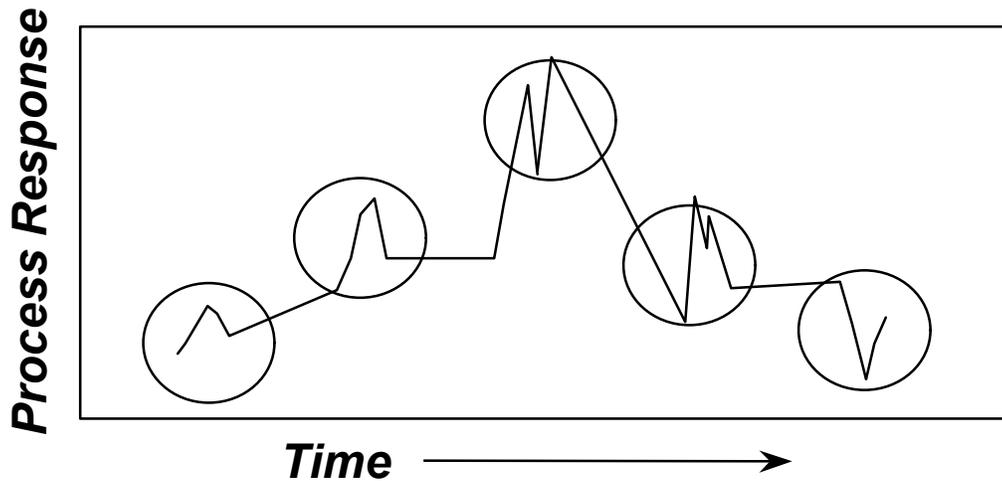
## Rational Subgrouping



***Rational Subgrouping attempts to take samples that include only common cause variation, within the samples. Special Cause Variation occurs between the samples.***



# Rational Subgrouping Principles



## RATIONAL SUBGROUPING

CHOOSE SUBGROUPS SO THAT:

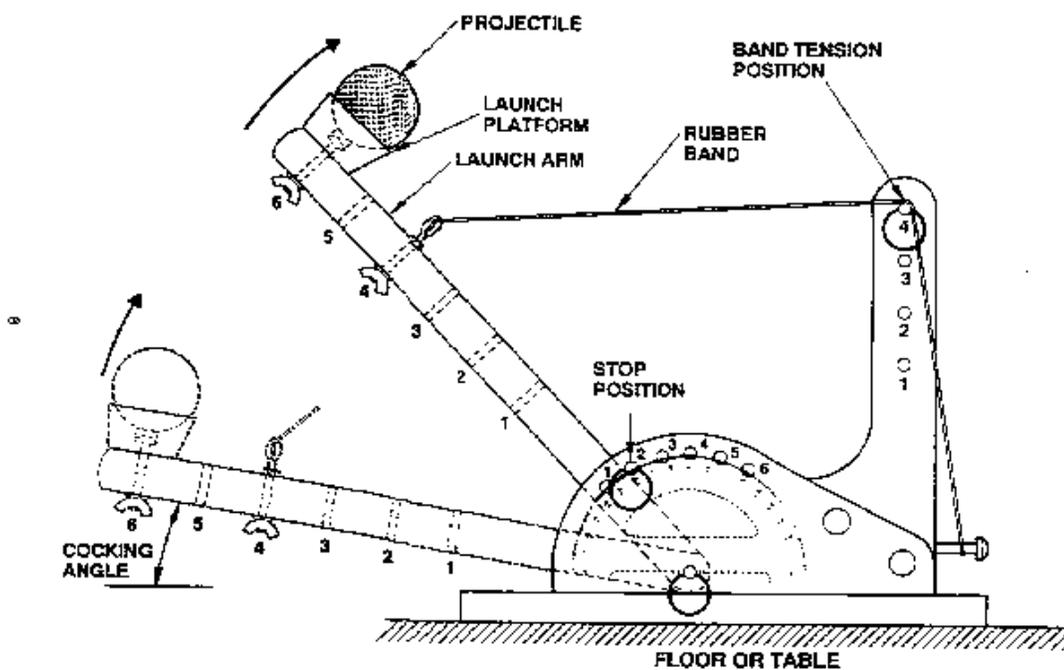
1. There is maximum chance for the measurements in each subgroup to be alike. A subgroup should only contain common cause variation.
2. There is maximum chance for subgroups to differ from one to the next. The difference between the subgroups is the special cause variation.

	Special Cause Variation	Common Cause Variation
Long term Capability	X	X
Short term Capability		X



## *Rational Subgrouping Exercise*

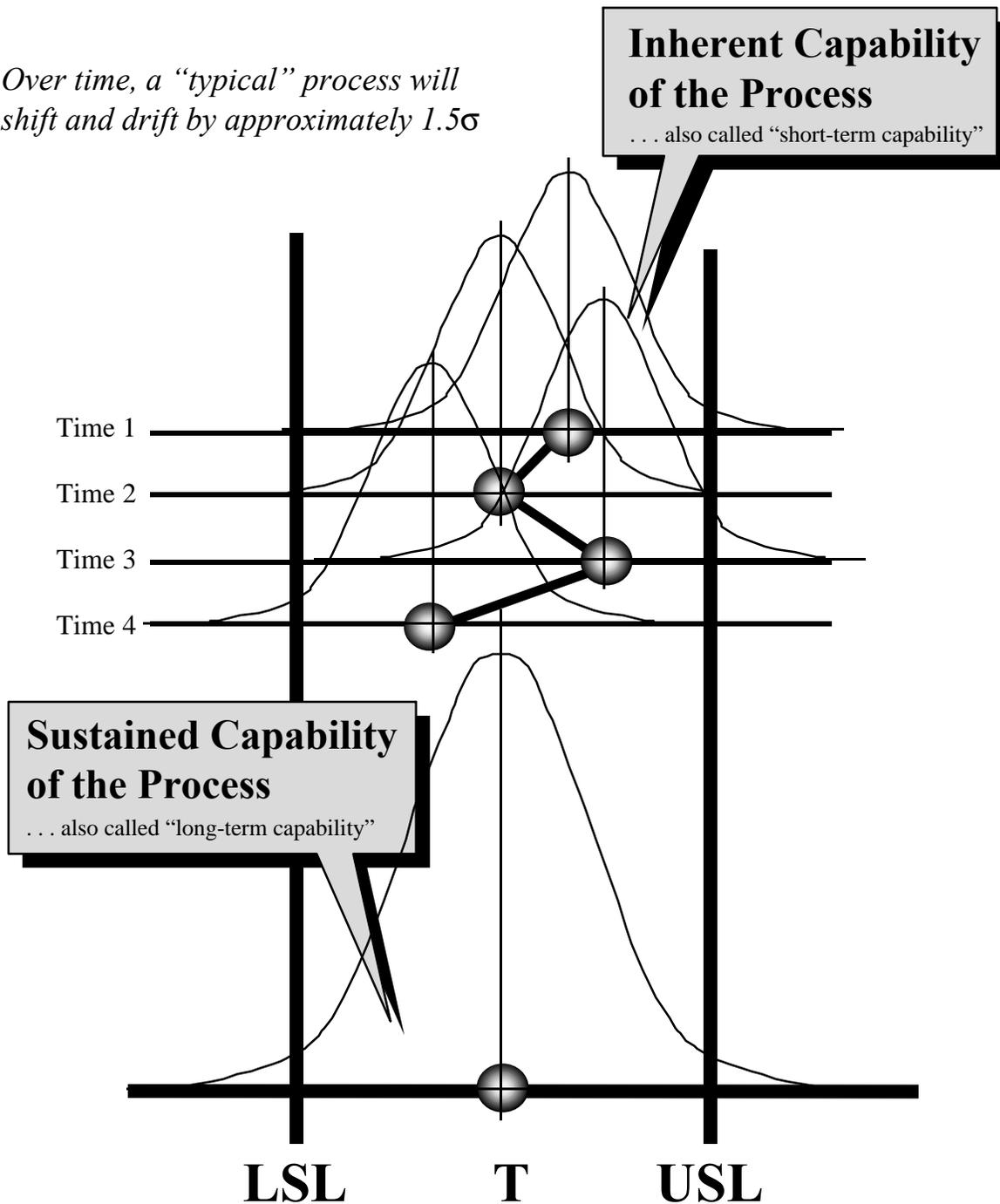
*Based upon our discussion of the sources of variation, how would you rationally subgroup the catapult data?*





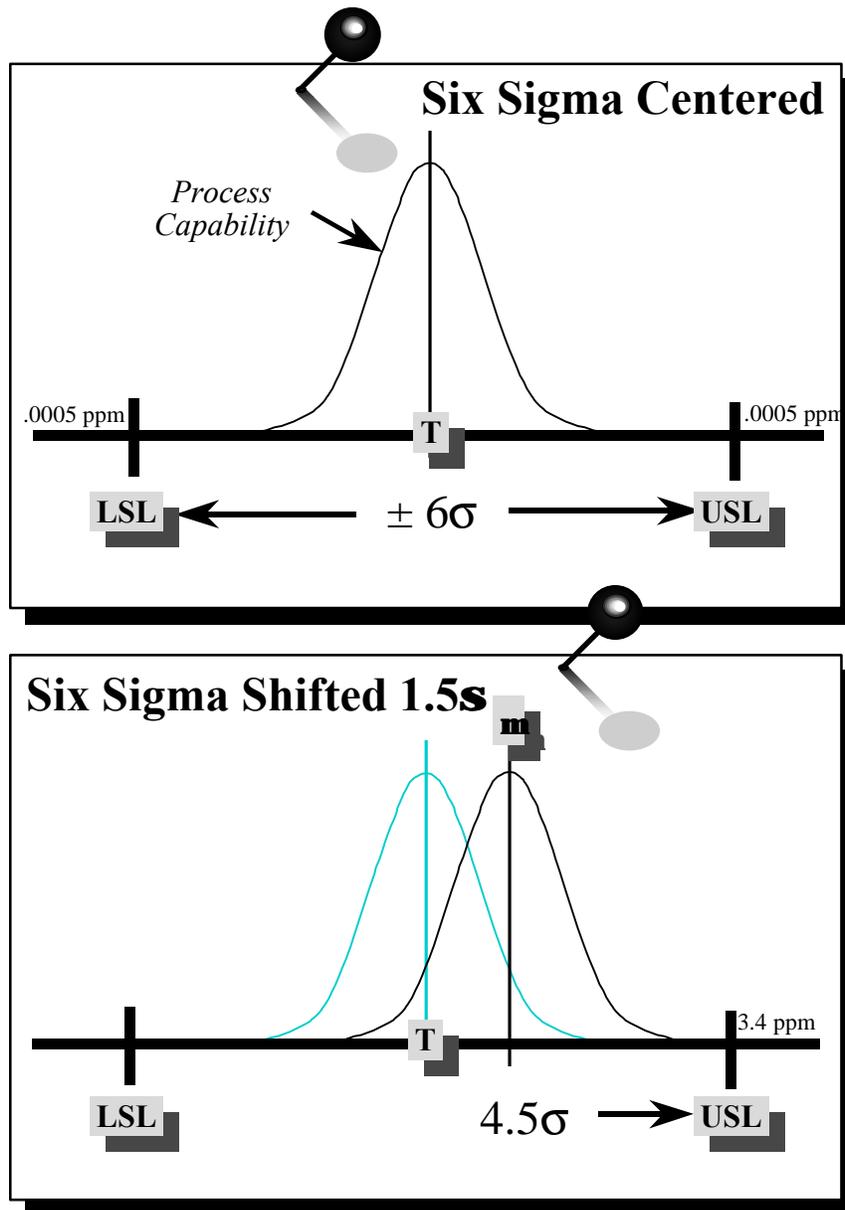
# Visualizing the Process Dynamics

*Over time, a “typical” process will shift and drift by approximately  $1.5\sigma$*





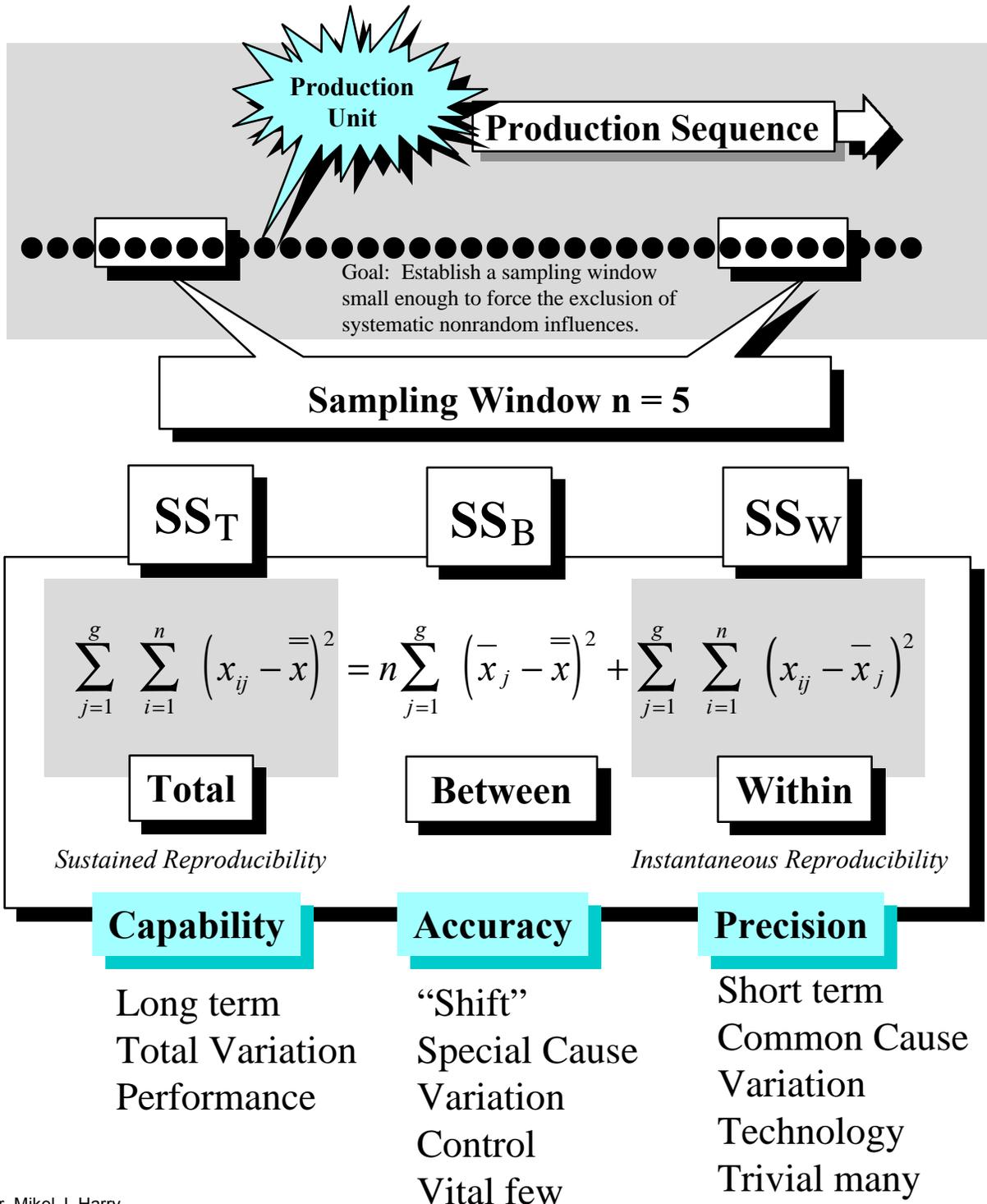
## Generalizing the Correction



The 1.5 $\sigma$  shift is used as a compensatory offset in the mean to generally account for dynamic nonrandom variations in process centering. It represents the average amount of change in a typical process over many cycles of that process.

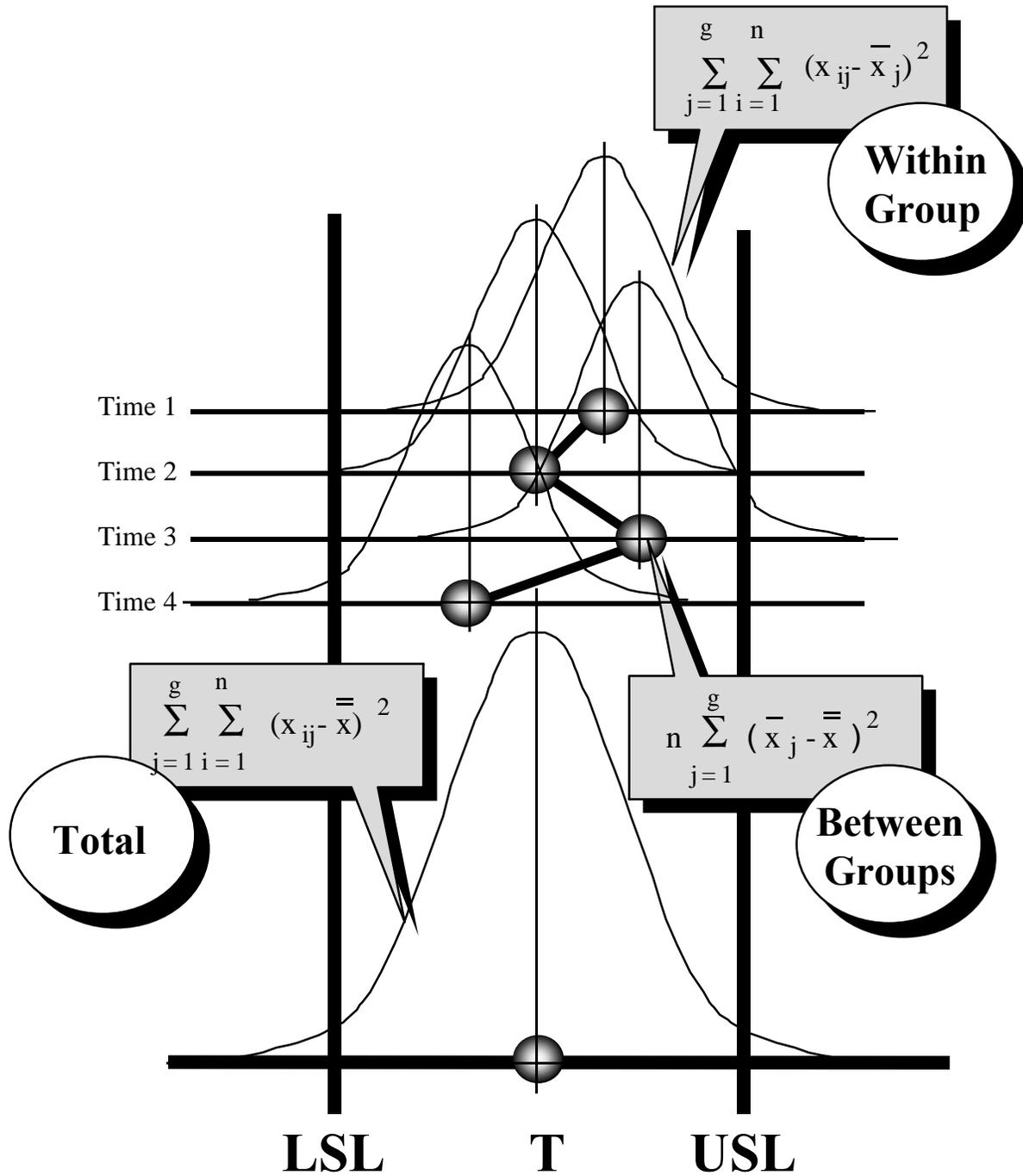


# The Components of Variation





# Visualizing the Components





# Example: Process Capability

## 3/4 the Components of Variation

	<u>Sample (i) = data points</u>					<u>Subgroup Average</u>	<u>Subgroup SS</u>
	<u>i=1</u>	<u>i=2</u>	<u>i=3</u>	<u>i=4</u>	<u>i=5</u>	$\bar{X}_j = \frac{\sum_{i=1}^n X_{ij}}{n}$	$SS_j = \sum_{i=1}^n (X_{ij} - \bar{X}_j)^2$
j=1	1	2	3	4	5	_____	_____
j=2	2	3	4	5	6	_____	_____
j=3	3	4	5	6	7	_____	_____
j=4	4	5	6	7	8	_____	_____
j=5	5	6	7	8	9	_____	_____
j=6	4	5	6	7	8	_____	_____
j=7	3	4	5	6	7	_____	_____
j=8	2	3	4	5	6	_____	_____

g = number of subgroups = 8, n = samples per group = 5

**Overall Average:**  $\bar{\bar{X}} = \frac{\sum (\bar{X}_j)}{g} = \underline{\hspace{2cm}}$

$= \frac{\sum_j \sum_i (x_{ij})}{ng} = \underline{\hspace{2cm}}$

**Overall SS:**  $SS_{Total} = \sum_j \sum_i (x_{ij} - \bar{\bar{X}})^2 = \underline{\hspace{2cm}}$

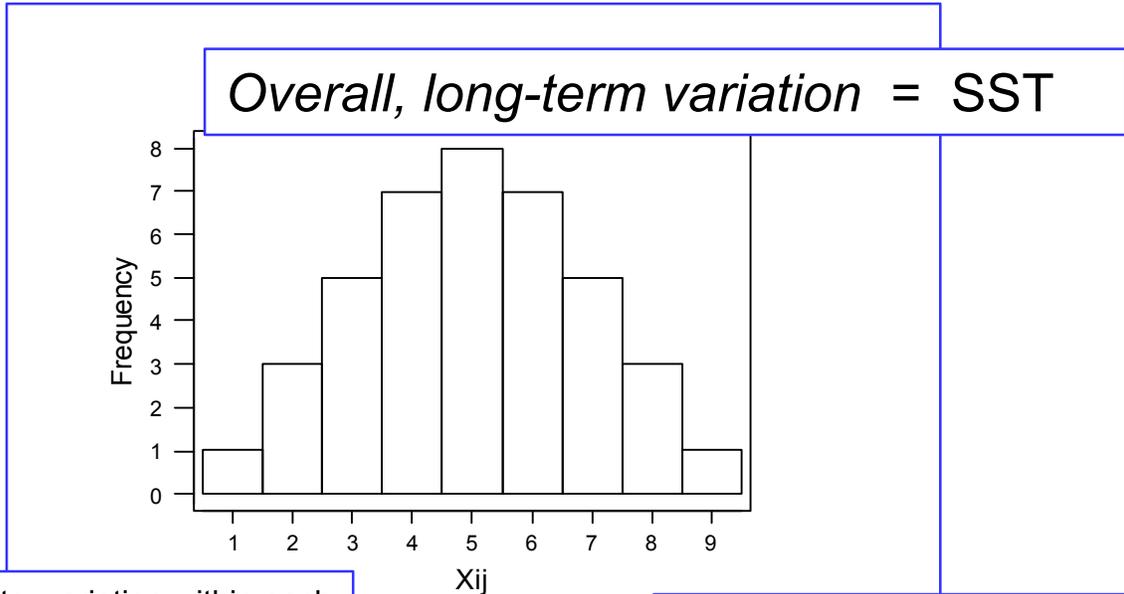
**Pooled Subgroup SS:**  $SS_{Within} = \sum_{j=1}^g \sum_i (x_{ij} - \bar{X}_j)^2 = \underline{\hspace{2cm}}$

**SS due to difference between groups:**  $SS_{Between} = n \sum_j (\bar{X}_j - \bar{\bar{X}})^2 = \underline{\hspace{2cm}}$

**Hint:**  $SS_{Between} = SS_{Total} - SS_{Within}$



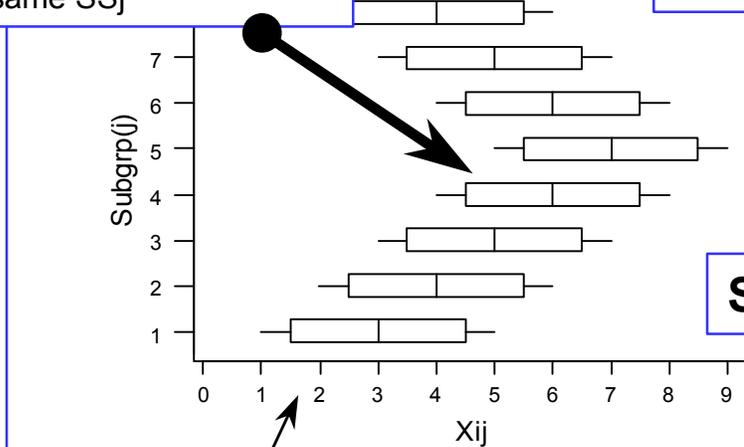
# Plot data: Histogram & Box Plots



Note: variation within each group is the same:

- same Range
- same subgroup Std Dev
- same SSj

has two components =  
*within group* and  
.... *between group*



$$SST = SSW + SSB$$

Any signs of “*shift and drift*”?  
Which “component” increases due to “*shift & drift*”?



## “Hand-calculations” using Minitab:

1.  $\bar{X} = 5.0$

[Calc > Column Statistics > Mean] or use  
Descriptive Statistics tool

Mean = 5.0 and **StdDev** =  $s_{LT} = 1.89$

2.  $SST = 140$

calculate  $Diff = (X_i - \bar{X})$  for all 40  
datapoints—store  $Diff$  in Minitab data sheet ...  
then Calc > Col. > Sum of Squares >  $Diff$   
to get  $SST$  ... a double-check is to square  $Diff$   
column and sum it.

3. **SSW** = 80—see work sheet and our hand calculation

4.  $SSB = SST - SSW = 140 - 80 = 60$   
**SSB = 60**



## “Hand-calculations” using Minitab:

**Question:** Is 60 a “significant” portion of SST—i.e., is there “significant” Special Cause Variation (Shift & Drift), in this case?

The ANOVA tool gives the answer:

Stat > ANOVA > OneWay

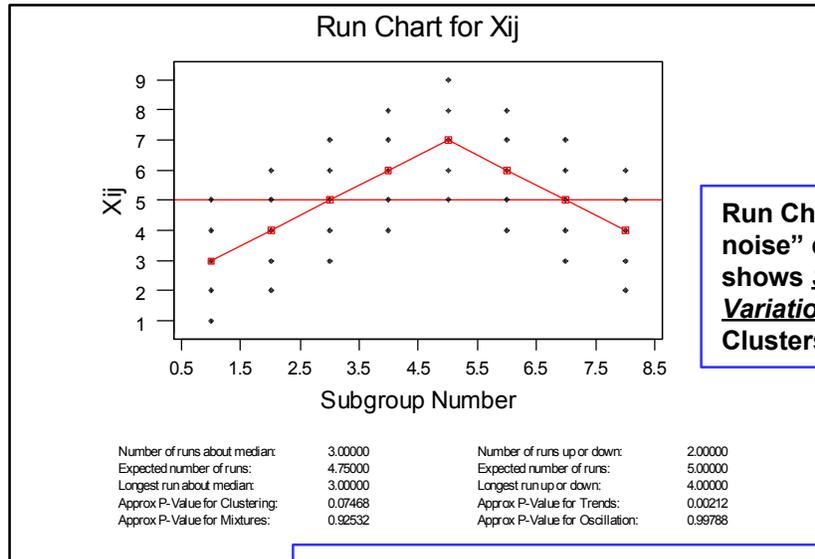
Xij	Subgrp(j)	Xbarbar	Diff=(Xi-Xbarbar)	Sqrd
1	1	5	-4	16
2	1	5	-3	9
3	1	5	-2	4
4	1	5	-1	1
5	1	5	0	0
2	2	5	-3	9
3	2	5	-2	4
4	2	5	-1	1
5	2	5	0	0
6	2	5	1	1
3	3	5	-2	4
4	3	5	-1	1
5	3	5	0	0
6	3	5	1	1
7	3	5		
4	4	5		
5	4	5		
6	4	5		

Sample columns from Minitab data sheet

... more data in file ...



# “Hand-calculations” and plots:



Run Chart, the “black noise” detection tool, shows **Special Cause Variation** = Trends & Clusters (due to trend)

ANOVA by Subgroup shows **Special Cause Variation** is significant—and here is **MSB + MSW = MST**

One-Way Analysis of Variance

Analysis of Variance for  $X_{ij}$

Source	DF	SS	MS	F	P
Subgrp(j)	7	60.00	8.57	3.43	0.008
Error	32	80.00	2.50		
Total	39	140.00			

Individual 95% CIs For Mean  
Based on Pooled St Dev

Level	N	Mean	St Dev
1	5	3.000	1.581
2	5	4.000	1.581
3	5	5.000	1.581
4	5	6.000	1.581
5	5	7.000	1.581
6	5	6.000	1.581
7	5	5.000	1.581
8	5	4.000	1.581

Pooled St Dev = 1.581

this is  $s_{ST}$

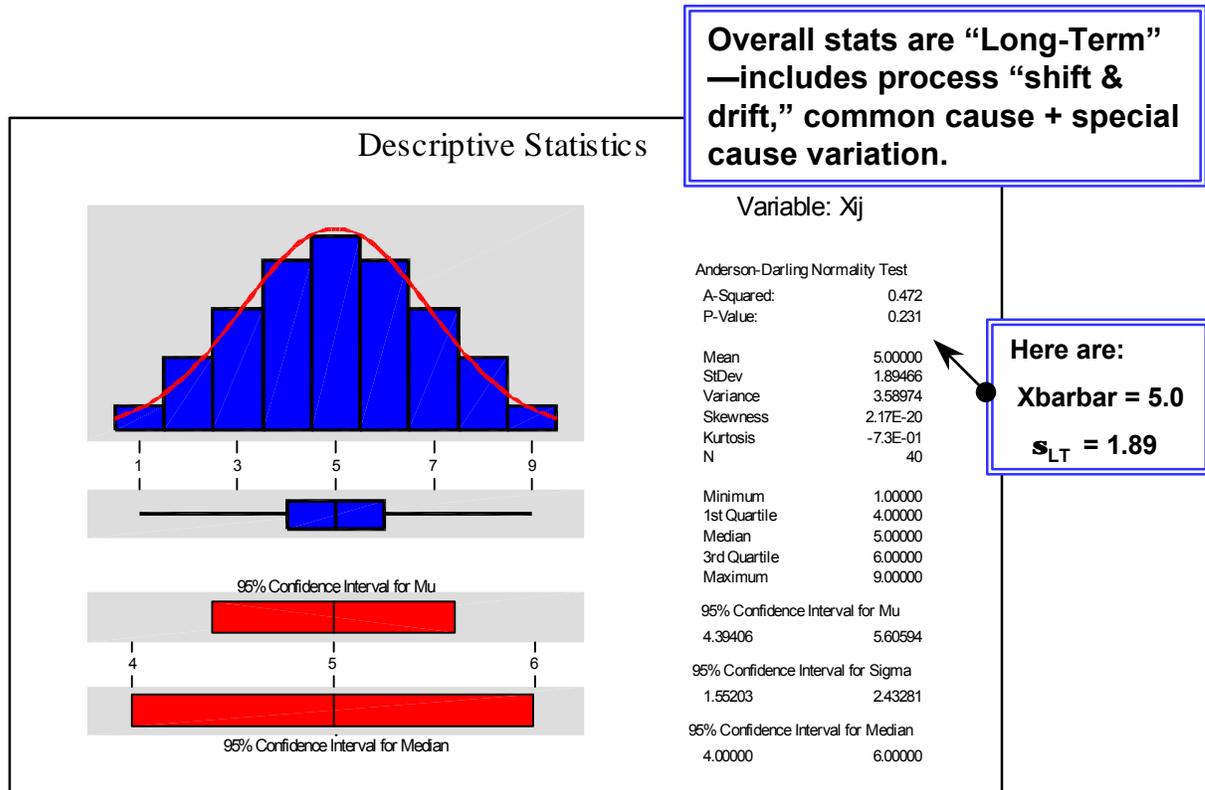
NOTE: in this case, all subgroups have the same spread:

- same Range
- same subgroup StdDev (Variance)
- same SSj
- same Confidence Interval width

[example was made this way]



## “Hand-calculations” and plots:



### Summary—we now have data on:

- $\bar{X} = 5.0$
- $s_{LT} = 1.89$
- $s_{ST} = 1.58$

also know that  $MST = 140$ ,  $MSB = 60$ ,  $MSW = 80$

... with  $MSB$  being a “significant” portion of  $MST$ —  
i.e., there is a “significant” amount of *Special Cause Variation* in this case.

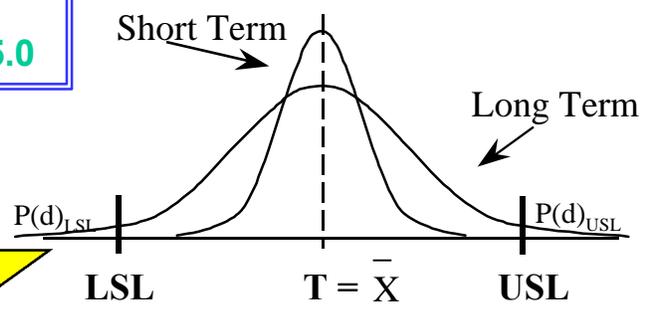
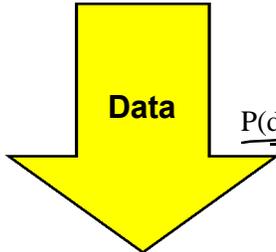
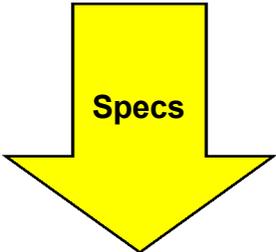
NEXT STEP: Calculate Z values and final, overall  $Z_{\text{benchmark}}$



# Capability Calculations = Z-Bench

**LSL = 0**  
**T = Nom = 5**  
**USL = 10**  
 **$s_{LT} = 1.89$**   
 **$s_{ST} = 1.58$**   
**Xbarbar = 5.0**

**Step 6: continued**



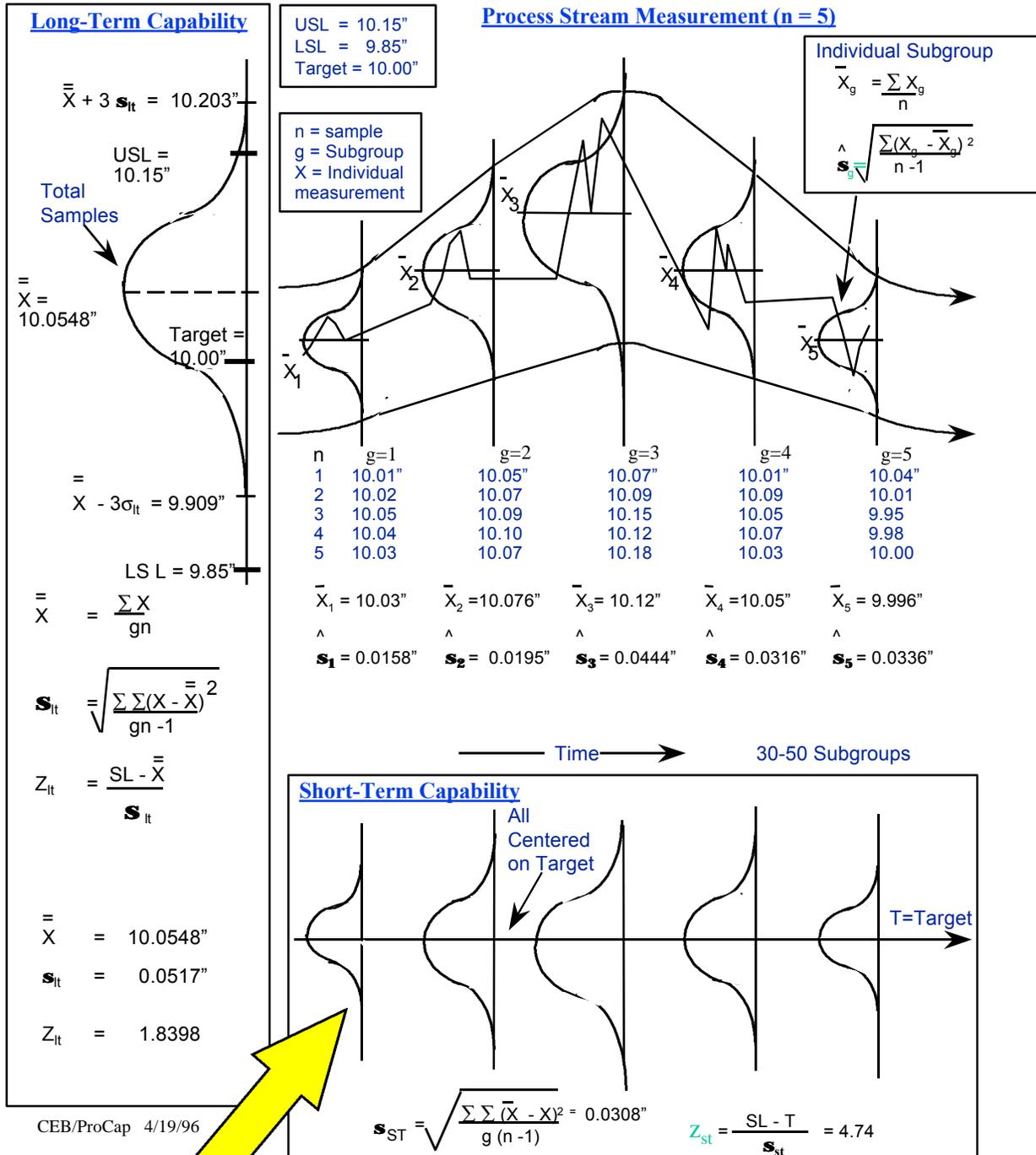
Z-Long Term:	Z-Short Term:
$Z_{USL} = \frac{USL - \hat{X}}{\hat{s}_{LT}}$ $Z_{LSL} = \frac{\hat{X} - LSL}{\hat{s}_{LT}}$	$Z_{USL} = \frac{USL - T}{\hat{s}_{ST}}$ $Z_{LSL} = \frac{T - LSL}{\hat{s}_{ST}}$
<p> <math>Z_{USL} = (10-5)/1.89 = 2.65</math> and  <math>Z_{LSL} = (5-0)/1.89 = 2.65</math>            [ Find P(d) for both Z's from Tables ]            and since <math>P(d)_{USL} = P(d)_{LSL} = 0.004</math>  <math>P(d)_{Total} = P(d)_{USL} + P(d)_{LSL}</math>  <math>P(d)_{Total} = 0.004 + 0.004 = \underline{0.008}</math> </p> <p> <math>Z_{B-LT}</math> [ from Z-table ] gives  <math>Z_{B-LT} = \underline{2.41}</math> </p>	<p> <math>Z_{USL} = \underline{\hspace{2cm}}</math>  <math>Z_{LSL} = \underline{\hspace{2cm}}</math>  <math>P(d)_{USL} = \underline{\hspace{2cm}}</math>  <math>P(d)_{LSL} = \underline{\hspace{2cm}}</math>  <math>P(d)_{Total} = P(d)_{USL} + P(d)_{LSL}</math>  <math>P(d)_{Total} = \underline{\hspace{2cm}}</math> </p> <p> <math>Z_{B-ST}</math> [ from Z-table ] = <math>\underline{\hspace{2cm}}</math> </p>



Now that we've done this by hand .... let's use Minitab Six Sigma Process Report tool set ...



# Exercise: Short- & Long-Term Process Capability $\frac{3}{4}$ using Minitab tools



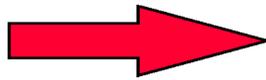
**Entitlement = On Target & Min Between-Group Variation**



# Sum of Squares & Standard Deviations

## One Sample

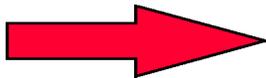
$$\hat{s} = \sqrt{\frac{\sum (X - \bar{X})^2}{n - 1}}$$



$$\hat{s} = \sqrt{\frac{\text{Sum of Squares}}{\text{Degrees of freedom}}}$$

## Long Term

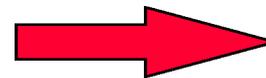
$$\hat{s}_{lt} = \sqrt{\frac{\sum_{j=1}^g \sum_{i=1}^n (x_{ij} - \bar{x})^2}{gn - 1}}$$



$$\hat{s}_{lt} = \sqrt{\frac{SS_{\text{Total}}}{\text{Degrees of freedom}}}$$

## Short Term

$$\hat{s}_{st} = \sqrt{\frac{\sum_{j=1}^g \sum_{i=1}^n (x_{ij} - \bar{x}_j)^2}{g(n - 1)}}$$



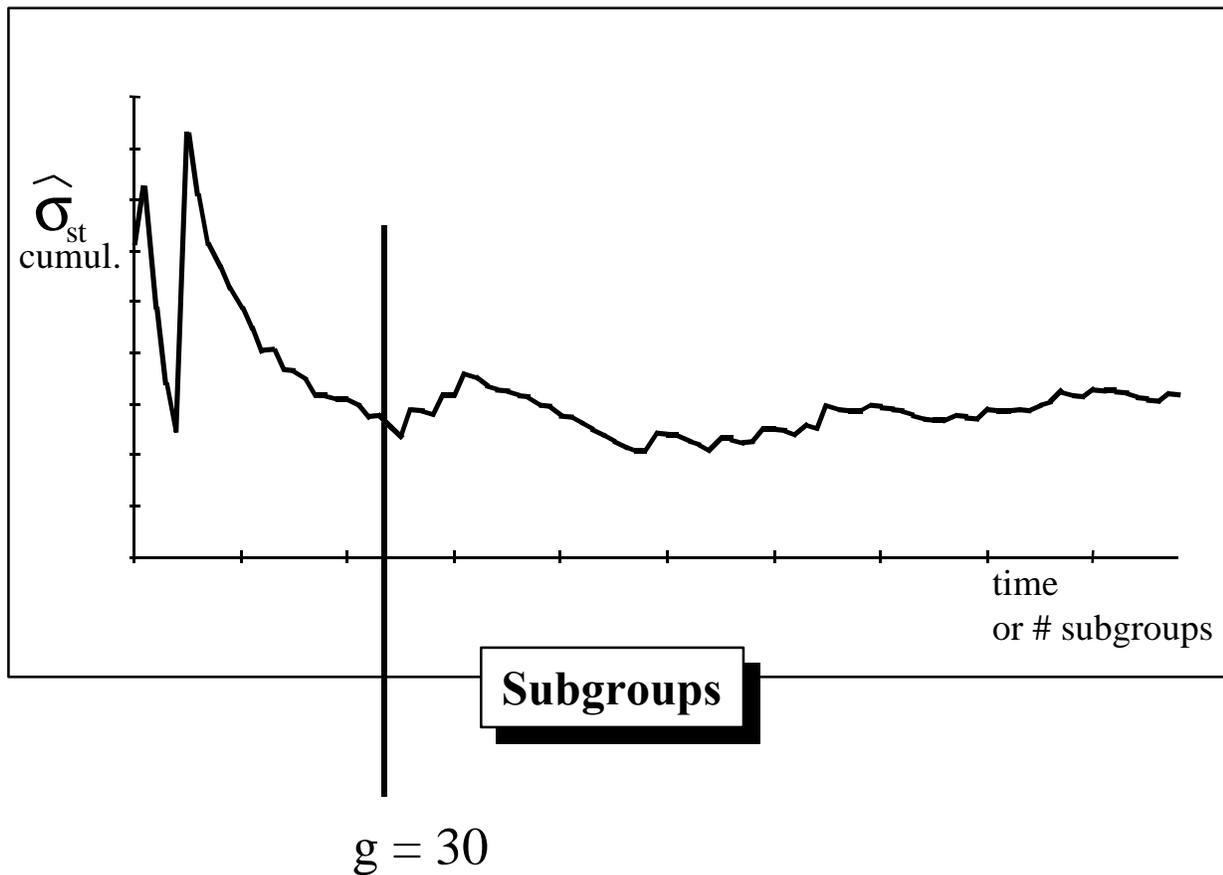
$$\hat{s}_{st} = \sqrt{\frac{SS_{\text{Within}}}{\text{Degrees of freedom}}}$$

***It's All Variation***



# How Many Subgroups Do You Need?

Keep taking data until  $\hat{\sigma}_{st}$  stabilizes





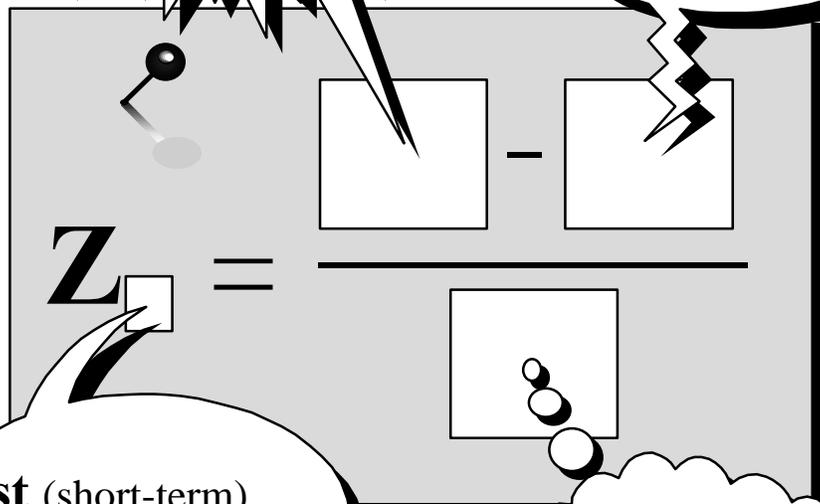
# The Universal Equation for Z

... so what are the possibilities?

$$Z = \frac{SL - \lambda}{\hat{\sigma}}$$

$$SL = \frac{USL}{LSL}$$

$$\lambda = \frac{T \text{ (Target)}}{\hat{m} \text{ (Mean)}}$$



$$Z = \frac{\quad - \quad}{\quad}$$

**st** (short-term)  
**lt** (long-term)

$\hat{S}_{st}$   
 $\hat{S}_{lt}$

and how do we choose the right one?



# Selecting an Appropriate Z

★ Eq. 8.1  $Z_{st} = \frac{SL - T}{\hat{\sigma}_{st}}$

This Z value is designated as Z.st. It describes how precise the process is at any given moment in time. For this reason, it is referred to as “instantaneous capability.” It is also called “short-term capability.” In context of the Six Sigma Program, it is the value used when referring to the “SIGMA” of process. It represents the true potential of the process technology to meet the given performance specification(s); I.e., what the process can do if everything is controlled to such an extent that only background noise is present, common cause variation. It reflects the process capability under the assumption of random variation and does not give consideration to the process center. This metric assumes the data were gathered in accordance to the principals and spirit of a “rational sampling” plan. For a unilateral tolerance with no target Eq. 2 should be used.

Eq. 8.3  $Z = \frac{SL - T}{\hat{\sigma}_{lt}}$

This Z value is designated as Z.lt.d. It is a measure of long-term process capability. It reflects the influence of white noise, as well as the dynamic variations due to nonrandom process centering error; i.e., shifts and drifts in the process mean across sampling subgroups. It assumes that the errors in process centering are dynamic and will eventually average out (over a great many cycles) to the target specification. In context of the Six Sigma Program, it is not often used, except in some design engineering applications. This metric assumes the data were gathered in accordance to the principals and spirit of a “rational sampling” plan. For a unilateral tolerance with no target value, this equation cannot be used. In such an event, Eq. 4 should be employed to estimate long-term process capability.

Eq. 8.2  $Z = \frac{SL - \hat{\mu}}{\hat{\sigma}_{st}}$

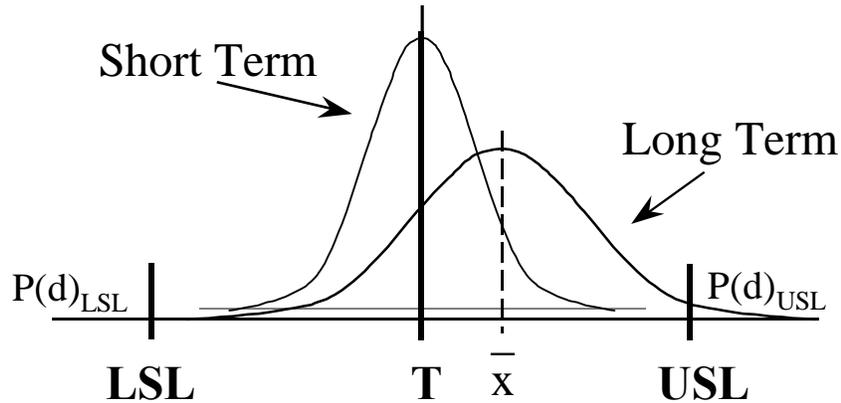
This Z value is designated as Z.lt. It is a measure of long-term capability and, when used properly, reflects process accuracy when compared to Z.st; e.g., Z.st - Z.lt = Z.shift. Expressed differently, it reflects how well the process remains centered over time. Of course, it ignores any nonrandom process centering errors which may occur between sampling intervals. This metric assumes the data were gathered in accordance to the principals and spirit of a “rational sampling” plan. However, in the instance of a unilateral tolerance with no target specification, the given Z value will reflect only short-term capability. In this circumstance, the mean becomes the target. Consequently, it will produce the same result as Eq. 1; therefore, it should be designated as Z.st and so interpreted.

★ Eq. 8.4  $Z_{lt} = \frac{SL - \hat{\mu}}{\hat{\sigma}_{lt}}$

This Z value is designated as Z.lt.s. It describes the sustained reproducibility of a process. Because of this, it is also called “long-term capability.” In context of the Six Sigma Program, it is the value used to estimate the long-term process “PPM.” It reflects the influence of special cause variation, dynamic nonrandom process centering error, and any static offset present in the process mean. From this perspective, it considers all of the “vital few” sources of manufacturing error. It is a measure of how well the process is controlled (over many cycles) when compared to Z.st. This metric assumes the data were gathered in accordance to the principals and spirit of a “rational sampling” plan. This equation is applicable to all types of tolerances.



# Z-Bench



Z-Long Term	Z-Short Term
$Z_{lt} = \frac{SL - \hat{m}}{\hat{\sigma}_{lt}}$	$Z_{st} = \frac{SL - T}{\hat{\sigma}_{st}}$
Z-Bench-Long Term	Z-Bench-Short Term
$Z_{USL} = \frac{USL - \hat{m}}{\hat{\sigma}_{lt}}$ $Z_{LSL} = \frac{\hat{m} - LSL}{\hat{\sigma}_{lt}}$ <p> <math>P(d)_{USL} =</math> from Z table  <math>P(d)_{LSL} =</math> from Z table         </p> $P(d)_{Total} = P(d)_{USL} + P(d)_{LSL}$ <p> <math>Z_{B-lt} =</math> from Z table         </p>	$Z_{USL} = \frac{USL - T}{\hat{\sigma}_{st}}$ $Z_{LSL} = \frac{T - LSL}{\hat{\sigma}_{st}}$ <p> <math>P(d)_{USL} =</math> from Z table  <math>P(d)_{LSL} =</math> from Z table         </p> $P(d)_{Total} = P(d)_{USL} + P(d)_{LSL}$ <p> <math>Z_{B-st} =</math> from Z table         </p>



# Minitab Six Sigma Process Report

MINITAB FILE: Catapult.mtw

Session

```
Worksheet size: 100000 cells
MTB > Retrieve "C:\quality\Catapult.mtw"
Retrieving worksheet from file: C:\qual...
Worksheet was saved on 1/28/1998
MTB >
```

	C3	C4	C5	C6	C7	C8	C9
	Oper 1	Oper 2	Oper 3	Oper 4	Oper 5	Dist 50	Oper 50
2	50.50	49.00	50.00	50.25	49.75	50.50	1
3	49.75	51.50	49.25	50.50	49.75	49.75	1
4							
5							
6							

**Six Sigma Process Report**

Data are arranged as

Single column: 'Dist 50'

Subgroup size: 10  
(use a constant or an ID column)

Subgroups across rows of:

Lower spec: 46

Upper spec: 54

Target: 50 (optional)

Select Help OK Cancel

1. Double Click →

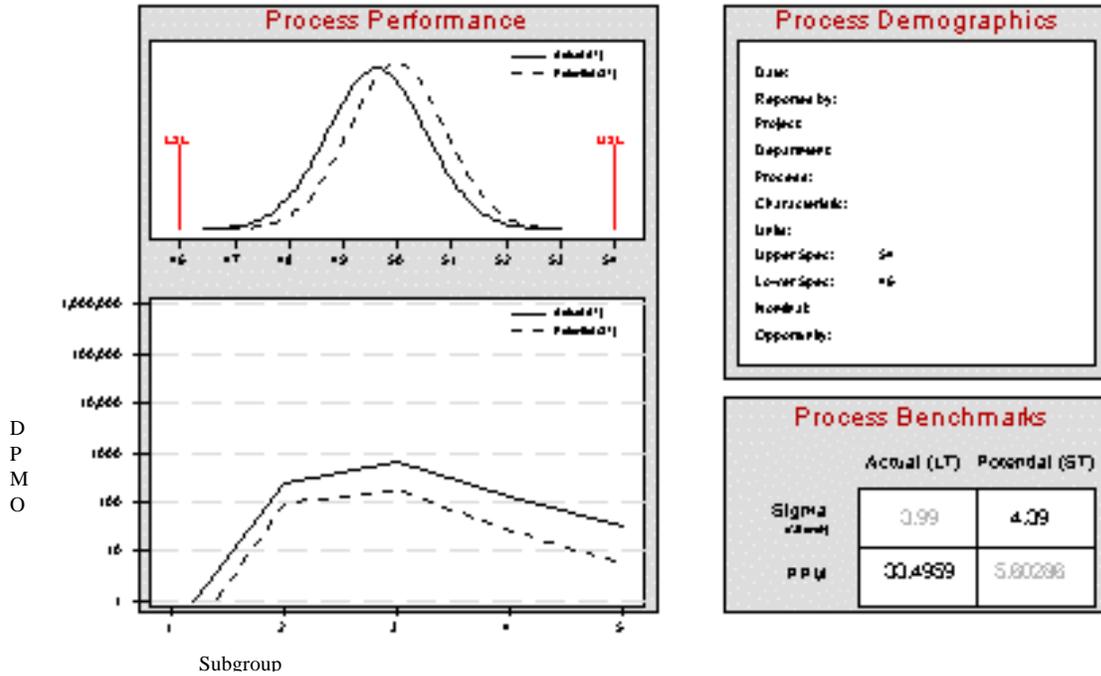
2. Type in Subgroup size "10"

3. Type in Lower and Upper specs and Target

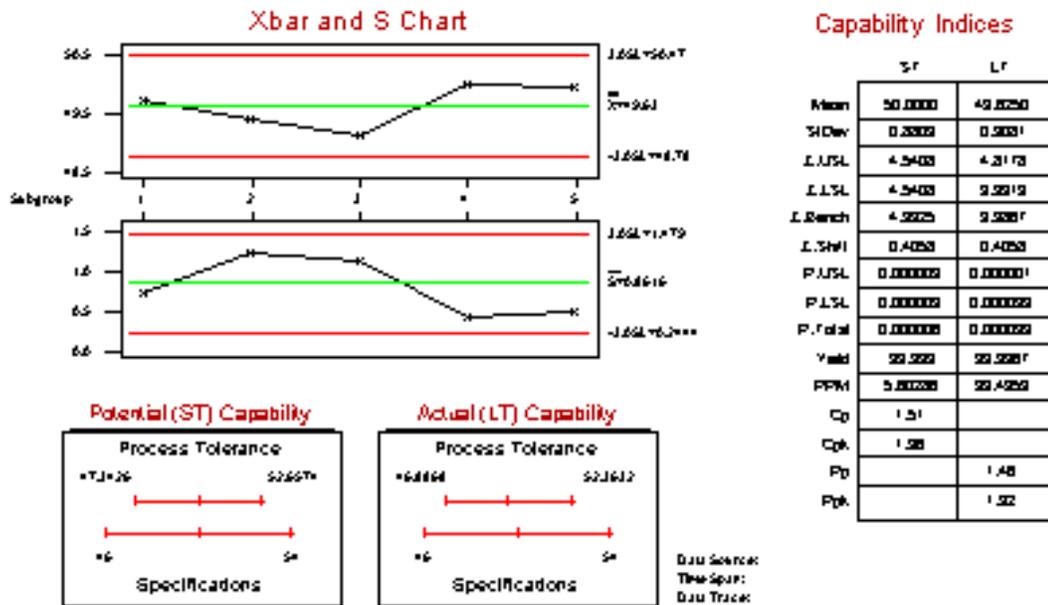


# Minitab Six Sigma Process Report

## Report 1: Executive Summary



## Report 2: Process Capability for Dist 50





## *Minitab Six Sigma Process Report*

### **Important Note:**

- *Using a subgroup size = 1 in the Minitab Process report results in misleading and, in most cases, wrong  $Z_{ST}$ ,  $Z_{SHIFT}$ , and  $DPMO_{ST}$  values.*
- *This discrepancy is due to the way that Minitab estimates  $\sigma_{ST}$ .*
- *Only the long term values are valid with a subgroup size = 1.*

**Subgroup Size = 1  
Makes Short Term Values Invalid**



## Is it Control or Technology?

### LONG TERM CAPABILITY

- $Z_{lt}$
- *Defined by technology and process control*
- *Real process performance*
- *6s means:  $Z_{lt} = 4.5$*

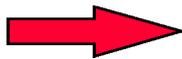
### SHORT TERM CAPABILITY

- $Z_{st}$
- *Limited by technology*
- *“Entitlement” - the best performance the process can have*
- *6s means:  $Z_{st} = 6.0$*

#### SHIFT:

- *Sigma Shift =  $Z_{Shift} = Z_{st} - Z_{lt}$*
- *Process control*

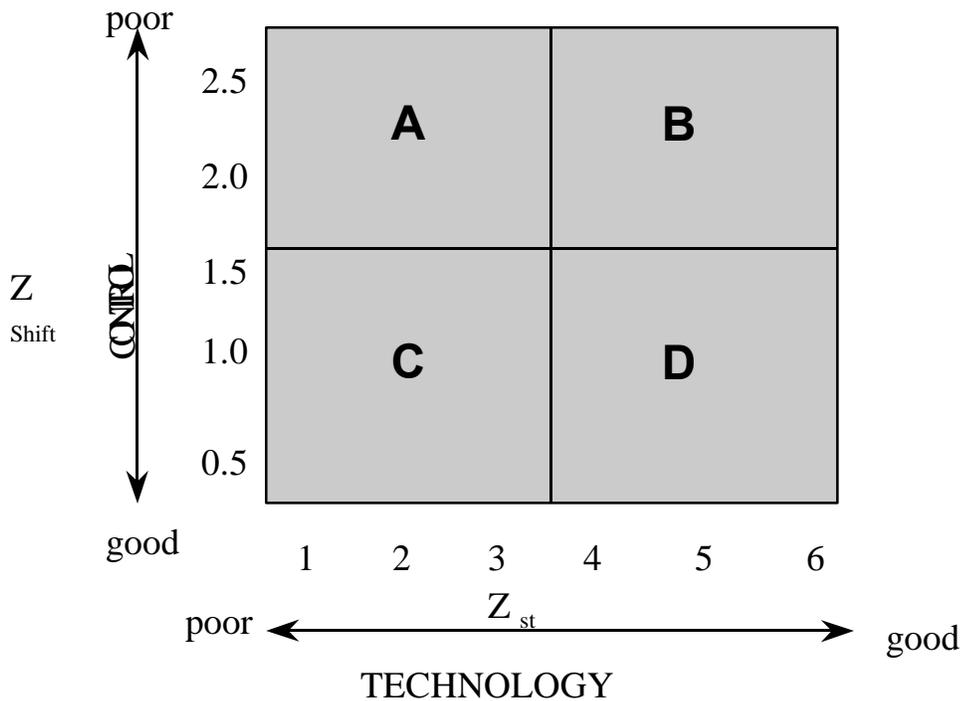
$Z_{st}$  vs.  $Z_{Shift}$



**Technology vs. Process Control**



## *A great way to tie it all together...*



**A - Poor control, poor technology**

**B - Must control the process better, technology is fine**

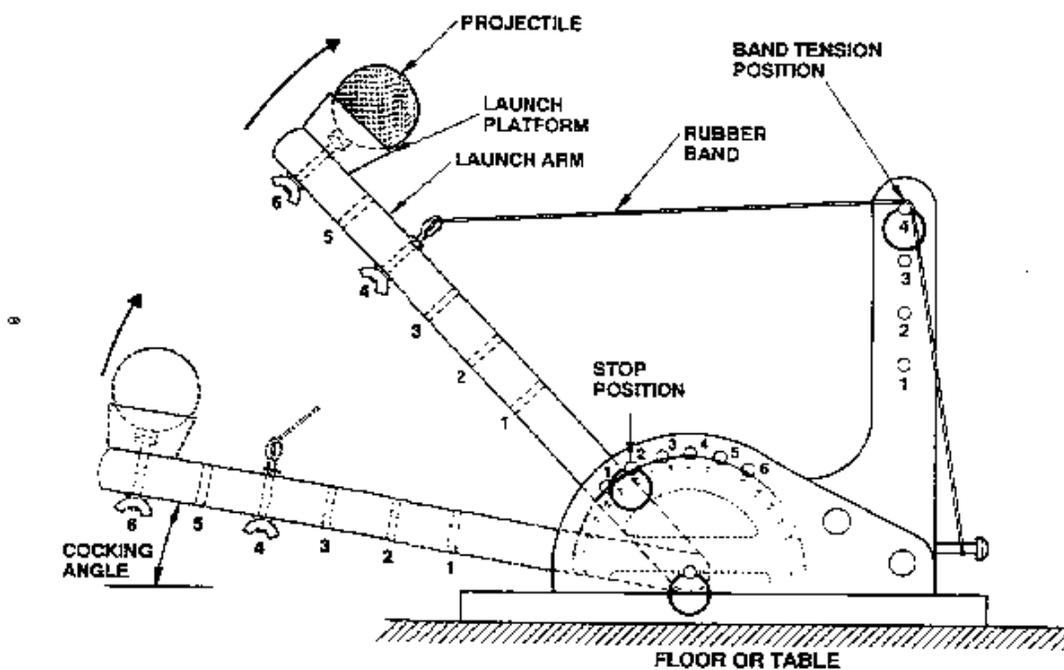
**C - Process control is good, bad technology**

**D - World class**



## Exercise

*Perform a capability analysis of your team's catapult data using the Minitab Six Sigma Process Report.*





## More Process Capability Terminology

	Short Term (Pooled) Standard Deviation	Long Term (Overall) Standard Deviation
Target	$C_p = \frac{\overset{\text{Closest}}{SL-T}}{3S_{st}}$	$P_p = \frac{\overset{\text{Closest}}{SL-T}}{3S_{lt}}$
Mean	$C_{pk} = \frac{\overset{\text{Closest}}{\bar{X}}}{3S_{st}}$	$P_{pk} = \frac{\overset{\text{Closest}}{\bar{X}}}{3S_{lt}}$



## Relating Back to Z

	Short Term (Pooled) Standard Deviation	Long Term (Overall) Standard Deviation
Target	Eq. 8.1 $Z_{st - \text{Closest SL}} = 3C_p$	Eq. 8.3 $P_p$
Mean	Eq. 8.2 $C_{pk}$	Eq. 8.4 $Z_{lt - \text{Closest SL}} = 3P_{pk}$



## *Take Aways—Step 4*

- *Continuous data is characterized by:*
  - *mean - numerical average, a measure of central tendency*
  - *median - “middle” observation if arranged in sequence*
  - *standard deviation - measure of variation, or dispersion*
  - *variance - square of standard deviation*
- *A normal curve may be used to describe a process that is experiencing only random variation. All normal distributions can be related to the standard normal distribution which has a mean of zero and a standard deviation of one.*
- *A sample is a subset of a population. In general, a good sample has the same characteristics as the population under study.*



## *Take Aways—Step 4*

- *Basic statistical summaries, histograms, dotplots, boxplots, and run charts are used to visualize data and better understand a process.*
- *The z-value is the number of standard deviations that will fit between the mean and the respective specification limit.*
- *The z-value corresponds to a probability of defect, or the area under the curve outside the specification limits.*
- *The z-value is a non-dimensional quantity that enables us to compare different processes - it represents the process capability.*



## *Take Aways—Step 4*

- *Define Defects, Units and Opportunities with your team. Be sure the definitions make sense and are consistent with similar processes. Benchmark where possible.*
  
- *Defects will be stated as Defects Per Million Opportunities. Discrete data is generally considered long term data.*
  
- *For discrete data, the L1 spreadsheet or Minitab Six Sigma Product Report will be used to calculate capability from defects and opportunities.*
  - Determine DPMO (which is long-term), then determine the corresponding Z value (LT capability).*



## *Take Aways—Step 4*

---

- *Often, you must assume a shift value (default 1.5) to estimate short term capability.*
- *Our customers experience the long term capability of the process.*
- *Rolled Throughput Yield ( $Y_{RT}$ ) illustrates how complexity (i.e. a multi-process system) affects quality.*
- *Six Sigma cannot be reached by inspection.*



## *Take Aways—Step 4*

- *Rational Subgrouping refers to grouping the data for analysis in a meaningful way to understand variation. Rational Subgrouping attempts to select groups of data such that mainly common cause variation is within groups, and mainly special cause variation is between groups.*
  - *Special Cause = Between group variation, due to assignable causes, non-random influences*
  - *Common Cause = Within group variation, inherent in a process, random influences*
  
- *The Sum of Squares (SS) reflects the different types of variation as described above. The Sum of Squares Total (SST) is equal to the Sum of Squares Between (SSB) plus the Sum of Squares Within (SSW).*



## *Take Aways—Step 4*

- *The Minitab Six Sigma Process Report is used to describe capability with continuous data.*
  - *displays the actual capability relative to the target distribution*
  - *by rationally subgrouping (subgroup size >1), long term capability, short term capability, and shift are calculated*
- *To minimize shift we need to reduce special cause variation.*
- *A subgroup that contains only common cause variation, or random variation, represents the **short term capability** of the process, or process entitlement (**Z Bench**).*
- *When a subgroup contains **both** common cause and special cause variation, this data represents the **long term capability of the process**. The shift (from  $Z_{ST}$  to  $Z_{LT}$ ) occurs over many cycles of the process.*



# Analyze Phase



**4. Establish Process Capability**

*Deliverable: Baseline Current Process*

*Tools:*

- *Basic statistics*
- *Graphical Analysis*
- *Sampling*
- *Continuous Zst, ZIt*
- *Normality*
- *Discrete Zst, ZIt*

**5. Define Performance Objectives**

*Deliverable: Statistically Define the Goal of the Project*

*Tools:*

- *Benchmarking*

**6. Identify Variation Sources**

*Deliverable: List of Statistically Significant X's, Chosen Based on Analysis of Historical Data*

*Tools:*

- *Process Analysis*
- *Graphical Analysis*
- *Hypothesis Testing*
- *Regression Analysis*
- *GLM*



# *Define Performance Objectives*



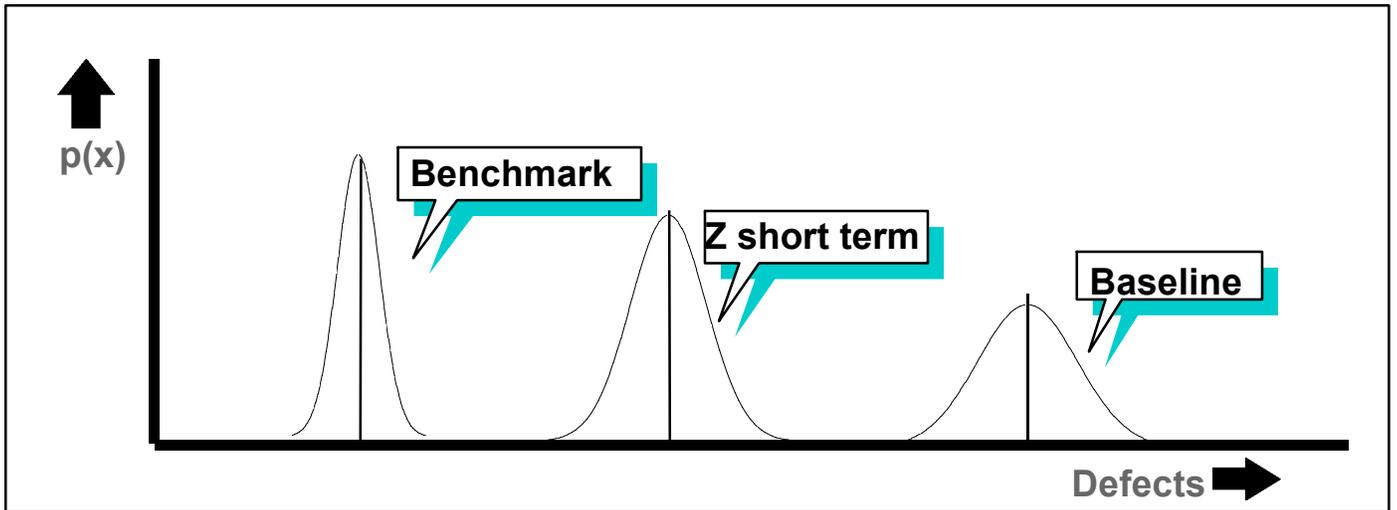
## *Step 5: Define Performance Objective*

---

- *By the end of Step 5, the BB/GB will have:*
  - *Statistically defined the goal of the project*



# Defining Performance Objectives



**Benchmark:** World-Class performance

**Z short term:** The level of performance a business should be able to achieve given the investments already made

**Baseline:** The current level of performance

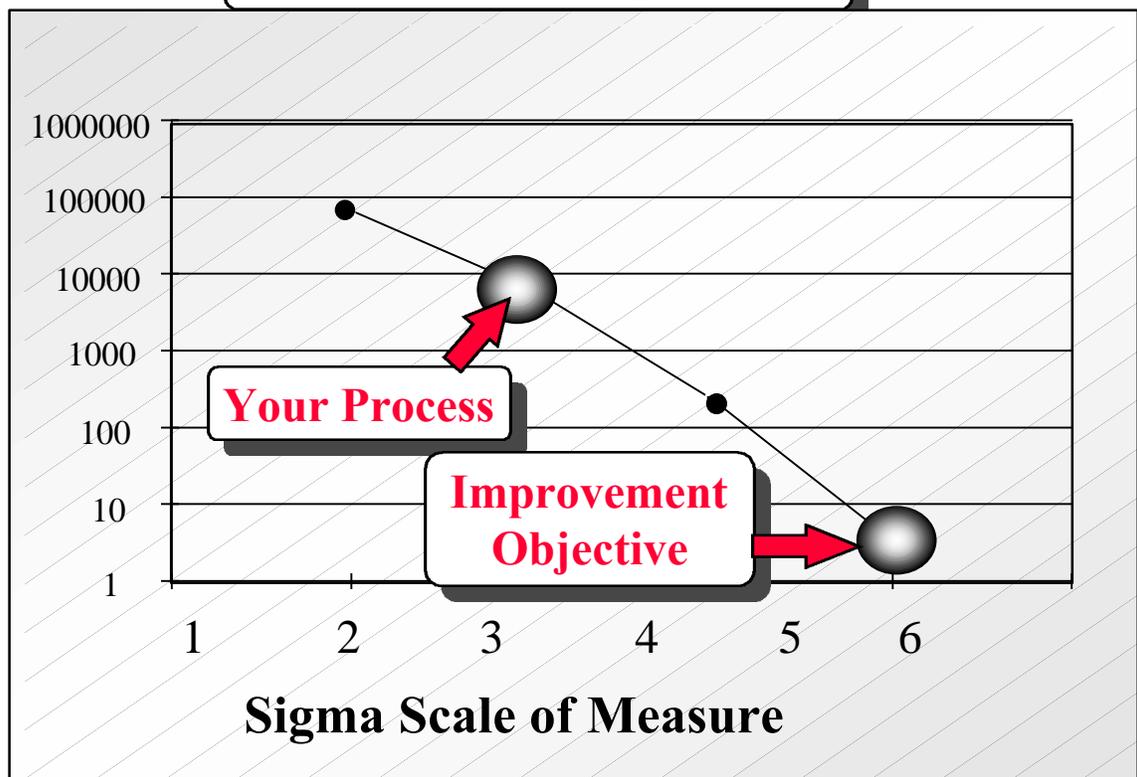
***Benchmarking sets the ultimate goal, while baselining takes current measurements to monitor a process***



## Define Improvement Objective For Y

- **Z short term:** Short-term performance of the process
- **Benchmarking:** get to best in class
- **Learning curve based:** get to 6 Sigma across all processes in 5 years
- **Defect Reduction:** e.g., eliminate 75% of defects

### The Basic Objective

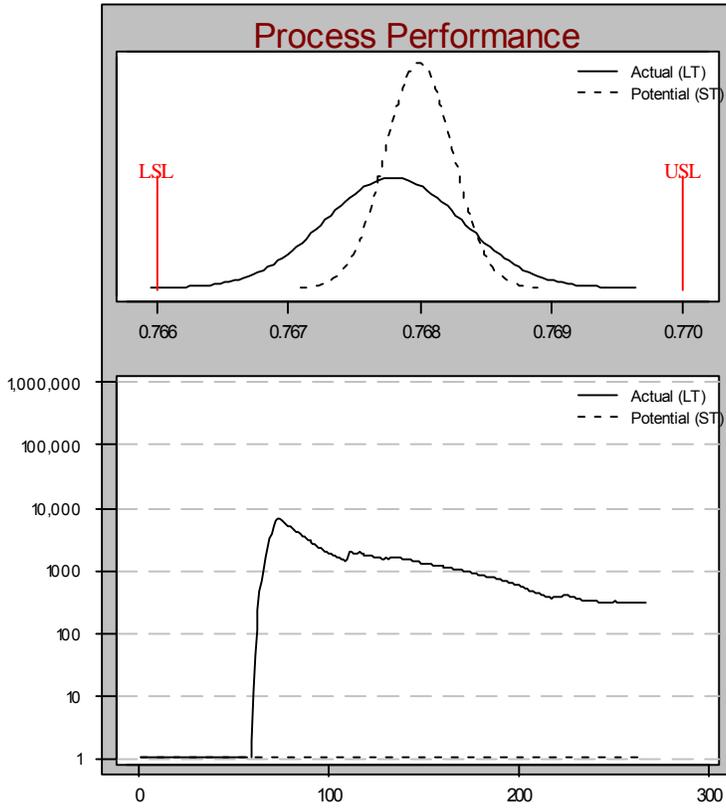


**Six Sigma teams set aggressive but achievable objectives**



# Process Capability

## Report 1: Executive Summary



### Process Demographics

Date:  
 Reported by:  
 Project:  
 Department:  
 Process:  
 Characteristic:  
 Units:  
 Upper Spec:  
 Lower Spec:  
 Nominal:  
 Opportunity:

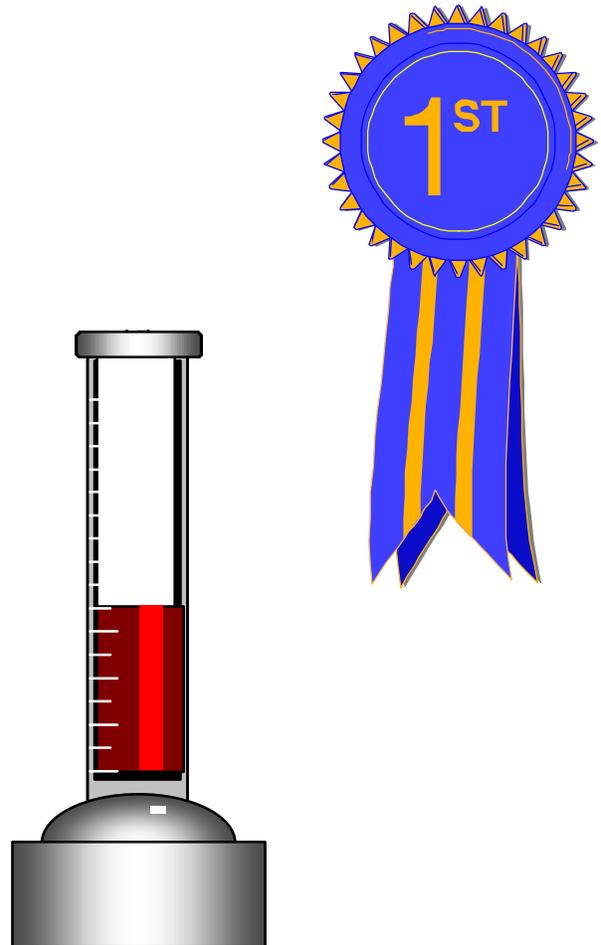
### Process Benchmarks

	Actual (LT)	Potential (ST)
Sigma (Z.Bench)	3.42	6.00
PPM	309	0



## *Nature of Benchmarking*

*Benchmarking is the process of continually searching for the best methods, practices and processes, and either adopting or adapting their good features and implementing them to become the “best of the best”*





# *Types of Benchmarking*

---

**Competitive  
Benchmarking**

**Product  
Benchmarking**

**Process  
Benchmarking**

**Best Practices  
Benchmarking**

**Strategic  
Benchmarking**

**Parameter  
Benchmarking**



## *Concept Versus Process*

*Benchmarking is **simple as a concept** but much more involved as a process. The ultimate payoff is that you can become the best of what you do, and continuously improve upon that superiority.*

*Benchmarking is a means of identifying best practices and using this knowledge to continuously improve our products, services, and systems so that we increase our capability to provide total customer satisfaction.*

*Benchmarking ensures that best practices from competitors or best-in-class companies will be identified. These in turn will point the way to needed improvements. It can help locate new techniques and technologies that are used by best-in-class companies, whether they are competitors or non-competitors.*

*Benchmarking will help a company to realize the value of having a marketing focus rather than strictly an internal one.*

*Benchmarking is a continuous process of measuring products, services, and practices against the toughest competitors and/or those companies renowned as the leaders.*

*Benchmarking is a process used to identify, establish, and achieve standards of excellence, standards based on the realities of the marketplace. It is a process to be used to manage on a continuous basis.*

*Benchmarking draws upon the integration of competitive information, practices, and performance into the decision-making and communication functions at all levels of the business.*

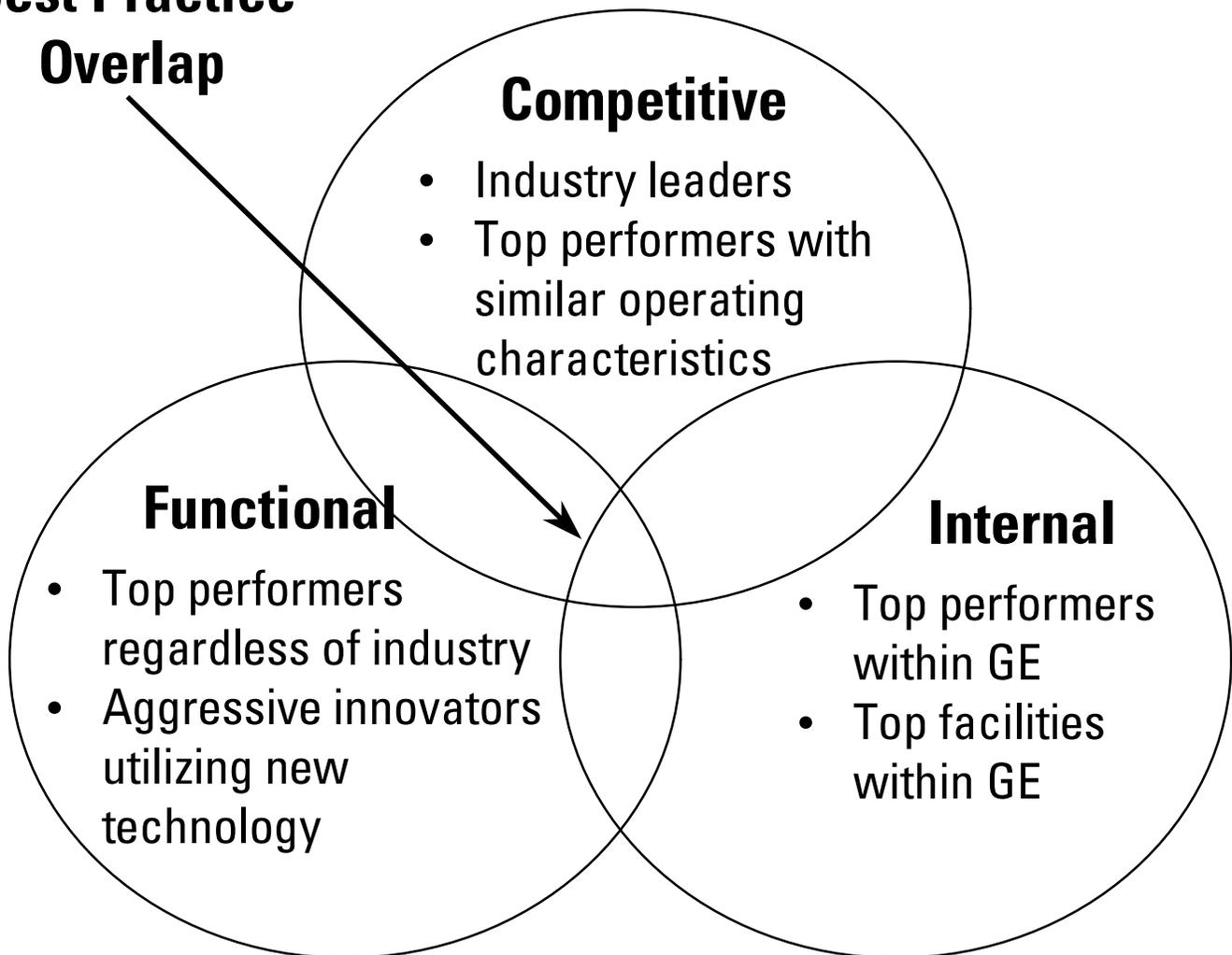
© 1994 Dr. Mikel J. Harry V3.0



# Benchmarking Methodology

## Types of Benchmarking

### Best Practice Overlap



## Understanding the benchmarking candidate pool



# Benchmarking Methodology

## Advantages and Disadvantages

Type	<i>Internal</i>	<i>Competitive</i>	<i>Functional</i>
<b>Definition</b>	<i>Similar activities within GE, but in different locations, departments, operating units, country, etc.</i>	<i>Direct competitors selling to same customer base</i>	<i>Organizations recognized as having world class processes regardless of industry</i>
<b>Examples</b>	<ul style="list-style-type: none"><li>• <i>Claims research cycle time for GE Plastics</i><ul style="list-style-type: none"><li>—GE Corporate Card</li><li>—GE Rewards Card</li><li>—GE Appliances</li><li>—GE Lighting</li></ul></li></ul>	<ul style="list-style-type: none"><li>• <i>GE Appliances</i><ul style="list-style-type: none"><li>—Maytag</li><li>—Whirlpool</li><li>—Siemens</li></ul></li></ul>	<ul style="list-style-type: none"><li>• <i>Warehousing</i><ul style="list-style-type: none"><li>—L.L. Bean</li></ul></li><li>• <i>Shipment status tracking</i><ul style="list-style-type: none"><li>—Federal Express</li></ul></li><li>• <i>Refueling cycle time</i><ul style="list-style-type: none"><li>—Indy car pit crews</li></ul></li></ul>
<b>Advantages</b>	<ul style="list-style-type: none"><li>• <i>Data should be easy to collect</i></li><li>• <i>Good results for diversified company such as GE</i></li></ul>	<ul style="list-style-type: none"><li>• <i>Information relevant to business results</i></li><li>• <i>Comparable practices/ technologies</i></li><li>• <i>May be a history of information gathering</i></li></ul>	<ul style="list-style-type: none"><li>• <i>High potential for discovery</i></li><li>• <i>Development of professional networks</i></li><li>• <i>Access to relevant databases</i></li><li>• <i>Provides best out of the box thinking</i></li></ul>
<b>Disadvantages</b>	<ul style="list-style-type: none"><li>• <i>Limited focus</i></li><li>• <i>Internal bias</i></li></ul>	<ul style="list-style-type: none"><li>• <i>Data collection difficulties</i></li><li>• <i>Ethical issues</i></li><li>• <i>Antagonistic attitudes</i></li></ul>	<ul style="list-style-type: none"><li>• <i>Difficulty transferring practices into different environment</i></li><li>• <i>Some information not transferable</i></li><li>• <i>Time-consuming</i></li></ul>

## Choosing the right form of benchmarking



# *Benchmarking Methodology*

## **What it is and What it isn't**

### ***Benchmarking Is...***

- *A continuous process*
- *A process of investigation that provides valuable information*
- *A process of learning from others; a pragmatic search for ideas*
- *A time consuming, labor-intensive process requiring discipline*
- *A viable tool that provides useful information for improving virtually any business process*

### ***Benchmarking Isn't...***

- *A one-time event*
- *A process of investigation that provides simple answers*
- *Copying; imitating*
- *Quick and easy*
- *A buzzword, a fad*



# *Benchmarking Methodology*

## ***Common Benchmarking Mistakes***

- 1. Internal process(es) is unexamined*
- 2. Site visits “feel good,” but don’t elicit data or ideas*
- 3. Questions and goals are vague*
- 4. The effort is too broad or has too many parameters*
- 5. The focus is not on processes*
- 6. The team is not committed to the effort*
- 7. Homework and/or advanced research isn’t assigned*
- 8. The wrong benchmarking partner is selected*
- 9. The effort fails to look outside the industry (outside the box)*
- 10. No follow-up action is taken*

***Be wary of the pitfalls***



# Benchmarking Methodology

## Checklist

### Identify Process to Benchmark

- *Select process and define defect and opportunities*
- *Measure current process capability and establish goal*
- *Understand detailed process that needs improvement*



### Select Organization to Benchmark

- *Outline industries/functions which perform your process*
- *Formulate list of world class performers*
- *Contact the organization and network through to key contact*

## **Benchmarking: A Six Step Process**



# Benchmarking Methodology

## Checklist

### Prepare for the Visit

- *Research the organization and ground yourself in their processes*
- *Develop a detailed questionnaire to obtain desired information*
- *Set up logistics and send preliminary documents to organization*



### Visit the Organization

- *Feel comfortable with and confident about your homework*
- *Foster the right atmosphere to maximize results*
- *Conclude in thanking organization and ensure follow-up if necessary*

## **Benchmarking: A Six Step Process**



# Benchmarking Methodology

## Checklist

### Debrief & Develop an Action Plan

- *Review team observations and compile report of visit*
- *Compile list of best practices and match to improvement needs*
- *Structure action items, identify owners and move into Improve phase*



### Retain and Communicate

- *Report out to business management and 6s leaders*
- *Post findings and/or visit report on local server/6s bulletin board*
- *Enter information on GE Intranet benchmarking project database*

## **Benchmarking: A Six Step Process**



## *Sources of Information*

---

*Library Database*  
*Internal Reviews*  
*Internal Publications*  
*Professional Associations*  
*Industry Publications*  
*Special Industry Reports*  
*Functional Trade Publications*  
*Seminars*  
*Industry Data Firms*  
*Industry Experts*  
*University Sources*  
*Company Watches*  
*Newspapers*  
*Advertisements*  
*Newsletters*  
*Original Research*  
*Customer Feedback*  
*Supplier Feedback*  
*Telephone Surveys*  
*Inquiry Service*  
*Networks*  
*World Wide Web*

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# *Project: A515.1-Pipe and Flexible Metal Hose Contamination*

---

## **Project Goal(s)**

- *Elimination of contamination in Gas Turbine Piping and Tubing*
  
- *Establish cleaning methods for Gas Turbine Piping and Tubing*
  
- *Reduce contamination related SNs (Service notices)*



## *Benchmarking Process*

---

- *Identified 5-10 national pipe fabrication shops from the Thomas Register*
- *Selected shops (2) that manufacture piping for the food industry (higher cleanliness standards than mfgs. Of oil field piping)*
- *Polled shops (telephone polls) to determine pipe cleaning practices*



## *Results*

---

■ *Identified one viable new cleaning method...The Compri Technic - Contamination Eliminator System*

— *Foam Projectiles launched through pipes and tubes with 85-150 PSI compressed air*

■ *Invited Compri Technic to Greenville for product demo with Greenville Engineering and Manufacturing Teams*



## *Next Steps*

---

- *Establish which Gas Turbine Piping Systems CE system can be used on*
- *Develop cleaning methods*
- *Beta Test Product at Greenville*

## ***Additional Exposure***

- *Schenectady CASE, and Greenville Maintenance department are now evaluating the product*
- *CE System has now been purchased by one GE Field installation site*



## *Benchmarking Example: 20 mins.*

- *For one or more projects in your group*
  - *Identify areas of your process that you would like to benchmark*
  - *Make a list of possible benchmarking partners*
  - *Indicate whether the partner is a competitor, non-competitor, in a similar market, or in a different market*

***One or two teams will be asked to report their findings.***



## *Take Aways—Step 5*

---

- *A performance objective is determined by using Z-short term, benchmarking, or defect reduction goals.*
- *Benchmarking is a process of identifying best practices, measuring our own practices against those best practices, and adapting the appropriate best practices to our own processes.*



# Analyze Phase



## 4. Establish Process Capability

*Deliverable: Baseline Current Process*

*Tools:*

- *Basic statistics*
- *Graphical Analysis*
- *Sampling*
- *Continuous Zst, ZIt*
- *Normality*
- *Discrete Zst, ZIt*

## 5. Define Performance Objectives

*Deliverable: Statistically Define the Goal of the Project*

*Tools:*

- *Benchmarking*

## 6. Identify Variation Sources

*Deliverable: List of Statistically Significant X's, Chosen Based on Analysis of Historical Data*

*Tools:*

- *Process Analysis*
- *Graphical Analysis*
- *Hypothesis Testing*
- *Regression Analysis*
- *GLM*



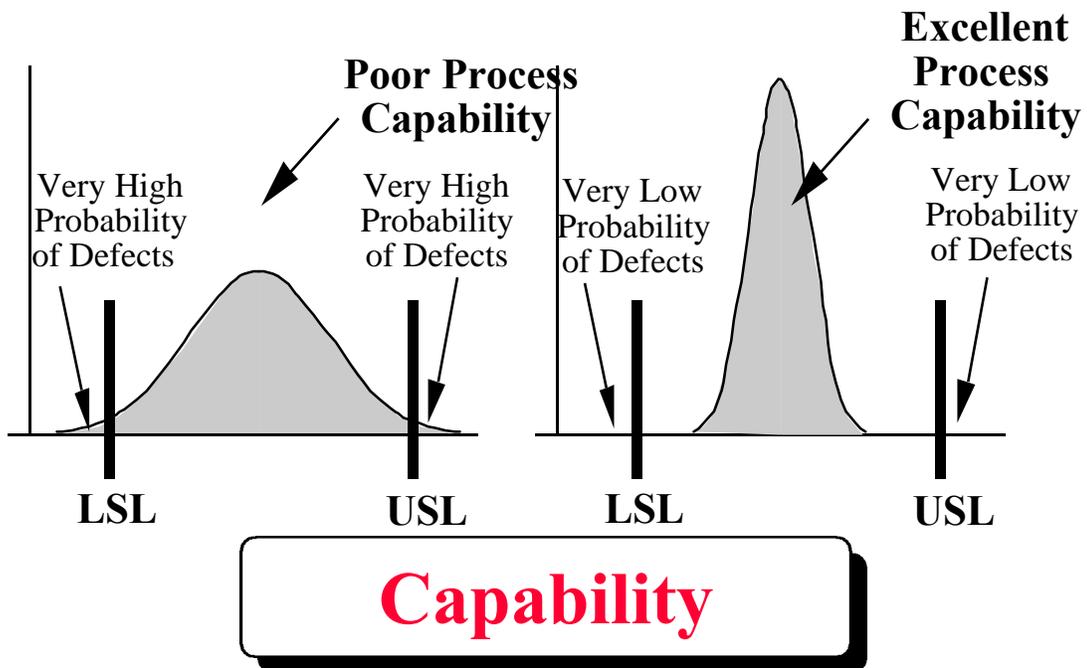
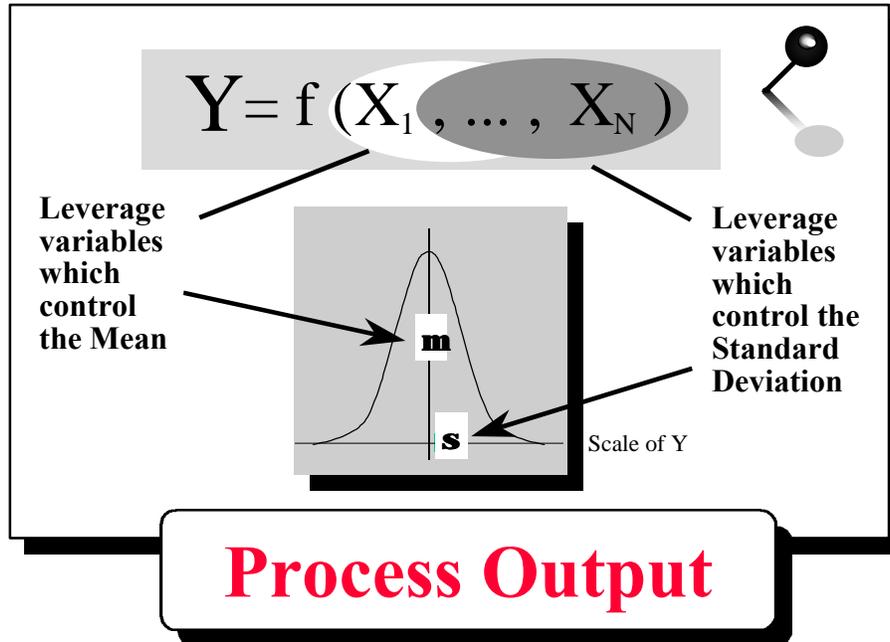
## *Step 6: Identify Variation Sources*

---

- *By the end of Step 6, the BB/GB will have:*
  - *Generated a list of Statistically Significant Xs based on analysis of historical data*
  - *Identified which Xs to further investigate in the Improve phase*
  - *Gained consensus with the project team on the list of Xs for investigation*
  - *Understand value added & non-value added analyses*



# The Focus of Improvement





## *Tools to Identify Variation Sources*

<b>Process Mapping</b>	<i>Understand process steps; narrow project focus</i>  <i>Steps: 1, 6, Improve Phase</i>
<b>FMEA</b>	<i>Identify and prevent failures; narrow project focus</i>  <i>Steps: 1, 6, 12</i>
<b>Cause &amp; Effect</b>	<i>Understand problem: narrow project focus</i>  <i>Step: 6</i>
<b>Pareto Chart</b>	<i>Prioritize items: narrow project focus</i>  <i>Step: 6</i>

***The same tools taught in step 1 can be used to identify variation sources***



# *Cause & Effect Diagram*



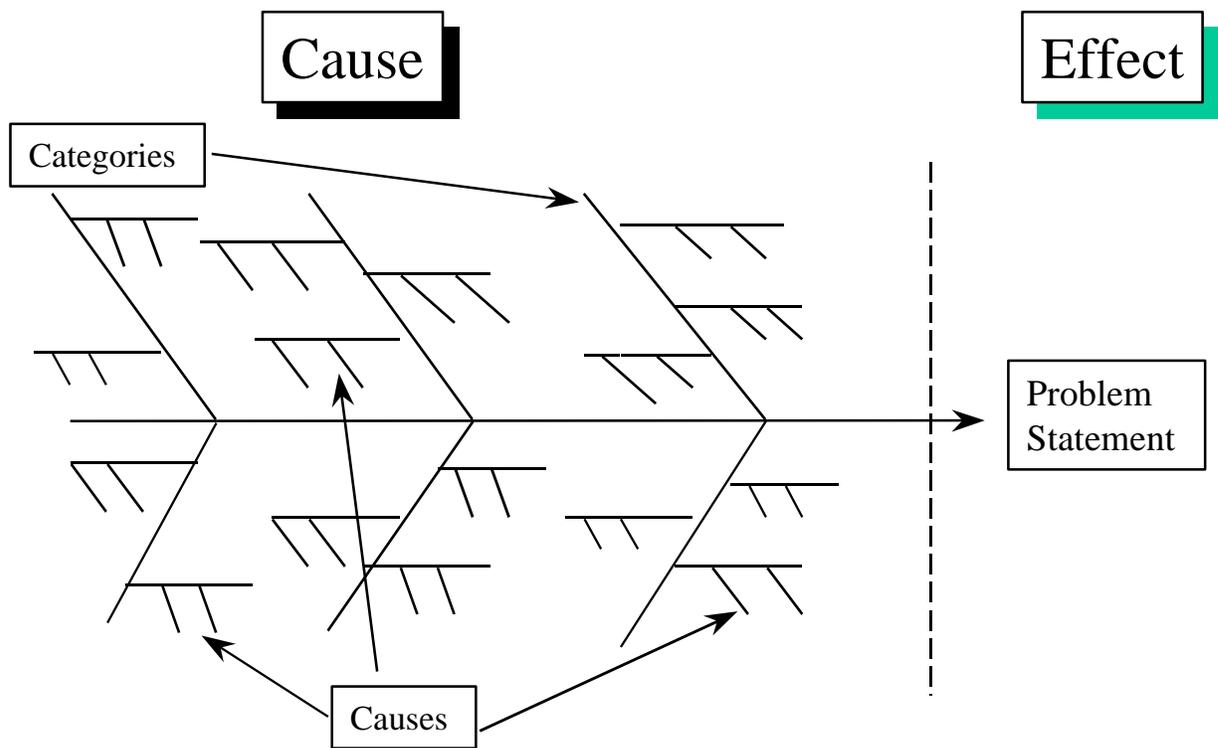
# Cause & Effect Diagram (Fishbone Diagram)

## ■ Purpose:

- To provide a visual display of all possible causes of a specific problem

## ■ When:

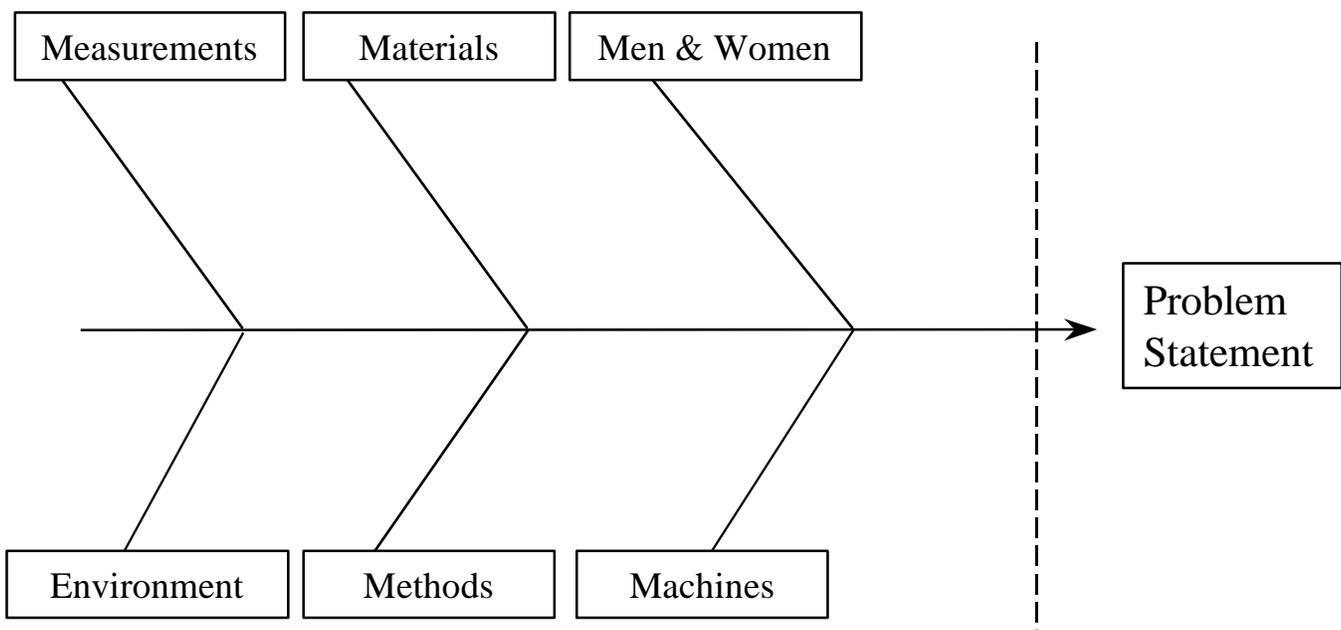
- To expand your thinking to consider all possible causes
- To gain group's input
- To determine if you have correctly identified the true problem





## *Building a Fishbone Diagram*

- 1** *Draw a blank diagram on a flip chart.*
- 2** *Define your problem statement.*



- 3** *Label branches with categories appropriate to your problem.*

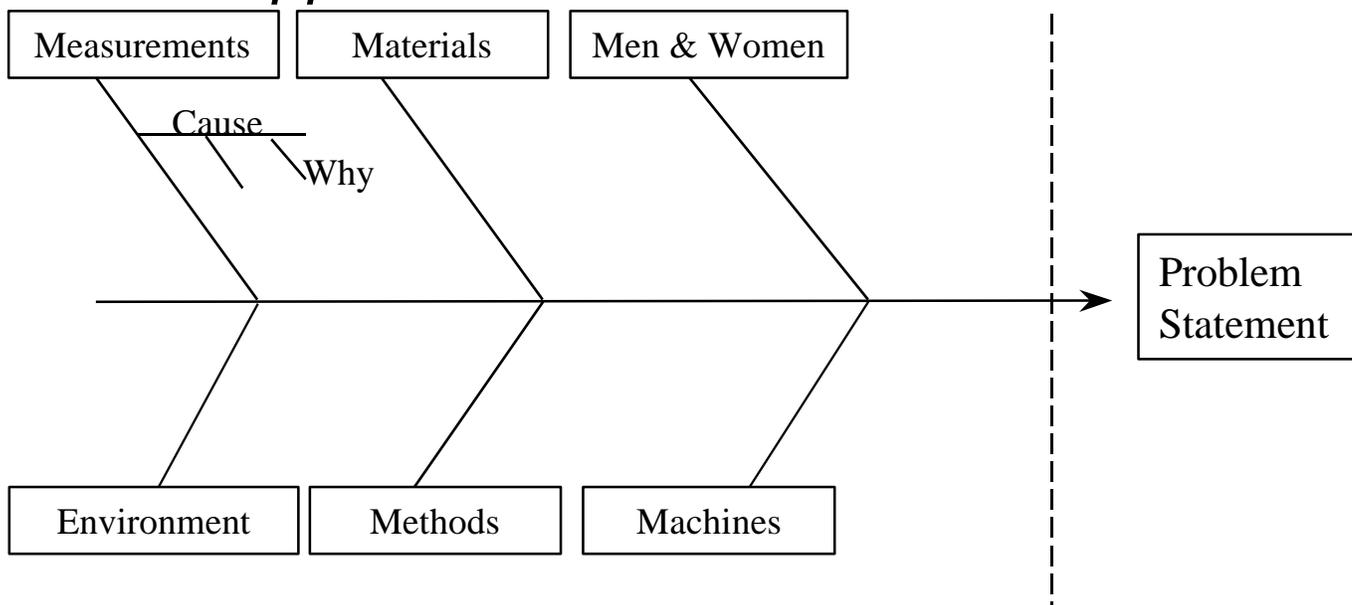
### **The 4 Ps**

Categories can also be **Policies**, **Procedures**, **People**, and **Plant** or any other category that will help people think creatively.



## *Building a Fishbone Diagram (cont.)*

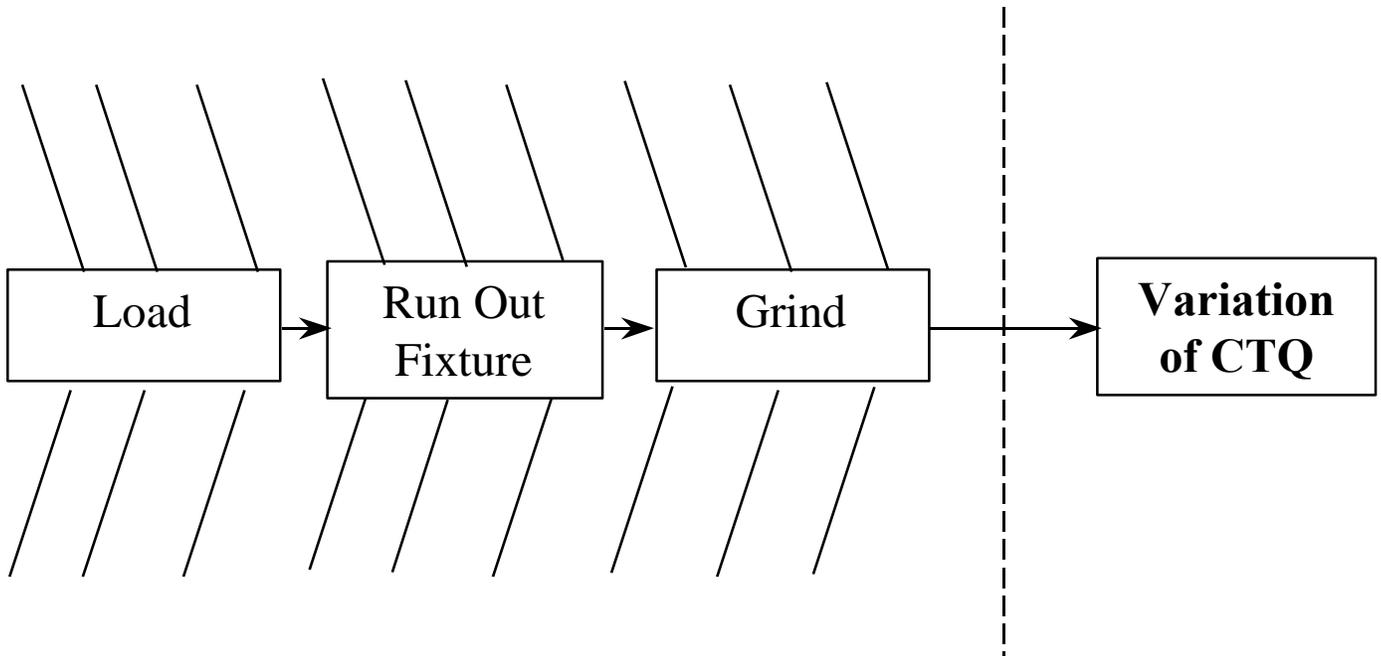
- 4** *Brainstorm possible causes and attach them to appropriate categories.*
- 5** *For each cause ask, “Why does this happen?”*



- 6** *Analyze results, any causes repeat?*
- 7** *As a team, determine the three to five most likely causes.*
- 8** *Determine which likely causes you will need to verify with data.*



# Process Fishbone Example



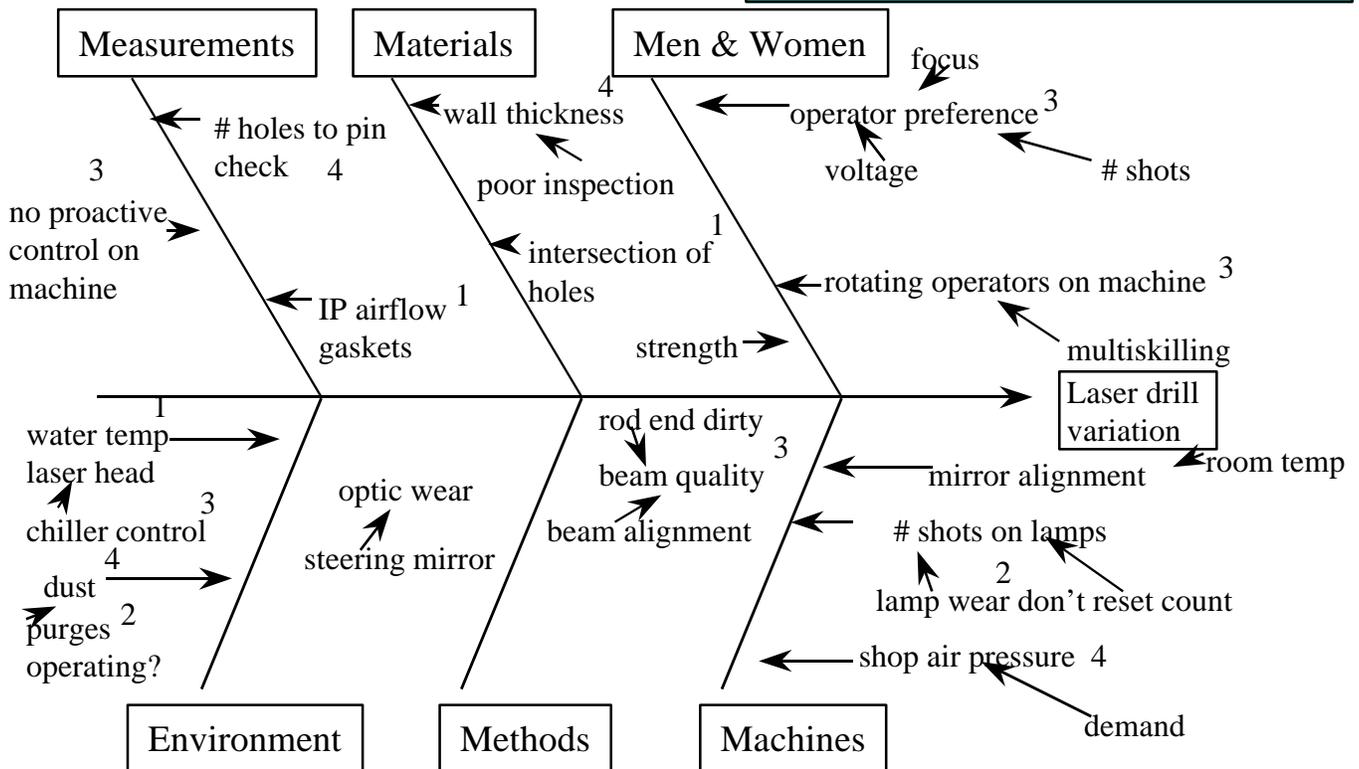


# Fishbone Example

- The following example was the result of brainstorming sources of variation in a laser drill operation.

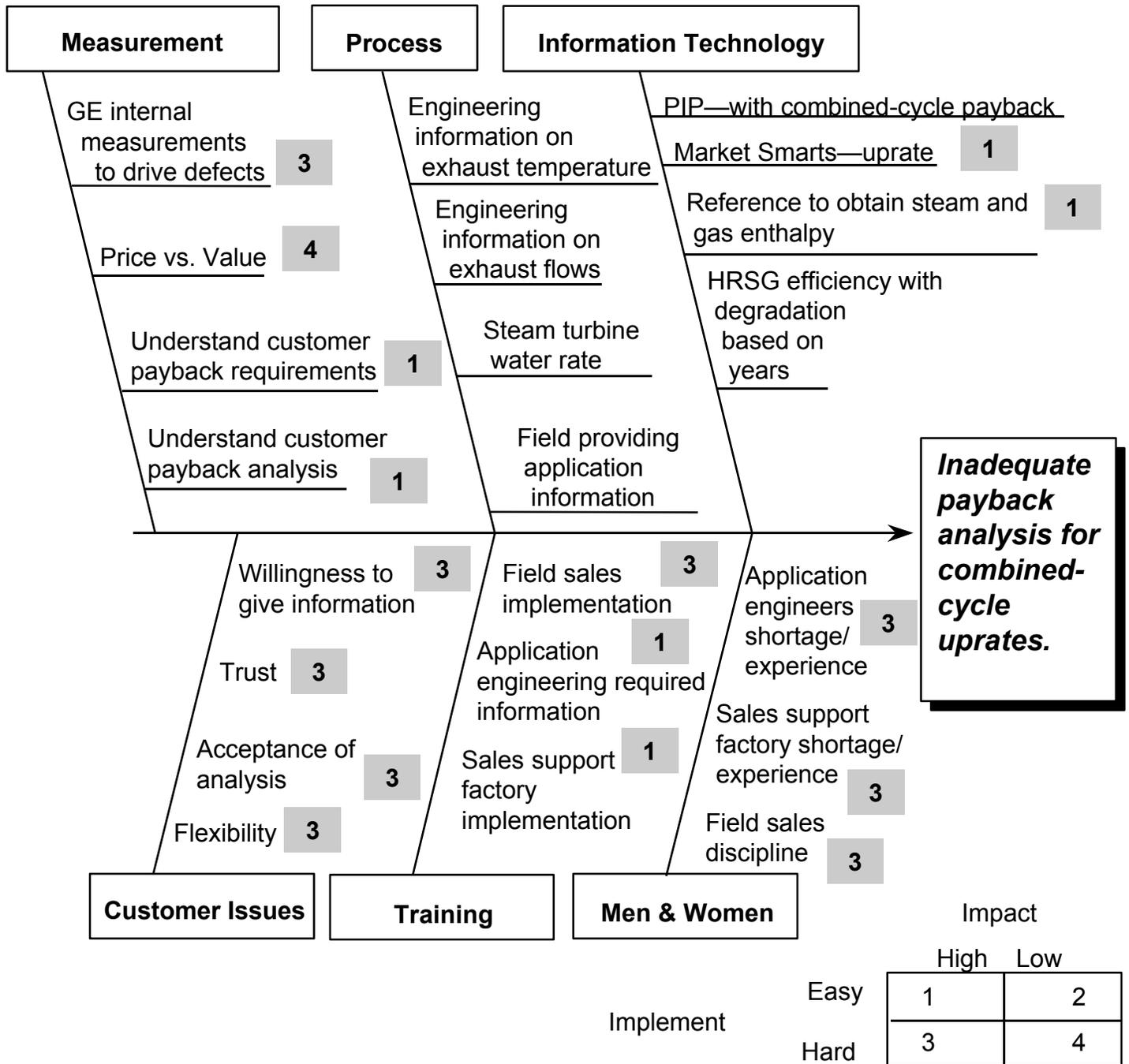
		Impact	
		High	Low
Implementation	Easy	1	2
	Hard	3	4

## Laser Drill Variation



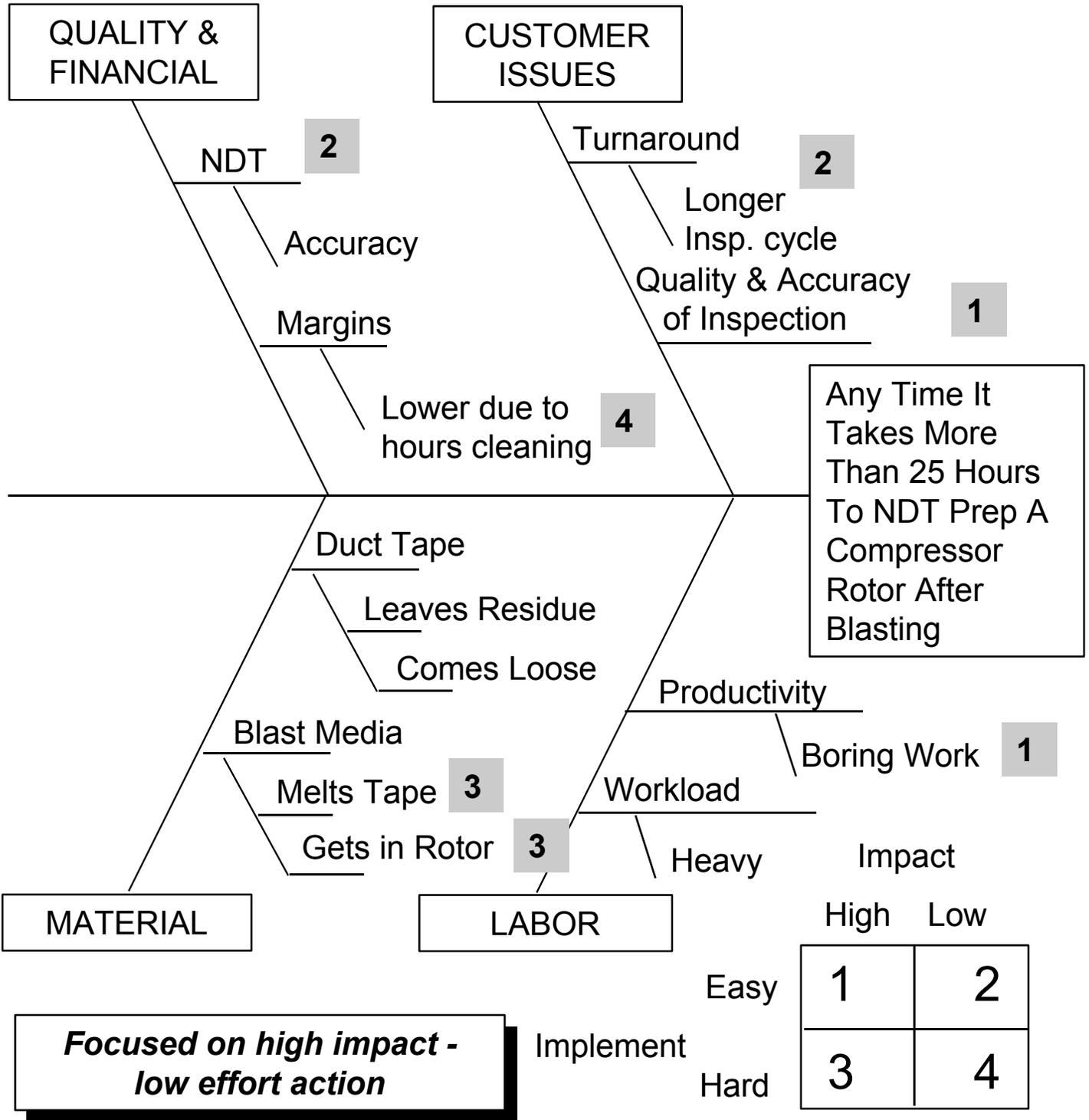


# S171.1 CC Payback Analysis: Fishbone





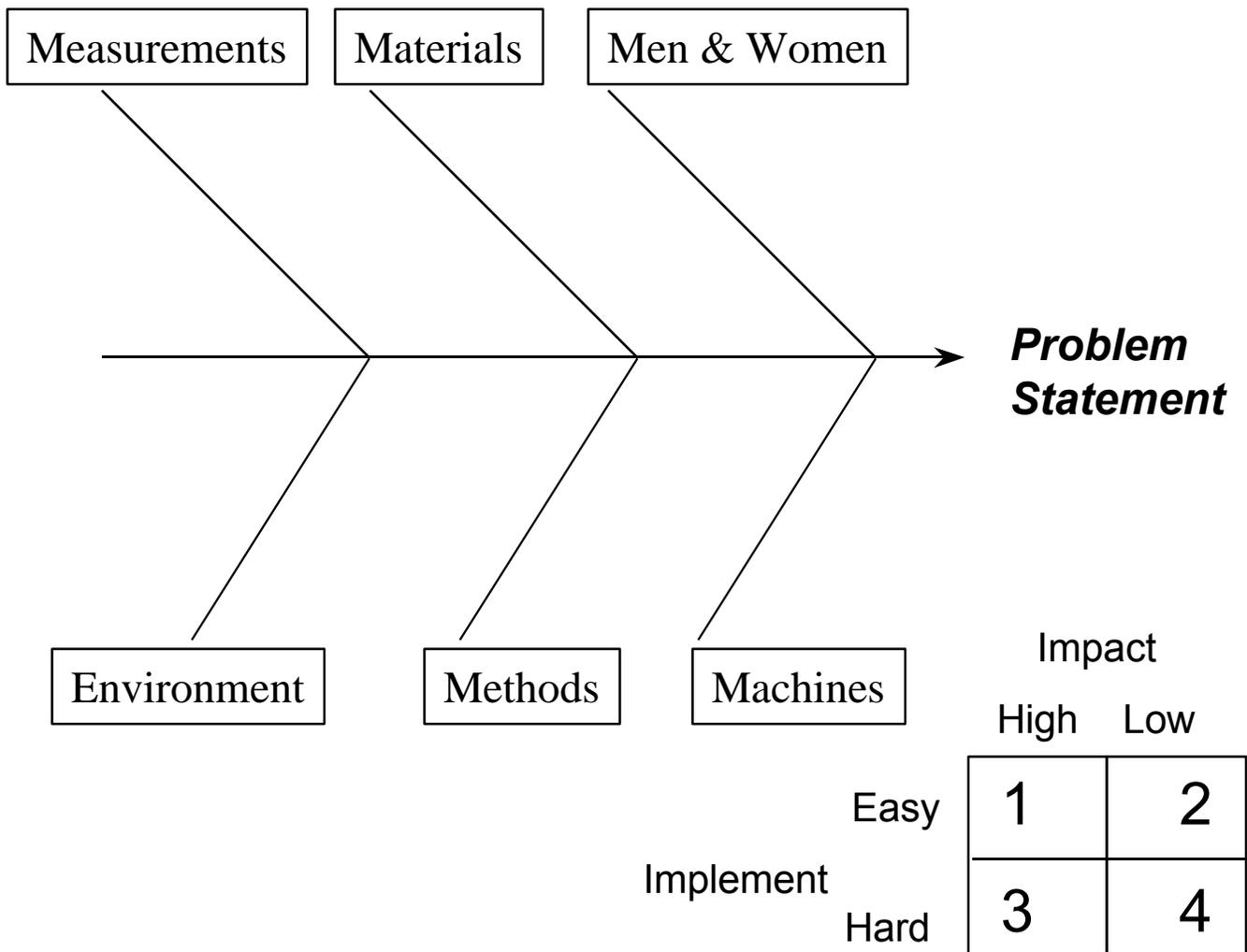
# B3095.1 Reducing Rotor Blasting Process Time: Fishbone





## *Fishbone Exercise: 20 mins.*

- *For one or more projects on your team, construct a Fishbone diagram.*
- *What are main causes of your problem?*
- *Use the four blocker to prioritize.*





# *Pareto Chart*



## *Pareto Chart*

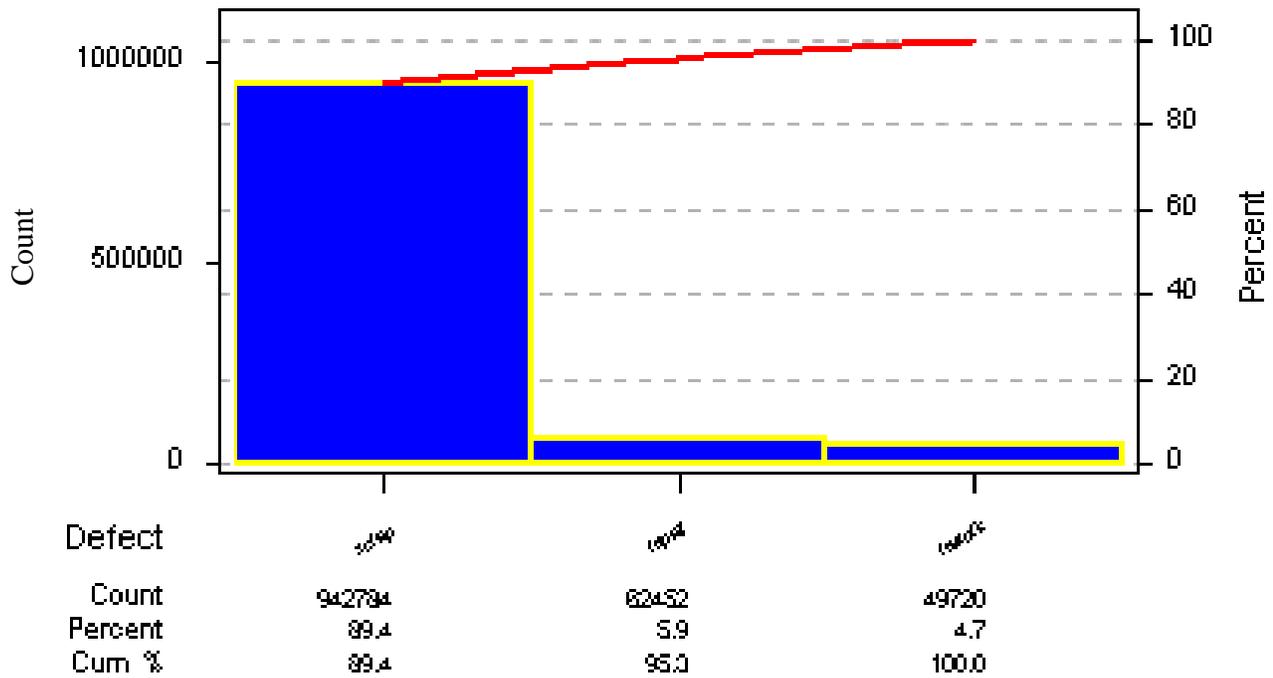
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- **Purpose:** *To separate the vital few from the trivial many in a process. To compare how frequently different causes occur or how much each cause costs your organization.*
  
- **When:** *To sort data for determining where to focus improvement efforts.*
  - *To choose which causes to eliminate first*
  - *To display information objectively to others*



# Pareto Principle

Manufacturing Losses by Type



**20% of causes account for 80% of the effect**



## *Building a Pareto Chart*

### *1. Collect data (checksheets, surveys)*

TYPE OPR	REWORK	REPAIR	SCRAP
MACH. SHAPE	1167	4969	270008
FINAL AIRFLOW	5266	10236	115342
COAT	0	43	127161
LASER HOLE	25869	23683	53047
X-RAY INSP	757	7529	93205
ES HOLE	1958	16	91379
INSP	564	7907	74390
BENCH	9563	1083	43464
EDM HOLE	2126	1095	46422
FINAL WATER FLOW	2450	5891	28366
TOTAL	49720	62452	942784

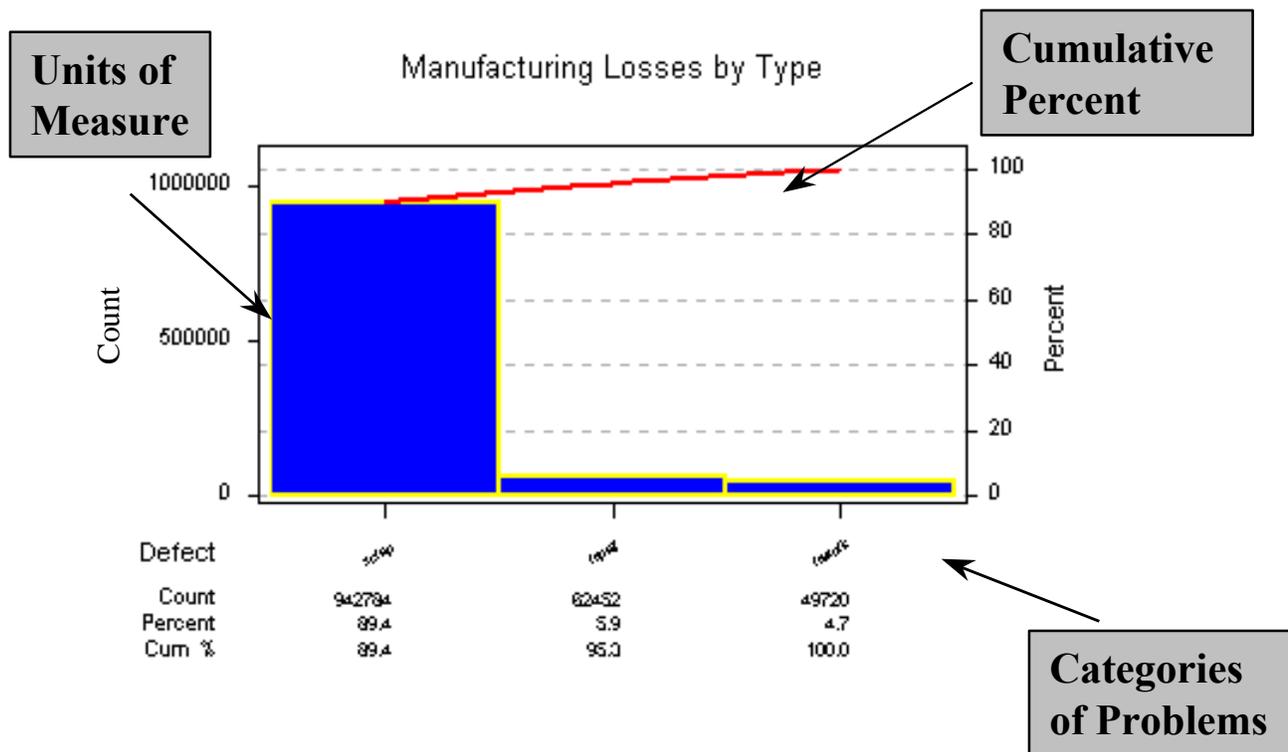
### *2. Total results and arrange data in descending order*

type	\$
scrap	942784
repair	62452
rework	49720



## *Building a Pareto Chart (cont.)*

### *3. Draw and label a Pareto Chart*



### *4. Analyze results*

### *5. Evaluate improvement effectiveness after change initiated by comparing before and after Pareto charts*



# Pareto Example

MINITAB FILE: Pareto.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the 'Quality Tools' sub-menu is selected, with 'Pareto Chart...' highlighted. The background worksheet contains the following data:

	C2	C3	C4	C7	C8
↓	REWORK	REPAIR	SCRAPE		
1	1167	4969	27000	\$	
2	5266	10236	11534	49720	
3	0	43	12716	62452	
4	25869	23683	5304	942784	
5	757	7529	9320		



# Pareto Chart Example (cont.)

## Input & Output

1. Double click on these items to input them to the appropriate fields.

Circles indicate where input is required.

2. Click "OK"

**Pareto Chart**

C1	TYPE	OPR
C2	REWORK	
C3	REPAIR	
C4	SCRAP	
C5	TOTAL	
C6	type	
C7	\$	

Chart defects data in:

BY variable in:  [optional]

Default (all on one page, same ordering of bars)

One chart per page, same ordering of bars

One chart per page, independent ordering of bars

Chart defects table

Labels in:

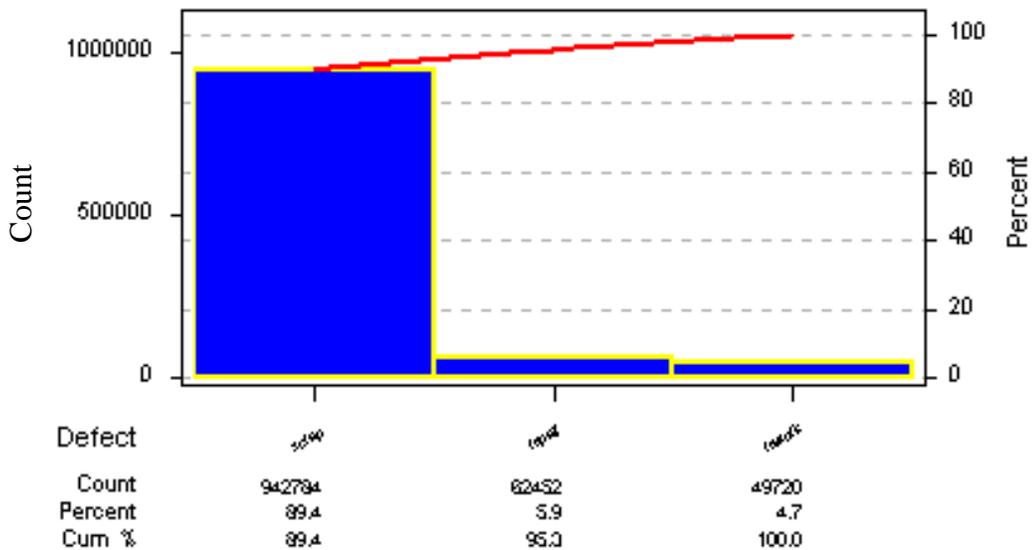
Frequencies in:

Combine defects after the first  % into one

Title:

Select Help OK Cancel

Manufacturing Losses by Type



**Where do most of the losses occur?**



---

## *Pareto Exercise*

---

- *The manufacturing losses data is contained in file Pareto.mtw*
- *Make a Pareto chart which breaks down the scrap category by type of operation.*



# Pareto Exercise Answer

1. Double click on these items to input them to the appropriate fields.

**Pareto Chart**

C1	TYPE OPR
C2	REWORK
C3	REPAIR
C4	SCRAP
C5	TOTAL
C6	type
C7	\$

Chart defects data in:

BY variable in:  (optional)

Default [all on one page, same ordering of bars]

One chart per page, same ordering of bars

One chart per page, independent ordering of bars

Chart defects table

Labels in:

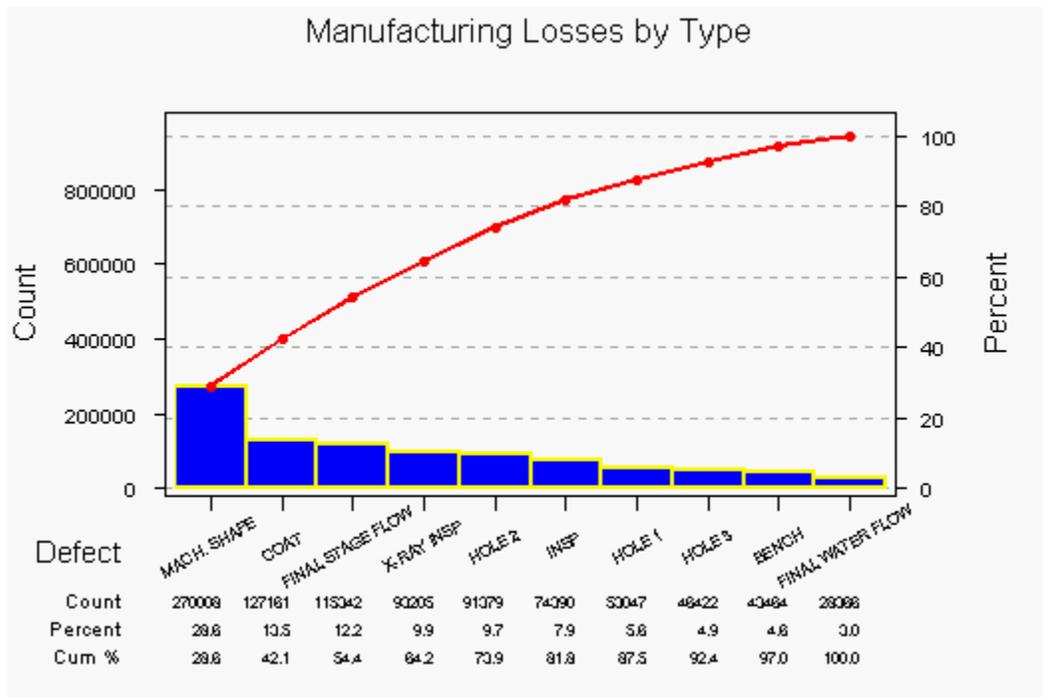
Frequencies in:

Combine defects after the first  % into one

Title:

Select Help OK Cancel

2. Click "OK"





## *Process Map Analysis*

- *Value Added analysis*
- *Non-value added analysis*



# *Process Map Analysis*

## *Types Of Analysis*



- *Moments of truth*  
– *What does the customer feel?*



- *Nature of work*



- *Flow of work*



## *Nature Of Work: Value Analysis*

### **Value-Added Work**

Steps That Are Essential Because They Physically Change The Product/Service, The Customer Is Willing To Pay For Them And Are Done Right The First Time.

### **Nonvalue-Added Work**

Steps That Are Considered Non-Essential To Produce and Deliver The Product Or Service To Meet The Customer's Needs And Requirements. Customer Is Not Willing To Pay For Step.

Steps That Are Not Essential To The Customer, But That Allow the Value-Adding Tasks To Be Done Better/Faster.

### **Value-Enabling Work**



## *Types Of Nonvalue-added Work*



**Internal Failure**



**Delay**



**External Failure**



**Preparation/Set-Up**



**Control/Inspection**



**Move**

***What Does the Customer Value?***



# *Flow Of Work*

---



**Process Time**



**+**

**Delay Time**



---

**Cycle Time**



## *Flow Of Work: Process Disconnects*



■ *Gaps*



■ *Redundancies*



■ *Implicit or unclear requirements*



■ *Tricky hand-offs*



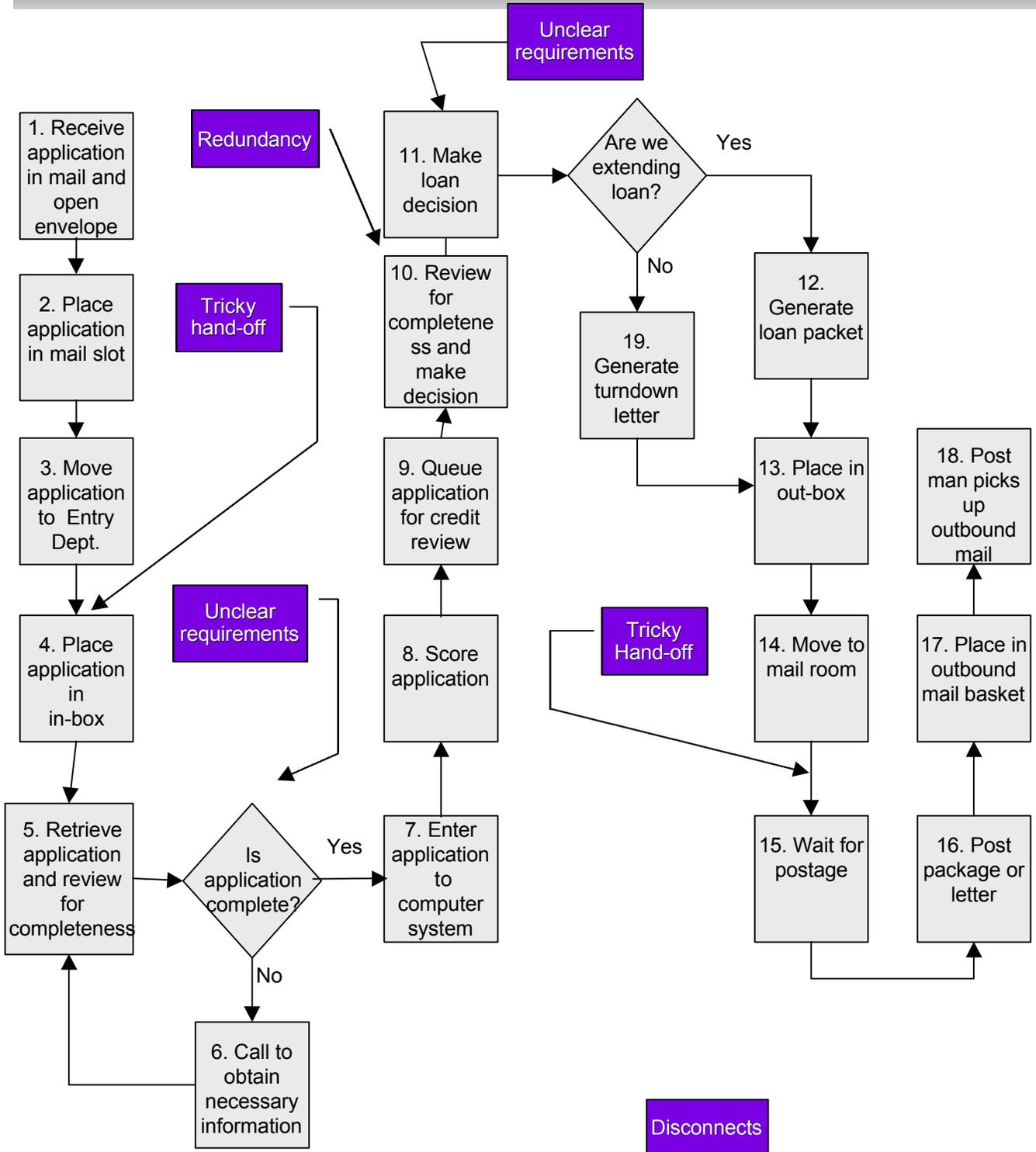
■ *Conflicting objectives*



■ *Common problem areas*



# Flow Of Work: "Be The Unit"





# Linking Value Analysis With Process Flow

## Summarized Analysis

Process Step	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	Total	% Total	% Steps
Est. Avg. Time (Mins)	1	120	15	120	3	180	7	1	120	5	10	15	90	15	120	2	120	5	8	957	100%	
Value-Added	✓						✓				✓	✓				✓		✓	✓	48	5.0%	
Nonvalue-Added																						
Internal Failure							✓													180	18.8%	
External Failure																						
Control/Inspection							✓				✓									8	.8%	
Delay		✓		✓						✓			✓		✓		✓			690	72.1%	
Prep/Set-Up																						
Move			✓											✓						30	3.1%	
Value-Enabling								✓												1	.1%	
Total																				957	100%	



# *Process Map Analysis*

## *How to Interpret*

- Use the matrix to direct your improvement efforts. Some questions you might ask as you examine the matrix include:
  - Where are the longest Cycle times? Are they value added steps? If not, can you eliminate or moderate them to save time?
  - What type of non-value added steps dominate your process? Inspection? Signature loops? Delays? How much time in your process do they account for? Can you eliminate or moderate them to save time?
  - What amount of time in your process comes from value enabling steps? Challenge the value enabling aspect of these steps. Ask yourself if there is any value to eliminating these steps if you could. If the answer is yes, then look for ways to persuade others to eliminate this step without causing problems in the process.



# Value And Cycle Matrix – Breakout Activity (25 Minutes)

## Desired Outcome

- Practice using a value and cycle time matrix for process analysis

What	How	Who	Time
Team Preparation	<ul style="list-style-type: none"><li>■ Choose facilitator and timekeeper.</li><li>■ Determine timing for each activity.</li></ul>	All	
Prepare Value And Cycle Time Analysis	<ul style="list-style-type: none"><li>■ Using the process from subprocess mapping breakout activity, number all subprocess steps.</li><li>■ Prepare a value and cycle time matrix (using the worksheet provided.)</li><li>■ Classify each step as value-added, nonvalue-added or value-enabling.</li><li>■ Estimate cycle times for each step.</li><li>■ Compute totals for each category.</li><li>■ Brainstorm next steps you would recommend for this project case example.</li></ul>	Facilitator	
Close Exercise	<ul style="list-style-type: none"><li>■ Choose spokesperson</li></ul>	All	



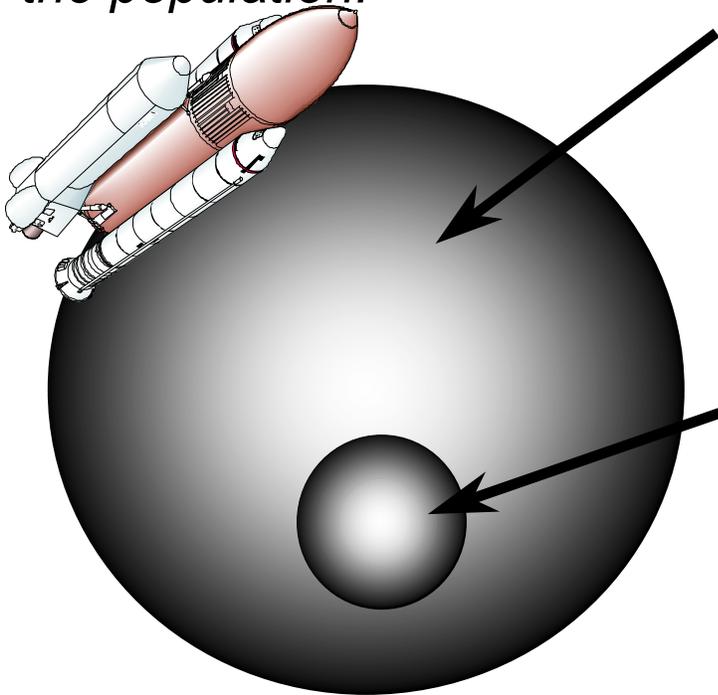
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# *Hypothesis Testing*



## *The Idea of Sampling*

*Based on the sample, we make decisions about the population.*



### **Population**

**(Universe):** A set of characteristics that defines membership in the complete set.

**Sample:** A subset of members that possesses the same characteristics as that of the population.

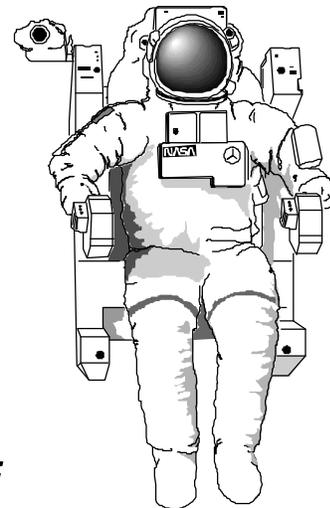
*Why should we take a sample?*

*Should the sample be random?*

*Is it possible to have sampling error?*

*How many samples should be taken?*

***What are some everyday examples of sampling?***



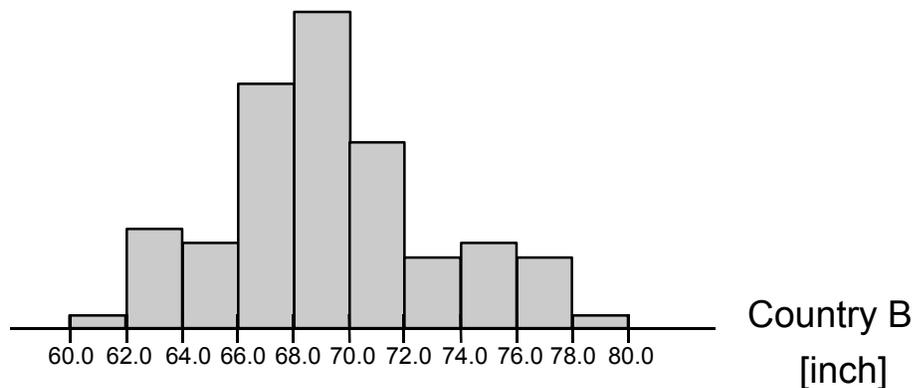
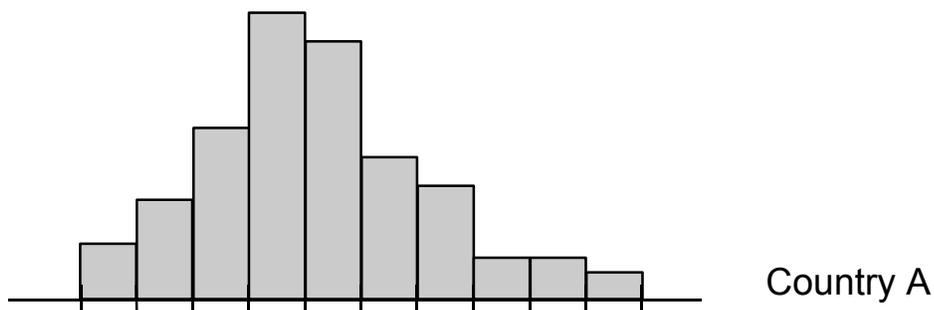


## Hypothesis Testing for Equal Means

The histograms below show the height of inhabitants of countries A and B.

Both samples are of size 100, the scale is the same, and the unit of measurement is inches.

Question: Is the population of country B, on average, taller than that of country A?





## *A Statistical Hypothesis*

*An assertion or conjecture about one or more parameters of a population(s).*

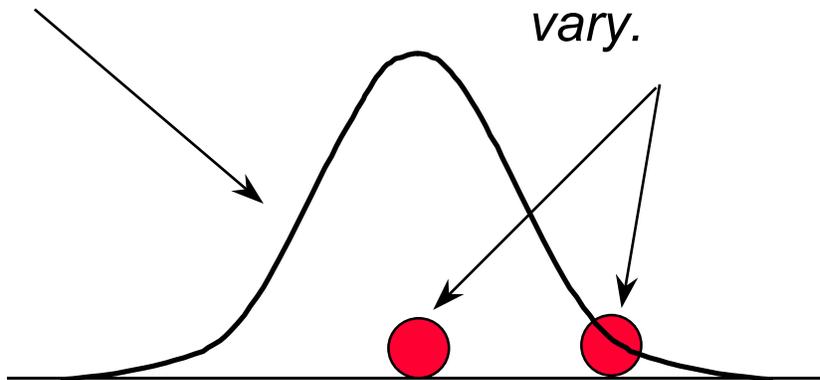
- *To determine whether it is true or false, we must examine the entire population. This is impossible!!*
- *Instead use a random sample to provide evidence that either supports or does not support the hypothesis.*
- *The conclusion is then based upon statistical significance.*
- *It is important to remember that this conclusion is an inference about the population determined from the sample data.*



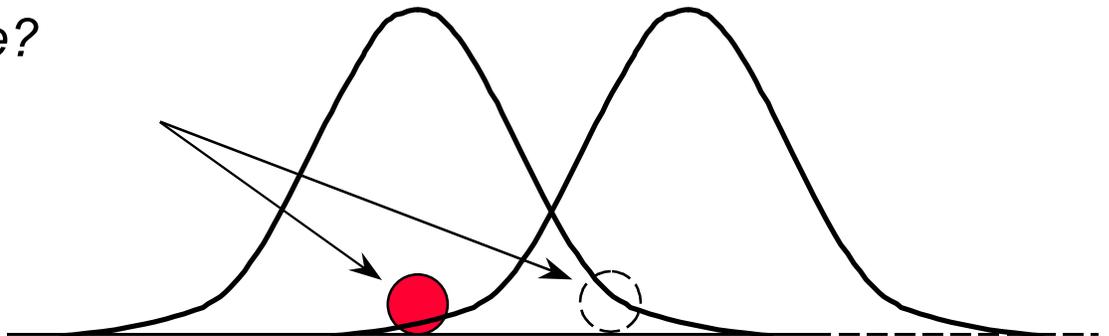
# Concept of Hypothesis Testing

1. All processes have variation.

2. Samples from one given process may vary.



3. How can we differentiate between sample based "chance" variation and a true process difference?





## *Why Do Hypothesis Testing?*

- 1. To improve processes, we need to identify factors which impact the mean or standard deviation.*
- 2. Once we have identified these factors and made adjustments for improvement, we need to validate actual improvements in our processes.*
- 3. Sometimes we cannot decide graphically or by using calculated statistics (sample mean and standard deviation) if there is a statistically significant difference between processes.*
- 4. In such cases the decision will be subjective.*
- 5. We perform a formal statistical hypothesis test to decide objectively whether there is a difference.*

***This way everyone makes the same decision.***



## *Hypothesis Testing Protocol*

- ***The hypotheses are always statements about the population parameters.***
- ***State your Null Hypothesis ( $H_o$ )***
  - $H_o$ : The height of citizens in country A is greater than or equal to the height of citizens in country B ( $\mu_A \geq \mu_B$ ).
- ***State your Alternative Hypothesis ( $H_a$ )***
  - $H_a$ : The height of citizens in country A is less than the height of citizens in country B ( $\mu_A < \mu_B$ ).
- ***Test Alternative Hypothesis with Statistical Test***
- ***Based on the test result, we reject or fail to reject the null hypothesis  $H_o$ .***



# *Nature of Hypotheses*



## ***Null Hypothesis ( $H_0$ ):***

- *Usually describes a status quo*
- *The one you assume unless otherwise shown*
- *The one you reject or fail to reject based upon evidence*
- *Signs used in Minitab:  
= or  $\geq$  or  $\leq$*

## ***Alternative Hypothesis ( $H_a$ ):***

- *Usually describes a difference*
- *Signs used in Minitab:  
 $\neq$  or  $<$  or  $>$*



## *Hypothesis Testing Guilty vs. Innocent Example*

*The USA justice system can be used to illustrate the concept of hypothesis testing.*

*In America we assume innocence until proven guilty. This corresponds to the null hypothesis.*

*It requires strong evidence, “beyond a reasonable doubt,” to convict the defendant. This corresponds to rejecting the null hypothesis and accepting the alternative hypothesis. **More specifically, we have significant evidence to support that a difference exists.***

$H_0$ : Person is not guilty.  
 $H_a$ : Person is guilty.

***What are the possible outcomes when the truth is known?***



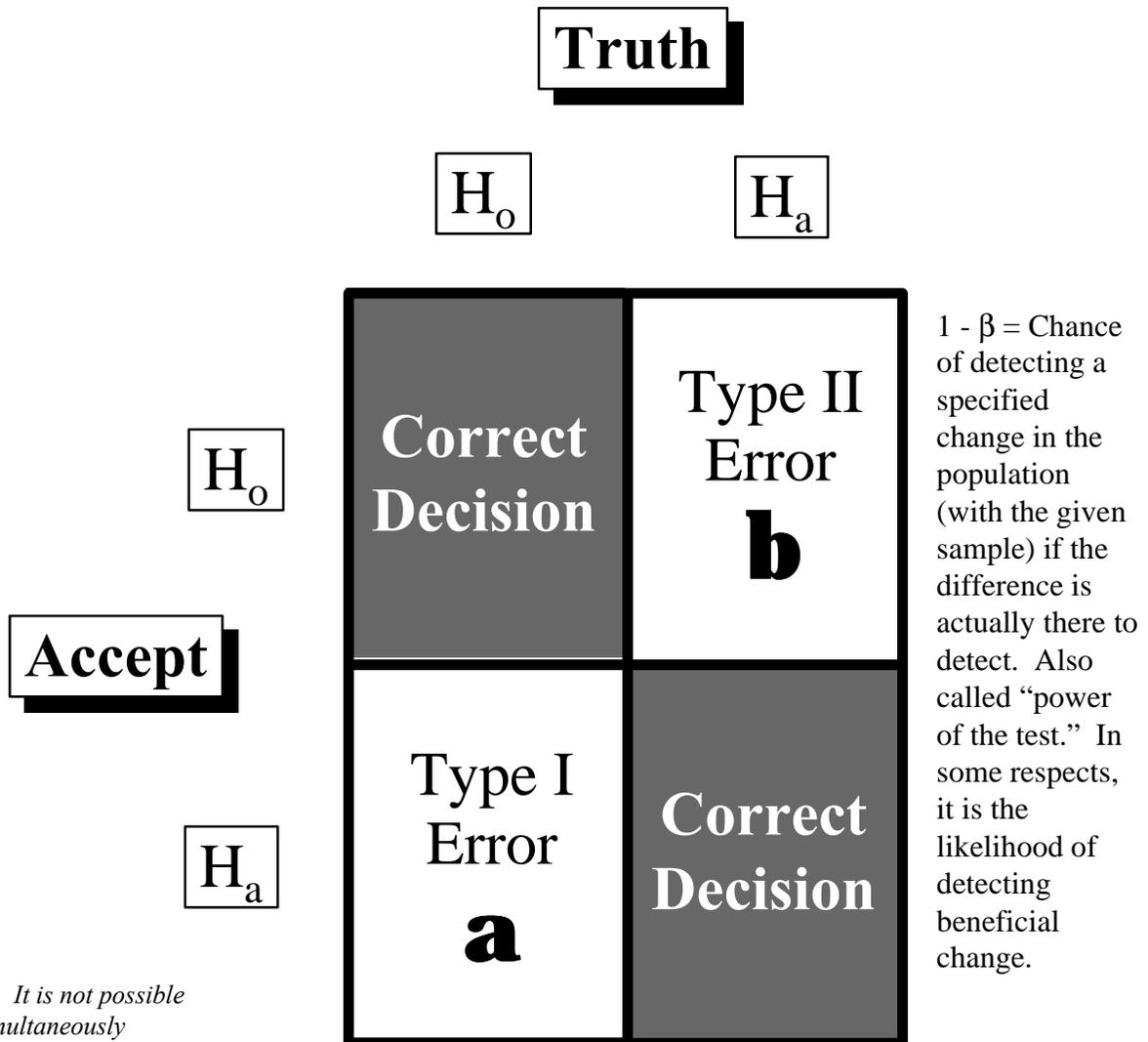
## Evaluation of Decision Error

Four possible outcomes that determine whether a decision is correct or in error:

		<b>Truth</b>	
		$H_0$	$H_a$
<b>Verdict</b>	$H_0$ <i>Set Free</i>	Innocent, Set Free	Guilty, Set Free
	$H_a$ <i>Jailed</i>	Innocent, Jailed	Guilty, Jailed



# Evaluation of Decision Error



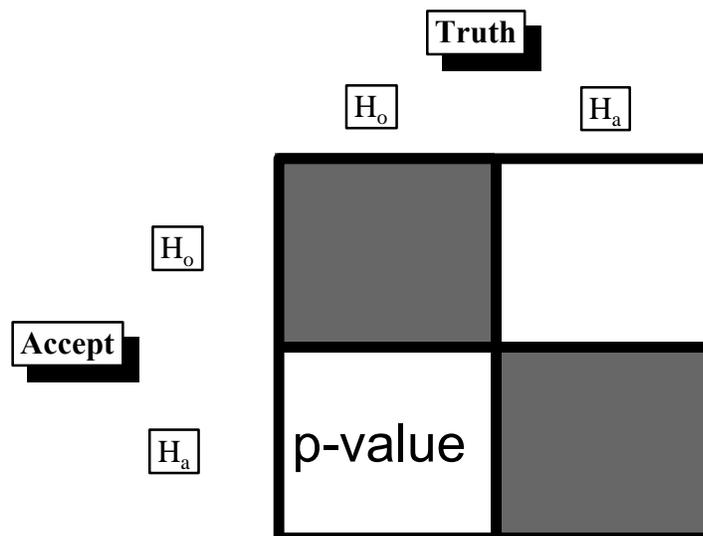
*Note: It is not possible to simultaneously commit a Type I and Type II decision error. In short, either an alpha or beta decision error can be made, but not both.*

$1 - \alpha$  = Confidence that an observed outcome in the sample is “real” (i.e., the outcome is not due to random sampling error and, therefore, reflects the true state-of-affairs in the population).



## The p-value

- Alpha is the maximum acceptable probability of being wrong if the alternative hypothesis is selected.
- The p-value is the probability that you will be wrong if you select the alternative hypothesis. This is a Type I error.
- Unless there is an exception based on engineering judgment, we will set an acceptance level of a Type I error at  $\alpha = 0.05$ .
- Thus, any p-value less than 0.05 means we reject the null hypothesis.

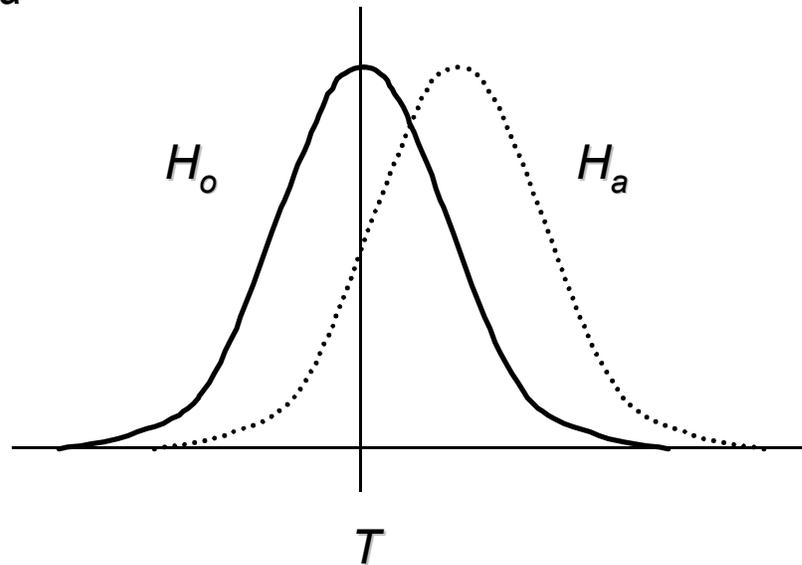




# One Sample Hypotheses

1.  $H_o : \mu = \text{constant} = T$

$H_a : \mu \neq \text{constant} = T$



2.  $H_o : \sigma^2 = \text{constant}$

$H_a : \sigma^2 \neq \text{constant}$



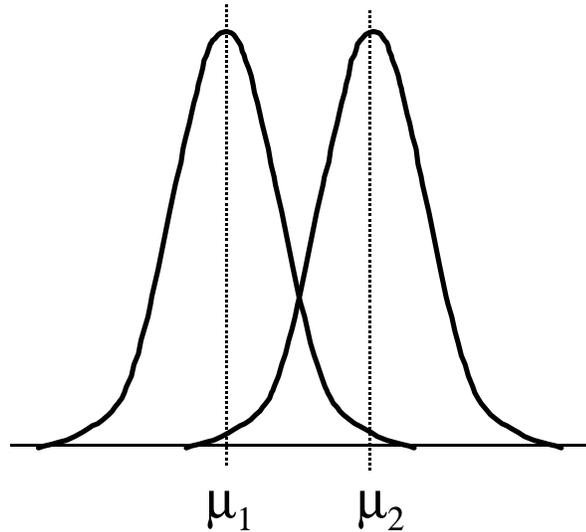
## Two Sample Hypotheses

3.  $H_o : m_1 = m_2$

$H_a : m_1 \neq m_2$

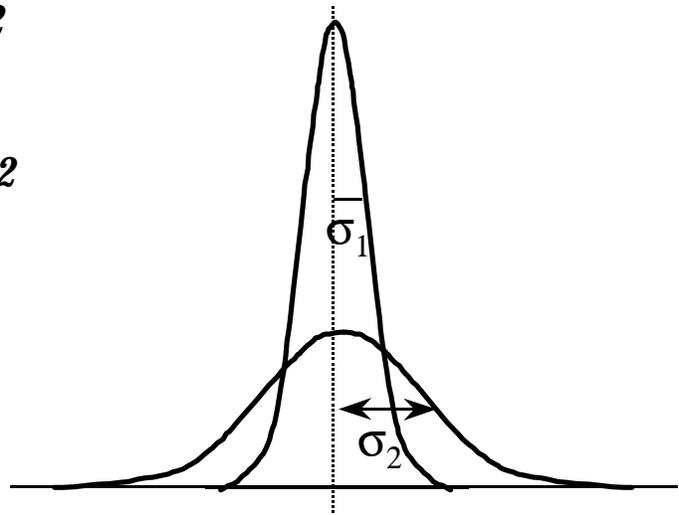
4.  $H_o : m_1 \leq m_2$

$H_a : m_1 > m_2$



5.  $H_o : s_1^2 = s_2^2$

$H_a : s_1^2 \neq s_2^2$

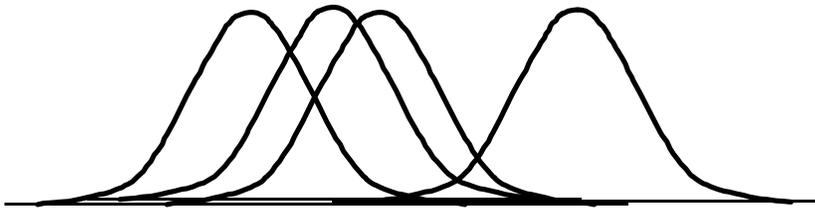




## *Multi Sample Hypotheses*

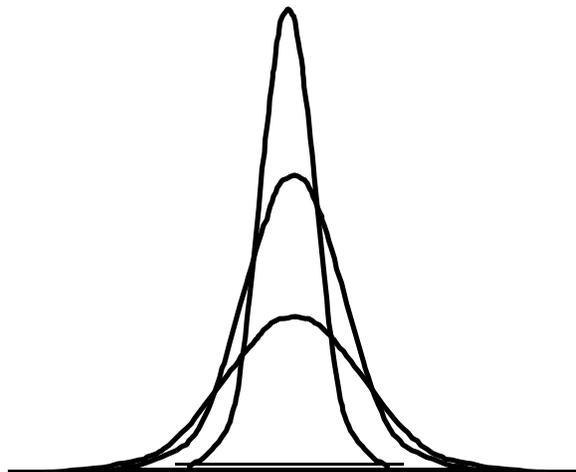
6.  $H_o : m_1 = m_2 = \dots = m_n$

$H_a : \text{at least one not equal}$



7.  $H_o : s_1^2 = s_2^2 = \dots = s_n^2$

$H_a : \text{at least one not equal}$

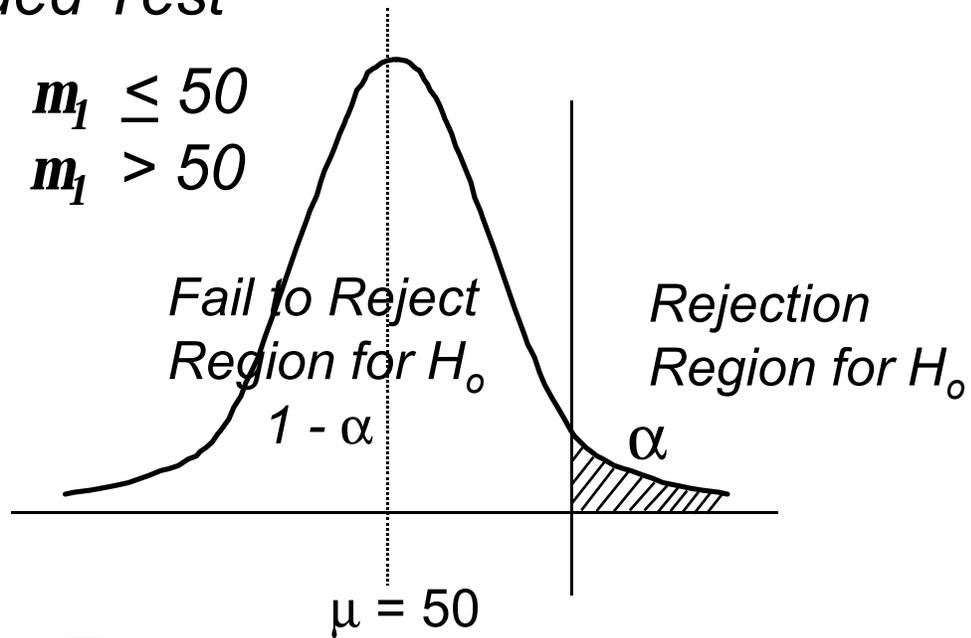




# Types of Hypothesis Tests

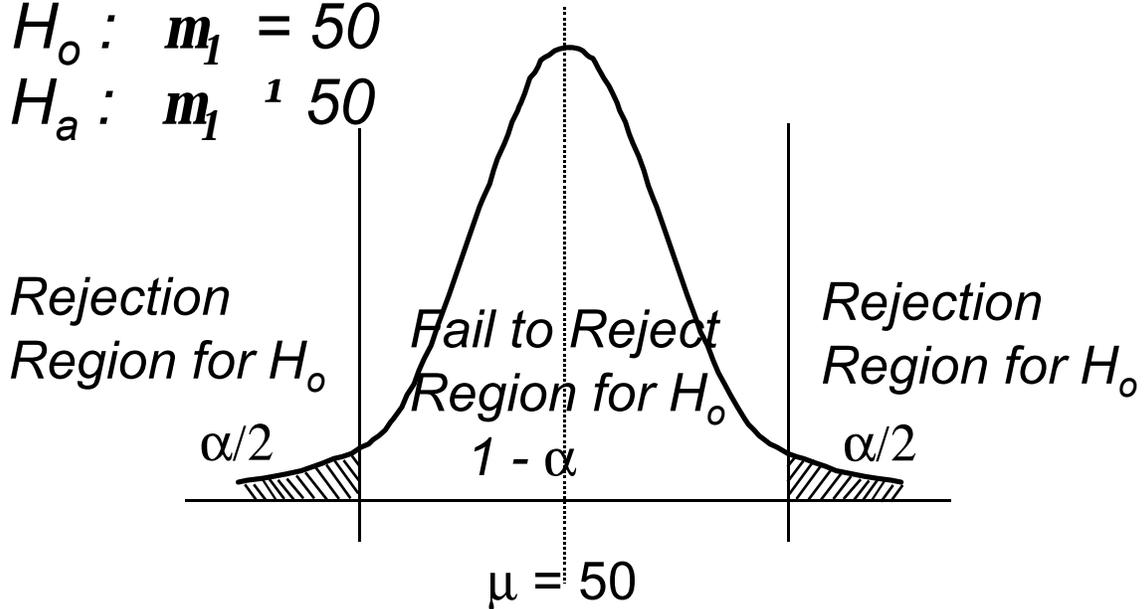
## One-Sided Test

Ex.  $H_0 : m_1 \leq 50$   
 $H_a : m_1 > 50$



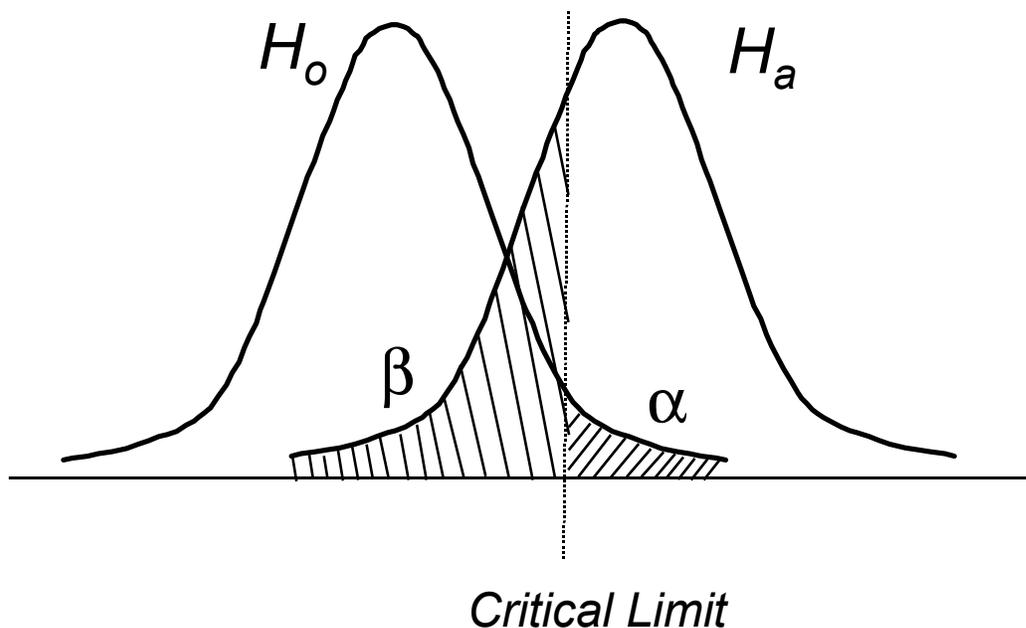
## Two-Sided Test

Ex.  $H_0 : m_1 = 50$   
 $H_a : m_1 \neq 50$





## *Relationship Between a & b*

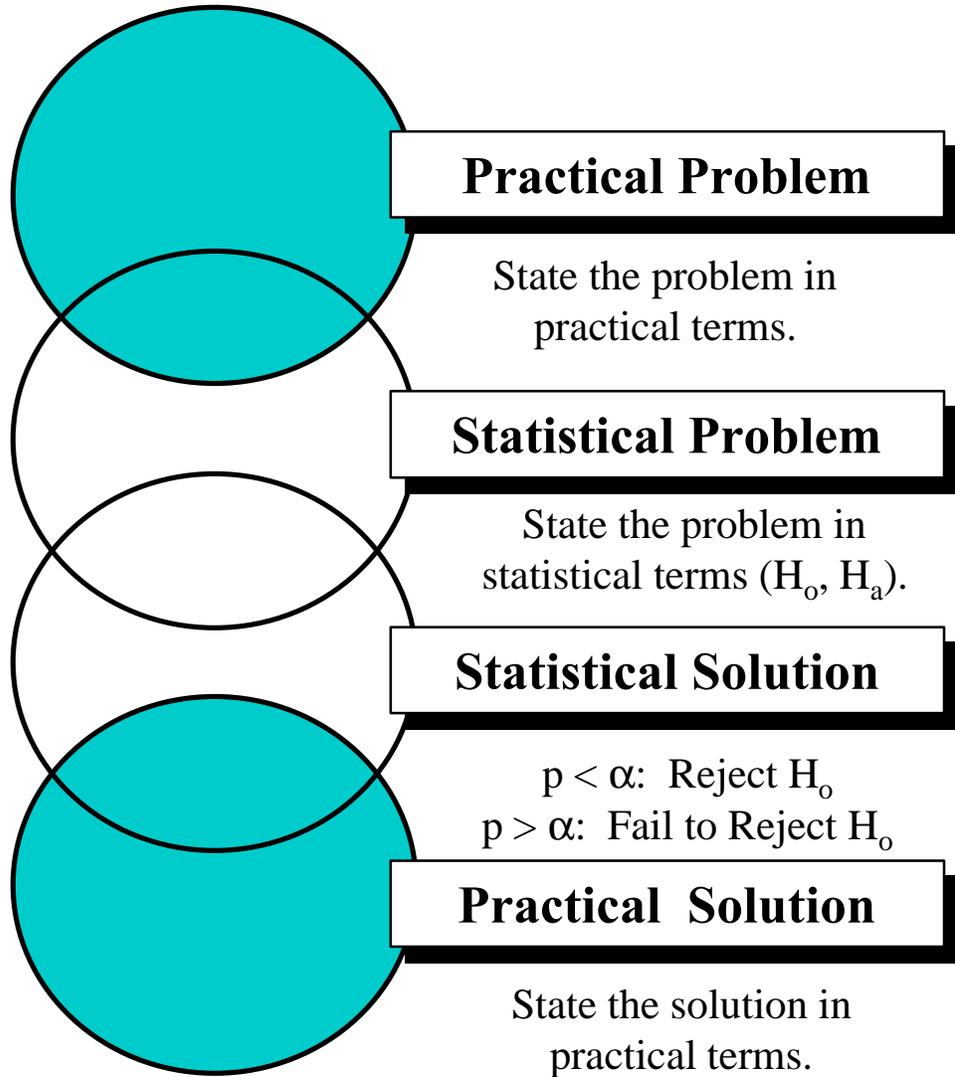
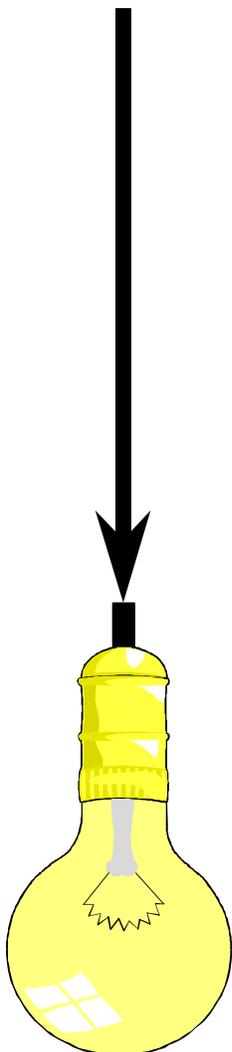


*a and b are represented by distinct regions. We cannot simultaneously make a Type I and Type II error.*



# Bridging the Real World

**Problem Solving Flow**





---

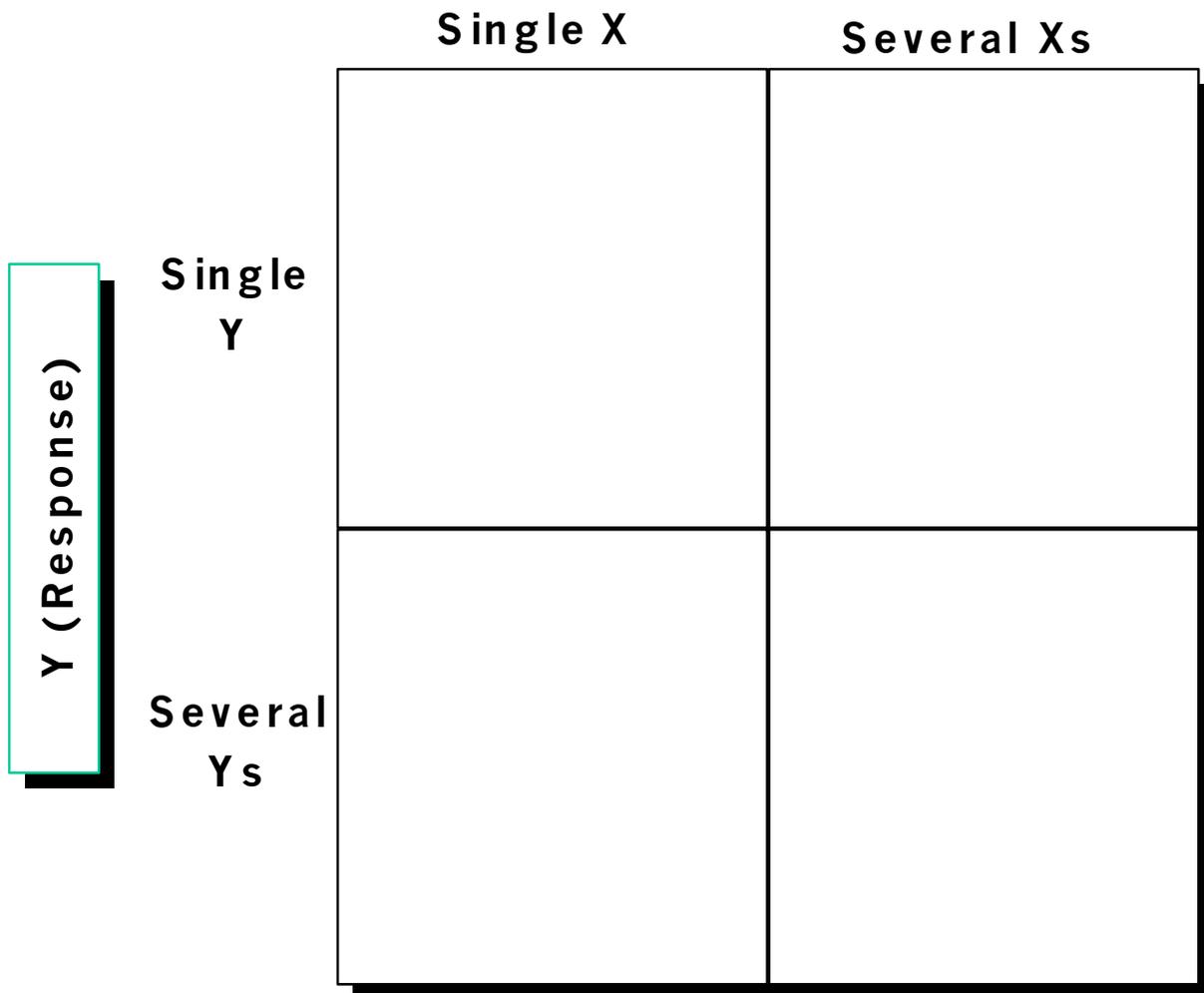
# *Data Analysis*



# Data Analysis:

## Bird's Eye View (Bird In Orbit)

X (Factor)





# Hypothesis Tests Summary

## Normal Data

### Variance Tests

$\chi^2$  - Compares a sample variance to a known population variance.

F-test- Compares two sample variances.

### Homogeneity of Variance

Bartlett's - Compares two or more sample variances

### Means Tests

t-Test 1-sample -Tests if sample mean is equal to a known mean or target.

t-Test 2-sample -Tests if two sample means are equal.

ANOVA One Way - Tests if two or more sample means are equal.

ANOVA Two Way- Tests if means from samples classified by two categories are equal.

Correlation- Tests linear relationship between two variables.

Regression - Defines the linear relationship between a dependent and independent variable. (Here, "Normality" applies to the residuals of the regression

## Non-normal Data

### Variance Tests

### Homogeneity of Variance

Levine's- Compares two or more sample variances.

### Medians Tests

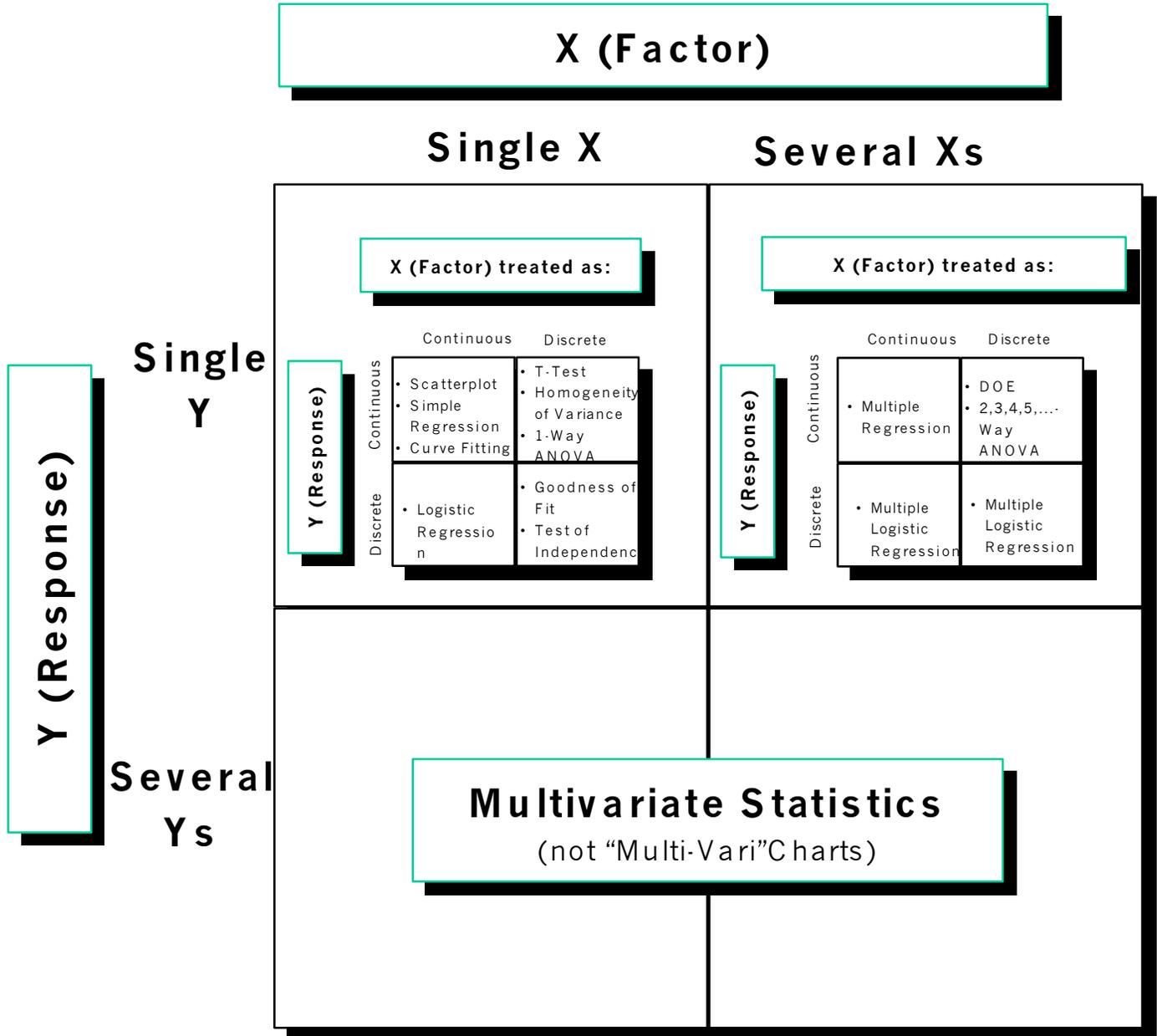
Mood's Median Test- Another test for two or more medians. More robust to outliers in data.

Correlation-Tests linear relationship between two variables.



# Data Analysis:

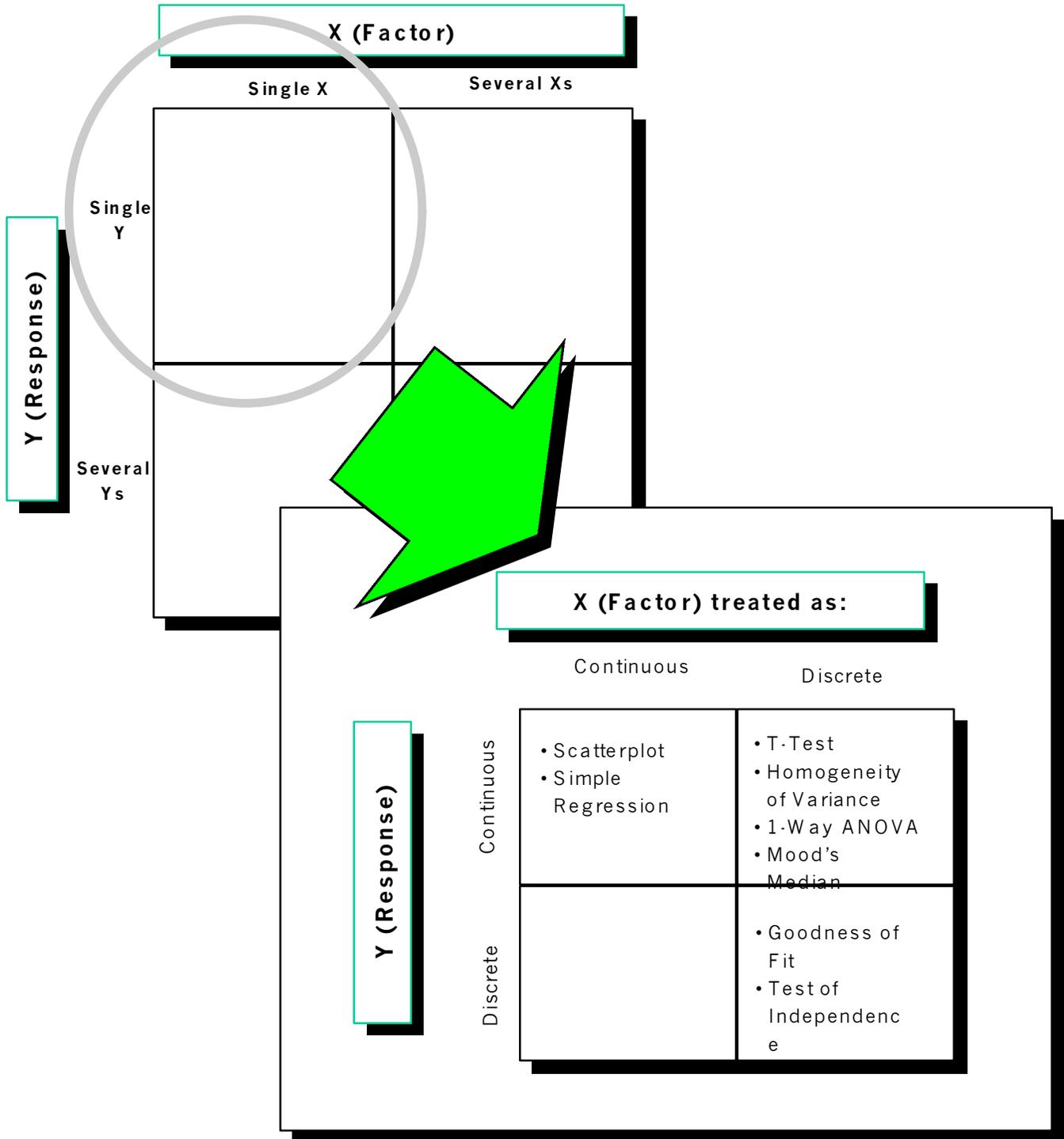
## Bird's Eye View



*Add non-parametrics to the table*

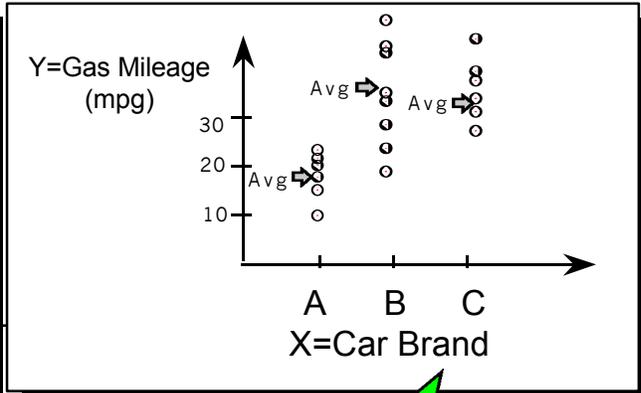
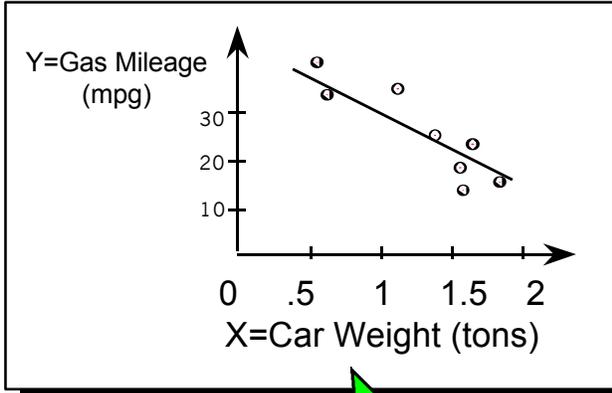


# What We Will Be Covering In Analyze





# Examples



X (Factor) treated as:

Continuous

Discrete

Y (Response)

Continuous

Discrete

- Scatterplot
- Simple Regression

- T-Test
- Homogeneity of Variance
- 1-Way ANOVA
- Mood's Median

- Goodness of Fit
- Test of Independence

Y= Injury Severity

Death	75	60	65
Major	160	115	175
Minor	100	65	135
None	15	10	25
	A	B	C
	X=Car Brand		



## *Data Analysis*

---

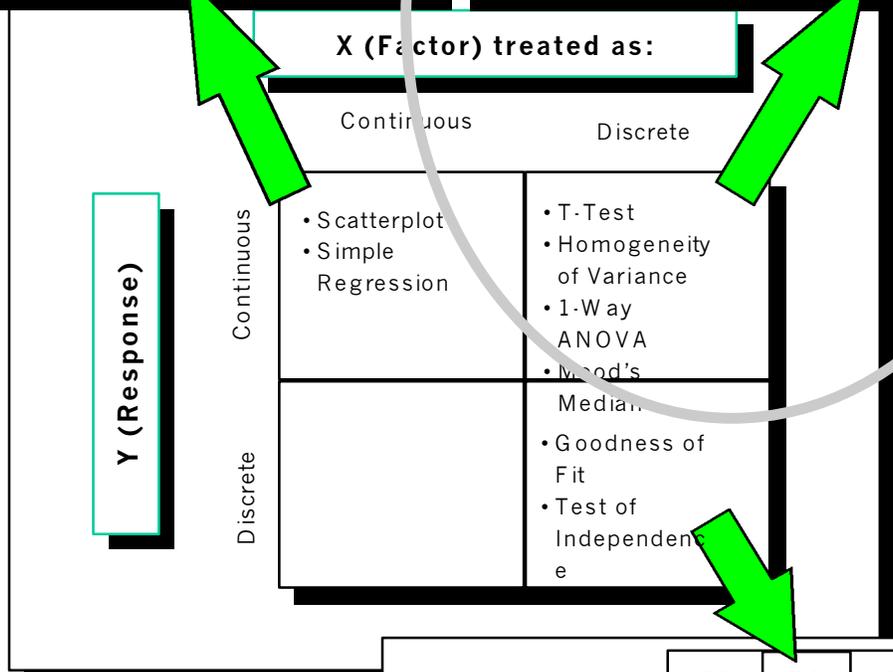
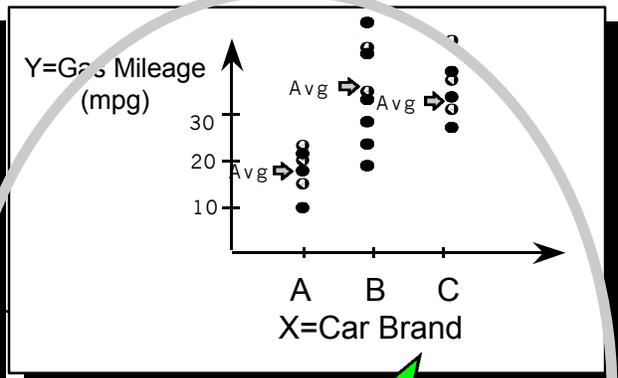
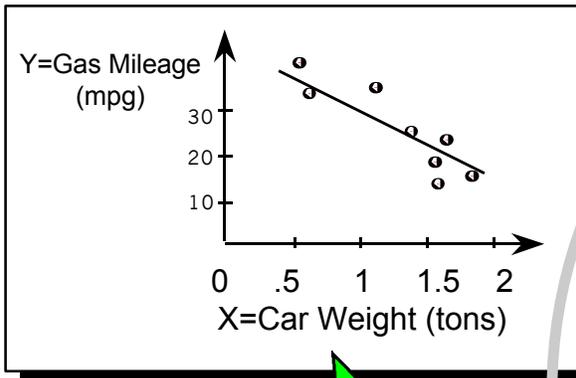
- *The tools that will be discussed during the Analyze portion of the training include the following:*
  - *1-Sample and 2-Sample T-tests*
  - *Homogeneity of Variance*
  - *1-Way ANOVA*
  - *Goodness of Fit test, Test of Independence (Chi-Square)*
  - *Scatterplot*
  - *Simple Regression*
  - *Non-parametric data*



# ***Hypothesis Testing: Continuous $Y$ ; Discrete $X$***



# Continuous Y; Discrete X



Y= Injury Severity

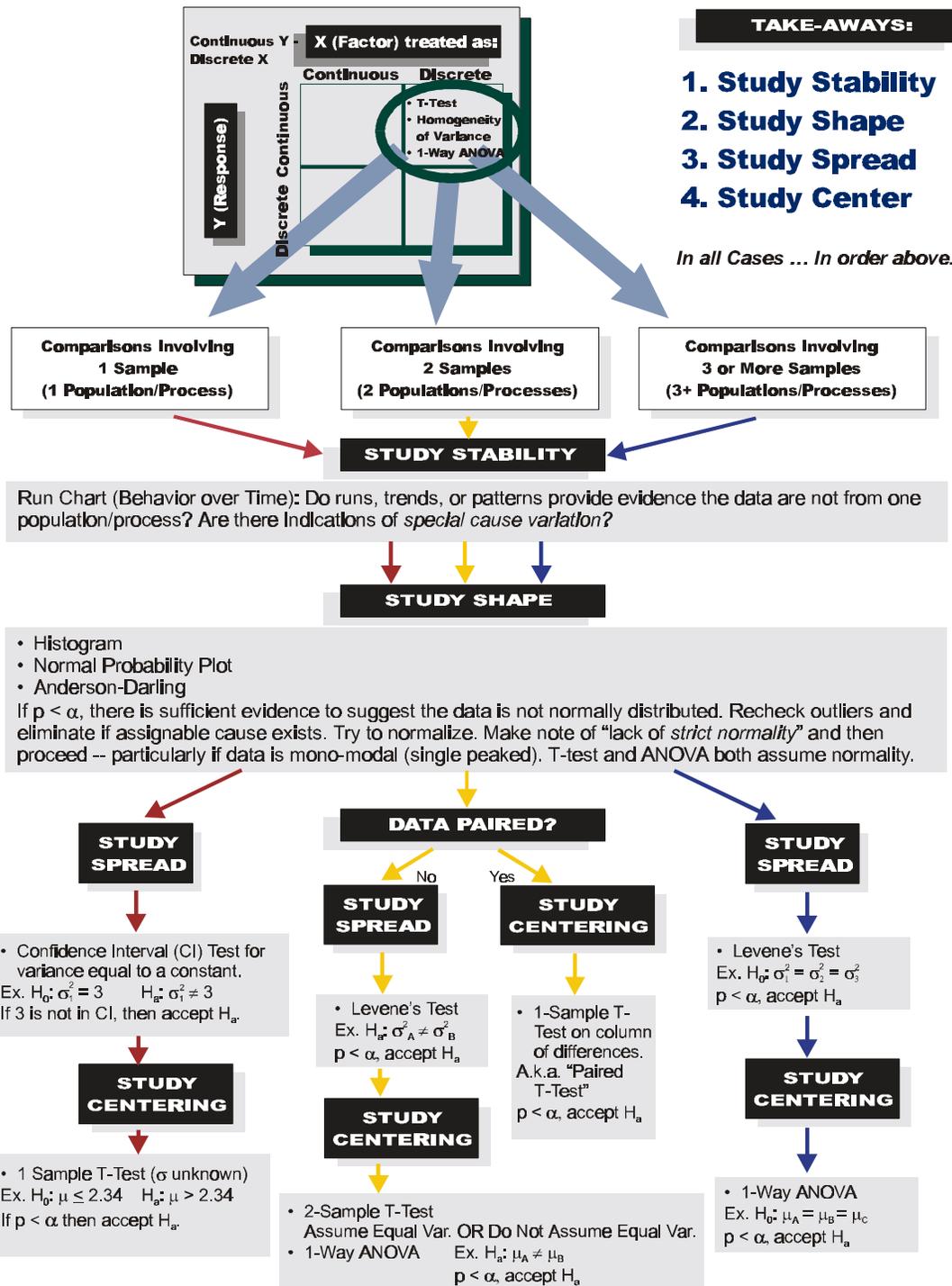
Death	75	60	65
Major	160	115	175
Minor	100	65	135
None	15	10	25
	A	B	C

X=Car Brand

Developed by W. Scott Lasater - GEMIS



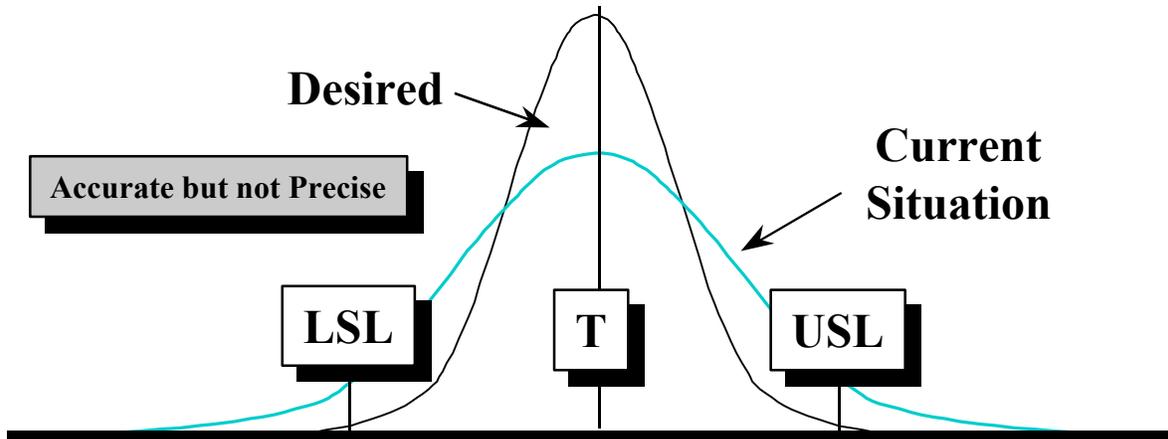
# Data Analysis



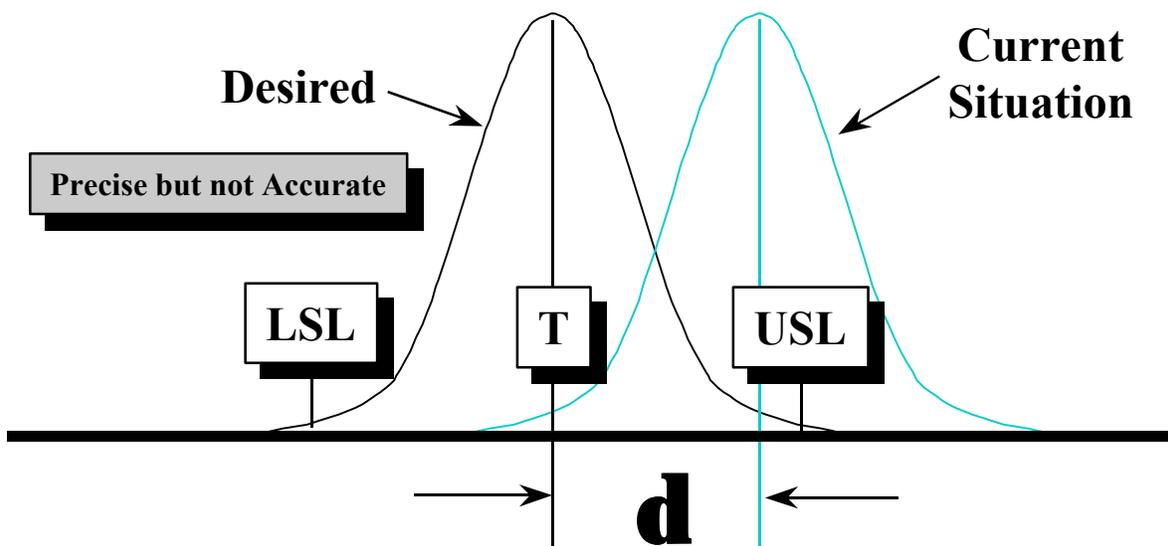


# The Nature of Statistical Problems

## Problem with Spread



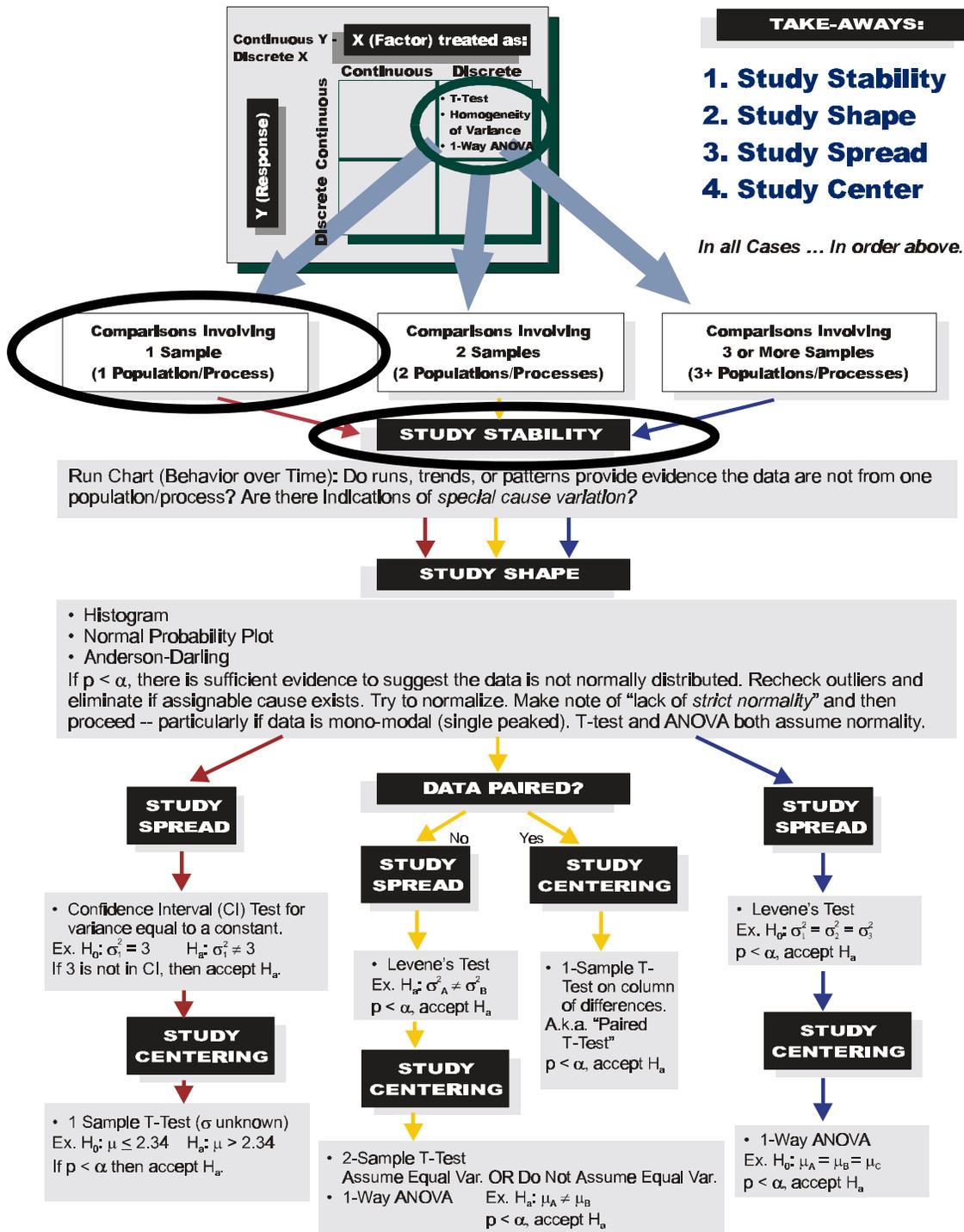
## Problem with Centering



**Goal: reduce variation & center process**



# Data Analysis





## Stability - Minitab Run Chart

*Is the first catapult operator on target?*

Step 1: *Check the stability of the first operator's process. Are there any trends in the data?*

MINITAB FILE: Catapult.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the path 'Quality Tools' > 'Run Chart...' is selected. The worksheet 'Catapult.mtw' is open, displaying data for three operators (C3, C4, C5) over 5 trials. The data is as follows:

	C3	C4	C5
↓	Oper 1	Oper 2	Oper 3
1	50.50	50.50	46.50
2	50.50	49.00	50.00
3	49.75	51.50	49.25
4	49.50	50.50	48.75
5	49.50	47.00	49.00

The status bar at the bottom of the window reads: "Draw a run chart with tests for randomness".



# Stability - Minitab Run Chart

1. Double Click.

2. Type in a "1."

**Run Chart**

C1	Target
C2	Angle
C3	Oper 1
C4	Oper 2
C5	Oper 3
C6	Oper 4
C7	Oper 5
C8	Dist 50
C9	Oper 50
C10	Dist 1&2
C11	Oper 1&2
C12	Dist 3&4
C13	Oper 3&4
C14	150
C15	155
C16	160
C17	165
C18	170
C19	Dist Ang

Data are arranged as

Single column: 'Oper 1'

Subgroup size: 1

(use a constant or an ID column)

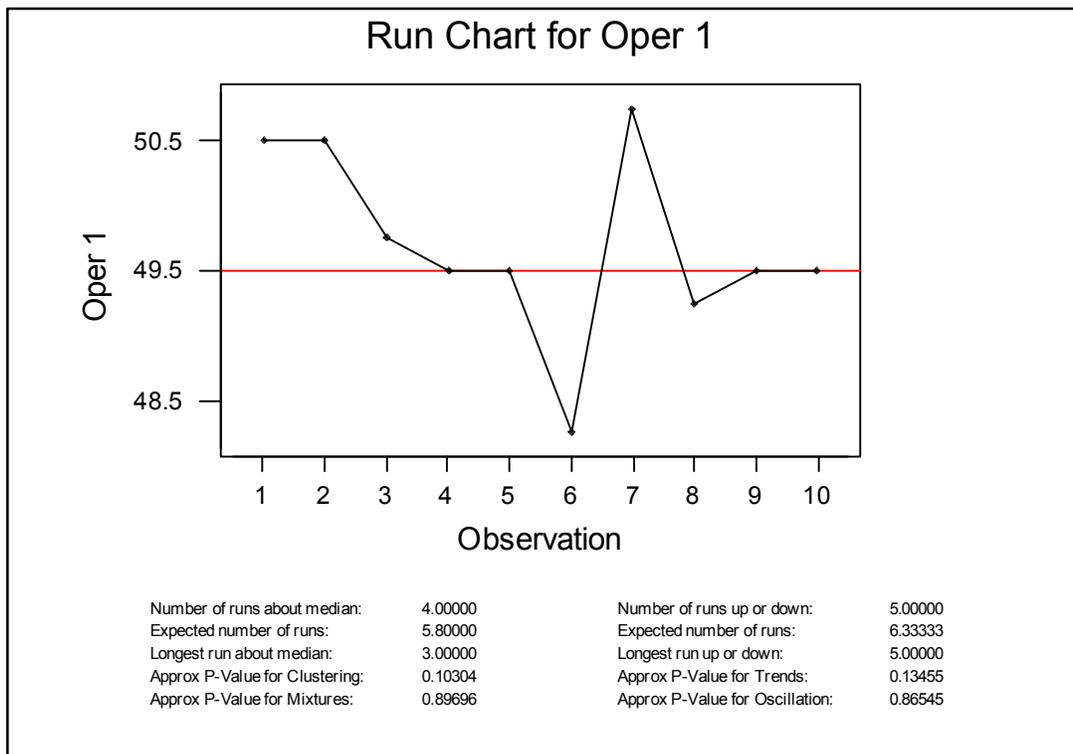
Subgroups across rows of:

For data in subgroups

Plot subgroup means

Plot subgroup medians

Buttons: Select, Help, Options..., OK, Cancel

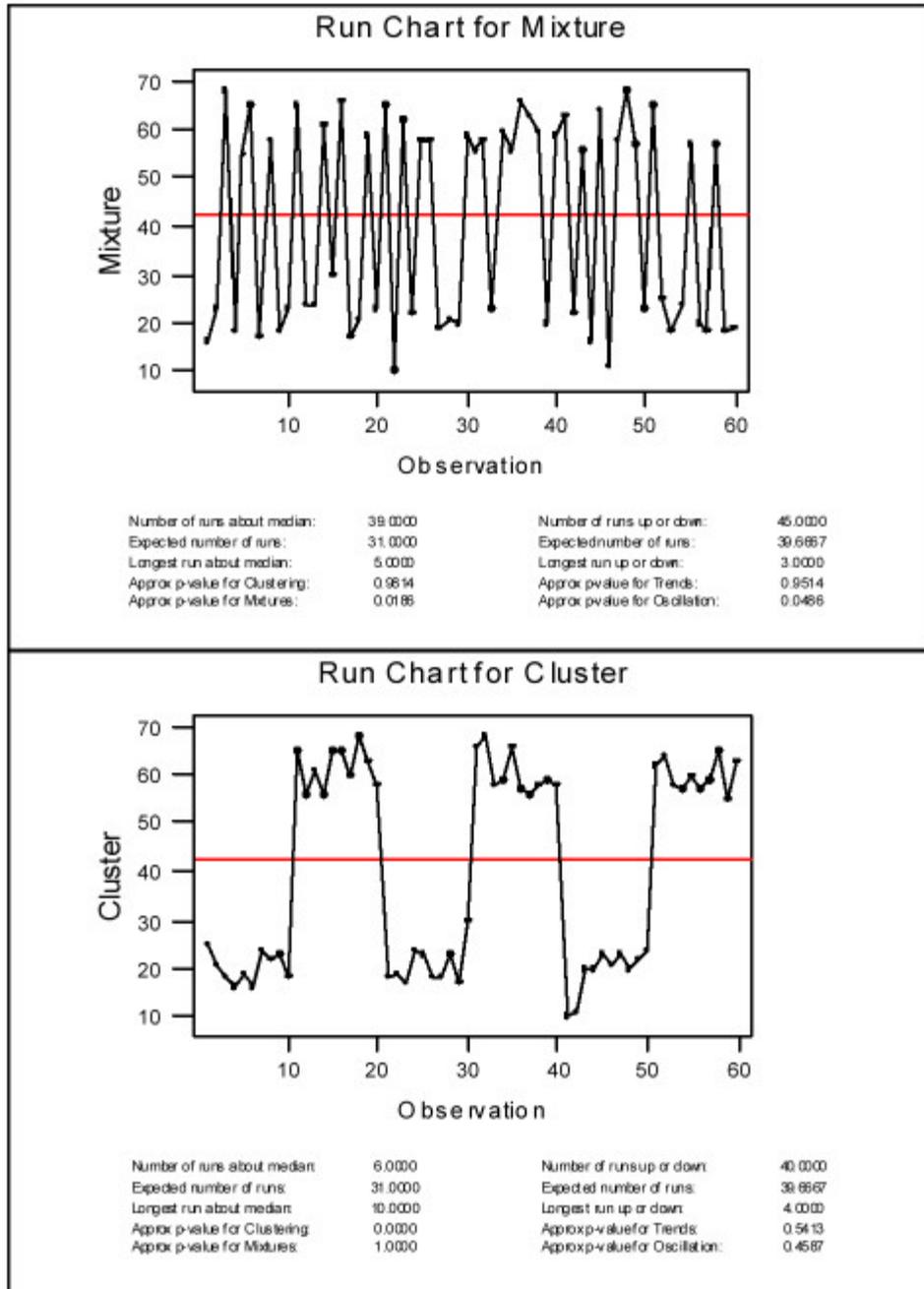




# Studying Stability

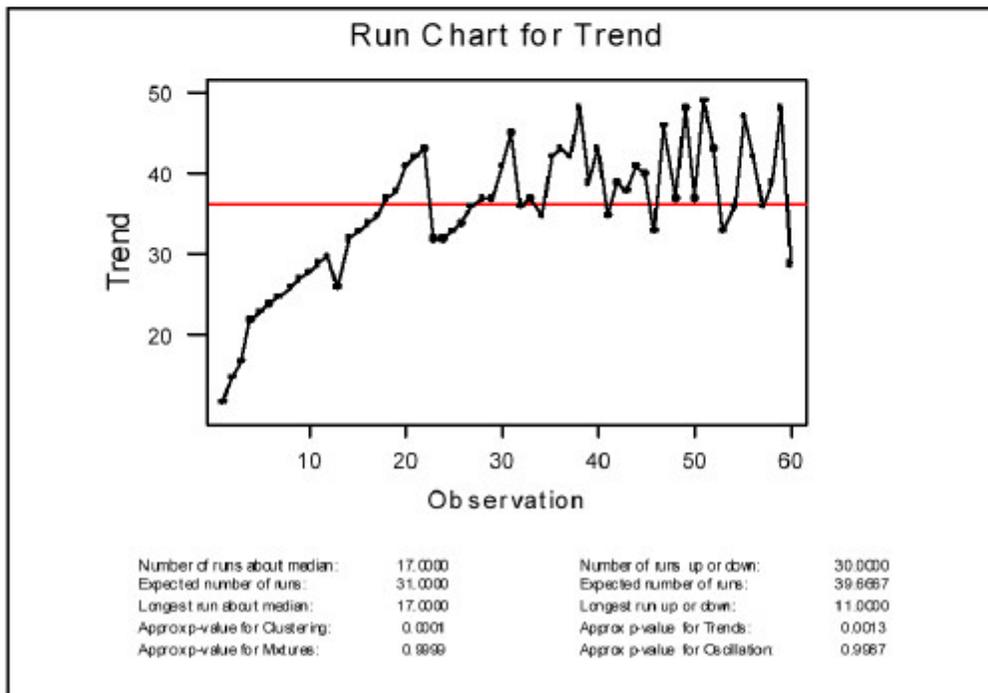
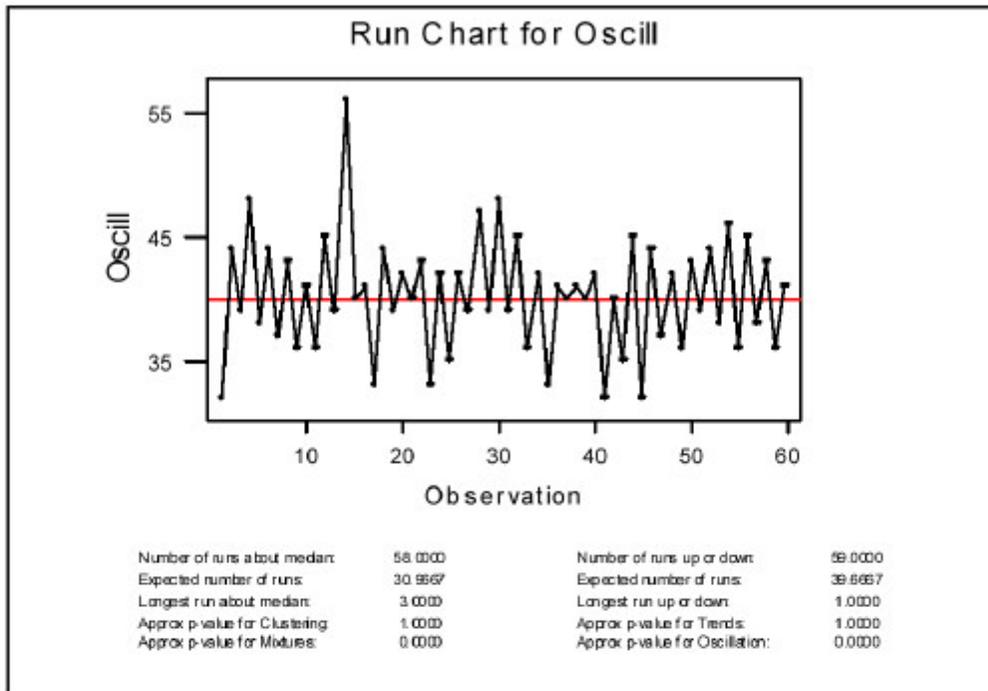
$H_0$ : Data is random, special causes not present

$H_a$ : Data is not random, special causes present



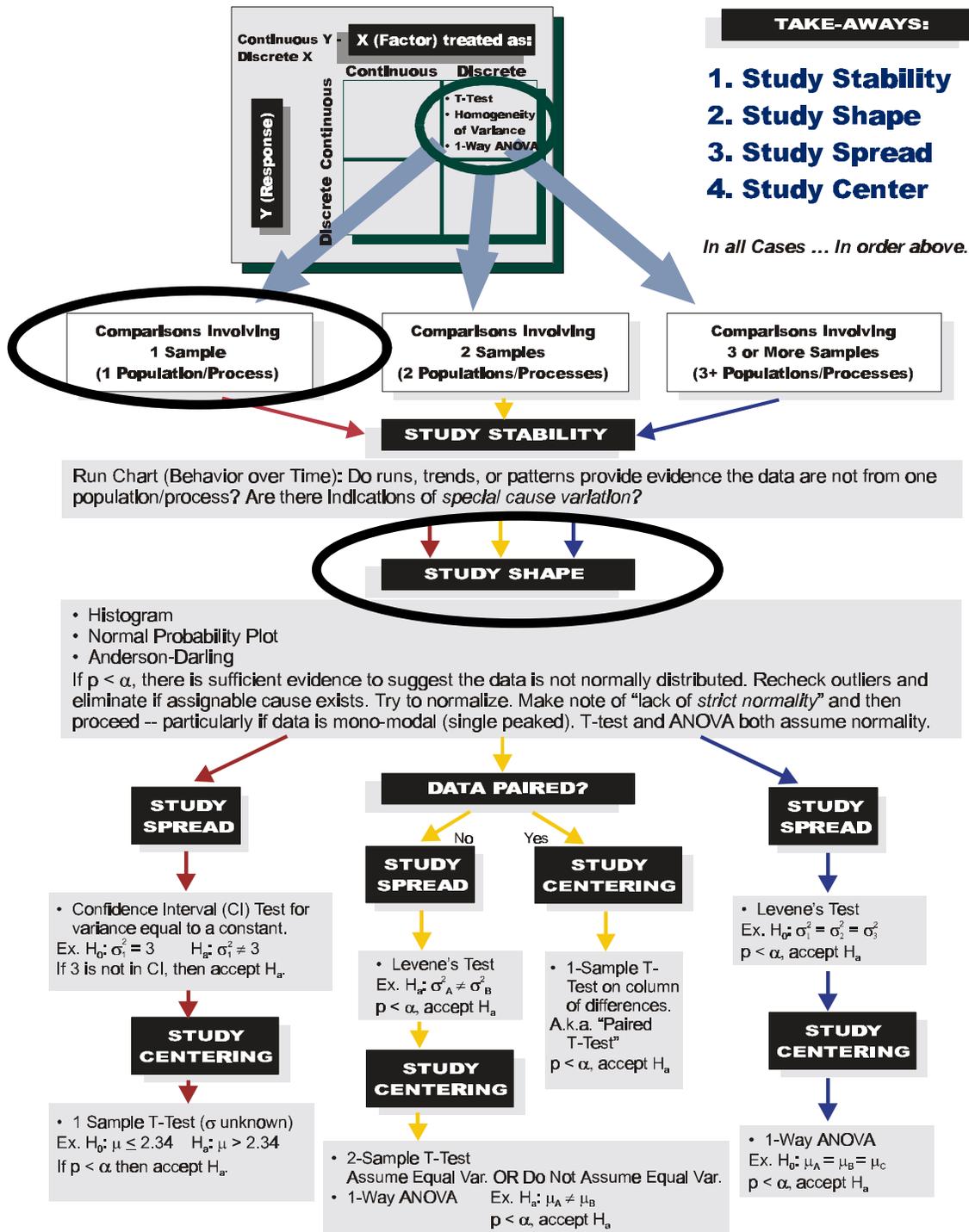


# Studying Stability





# Data Analysis





## Shape - Minitab Descriptive Statistics

Step 2: Check the shape of the data. Is the data normally distributed?

MINITAB FILE: Catapult.mtw

Calculate descriptive statistics and display in the Session window

	C3	C4	C5	C6	C7	C8	C9
↓	Oper 1	Oper 2	Oper 3	Oper 4	Oper 5	Dist 50	Oper 50
1	50.50	50.50	46.50	49.00	50.00	50.50	1
2	50.50	49.00	50.00	50.25	49.75	50.50	1
3	49.75	51.50	49.25	50.50	49.75	49.75	1
4	49.50	50.50	48.75	49.75	50.00	49.50	1
5	49.50	47.00	49.00	50.00	48.75	49.50	1

$H_o$ : The data is normally distributed

$H_a$ : The data is not normally distributed



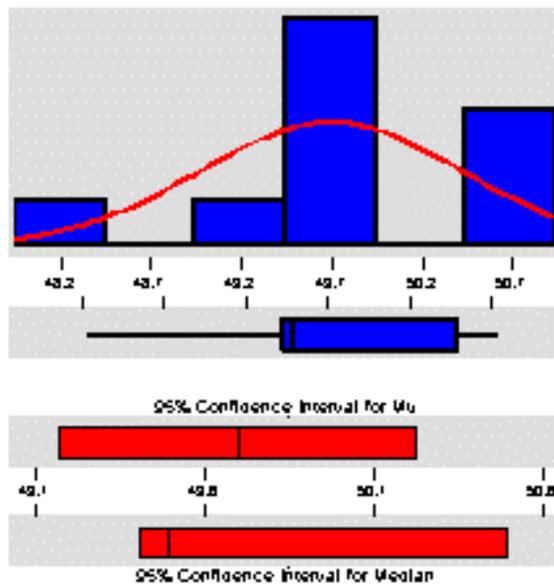
# Descriptive Statistics - Input & Output

1. Double Click

2. Click on Graphs, Select Graphical Summary

3. Click OK

Descriptive Statistics



Variable: Oper 1

Anderson-Darling Normality Test

A-Square:	0.556
P-Value:	0.113
Mean	49.7000
StDev	0.7341
Variance	0.538880
Skewness	-3.5E-01
Kurtosis	0.547827
n	10
Minimum	48.2500
1st Quartile	49.4375
Median	49.5000
3rd Quartile	50.5000
Maximum	50.7500
95% Confidence Interval for $\mu$	49.1740 50.2251
95% Confidence Interval for Sigma	0.5040 1.3402
95% Confidence Interval for Median	49.4144 50.5000

Data is Normal if  $p\text{-value} > .05$ . (Fail to Accept  $H_A$ )



---

## *Nonparametric Tests*

---

What if I don't have normal  
data?!!



## *Objectives*

---

- *Know how to determine normality*
- *Understand the possible causes of non-normal data*
- *Understand how to run and interpret results from the following test:  
Mood's Median Test*



## Overview

---

- *Dealing with Non-normal Data*
- *Mood's Median Test*
- *Homogeneity of Variance*



# Statistical Test Choices

NOTE: Temperature, Cycle Time, and Length are variables. Operators, Suppliers, and Customers are not.

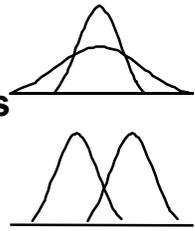
Is your "Y" variable data or attribute data?

Variable  
Is the input being tested also a variable?

Attribute

Construct a multi-vari of the data

Determine what should be tested for:  
 1. a difference in the variation of the different subgroups  
 OR  
 2. a difference in the "centering" of the different subgroups



Regression  
Correlation

Chi-square  
Binomial

Variation		"Centering"	
Normal Subgroups	Non-normal Subgroups	Normal Subgroups	Non-normal Subgroups
HOV Bartlett's	HOV Levene's	1 Sample T-test	Mood's Median
HOV F-test		2 Sample T-test	
		Oneway ANOVA	



# Hypothesis Tests Summary

## Normal Data

### Variance Tests

$\chi^2$  - Compares a sample variance to a known population variance.

F-test- Compares two sample variances.

### Homogeneity of Variance

Bartlett's - Compares two or more sample variances

### Means Tests

t-Test 1-sample -Tests if sample mean is equal to a known mean or target.

t-Test 2-sample -Tests if two sample means are equal.

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ANOVA Two Way- Tests if means from samples classified by two categories are equal.

Correlation- Tests linear relationship between two variables.

Regression - Defines the linear relationship between a dependent and independent variable. (Here, "Normality" applies to the residuals of the regression)

## Non-normal Data

### Variance Tests

### Homogeneity of Variance

Levine's- Compares two or more sample variances.

### Medians Tests

Mood's Median Test- Another test for two or more medians. More robust to outliers in data.

Correlation-Tests linear relationship between two variables.



## *Dealing with Non Normal Data*

---

Check and be sure the data is truly non-normal. Non-normality can eliminate a number of helpful tools from consideration.

- Perform a **Normality Test** (to verify that it's truly non-normal)
- Consider whether you have sufficient resolution (How finely divided is the scale of your measurement - should you use minutes instead of hours? Thousandths of an inch instead of sixteenths of an inch?)
- Check data for (typographical) errors. Investigate outliers.
- Be cautious of small sample sizes (i.e.  $<30$ ). A small sample from a normal population will sometimes test as non-normal - be aware of how many data points you have.
- Attempt to transform the data. Common transforms include:
  - finding the square root of all data points
  - finding the log of all data points
  - finding the square of all data points
- If the data is still non-normal, use the **Non Parametric tools**



## ***DEFINITIONS***

---

- **Mean** - *Arithmetic average of the data. Sum of all the data points divided by the number of data points.*
- **Median** - *Value of middle data point when the data are sorted or ranked.*
- **Mode** - *The most often occurring value in the data set.*

### *Example:*

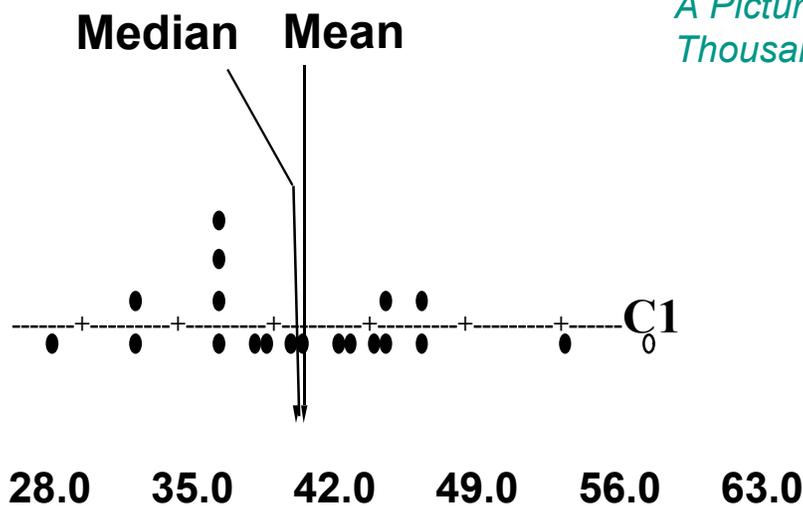
*Data was collected weekly for the inventory level of a certain type of bearing in a stockroom. What is the mean and median for this set of data?*

*30, 37, 25, 35, 42, 35, 35, 47, 45, 60,  
39, 45, 30, 38, 35, 40, 44, 55, 47, 43*



Enter data into Minitab worksheet then  
Graph>Character Graphs>Dotplot

## Character Dotplot



*A Picture is Worth a  
Thousand Words*

**Mean = 40.35      Median = 39.5**

The **mean** can be influenced (or leveraged) considerably by outliers because when you calculate a mean, you factor in the actual *values* of outliers.

The **median**, on the other hand, assigns equal weight to all observations regardless of actual outlier values because it is concerned only with the value which has an equal number of observations above and below it.

*(If the response of 60 (white dot) were 6000, the mean would change, but the median would not.)*



## Mood's Median Test

Project #1204-1 - Scrap Requisitions were processed greater than 29 days 30% of the time. One of the Families of Variation was “Business to Business”. Is there a difference in the medians between Locomotive, Control, and Propulsion.

### Locomotive

5, 122, 8, 22, 9,  
22, 22, 21, 21, 18,  
46, 43, 33, 19, 16,  
15, 12, 12, 12, 36,  
45, 57, 32, 104,  
11, 57, 36, 6, 7,  
111, 36, 49, 29,  
28, 43, 72, 48, 27,  
10, 8, 7, 7, 48, 30,  
14, 52, 44, 41, 31,  
22, 10, 10, 9, 7,  
70, 21, 15, 15 6,  
47, 35, 35, 34, 25,  
22, 21, 14, 12, 6,  
6, 5, 22

### Control

20, 18, 9, 37, 8,  
30, 15, 26, 26,  
26, 26, 26, 14,  
36, 28, 13, 22,  
126, 119, 222,  
119, 119, 119,  
119, 119, 83, 10,  
13, 5, 55, 15, 8,  
22, 22, 22,  
22, 21, 21, 15,  
15, 22, 8, 13, 42,  
14, 13, 12, 12, 8

### Propulsion

20, 14, 23, 23,  
23, 17, 17, 15,  
15, 15, 18, 22,  
22, 41, 8, 6, 6,  
6, 22, 8, 14,  
14, 34, 14, 14,  
14, 33, 32, 21,  
20, 20, 20, 21,  
19, 12, 12, 12

**Each data point represents the number of days to process a single Scrap Requisition.**



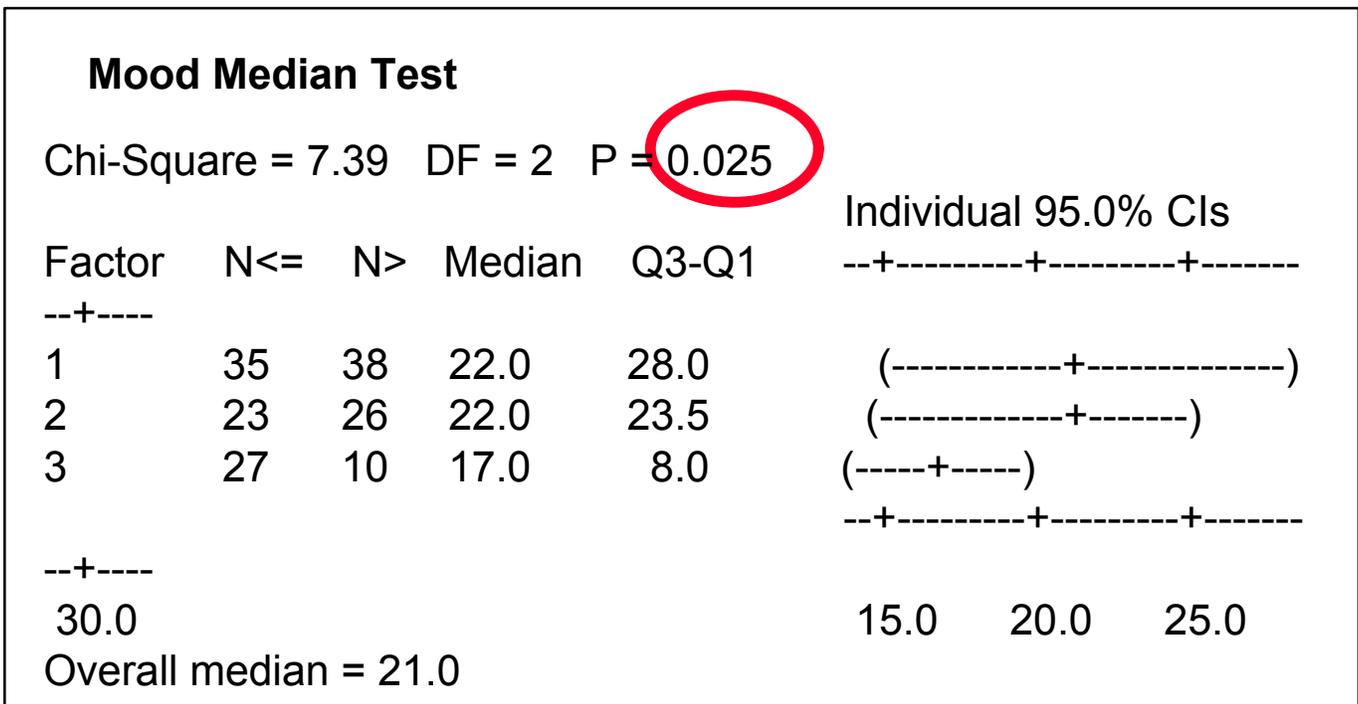
# Mood's Median Test

Mood's Median test requires that all the data be in one column with factors (or subscripts) in a second column.

If your data is not already stacked, you can stack it using: Manip>Stack/Unstack>Stack.

## In Minitab:

- Open File>New Worksheet      Copy in data
- **Stat>Nonparametrics>Mood's Median Test**
- Response: **Days**    Factor: **Business**



Since  $p < 0.05$  we reject  $H_0$ . There is a difference between Businesses



## *Non-normal data test*

---

### ***Mood's Median Test***

**This test will show Statistical Significance.**

**It will not show Practical Significance.**

**It is recommended to do a Multi-vari  
to see the Practical Significance.**



## Summary

---

- An **Anderson-Darling Normality Test** is used to determine if your data is normal.
- A **Mood's Median Test** is used to test **two or more** population medians. Test is robust to outliers or errors in data. (shows statistical significance)



## *Nonparametric Tests*

---

- One of the advantages of nonparametric tests is that they assume no knowledge about the underlying distributions. They often use an analysis of the ordered ranks of the data
- A disadvantage of nonparametric tests is that they are less powerful (it takes more data to find the same size difference) than the equivalent t-tests and ANOVA tests
- The **Mood's Median Test** is a nonparametric test which is very similar to the One-Way ANOVA test



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# *Data Transformation*

---

***When do I Transform my Data?***



## *Transformation*

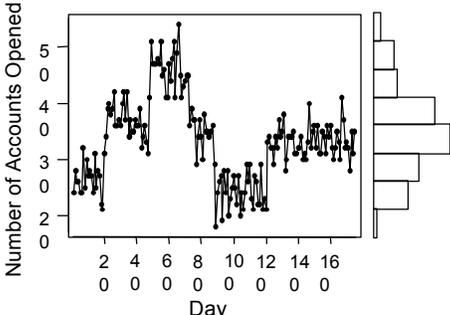
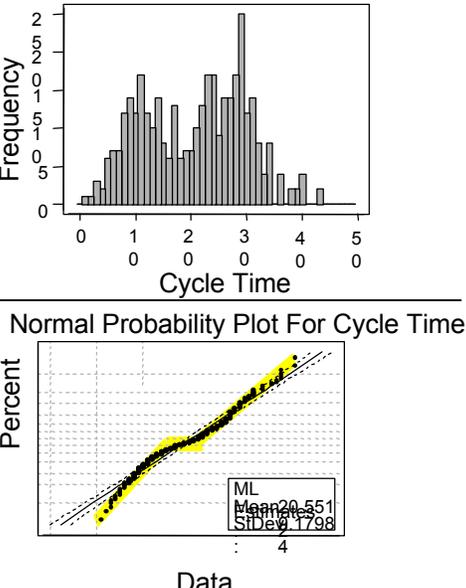
---

■ *Typically, it is not recommended that you transform your data for several reasons:*

1. You lose touch with the physical process.
2. It is a very rare instance when this is appropriate and it tends to be more of an art than a science.



# When Not To Use A Transformation

Situation	How to Check for the Situation	What To Do Instead
<p><b>1.</b> Data has cycles, shifts or trends  (Data Is Not Stable)</p>	<p>Plot the data in time order using a run chart or control chart.</p>  <p>Here the histogram is skewed, but the data is not fundamentally non-normal.</p>	<ul style="list-style-type: none"> <li>■ Understand the nature of the special causes.</li> <li>■ Use scatterplots and segmentation analysis to look for factors that correlate with or explain the shifting.</li> <li>■ Test only stable segments for normality.</li> </ul>
<p><b>2.</b> Data is different for different groups</p>	<p>Histogram will sometimes show multiple humps (bimodal or multimodal.) Normal probability plot will show multiple sloped line segments attached with flat line segments.</p>  <p>Normal Probability Plot For Cycle Time</p> <p>Percent</p> <p>ML Mean 20.551 StdDev 1.798</p> <p>Data</p>	<ul style="list-style-type: none"> <li>■ Segment or Stratify the data.</li> <li>■ Look for differences between groups using stratified histograms and ANOVA.</li> <li>■ Test for normality only within groups.</li> </ul>



# When Not To Use A Transformation

Method	How to Check for the Situation	What To Do Instead
<p><b>3. Data is symmetric</b></p> <p><b>3A.</b> With less in the tails than the normal distribution</p>	<p>Normal probability plot is straight except for points at either end are "inside" the line.</p> <p>Normal Probability Plot for Usage</p>	<ul style="list-style-type: none"> <li>■ Generally OK to ignore this departure from normality.</li> </ul>
<p><b>3B.</b> With more in the tails than the Normal distribution</p>	<p>Normal probability plot is straight except for points at either end are "outside" the line.</p> <p>Normal Probability Plot for Rework Volume</p>	<ul style="list-style-type: none"> <li>■ May have special causes (outliers) in the data.</li> <li>■ See Situation 1.</li> </ul>
<p><b>4. Data is rounded to the nearest integer</b></p>	<p>Normal probability plot has vertical bars. Test of normality will fail.</p> <p>Normal Probability Plot For Days To Close</p>	<ul style="list-style-type: none"> <li>■ As long as there are 5 or more distinct values and the data are symmetric, treat as normal.</li> </ul>



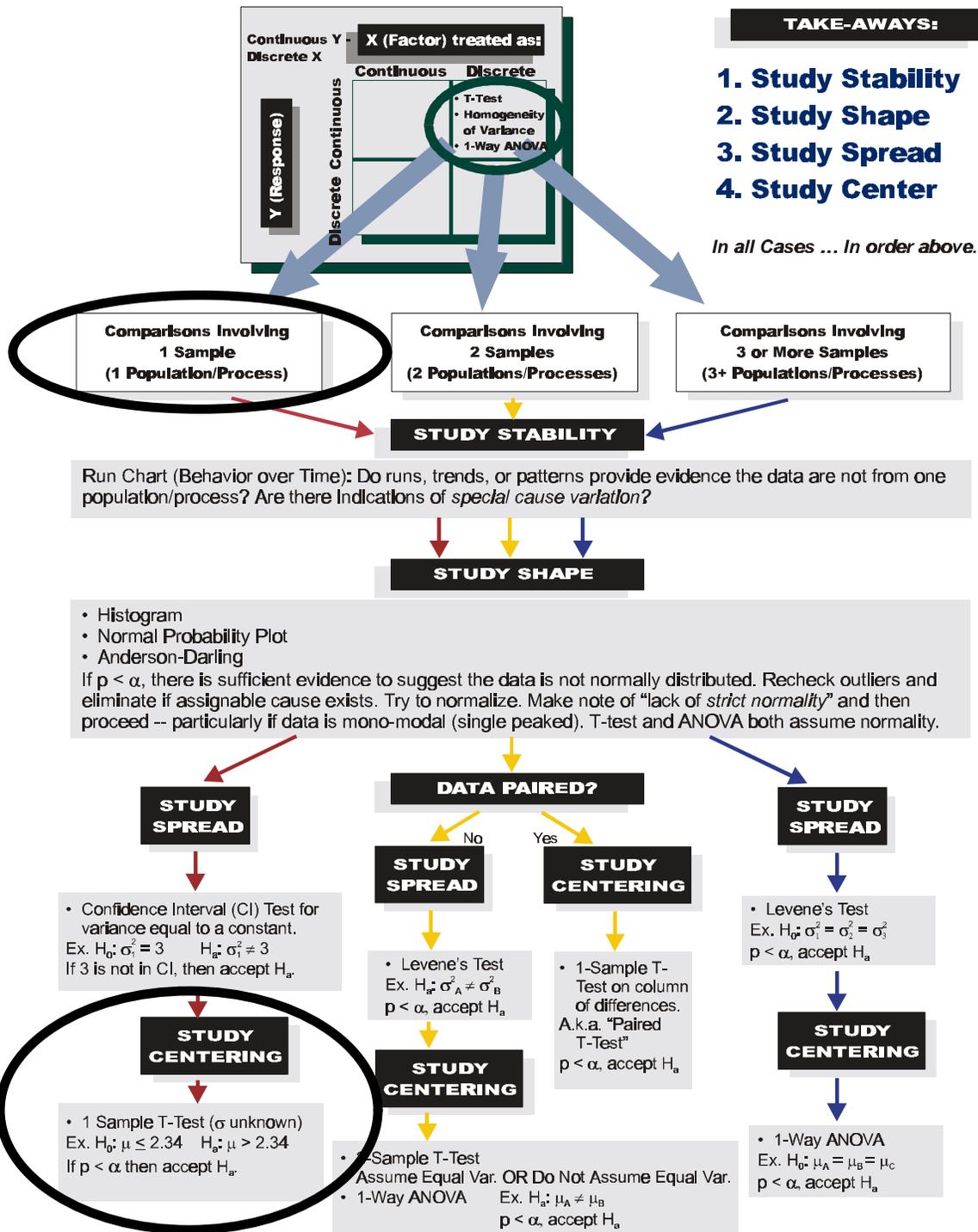
## *When To Use A Transformation*

---

- *There are specific methods to use in transforming data.*
- *If you think that you need this for your data, see your MBB.*



# Data Analysis





## Example 1a, Hand Calculation

Target = .96960	
$\bar{X} = .96953$	$s = .00017$
$n = 30$	$n-1 = \text{dof} = 29$
$\alpha = .05$	$\alpha/2 = .025$
$t_{\alpha/2, n-1} = t_{.025, 29} = 2.045$ (from t Distribution table)	

Equation for the confidence interval about the sample mean:

$$\bar{X} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} < \mu < \bar{X} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$$

---

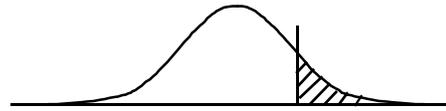
$$.96953 - 2.045 * \frac{.00017}{\sqrt{30}} < \mu < .96953 + 2.045 * \frac{.00017}{\sqrt{30}}$$

$$.96946 < \mu < .96959$$

**Conclusion:** Because the target is not in the acceptance region for  $H_0$ , we conclude the process is off target.  
Reject  $H_0$ .



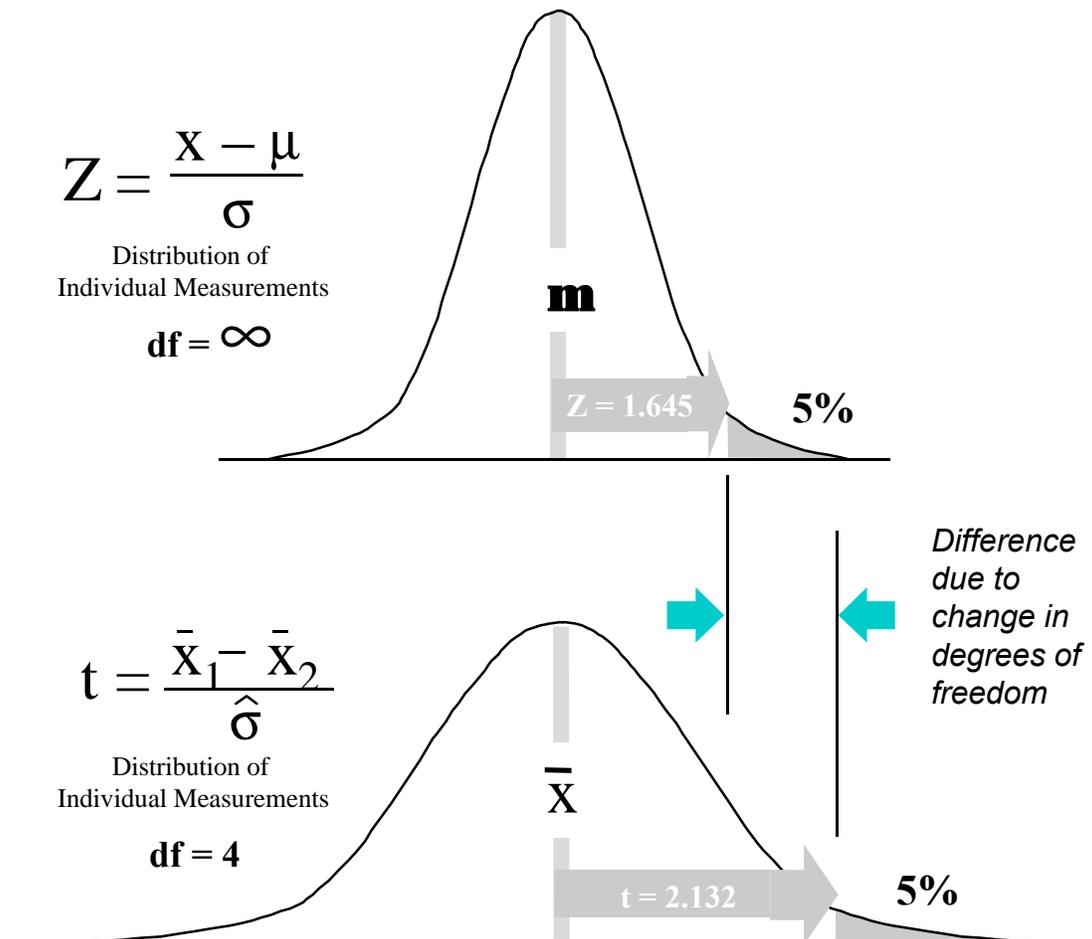
# t Distribution



<b>a</b>	<b>.400</b>	<b>.300</b>	<b>.200</b>	<b>.100</b>	<b>.050</b>	<b>.025</b>	<b>.010</b>	<b>.005</b>
<b>1 - a</b>	<b>.600</b>	<b>.700</b>	<b>.800</b>	<b>.900</b>	<b>.950</b>	<b>.975</b>	<b>.990</b>	<b>.995</b>
<b>n - 1</b>								
1	0.325	0.727	1.376	3.078	6.314	12.706	31.821	63.657
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750
40	0.255	0.529	0.851	1.303	1.684	2.021	2.423	2.704
60	0.254	0.527	0.848	1.296	1.671	2.000	2.390	2.660
120	0.254	0.526	0.845	1.289	1.658	1.980	2.358	2.617
∞	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576



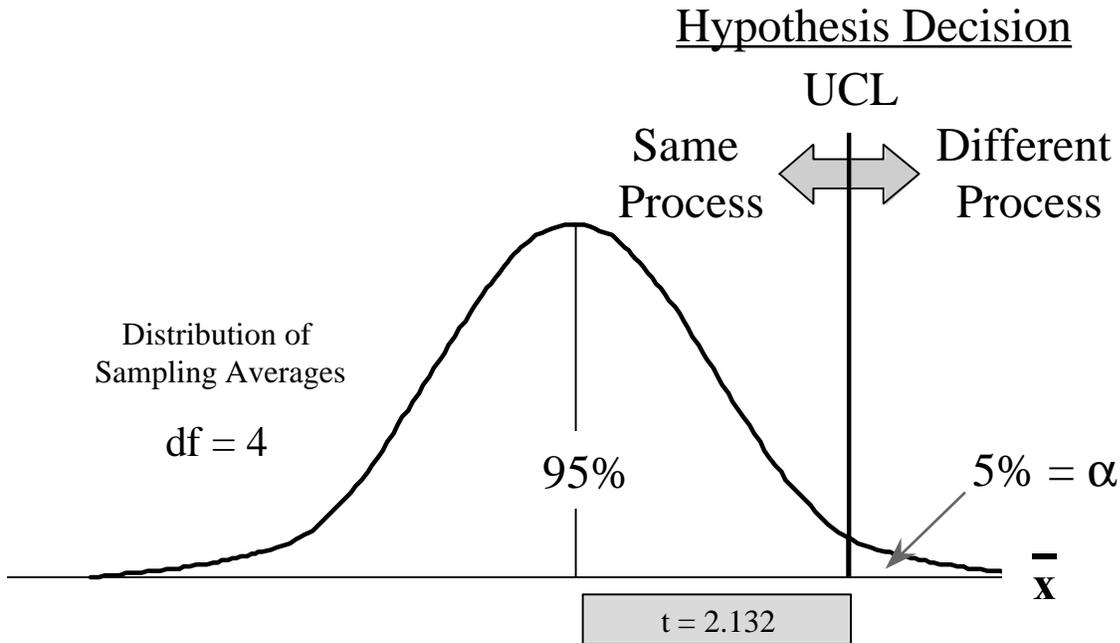
## Nature of the t Distribution



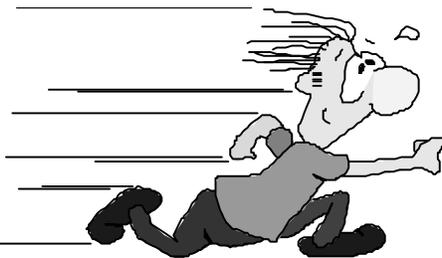
*If the population distribution is unknown, we may estimate it with a random sample. When the sample size is infinite, there is no uncertainty of estimation; hence, we apply the normal (Z) distribution to discover a given probability of chance occurrence. However, as the sample size declines, our uncertainty increases; consequently, we must expand the range of prediction for the same probability. In other words, we must correct Z for the loss in degrees of freedom.*



# One-Sided Use of the *t* Distribution



Confidence Interval (Acceptance Region for $H_0$ )	Risk
---	------



$$UCL = \bar{X} + t_{\alpha} \frac{\hat{\sigma}}{\sqrt{n}}$$

$$UCL = \bar{X} + 2.132 \frac{\hat{\sigma}}{\sqrt{5}}$$

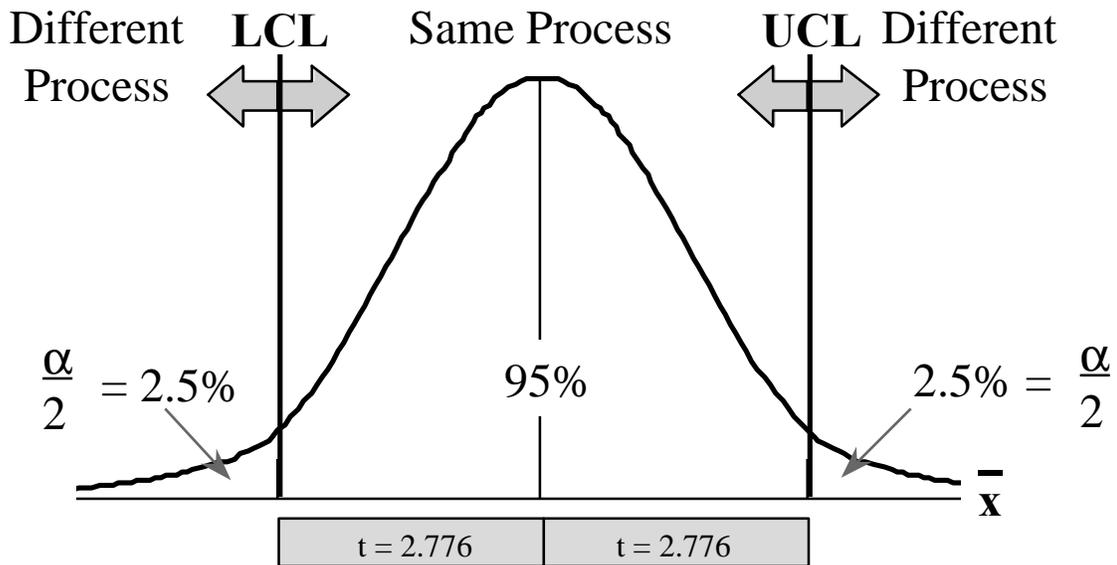
**When  $H_0$  is true, there is 95% certainty that the true population mean will be less than the UCL (Upper Confidence Limit). If we observe a sampling average greater than UCL, we may conclude that such an event could only occur 5 out of 100 by random chance (sampling variations).**



# Two-Sided Use of the *t* Distribution

Distribution of  
Sampling Averages

df = 4



Risk	Confidence Interval (Acceptance Region for $H_0$ )	Risk
------	---	------

$$LCL = \bar{x} - t_{\alpha/2} \frac{\hat{\sigma}}{\sqrt{n}}$$

$$UCL = \bar{x} + t_{\alpha/2} \frac{\hat{\sigma}}{\sqrt{n}}$$

$$LCL = \bar{x} - 2.776 \frac{\hat{\sigma}}{\sqrt{5}}$$

$$UCL = \bar{x} + 2.776 \frac{\hat{\sigma}}{\sqrt{5}}$$

**There is 95% certainty that the true population mean will be contained within the given confidence interval. If we observe a sampling average greater than UCL or less than LCL, we may conclude that such an event could only occur 5 out of 100 by random chance (sampling variations).**



# Example 2, Minitab Calculation Confidence Interval of Mean

MINITAB FILE: Catapult.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the path 'Basic Statistics' > '1-Sample t...' is selected. A dialog box for the '1-Sample t' test is visible, showing a mean of 0.734 and a standard deviation of 0.2. Below the menu, a data table is displayed with columns C3 through C9 and rows 1 through 5. The status bar at the bottom indicates the operation: 'Perform a one-sample t-test and compute a confidence interval'.

	C3	C4	C5	C6	C7	C8	C9
↓	Oper 1	Oper 2	Oper 3	Oper 4	Oper 5	Dist 50	Oper 50
1	50.50	50.50	46.50	49.00	50.00	50.50	1
2	50.50	49.00	50.00	50.25	49.75	50.50	1
3	49.75	51.50	49.25	50.50	49.75	49.75	1
4	49.50	50.50	48.75	49.75	50.00	49.50	1
5	49.50	47.00	49.00	50.00	48.75	49.50	1



# Confidence Interval of Mean - Minitab Input and Output

1. Double Click →

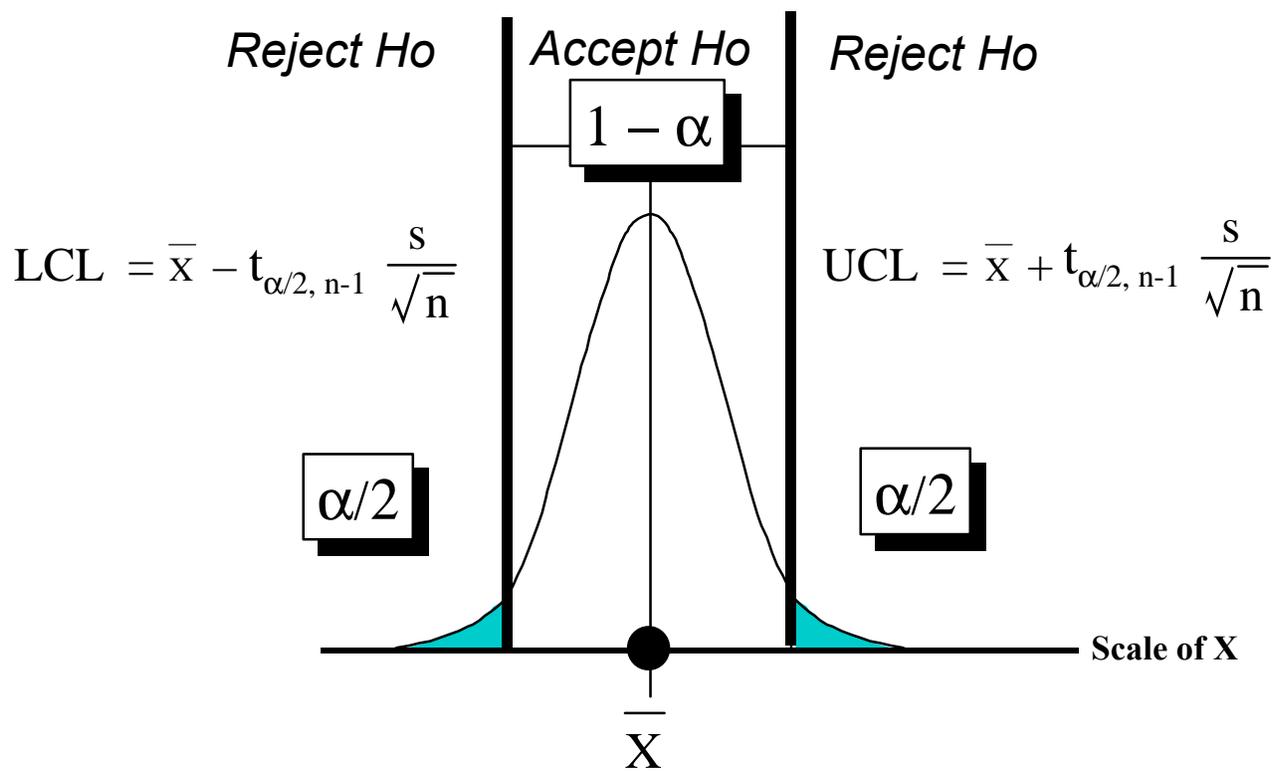
2. Click OK

Variable	N	Mean	StDev	SE Mean	95.0 % CI
Oper 1	10	49.700	0.734	0.232	( 49.175, 50.225)

*Since the Confidence Interval contains the target, 50 inches, we do not have enough evidence to show that the process mean is off center. Thus we accept  $H_0$ .*



# Single Sample Test for a Mean Equaling a Target (Two-Sided)





## *The p-Value? (Review)*

- *Alpha is the maximum acceptable probability of being wrong if the alternative hypothesis is selected.*
- *The p-value is the probability that you will be wrong if the alternative hypothesis is selected. This is a Type I error.*
- *Unless there is an exception based on engineering judgment, we will set an acceptance level of a Type I error at  $\alpha = 0.05$ .*
- *Thus, any p-value less than 0.05 means we reject the null hypothesis.*

$p < \alpha$ : Reject  $H_0$

$p > \alpha$ : Accept  $H_0$



# Example 2b, Minitab Calculation Test of Mean (p-Value)

MINITAB FILE: Catapult.mtw

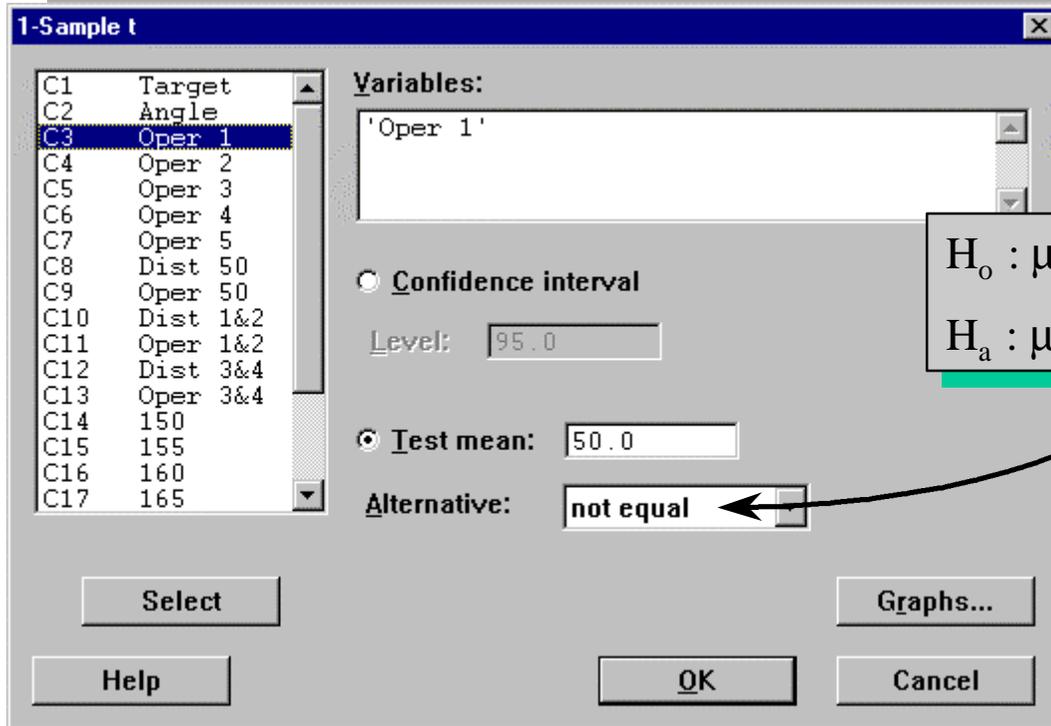
The screenshot shows the Minitab software interface. The 'Stat' menu is open, and '1-Sample t...' is selected. The data table below shows the following values:

	C3	C4	C5	C6	C7	C8	C9
↓	Oper 1	Oper 2	Oper 3	Oper 4	Oper 5	Dist 50	Oper 50
1	50.50	50.50	46.50	49.00	50.00	50.50	1
2	50.50	49.00	50.00	50.25	49.75	50.50	1
3	49.75	51.50	49.25	50.50	49.75	49.75	1
4	49.50	50.50	48.75	49.75	50.00	49.50	1
5	49.50	47.00	49.00	50.00	48.75	49.50	1

Perform a one-sample t-test and compute a confidence interval



# Test of Mean - Minitab Input and Output



## Minitab Output

$p = 0.23 > 0.05 = \alpha$   
Accept  $H_0$

The Minitab Session window displays the following output:

T-Test of the Mean

Test of  $\mu = 50.000$  vs  $\mu \text{ not } = 50.000$

Variable	N	Mean	StDev	SE Mean	T	P
Oper 1	10	49.700	0.734	0.232	-1.29	0.23

A callout box points to the P-value of 0.23 in the output table.



## *Exercise 1*

### *Single Sample Test for a Mean Two-Sided*

---

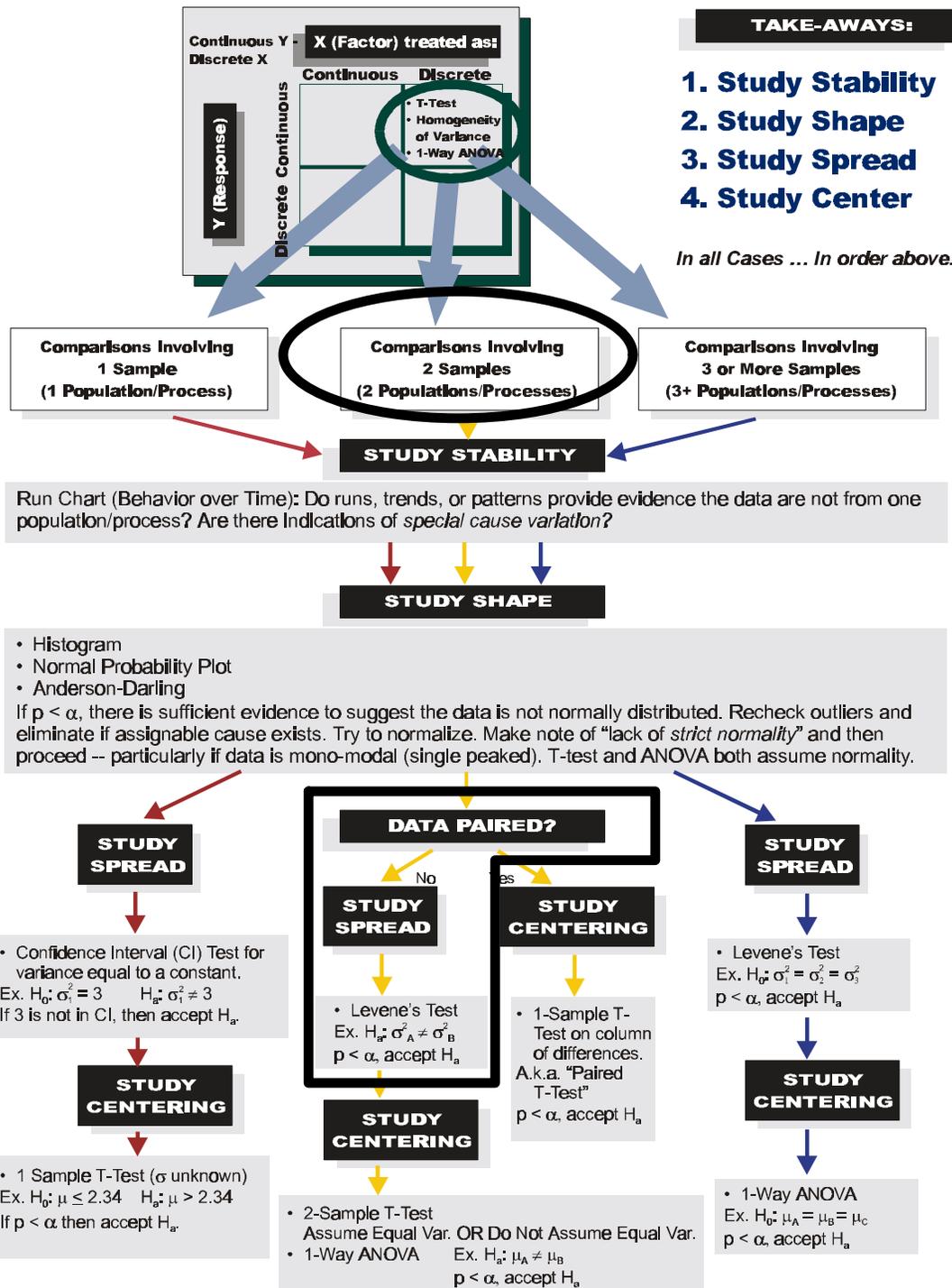
- *Use your team's catapult data. Evaluate each operator.*
- *If the target is 50 inches, is the process off center?*
- *State your hypotheses first.*

$p < \alpha$ : Reject  $H_0$

$p > \alpha$ : Accept  $H_0$



# Data Analysis



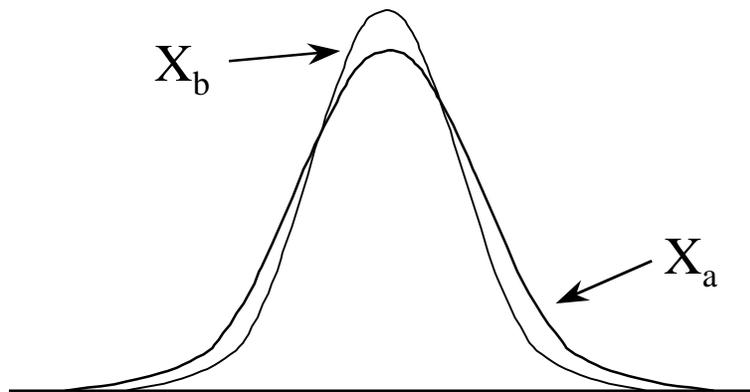


## *Test on Variances*

*Many times we want to know if we have succeeded in reducing the variation of a process or we might want to know if a change in a variable,  $X$ , changes the variation in the output,  $Y$ .*

*Knowing if the variances of two (or more) samples are different is also a prerequisite to the test on means.*

### **Are these Variances Different?**



*The hypothesis test for comparing variances is the Homogeneity of Variance Test.*



## Homogeneity of Variance Test

You want to compare the variability of operator 1 and 2. You have established that each process is stable and normally distributed. Perform the Homogeneity of Variance Test on Operator 1 and 2.

MINITAB FILE: Catapult.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the path 'ANOVA' > 'Homogeneity of Variance...' is selected. Below the menu, a data table is visible with columns C3 through C9 and rows 1 through 5. The status bar at the bottom indicates 'Perform Bartlett's and Levene's tests for homogeneity of variance'.

	C3	C4	C5	C6	C7	C8	C9
↓	Oper 1	Oper 2	Oper 3	Oper 4	Oper 5	Dist 50	Oper 50
1	50.50	50.50	46.50	49.00	50.00	50.50	1
2	50.50	49.00	50.00	50.25	49.75	50.50	1
3	49.75	51.50	49.25	50.50	49.75	49.75	1
4	49.50	50.50	48.75	49.75	50.00	49.50	1
5	49.50	47.00	49.00	50.00	48.75	49.50	1



## Test on Variances, Example - Input

**Homogeneity of Variance Test**

C1	Target
C2	Angle
C3	Oper 1
C4	Oper 2
C5	Oper 3
C6	Oper 4
C7	Oper 5
C8	Dist 50
C9	Oper 50
C10	Dist 1&2
<b>C11</b>	<b>Oper 1&amp;2</b>
C12	Dist 3&4
C13	Oper 3&4
C14	150
C15	155
C16	160
C17	165
C18	170
C19	Dist Ang

**Response:** 'Dist 1&2'

**Factors:** 'Oper 1&2'

**Confidence level:** 95.0

**Title:**

**Select** **Storage...**

**Help** **OK** **Cancel**

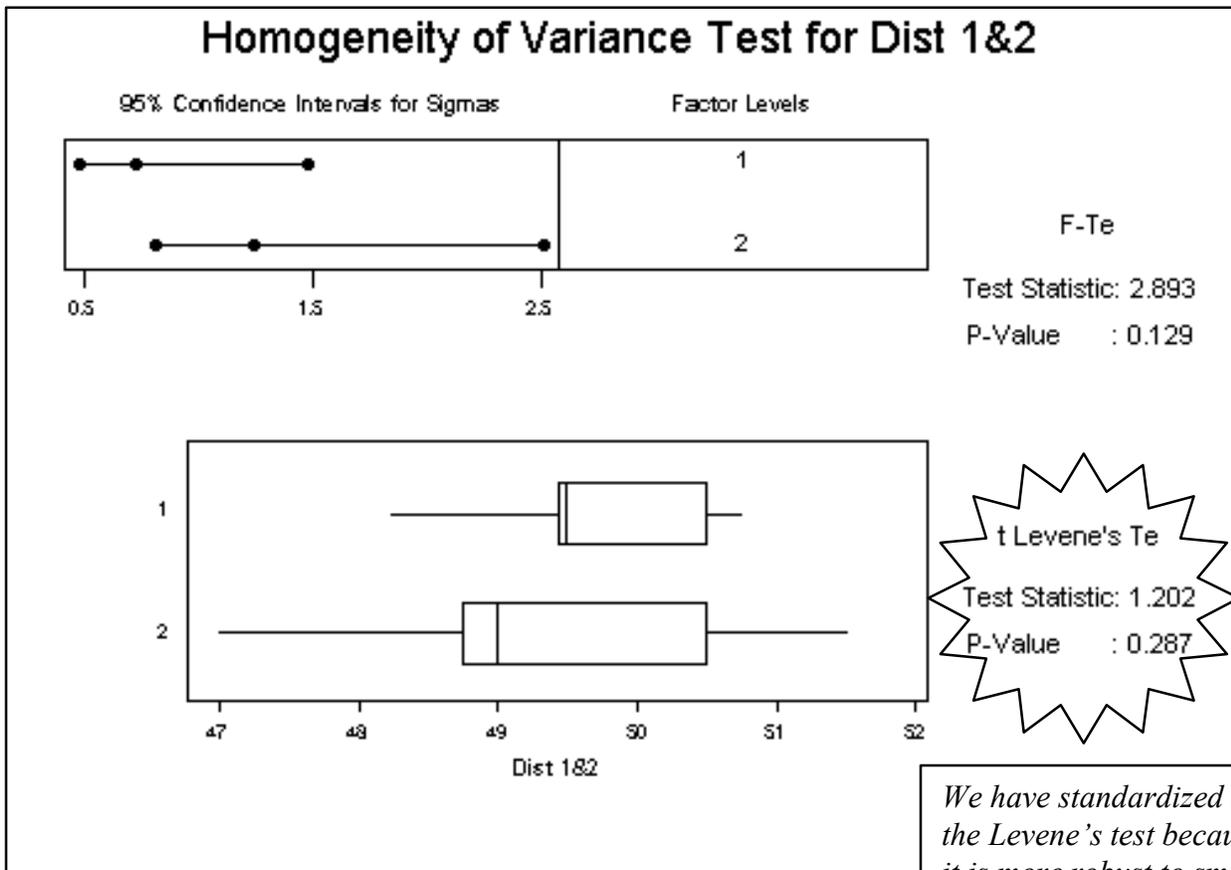
$$H_0 : \sigma_1^2 = \sigma_2^2 \quad \alpha = .05$$
$$H_a : \sigma_1^2 \neq \sigma_2^2$$



# Test on Variances, Example - Output

$$H_0: \sigma_1^2 = \sigma_2^2$$

$$\alpha = .05$$



*We have standardized on the Levene's test because it is more robust to small sample sizes and non-normality of samples.*

$p < \alpha$ : Reject  $H_0$   
 $p > \alpha$ : Accept  $H_0$



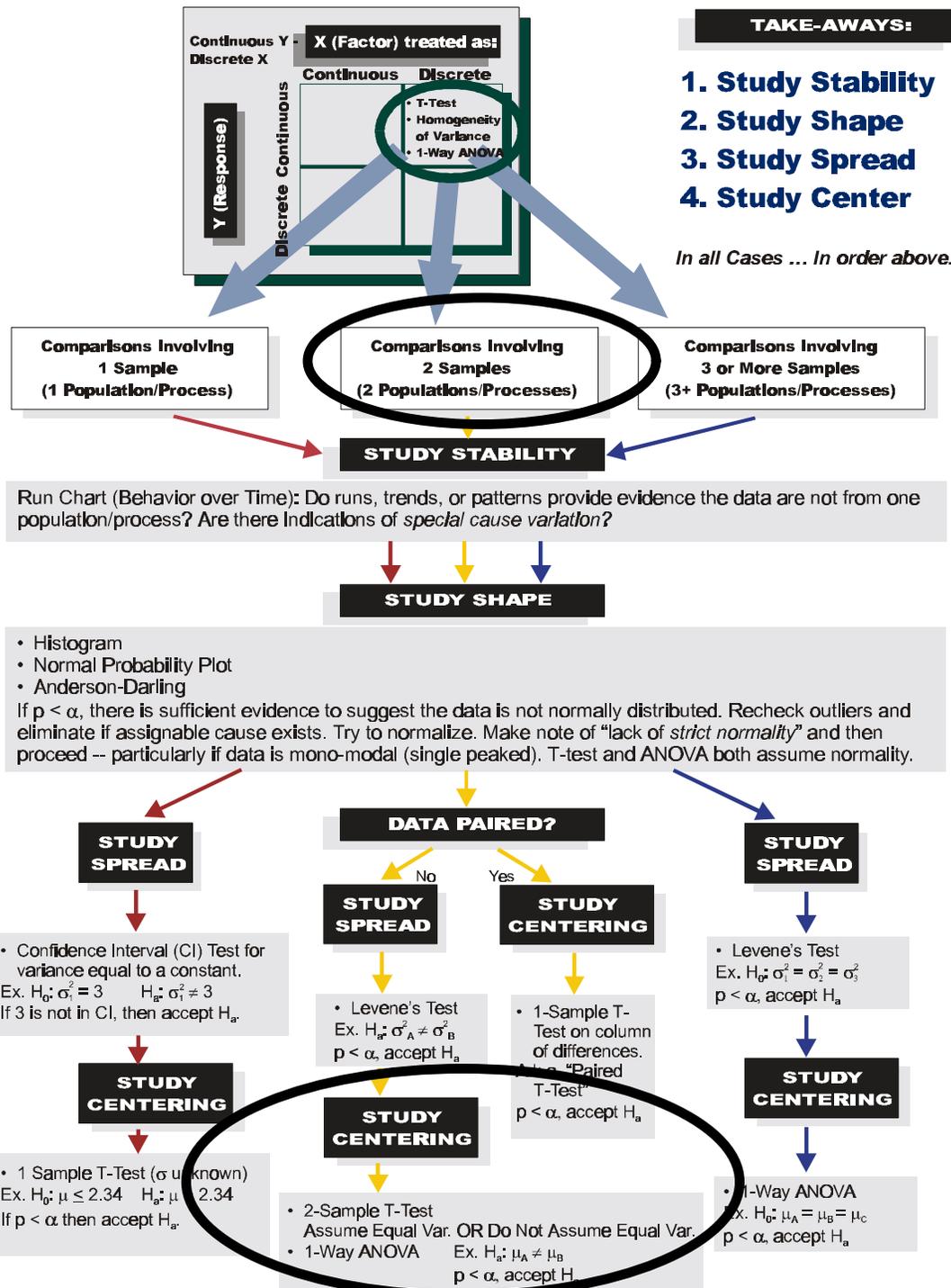
## *Exercise 2: Test on Variances*

---

- *Use your team's catapult data.*
- *Run a test to see if the variances could be the same for operator 1 and operator 2.*
- *To run this test, you need to stack the distances of operators 1 and 2. (Do not forget the subscripts.)*
- *Define your hypotheses first.*



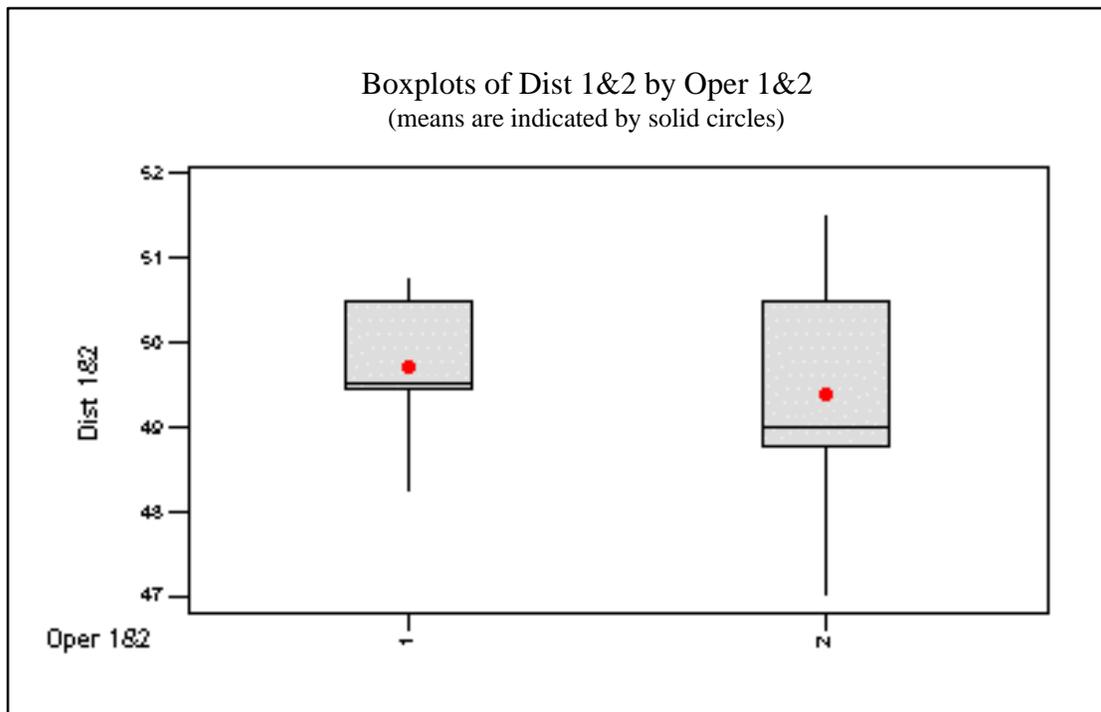
# Data Analysis





## Two-Sample t-Test: Testing for Equal Means Two-Sided

*Do we have enough evidence to say that the performance of the two operators is different?*



$$H_0 : \mu_1 = \mu_2$$

Null hypothesis

$$H_a : \mu_1 \neq \mu_2$$

Alternative hypothesis -  
what you wish to prove.



# Two-Sample t-Test in Minitab

MINITAB FILE: Catapult.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the path 'Stat > Basic Statistics > 2-Sample t...' is highlighted. The 'Session' window shows the following data:

Level	N
1	10
2	10

Pooled StDev = MTB >

The main data table is as follows:

	C3	C4	C5	C6	C7	C8	C9
↓	Oper 1	Oper 2	Oper 3	Oper 4	Oper 5	Dist 50	Oper 50
1	50.50	50.50	46.50	49.00	50.00	50.50	1
2	50.50	49.00	50.00	50.25	49.75	50.50	1
3	49.75	51.50	49.25	50.50	49.75	49.75	1
4	49.50	50.50	48.75	49.75	50.00	49.50	1
5	49.50	47.00	49.00	50.00	48.75	49.50	1

At the bottom of the window, the instruction reads: 'Perform a two-sample t-test and compute a confidence interval for the difference in means'.



## Standardization of Hypothesis Statements

The hypothesis you are trying to show should be stated in  $H_a$ .

$$H_o: \mu_{\text{oper1}} = \mu_{\text{oper2}} \quad \alpha = .05; 1-\alpha = .95$$
$$H_a: \mu_{\text{oper1}} \neq \mu_{\text{oper2}}$$

2-Sample t

Samples in one column

Samples:

Subscripts:

Samples in different columns

First:

Second:

Alternative:

Confidence level:

Assume equal variances

Select

Help

OK

Cancel

Graphs...

***This is how Minitab needs the input.***



# Minitab Output

```
Session
Two Sample T-Test and Confidence Interval

Two sample T for Oper 1 vs Oper 2

      N      Mean     StDev   SE Mean
Oper 1  10    49.700    0.734    0.23
Oper 2  10    49.38     1.25     0.39

95% CI for mu Oper 1 - mu Oper 2: ( -0.64,  1.29)
T-Test mu Oper 1 = mu Oper 2 (vs not =): T = 0.71  P = 0.49  DF = 18
Both use Pooled StDev = 1.02
```

## **Confidence Interval:**

95% C.I. contains 0.0 Accept  $H_0$

## **p-Value:**

$p = 0.49 > .05 = \alpha$  Accept  $H_0$

→  $H_0: \mu_1 = \mu_2$   
 $H_a: \mu_1 \neq \mu_2$

**Cannot conclude the operators are different.**

**Both tests result in the same conclusion!**



## *Exercise 3*

- *Using your team's data, conduct a two-sample t-test on the means for Operator 1 and 2. Can you assume equal variances?*
- *Are the means for the two operators different?*
- *State your hypotheses first.*



## Two Sample t-Test for One Mean Greater Than the Other (One-Sided)

For many problems, we want to know if the change we made caused an improvement.

For the catapult data, does the operator with the longest average distance statistically launch the projectile farther than the operator with the shortest average distance?

MINITAB FILE: Catapult.mtw

$$H_o : \mu_L \leq \mu_s \quad \alpha = .05$$

$$H_a : \mu_L > \mu_s$$

The screenshot shows the MINITAB software interface. The 'Stat' menu is open, and the path 'Stat > Basic Statistics > 2-Sample t...' is highlighted. Below the menu, a data table is visible with columns C3 through C9 and rows 1 through 5. The data values are as follows:

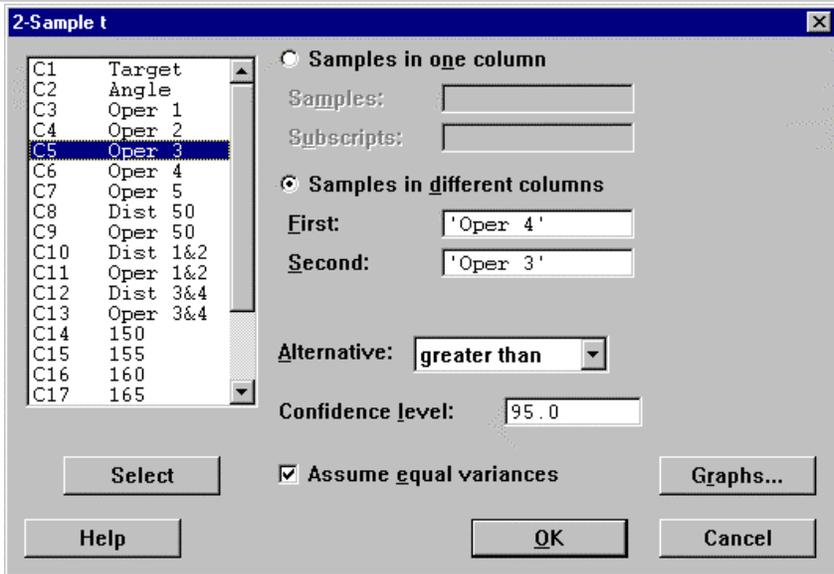
	C3	C4	C5	C6	C7	C8	C9
	Oper 1	Oper 2	Oper 3	Oper 4	Oper 5	Dist 50	Oper 50
1	50.50	50.50	46.50	49.00	50.00	50.50	1
2	50.50	49.00	50.00	50.25	49.75	50.50	1
3	49.75	51.50	49.25	50.50	49.75	49.75	1
4	49.50	50.50	48.75	49.75	50.00	49.50	1
5	49.50	47.00	49.00	50.00	48.75	49.50	1



# Two Sample t-Test - Minitab Input and Output

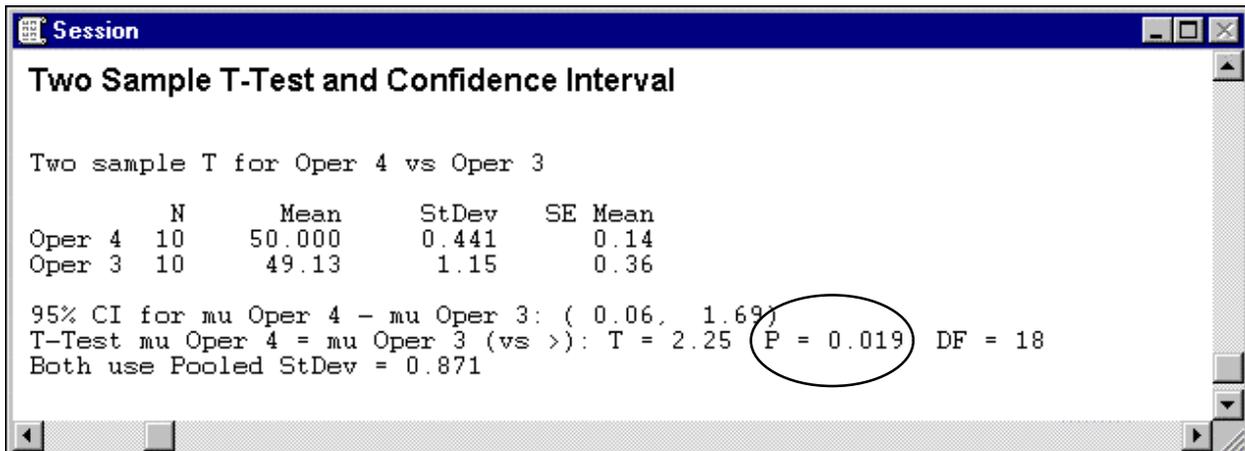
$$H_o: \mu_4 \leq \mu_3$$

$$H_a: \mu_4 > \mu_3$$



The dialog box shows the following settings:

- Method:  Samples in one column,  Samples in different columns
- First: 'Oper 4'
- Second: 'Oper 3'
- Alternative: greater than
- Confidence level: 95.0
- Assume equal variances



Two Sample T-Test and Confidence Interval

Two sample T for Oper 4 vs Oper 3

	N	Mean	StDev	SE Mean
Oper 4	10	50.000	0.441	0.14
Oper 3	10	49.13	1.15	0.36

95% CI for mu Oper 4 - mu Oper 3: ( 0.06, 1.69)

T-Test mu Oper 4 = mu Oper 3 (vs >): T = 2.25 P = 0.019 DF = 18

Both use Pooled StDev = 0.871

$p < \alpha$ : Accept  $H_a$  ←

$p > \alpha$ : Reject  $H_a$

*We have enough evidence (statistical significance) to say that the performance of Operator 4 results in a longer average distance than that of Operator 3.*



## *Exercise 4*

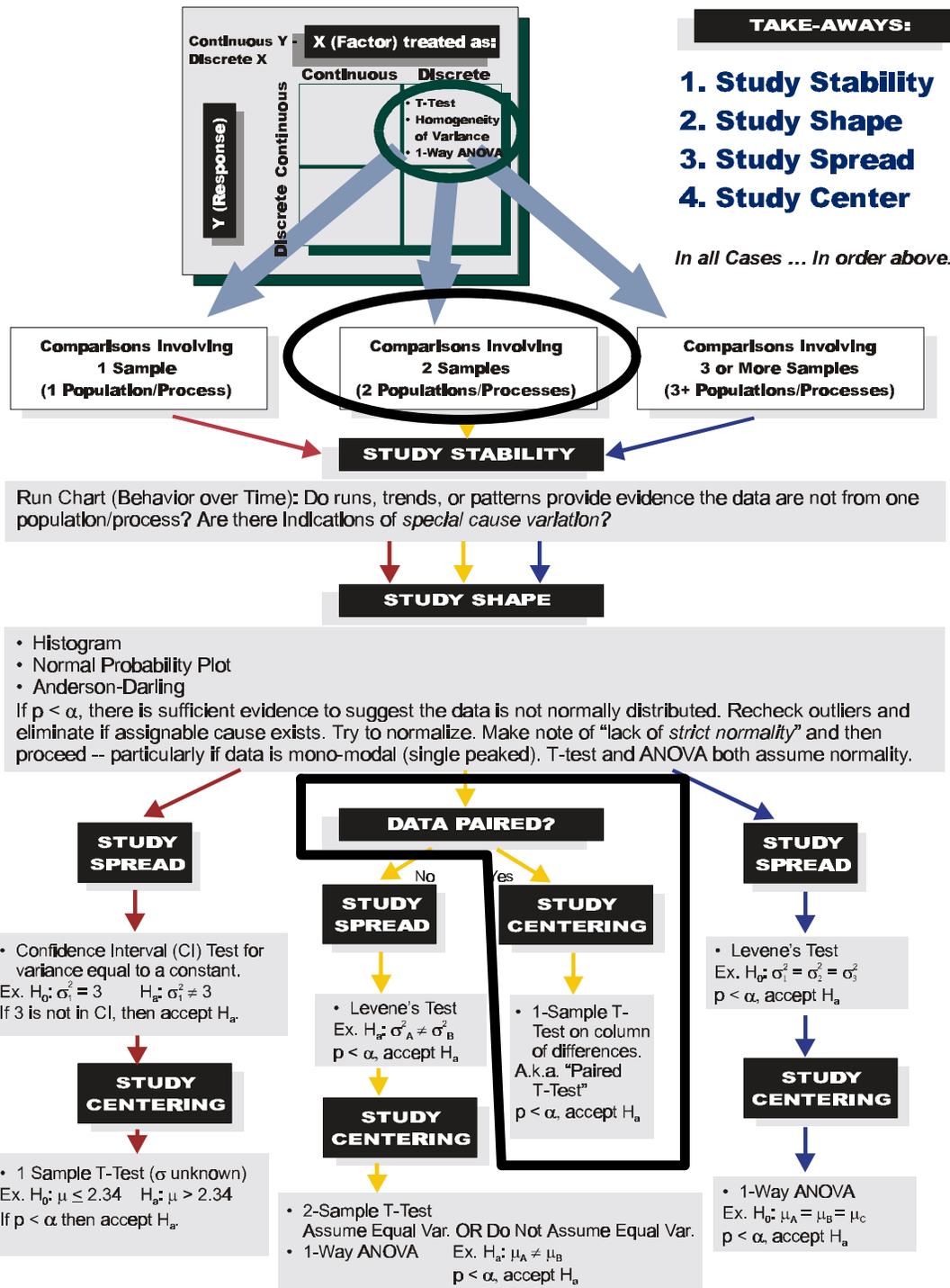
- *Using your team's catapult data, conduct a two-sample t-test on the means for your longest and shortest average distance operators. Is your data normally distributed for both samples? Can you assume equal variances?*
- *State your hypotheses first.*

$p < \alpha$ : Reject  $H_0$

$p > \alpha$ : Accept  $H_0$



# Data Analysis





## What is Paired Data?

**Y = Tensile Strength**

**X = Baking Temp.**

### Not Paired (Independent)

- 8 Sheets
- Randomly divided into 2 piles of 4

### Paired

- 4 Sheets
- Each cut into 2 halves

<u>Sheet</u>	<u>600°</u>	<u>Sheet</u>	<u>900°</u>
1	75	5	167
2	160	6	85
3	100	7	55
4	15	8	105

	<u>600°</u>	<u>900°</u>
Sheet 1	73	77
Sheet 2	161	175
Sheet 3	102	135
Sheet 4	14	25

**Better, since unwanted source of variation (sheet to sheet) is removed.**



## Paired Sample

Sometimes it is necessary to test for an effect on a population made up of non-identical specimens. For example, suppose a sparkplug manufacturer has developed a new sparkplug design and wishes to see if the new design causes a statistically significant difference in performance in several different engine designs.

A variety of automobiles are selected from among those owned by the manufacturer's employees. Unused spark plugs of the "old" design are installed in each engine, and the power of each engine is measured on a chassis dynamometer. Then unused spark plugs of the "new" design are installed and the dynamometer test is repeated.

The results are as follows:

<u>Automobile</u>	<u>Horsepower at 4000 rpm</u>		<u>Horsepower Difference, <math>d_i</math></u>
	<u>"Old" Plug Design</u>	<u>"New" Plug Design</u>	
Acura Integra (4 cylinder)	75	81	6
Chrysler Minivan (8 cylinder)	159	166	7
Ford F-150 Pickup (6 cylinder)	115	116	1
Pontiac Bonneville (8 cylinder)	191	201	10
Volvo 850 (5 cylinder)	123	119	-4
VW Passat (4 cylinder)	90	88	-2



## Paired Sample t-Test (cont.)

Clearly a two-sample t-test cannot be used to compare the difference between the “old” and “new” sample means directly, because the differences among engines is much greater than the performance change that may be due to sparkplug design.

The variation among engines can be approximately eliminated by analyzing the difference in performance for each engine. Create a new statistic  $d_i = \text{Horsepower}_{\text{new}} - \text{Horsepower}_{\text{old}}$  for each automobile. Now a one-sample t-test can be performed to test the alternative hypothesis that the plug design, on average, changes the horsepower.

$H_o$  : Plug design does not make a difference

$H_a$  : Plug design makes a difference

$$\bar{D} = \sum_{i=1}^n d_i / n$$
$$s_d = \sqrt{\frac{\sum_{i=1}^n (d_i - \bar{D})^2}{n-1}}$$

$$\text{Acceptance Region: } \bar{D} - t_{\alpha/2, n-1} \frac{s_d}{\sqrt{n}} < T < \bar{D} + t_{\alpha/2, n-1} \frac{s_d}{\sqrt{n}}$$



## Hand Calculation

*In the example used to develop the concept of the paired-sample test on the previous pages, did the new sparkplug design significantly affect engine performance at the 0.05 level of significance?*

$H_o$  : Plug design does not make a difference

$H_a$  : Plug design makes a difference

$$\bar{D} = \sum_{i=1}^n d_i / n = 3.00$$

$$\alpha = 0.05$$

$$s_d = \sqrt{\frac{\sum_{i=1}^n (d_i - \bar{D})^2}{n-1}} = 5.51$$

$$t_{\alpha/2, n-1} = 2.57$$

Acceptance region:  $-2.79 < T < 8.79$

*If there was no difference between the old and new plugs, you would expect there to be no difference between the two populations.*

*Since the value, 0, falls within the acceptance region, we conclude that the new spark plugs do not result in a significant difference in engine performance.*



## *Minitab Calculation Paired T-test*

MINITAB FILE: (Create Your Own)

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and 'Basic Statistics' is selected, leading to the '1-Sample t...' option. The worksheet below has the following data:

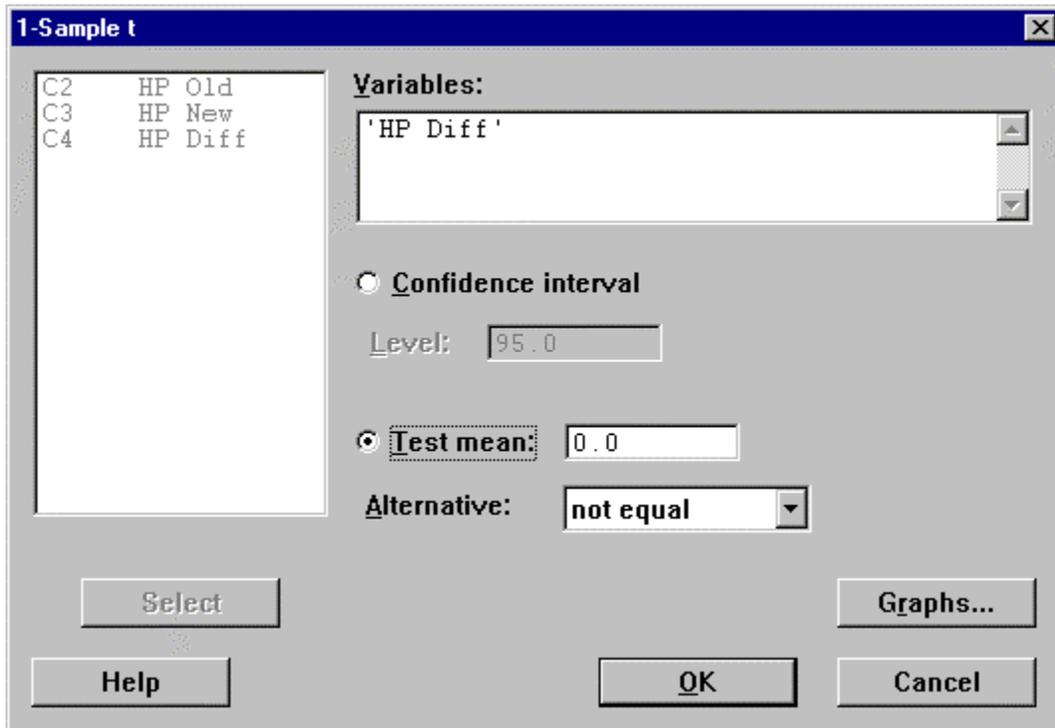
	C1-T	C2	C3	C4	C5	C6	C7
↓	Car	HP Old	HP New	HP Diff			
3	Ford P/U	115	116	1			
4	Pontiac	191	201	10			
5	Volvo	123	119	-4			
6	VW	90	88	-2			
7							

Perform a one-sample t-test and compute a confidence interval

$H_0$ : Plug design does not make a difference  
 $H_a$ : Plug design makes a difference



# Minitab Input & Output



Variable	N	Mean	StDev	SE Mean	T
HP Diff	6	3.00	5.51	2.25	1.33

P-Value  
0.24

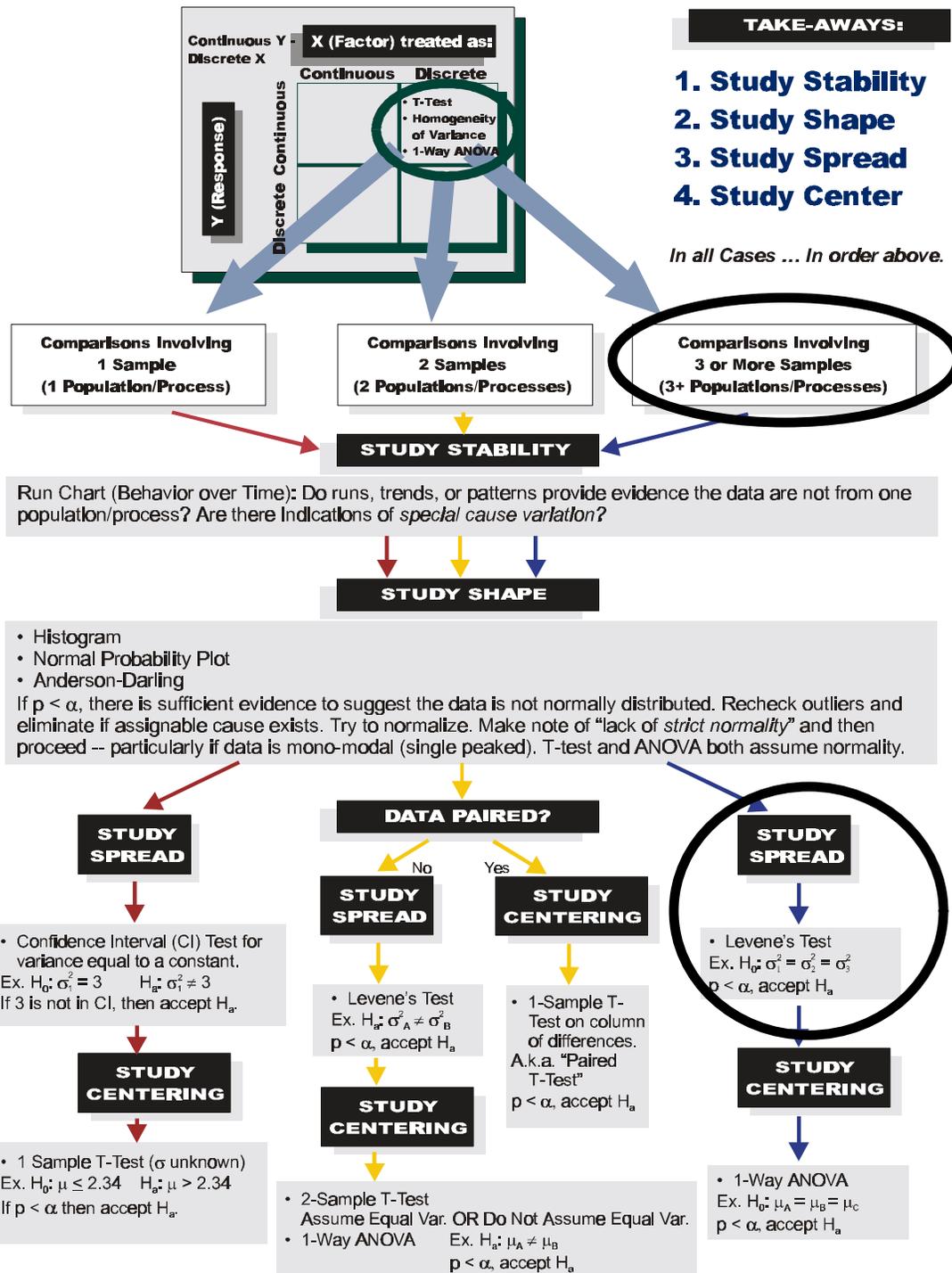
**$\alpha = 0.05$**

$p < \alpha$ : Reject  $H_0$

$p > \alpha$ : Accept  $H_0$



# Data Analysis





## *Test on Variances Example*

*You want to compare the performance of all the operators. You have established that each process is stable and normally distributed. Before you compare their means you must determine if their variances are equal.*

$$H_0: \sigma_1^2 = \sigma_2^2 \dots = \sigma_5^2 \quad \alpha = .05$$

$H_a$ : At least one  $\sigma^2$  is different



# Variances, Minitab Menu Commands

MINITAB FILE: Catapult.mtw

The screenshot shows the Minitab interface with the following data in the worksheet:

	C3	C4	C5	C6	C7	C8	C9
↓	Oper 1	Oper 2	Oper 3	Oper 4	Oper 5	Dist 50	Oper 50
1	50.50	50.50	46.50	49.00	50.00	50.50	1
2	50.50	49.00	50.00	50.25	49.75	50.50	1
3	49.75	51.50	49.25	50.50	49.75	49.75	1
4	49.50	50.50	48.75	49.75	50.00	49.50	1
5	49.50	47.00	49.00	50.00	48.75	49.50	1

Stat > ANOVA > Homogeneity of Variance...

Perform Bartlett's and Levene's tests for homogeneity of variance



## Variations, Minitab Input

Homogeneity of Variance Test

Response: 'Dist 50'

Factors: 'Oper 50'

Confidence level: 95.0

Title:

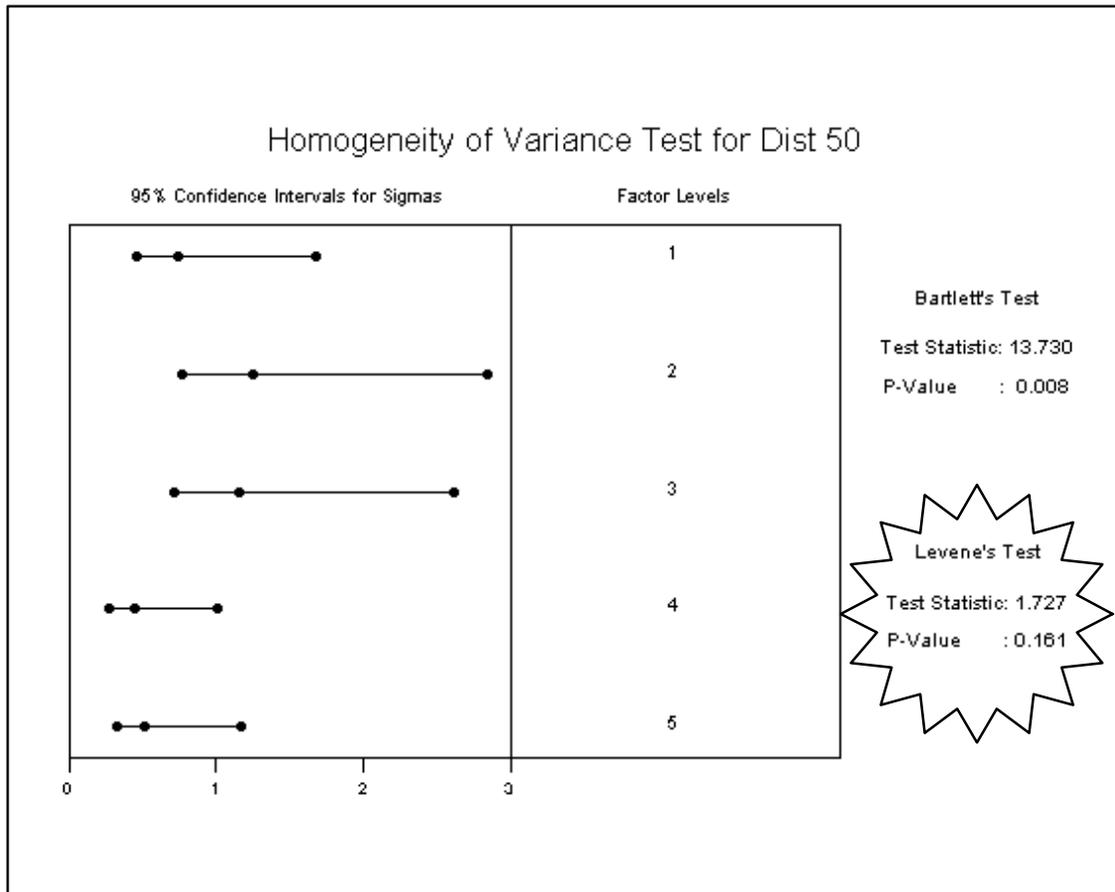
Select Storage... Help OK Cancel

$H_o$ : All variances  $s_i^2$ ,  $i = 1, 2, \dots, 5$  are the same.

$H_a$ : At least one variance,  $s_i^2$ , is different from the rest of the group.



## Variations, Minitab Output



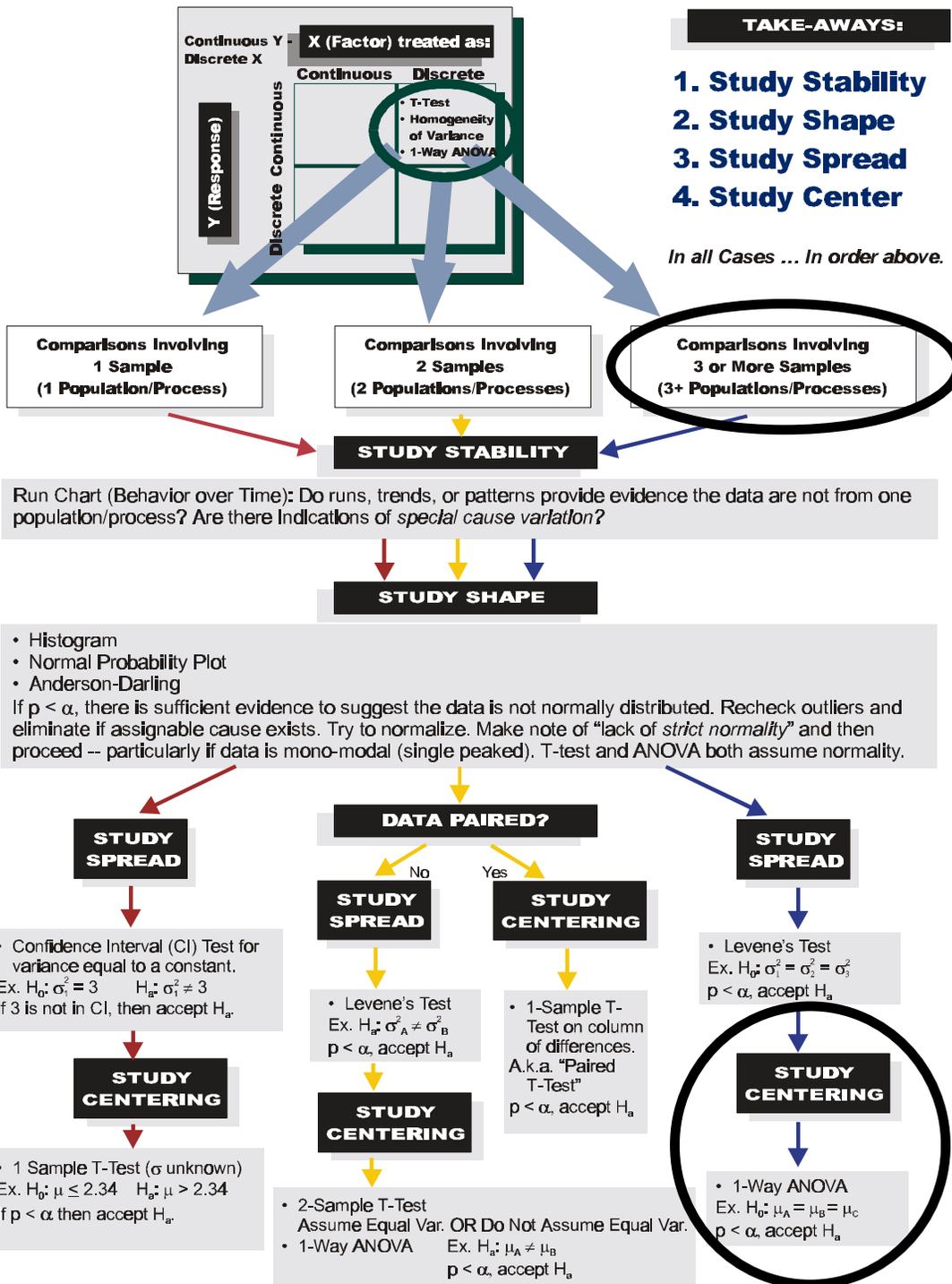
$p < \alpha$ : Reject  $H_0$

$p > \alpha$ : Accept  $H_0$

*We do not have enough evidence to accept that any one operator's variance is significantly different from the others.*



# Data Analysis





# *One-Way Analysis-of-Variance (ANOVA)*

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

$H_a$ : At least one  $\mu$  different from the others

*When comparing two or more means, we could use a t-test and compare all possible pairs. Suppose we wanted to compare four means. This would require six different t-tests. For each test we would have a a (5%) chance of a Type I error and would not know the overall risk.*

*The one-Way ANOVA enables us to control the overall risk of a Type I error and leads to a single test statistic for comparing all the means.*

***ANOVA is a Test to Compare Averages***

*(The name is misleading!)*



## *One-Way Analysis-of-Variance*

---

*One-way analysis-of-variance, or one-way ANOVA as it is often referred to, is a statistical method used to test the relationship between a given dependent variable and a single independent variable classified into two or more groups. To be specific, this procedure will ascertain whether or not the response means associated with the groups are drawn from the same population.*

*The analysis involves arithmetically decomposing the total observed variation into two components. One component represents the response variation strictly attributable to the independent variable, while the other represents residual variation, or background noise as it is sometimes called.*

*With these estimates of variability, the one-way procedure evaluates the probability of equal component variances. If the probability exceeds a given threshold value, the alternative hypothesis of statistically significant difference (in component variances) would be accepted. Under this condition, it would be concluded that the observed variation in group means did not result from chance sampling variations.*



## One-Way ANOVA Example

Is there any indication that the averages of the five catapult operators are different? Which operators are different?

$$H_o: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

$H_a$ : at least one  $\mu$  different from the others

MINITAB FILE: Catapult.mtw

The screenshot shows the MINITAB software interface. The 'Stat' menu is open, and the path 'ANOVA' > 'One-way...' is selected. The worksheet 'Catapult.mtw' is open, showing a table with columns C17 through C23 and rows 1 through 5. The status bar at the bottom indicates 'Perform a one-way analysis of variance on stacked data'.

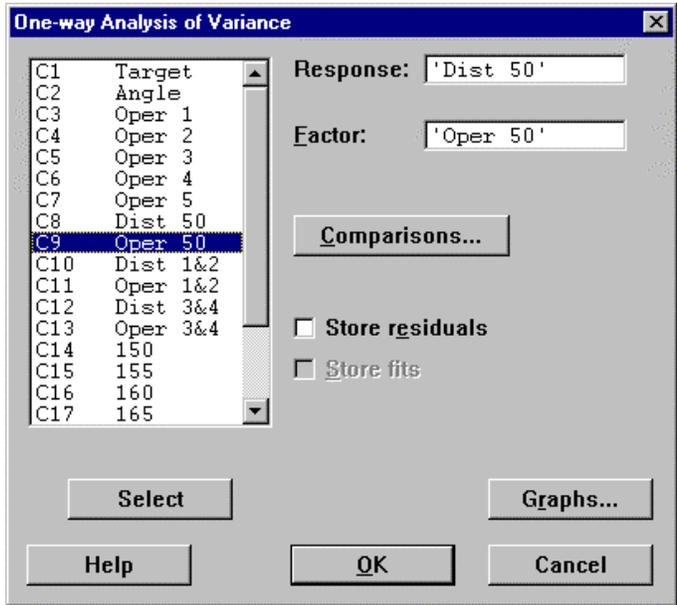
	C17	C18	C19	C20	C21	C22	C23
↓	165	170	Dist Ang	C Angle	D150&170	150&170	
1	53.00	62.25	26.00	150	26.00	1	
2	52.50	62.50	26.00	150	26.00	1	
3	53.00	62.00	26.75	150	26.75	1	
4	53.75	63.25	27.50	150	27.50	1	
5	53.25	63.00	27.00	150	27.00	1	



# One-Way ANOVA - Minitab Results

1. Double Click on Dist 50

2. Double Click on Oper50



3. Click OK

$p > \alpha \therefore$  accept  $H_0$   
There is no difference in means.

## One-way Analysis of Variance

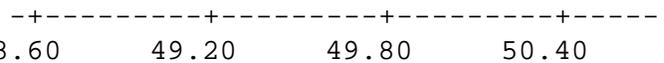
Analysis of Variance for Dist 50

Source	DF	SS	MS
Oper 50	4	5.487	1.372
Error	45	34.919	0.776
Total	49	40.406	

F	P
1.77	0.152

Individual 95% CIs For Mean  
Based on Pooled StDev

Level	N	Mean	StDev
1	10	49.700	0.734
2	10	49.375	1.249
3	10	49.125	1.150
4	10	50.000	0.441
5	10	49.925	0.514



Pooled StDev = 0.881



## *Exercise 6: One-Way ANOVA*

---

- *Use your team's catapult data.*
- *Is there an indication that any one operator's average distance is different from the others?*
  - *1. State your hypotheses*
  - *2. Do a One-Way ANOVA*
  - *3. What can you conclude from the results?*



---

# *Understanding Sample Size and Risk*



## Continuous Data Sample Size Formula

Using the Confidence Interval Formula for Continuous Data to Derive Sample Size Formula

$$CI_{\bar{X}} \longrightarrow \bar{X} \pm Z_{\alpha/2} \cdot \underbrace{\frac{\sigma}{\sqrt{n}}}_{\text{Standard error}}$$

$$\text{Given} \longrightarrow \bar{X} \pm \Delta$$

$$\text{Solve for } n \longrightarrow \Delta = Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$\sqrt{n} = \frac{Z_{\alpha/2} \cdot \sigma}{\Delta}$$

$$n = \left( \frac{Z_{\alpha/2} \cdot \sigma}{\Delta} \right)^2$$



## *How To Estimate $s$ When It Is Unknown*

### *3 Ways To Estimate $s$*

#### **■ Use an existing $\bar{X}$ or $R$ chart**

- $\hat{s} = \bar{R}/d_2$  where  $d_2$  is control chart factor
- $\hat{s} = (UCL - \bar{X})/3$
- Collect a small pre-sample & calculate  $s$  ( $n = 30$ )
- Ask subject matter experts to take an educated guess at the plausible range of data
  - $\hat{s} = (\text{Highest known value} - \text{Lowest known value})/6$



## Ways To Determine $D$

- Use business knowledge
- Take the  $D$  from the sample size you can afford; for example with 95% confidence

$$\Delta = \left( \frac{1.96 (\sigma)}{\sqrt{n}} \right) \text{ if } n = 15 \text{ \& } s = 3$$

$$\Delta = \left( \frac{1.96 \times 3}{3.87} \right) = 1.51$$

- Consider the minimum resolution of measurement equipment. For example, don't set  $D$  to minutes if you can only measure hours



## *Exercises With The Formulas*

### **Assume Sample Size Is Less Than 5% Of The Population Size**

#### **Instructions**

*In your table team, answer the assigned questions and be prepared to report out your answers. Use Minitab when possible. Assume confidence level = 95%*

- 1. We want to estimate the average cycle time within 2 days. A preliminary estimate of the population standard deviation is 8 days. How many observations should we take?*
- 2. We only can get our hands on 36 observations. How precisely can we estimate average performance, in terms of the population standard deviations?*

#### **Time**

*15 minutes*



# Error Review

**Type I error:** Reject  $H_0$  when  $H_0$  is true

**a** = The probability of making a Type I error

$1 - \mathbf{a}$  = The probability of accepting  $H_0$  when  $H_0$  is true

**Type II error:** Accept  $H_0$  when  $H_a$  is true

**b** = The probability of making a Type II error

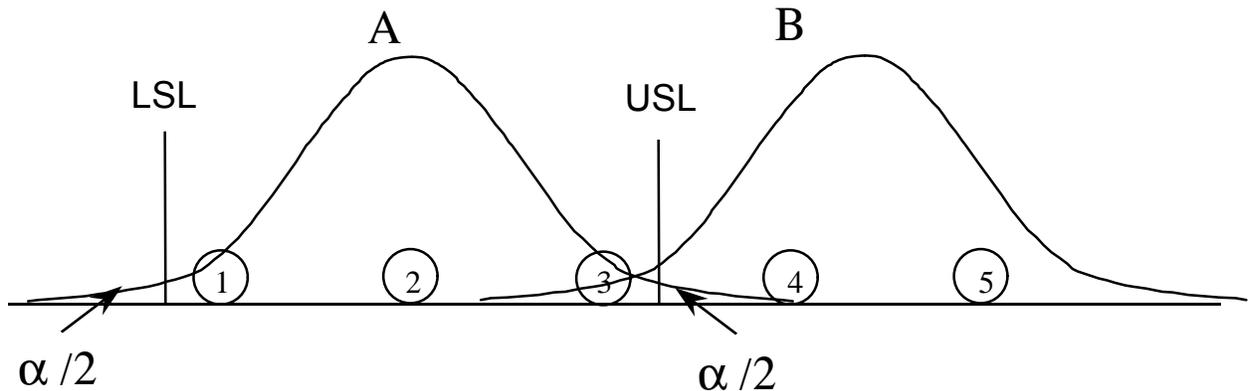
**Power** =  $1 - \mathbf{b}$  = The probability of accepting  $H_a$  when  $H_a$  is true

		<b>Truth</b>	
		$H_0$	$H_a$
<b>Verdict</b>	$H_0$ Set Free	Innocent, Set Free	Guilty, Set Free
	$H_a$ Jailed	Innocent, Jailed	Guilty, Jailed

		<b>Truth</b>	
		$H_0$	$H_a$
<b>Accept</b>	$H_0$	Correct Decision	Type II Error <b>b</b>
	$H_a$	Type I Error <b>a</b>	Correct Decision



## Making Decisions



Which distribution (A or B) did these samples come from?

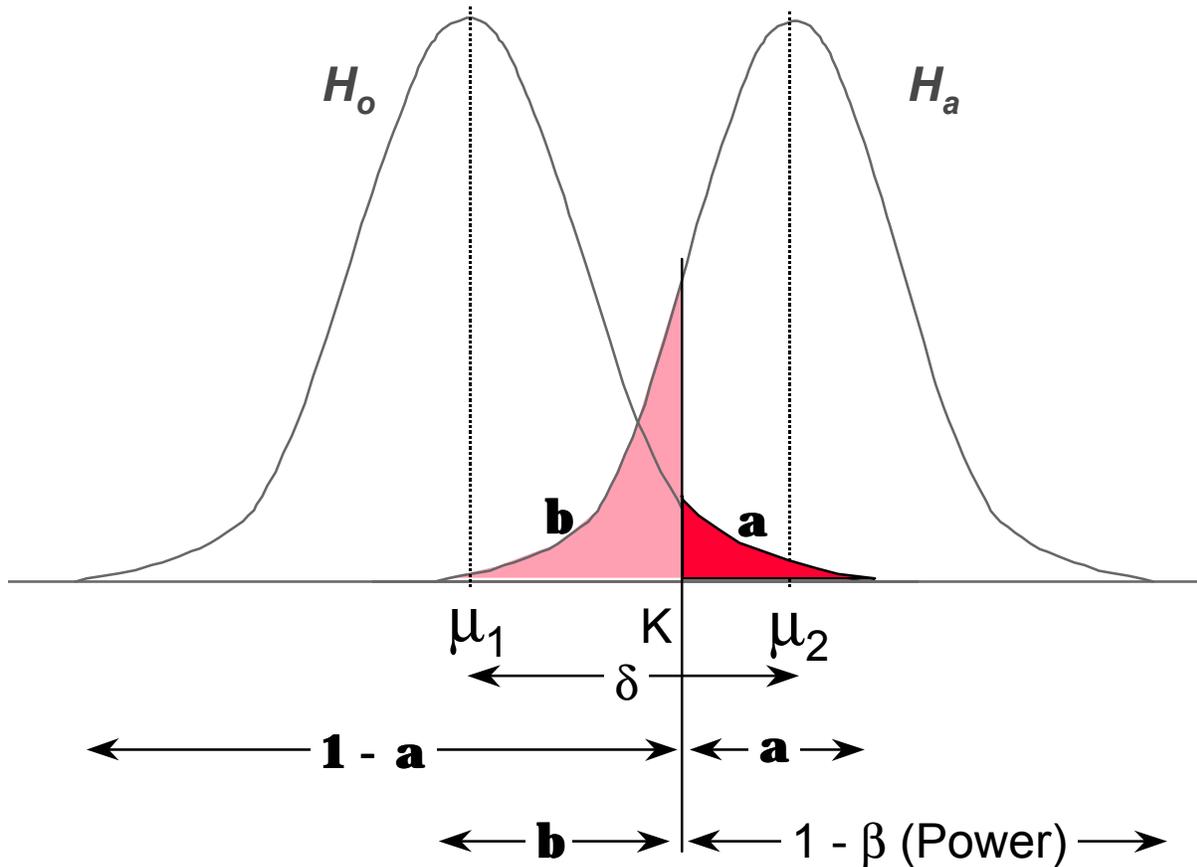
<u>Sample</u>	<u>Distribution</u>
1	_____
2	_____
3	_____
4	_____
5	_____

Could sample 3 have come from distribution B? If it did, we just made a Type II error. We missed an opportunity.

How can we improve our decision making capability?



# Power of a Test



		Truth	
		$H_0$	$H_a$
Accept	$H_0$	Correct Decision	Type II Error <b>b</b>
	$H_a$	Type I Error <b>a</b>	Correct Decision

**Power = f(d, s, n, a) = 1 - b**

$\delta$	↑	power	↑
$\sigma$	↓	power	↑
$n$	↑	power	↑
$\alpha$	↑	power	↑



# Sample Size for the One Sample t-Test

MINITAB FILE: (Create Your Own)

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the path 'Stat > Power and Sample Size > 1-Sample t...' is highlighted. The 'Worksheet size:' field in the Session window is empty. The main worksheet area shows a grid with columns C1 through C9 and rows 1 through 8. The status bar at the bottom reads 'Calculate the power and sample size for a one-sample t-test'.



## Sample Size for the One Sample t-Test

**Power and Sample Size for 1-Sample t** [X]

Calculate power for each sample size  
Sample sizes:   
Difference:

Calculate power for each difference  
Differences:   
Sample size:

Calculate sample size for each power value  
Power values:   
Difference:

Sigma:

Options...

Help OK Cancel

1. Enter various power values

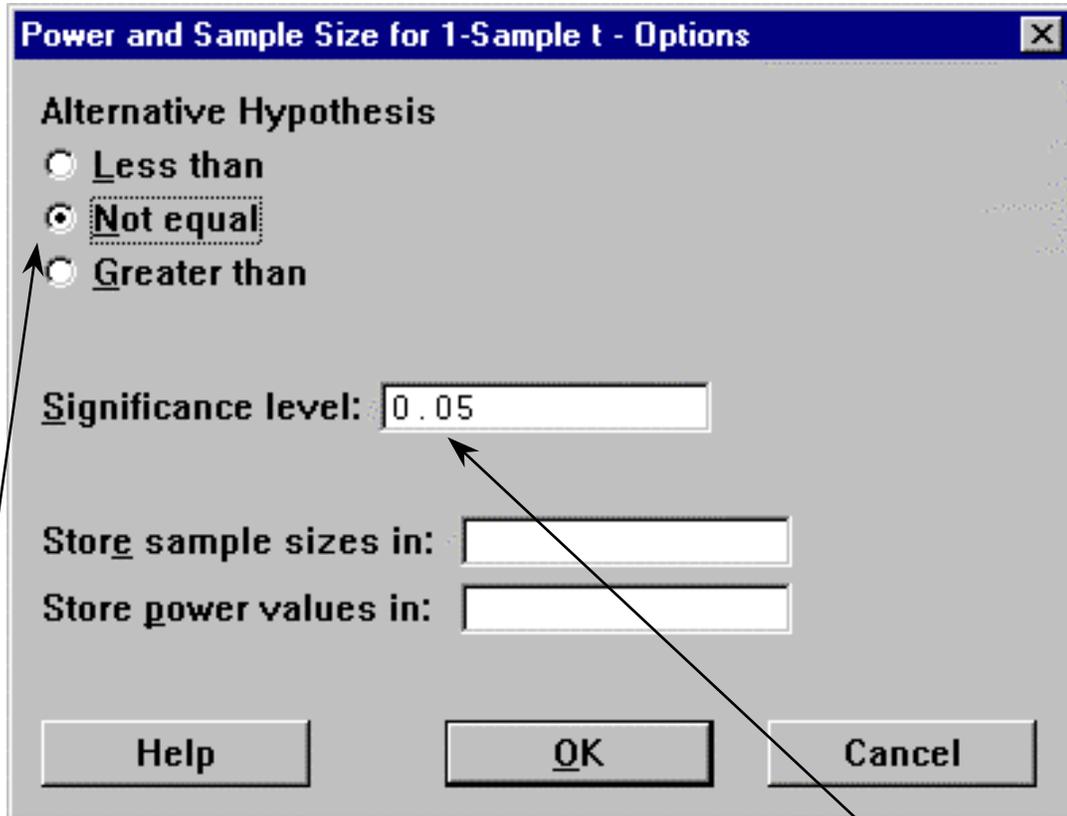
2. Enter standard deviation.

3. Enter difference to be detected, i.e. difference between target and sample mean.

4. Select options.



## Sample Size for the One Sample t-Test



5. Select alternative hypothesis.

7. Select  
OK

6. Select alpha  
(usually .05).



# Sample Size for the One Sample t-test

MINITAB - Untitled - [Session]

File Edit Manip Calc Stat Graph Editor Window Help Six Sigma

Power and Sample Size

1-Sample t Test

Testing mean = null (versus not = null)  
Calculating power for mean = null + 0.5  
Alpha = 0.05 Sigma = 1

Sample Size	Target Power	Actual Power
18	0.5000	0.5164
22	0.6000	0.6092
27	0.7000	0.7058
34	0.8000	0.8078
44	0.9000	0.9000
54	0.9500	0.9502

Undo or redo the last command

***For a one sample t-test, 44 samples are needed for 90% power.***



## *Example: Sample Size for the Two Sample t-test*

*Given the problem of trying to determine if the average distance of operator 1 is different than operator 2, what is the power of the test? (use the data from Example 3)*

$$\text{difference} = 49.7 - 49.37 = .33$$

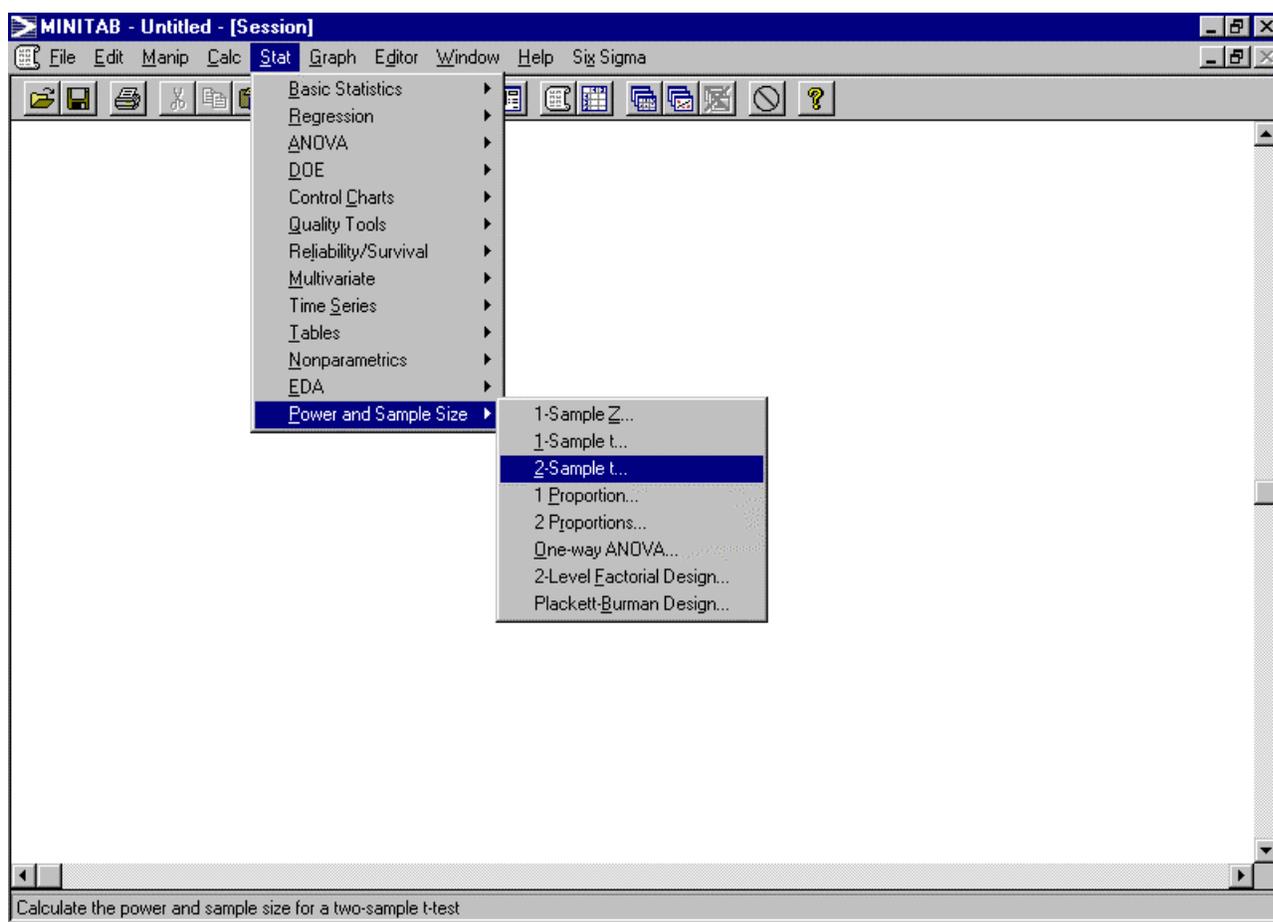
$$s_p = 1.02$$

$$n = 10$$



# Example: Sample Size for the Two Sample t-Test

MINITAB FILE: (Create Your Own)





# Example: Sample Size for the Two Sample t-Test

Power and Sample Size for 2-Sample t

Calculate power for each sample size  
Sample sizes: 10  
Difference: .33

Calculate power for each difference  
Differences:   
Sample size:

Calculate sample size for each power value  
Power values:   
Difference:

Sigma: 1.02

Options...

Help OK Cancel

Power and Sample Size for 2-Sample t - Options

Alternative Hypothesis

Less than  
 Not equal  
 Greater than

Significance level: 0.05

Store sample sizes in:

Store power values in:

Help OK Cancel



# Example: Sample Size for the Two Sample t-Test

```
MINITAB - Untitled - [Session]
File Edit Manip Calc Stat Graph Editor Window Help Six Sigma
Power and Sample Size
2-Sample t Test
Testing mean 1 = mean 2 (versus not =)
Calculating power for mean 1 = mean 2 + 0.33
Alpha = 0.05 Sigma = 1.02
Sample
  Size  Power
   10  0.1053
```

*The power of the test is quite poor - 10.5%*



## *Exercise 7*

---

- *Suppose that you are trying to show that  $H_a: \mu_1 > \mu_2$  and are interested in detecting a difference between samples of 2 inches or more.*
  - *What would your sample size need to be to have a test with 90% power?*
  - *You also believe that the pooled standard deviation is approximately 1 inch. Use  $\alpha = .05$*



## Summary of Hypothesis Tests

### ■ Run Charts

- $H_o$ : There are no trends, cluster, mixtures, or oscillations
- $H_a$ : There exist trends, clusters, mixtures, or oscillations

### ■ Normality Test

- $H_o$ : Data is normally distributed
- $H_a$ : Data is not normally distributed

### ■ Homogeneity of Variance

- $H_o: \sigma_1^2 = \sigma_2^2$                        $H_a: \sigma_1^2 \neq \sigma_2^2$

### ■ 1-sample t-test

- $H_o: \mu = 50$                                $H_a: \mu \neq 50$

### ■ 2-sample t-test

- $H_o: \mu_1 = \mu_2$                                $H_a: \mu_1 \neq \mu_2$

### ■ ANOVA

- $H_o: \mu_1 = \mu_2 = \mu_3 = \mu_4$                $H_a$ : At least one  $\mu$  is different from the others.



## *Take Aways—Hypothesis Testing Continuous Y; Discrete X*

- *The four basic steps in analyzing your data:*
  - *stability—looks for trends, clusters, oscillations, or mixtures (run chart)*
  - *shape—determines whether the data is normally distributed (Anderson-Darling Test)*
  - *spread—determines the variation of the process (Confidence Interval, Levene's Test)*
  - *centering—determines the mean of the process (1 or 2 Sample T-test, 1-Way ANOVA)*
  
- *95% confidence gives a range for accepting  $H_0$ .*
  - *If  $H_0 : \mu = \text{constant} = T$ , and the target ( $T$ ) is not included within the confidence range, it can be concluded that the process is off target.*



## *Take Aways—Hypothesis Testing Continuous Y; Discrete X*

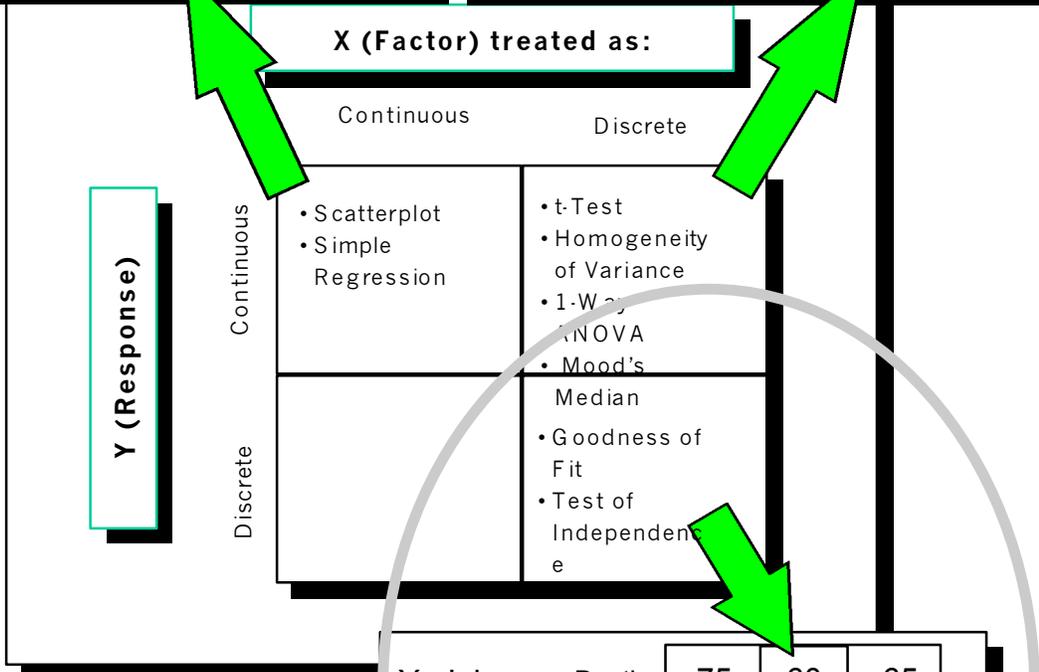
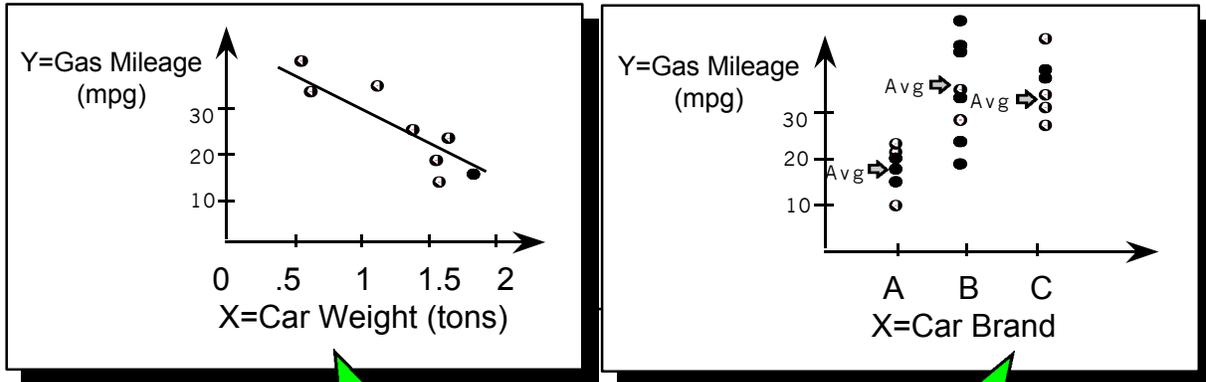
- *The p-value is the probability that you will be wrong if you select the alternative hypothesis. .*
  - *The p-value is the most common decision criteria that will be used in our analyses*
- *The power of a test, 1-Beta, is the probability of rejecting  $H_a$  when  $H_a$  is true.*
  - *the probability of correctly detecting that a change has occurred*



# *Hypothesis Testing: Discrete $Y$ ; Discrete $X$*



# Discrete Y; Discrete X



Y= Injury Severity

Death	75	60	65
Major	160	115	175
Minor	100	65	135
None	15	10	25
	A	B	C

X=Car Brand

Developed by W. Scott Lasater - GEMIS



## *Chi-Square Tests*

### The Chi-Square Tests

Used for:

- 1 - Goodness-of-Fit Test: To test if an observed set of data fits a model (an expected set of data)
- 2 - Test of Independence: To test hypothesis of several proportions (contingency table)

**It's for discrete data, any number of categories**

For all cases,  $H_0$ : no difference in data  
 $H_a$ : difference exists

$\chi^2$



## The Nature and Use of Chi-Square

**Suppose we flip a coin  $N = 100$  times and observe 63 heads and 37 tails. Could this ratio of heads to tails occur by chance or should we conclude the coin is somehow biased ?**

$$\chi^2 = \sum_{j=1}^g \frac{(f_o - f_e)^2}{f_e}$$

$g$  = number of categories

$f_o$  = observed frequency

$f_e$  = expected frequency



	Observed ( $f_o$ )	Expected ( $f_e$ )	$\frac{(f_o - f_e)^2}{f_e}$
Heads	63	50	3.38
Tails	37	50	3.38

$$\chi^2 = 6.76$$



## *The Goodness-of-Fit Test*

*To determine whether a sample comes from a population with a known distribution, calculate chi-square as shown on the previous page.*

*$H_0$  = good fit (observed distribution is consistent with random outcomes from the expected model)*

*$H_a$  = sampled population does not have the expected distribution*

*If the observed frequencies are close to the expected frequencies, chi-square is small. So a low chi-square leads to rejecting  $H_a$ .*

***If chi-square is greater than a critical value, the null hypothesis is rejected.***

*The critical values of chi-square for various confidence levels are given in the table, “Chi-Square Distribution.”*

*Each of the expected frequencies must be at least 5 for this test to be valid.*



## *Degrees of Freedom*

---

*The chi-square distribution has “degrees of freedom” as a parameter.*

*For a goodness-of-fit test, the number of degrees of freedom is  $k - 1$ , where  $k$  is the number of possible outcomes.*

*Thus for the penny example,*

$$\text{dof} = 2 - 1 = 1$$



## *The Statistical Test*

---

*If the chi-square statistic is less than the critical value in the table, the null hypothesis (good fit) is accepted. If not, the alternative hypothesis (that the data are not typical of the expected distribution) is accepted. For the penny example:*

*Critical value ( $\alpha = 0.05$ , dof = 1) = 3.841*

*Since the statistic from the data, 6.76, is greater than the critical value, the alternative hypothesis is accepted: the penny tested had a bias.*



# Chi-Square Distribution

df	.250	.100	.050	.025	.010	.005	.001	Alpha
1	1.323	2.706	3.841	5.024	6.635	7.879	10.828	
2	2.773	4.605	5.991	7.378	9.210	10.597	13.816	
3	4.108	6.251	7.815	9.348	11.345	12.838	16.266	
4	5.385	7.779	9.488	11.143	13.277	14.860	18.467	
5	6.626	9.236	11.070	12.832	15.086	16.750	20.515	
6	7.841	10.645	12.592	14.449	16.812	18.548	22.458	
7	9.037	12.017	14.067	16.013	18.475	20.278	24.322	
8	10.219	13.362	15.507	17.535	20.090	21.955	26.125	
9	11.389	14.684	16.919	19.023	21.666	23.589	27.877	
10	12.549	15.987	18.307	20.483	23.209	25.188	29.588	
11	13.701	17.275	19.675	21.920	24.725	26.757	31.264	
12	14.845	18.549	21.026	23.337	26.217	28.300	32.909	
13	15.984	19.812	22.362	24.736	27.688	29.819	34.528	
14	17.117	21.064	23.685	26.119	29.141	31.319	36.123	
15	18.245	22.307	24.996	27.488	30.578	32.801	37.697	
16	19.369	23.542	26.296	28.845	32.000	34.267	39.252	
17	20.489	24.769	27.587	30.191	33.409	35.718	40.790	
18	21.605	25.989	28.869	31.526	34.805	37.156	43.312	
19	22.718	27.204	30.144	32.852	36.191	38.582	43.820	
20	23.828	28.412	31.410	34.170	37.566	39.997	45.315	
21	24.935	29.615	32.671	35.479	38.932	41.401	46.797	
22	26.039	30.813	33.924	36.781	40.289	42.796	48.268	
23	27.141	32.007	35.172	38.076	41.638	44.181	49.728	
24	28.241	33.196	36.415	39.364	42.980	45.558	51.179	
25	29.339	34.382	37.652	40.646	44.314	46.928	52.620	
26	30.434	35.563	38.885	41.923	45.642	48.290	54.052	
27	31.528	36.741	40.113	43.194	46.963	49.645	55.476	
28	32.620	37.916	41.337	44.461	48.278	50.993	56.892	
29	33.711	39.087	42.557	45.722	49.588	52.336	58.302	
30	34.800	40.256	43.773	46.979	50.892	53.672	59.703	
40	45.616	51.805	55.758	59.342	63.691	66.766	73.402	
50	56.334	63.167	67.505	71.420	76.154	79.490	86.661	
60	66.981	74.397	79.082	83.298	88.379	91.952	99.607	
70	77.577	85.527	90.531	95.023	100.425	104.215	112.317	
80	88.130	96.578	101.879	106.629	112.329	116.321	124.839	
90	98.650	107.565	113.145	118.136	124.116	128.299	137.208	
100	109.141	118.498	124.342	129.561	135.807	140.169	149.449	



# Chi-Square Distribution

df	.995	.990	.975	.950	.900	.750	.500	Alpha
1	.000039	.000160	.000980	.003930	.015800	.101500	.455000	
2	0.010	0.020	0.051	0.103	0.211	0.575	1.386	
3	0.072	0.115	0.216	0.352	0.584	1.213	2.366	
4	0.207	0.297	0.484	0.711	1.064	1.923	3.357	
5	0.412	0.554	0.831	1.145	1.610	2.675	4.351	
6	0.676	0.872	1.237	1.635	2.204	3.455	5.348	
7	0.989	1.239	1.690	2.167	2.833	4.255	6.346	
8	1.344	1.646	2.180	2.733	3.490	5.071	7.344	
9	1.735	2.088	2.700	3.325	4.168	5.899	8.343	
10	2.156	2.558	3.247	3.940	4.865	6.737	9.342	
11	2.603	3.053	3.816	4.575	5.578	7.584	10.341	
12	3.074	3.571	4.404	5.226	6.304	8.438	11.340	
13	3.565	4.107	5.009	5.892	7.042	9.299	12.340	
14	4.075	4.660	5.629	6.571	7.790	10.165	13.339	
15	4.601	5.229	6.262	7.261	8.547	11.036	14.339	
16	5.142	5.812	6.908	7.962	9.312	11.912	15.338	
17	5.697	6.408	7.564	8.672	10.085	12.792	16.338	
18	6.265	7.015	8.231	9.390	10.865	13.675	17.338	
19	6.844	7.633	8.907	10.117	11.651	14.562	18.338	
20	7.434	8.260	9.591	10.851	12.443	15.452	19.337	
21	8.034	8.897	10.283	11.591	13.240	16.344	20.337	
22	8.643	9.542	10.982	12.338	14.041	17.240	21.337	
23	9.260	10.196	11.688	13.091	14.848	18.137	22.337	
24	9.886	10.856	12.401	13.848	15.659	19.037	23.337	
25	10.520	11.524	13.120	14.611	16.473	19.939	24.337	
26	11.160	12.198	13.844	15.379	17.292	20.843	25.336	
27	11.808	12.879	14.573	16.151	18.114	21.749	26.336	
28	12.461	13.565	15.308	16.928	18.939	22.657	27.336	
29	13.121	14.256	16.047	17.708	19.768	23.567	28.336	
30	13.787	14.953	16.791	18.493	20.599	24.478	29.336	
40	20.707	22.164	24.433	26.509	29.051	33.660	39.335	
50	27.991	29.707	32.357	34.764	37.689	42.942	49.335	
60	35.535	37.485	40.482	43.188	46.459	52.294	59.335	
70	43.275	45.442	48.758	51.739	55.329	61.698	69.334	
80	51.172	53.540	57.153	60.391	64.278	71.145	79.334	
90	59.196	61.754	65.647	69.126	73.291	80.625	89.334	
100	67.328	70.065	74.222	77.929	82.358	90.133	99.334	



## *The Statistical Test (cont.)*

---



*Alternately, a p-value (the probability of being wrong if accepting the alternative hypothesis) can be computed in EXCEL as follows:*

*= CHIDIST(statistic,dof)*



*= CHIDIST(6.76,1)*

*Excel will return the p-value, in this case 0.0093. Since the probability of being wrong is less than 0.05, we accept the alternative hypothesis: the penny had a bias.*



## *Test for Independence*

*Suppose a bill has been introduced in Congress to raise the Interstate speed limit to 75 mph. A polling organization polls 45 Republicans and 55 Democrats, and finds 30 Republicans and 35 Democrats oppose the bill. Is there a significant effect of party affiliation on preference for the bill?*

$H_o$  = no difference between parties

$H_a$  = one party likes the bill significantly more than the other

*We organize the data in a “contingency table”*

	Republicans	Democrats	
Oppose	$f_o = 30$	$f_o = 35$	Total = 65
Favor	$f_o = 15$	$f_o = 20$	Total = 35
	Total = 45	Total = 55	



## *Test for Independence (cont.)*

*Now calculate the expected distribution, given there is no party preference. For example, the expected frequency of Republicans opposing the bill would be:*

$$\frac{\text{Total people opposing bill}}{\text{Total people}} \times \text{Total Republicans}$$
$$= (65/100) \times 45 = 29.25$$

*Note that for each cell, this is the total frequency for the row, times the total frequency for the column, divided by the total population (see chart on following page)*



## *How to calculate the expected frequency and Degrees of Freedom*

$$f_e = \frac{f_{\text{row}} \times f_{\text{col}}}{N}$$

	Republicans	Democrats	
Oppose	$f_e = 29.25$	$f_e = 35.75$	Total = 65
Favor	$f_e = 15.75$	$f_e = 19.25$	Total = 35
	Total = 45	Total = 55	

*When performing a chi-square test on a contingency table, the number of degrees of freedom equals the number of rows in the table minus one, times the number of columns in the table minus one.*

$$dof = (2-1) \times (2-1) = 1$$



# Minitab Example

## MINITAB FILE: (Create Your Own)

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the 'Tables' option is selected, which has opened a sub-menu with 'Chi-Square Test...' highlighted. Below the menu, a worksheet titled 'Worksheet 1 \*\*\*' is visible, containing a two-way contingency table with columns C1-T, C2, C3, C4, C5, C6, C7, C8, and C9. The data is as follows:

	C1-T	C2	C3	C4	C5	C6	C7	C8	C9
↓		Republic	Democrat						
1	Oppose	30	35						
2	Favor	15	20						
3									
4									
5									
6									
7									
8									

At the bottom of the window, a status bar reads: 'Perform a chi-square test for association (non-independence) in a two-way contingency table'.

$H_0$ : *There is no difference between party affiliation and preference on the bill*

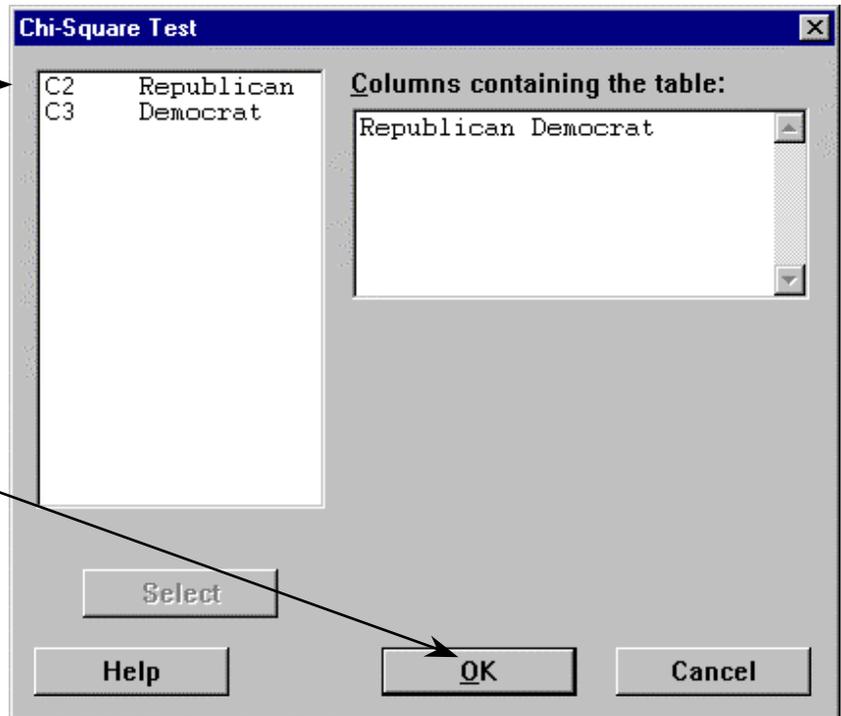
$H_a$ : *There is a difference between party affiliation and preference on the bill*



## Minitab Input and Results

1. Double click on both columns

2. Click OK



The calculated chi-square is less than the critical value of 3.841, so we accept  $H_0$ .

The p-value is greater than .05 so we are within our 95% confidence zone of good fit.

*There is no apparent relationship between political party affiliation and preference on the speed limit bill.*

Expected counts are printed below observed counts

	Republic	Democrat	Total
1	30 29.25	35 35.75	65
2	15 15.75	20 19.25	35
Total	45	55	100

Chi-Sq = 0.019 + 0.016 + 0.036 + 0.029 = 0.100  
DF = 1, P-Value = 0.752

Minitab calculates the expected frequency ( $f_e$ )



## L1 Example Using Chi-Square

To determine if location has an impact on proposal hit rate you compare the capability of “Location 1” and “Location 2” by using the L1 spreadsheet.

A **unit** is a proposal.

Each proposal has one **opportunity**.

A **defect** is a losing proposal.

Product/CEO/ Process	Defects	Unit	Opt	Total Opt	DPU	DPO	DPMO	Shift	Long Term Capabilit	Sigma
	<u>D</u>	<u>U</u>	<u>OP</u>	<u>TOP</u>	<u>DPU</u>	<u>DPO</u>	<u>DPMO</u>	<u>Shift</u>	<u>Sigma-L</u>	<u>Z.B</u>
Location 1	14	52	1	52	0.2692	0.269231	269231	1.5	0.61	2.11
Location 2	6	43	1	43	0.1395	0.139535	139535	1.5	1.08	2.58
<b>Grand Total</b>	20			95		0.210526	210526	1.5	0.80	2.30

**Does “Location 2” really have a better proposal hit rate than “Location 1?”**



# L1 Chi-Square Example Using Minitab

MINITAB FILE: (Create Your Own)

Perform a chi-square test for association (non-independence) in a two-way contingency table

	C1-T	C2	C3				C7
↓		Wins	Losses				
1	Location 1	38	14				
2	Location 2	37	6				
3							
4							
5							

$H_o$ : Location does not impact the ability to win/lose a proposal

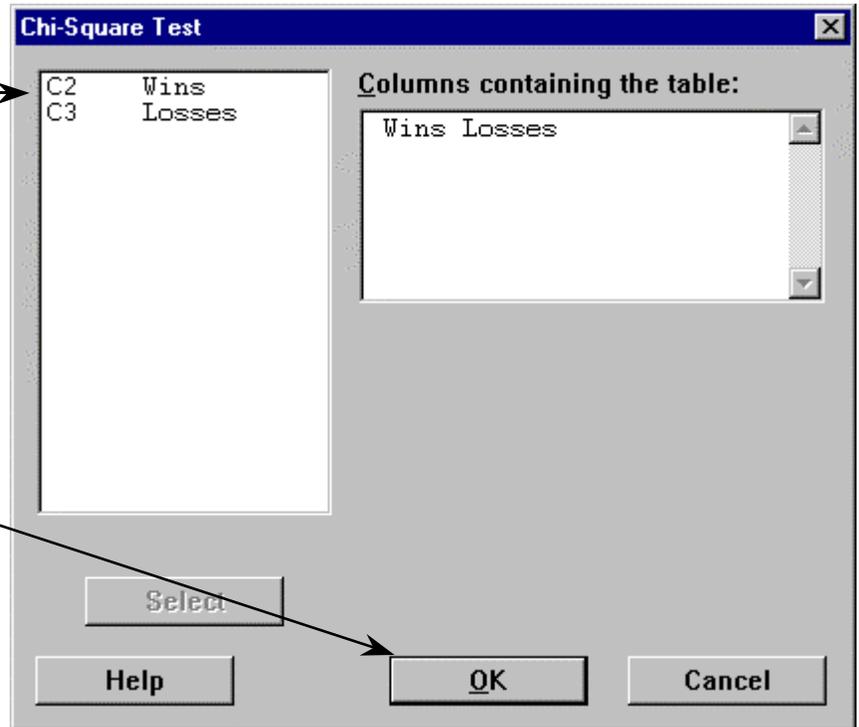
$H_a$ : Location impacts the ability to win/lose a proposal



## L1 Example Input & Results

1. Double click on both columns

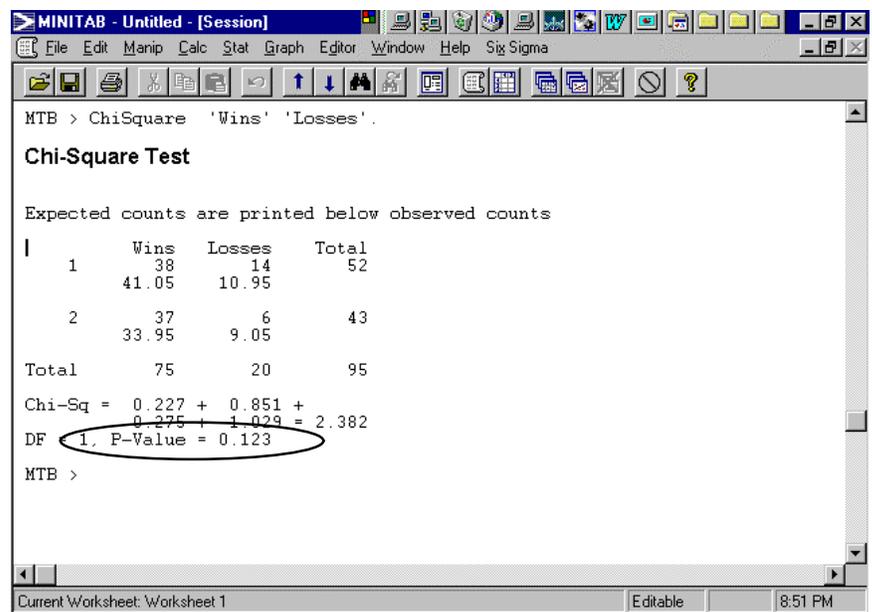
2. Click OK



Since the  $p$ -value is greater than .05 we accept  $H_0$ .

There is no statistically significant relationship between location and proposal hit rate.

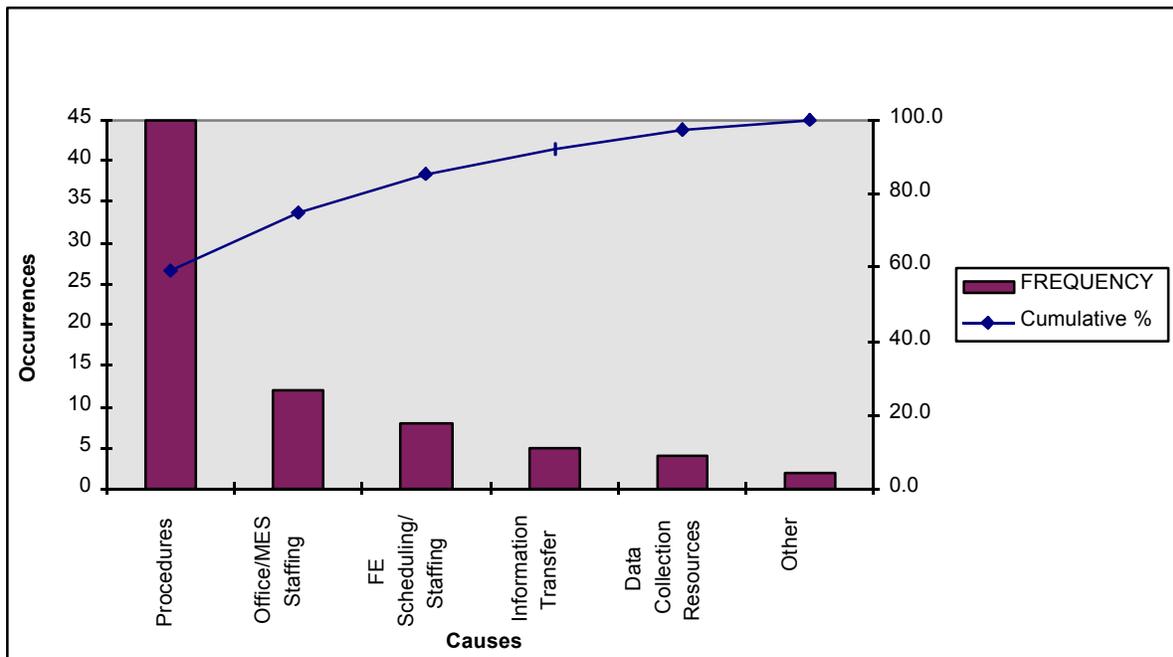
You would run a 12.3% risk of making an incorrect decision if you concluded that "Location 2" had a better proposal hit rate than "Location 1."





# Chi-Square Test

Product/CEO/Process	Defects	Unit	Opt	Total Opt	DPU	DPO	DPMO	Shift	Long Term Capability	Sigma
	<u>D</u>	<u>U</u>	<u>OP</u>	<u>TOP</u>	<u>DPU</u>	<u>DPO</u>	<u>DPMO</u>	<u>Shift</u>	<u>Sigma-L</u>	<u>Z.B</u>
Loan/Borrowed FE/TA (Actual)	76	1	215	215	76.0000	0.353488	353488	1.5	0.38	1.88
Target	8	1	215	215	8.0000	0.037209	37209	1.5	1.78	3.28
Actual	0	1	12	12	0.0000	0.000000	0	1.5	1.59	3.09
<b>Grand Total</b>	<b>84</b>			<b>442</b>		<b>0.190045</b>	<b>190045</b>	<b>1.5</b>	<b>0.88</b>	<b>2.38</b>





# The Confidence Interval Formula

## Using The Confidence Interval Formula For Attributes Data To Derive Sample Size Formula

Confidence interval for attribute data

$$P_B \pm Z_{\alpha/2} \sqrt{\frac{P_B (1-P_B)}{n}}$$

Given  $\rightarrow P_B \pm \Delta$

$$\text{Solve for } n \rightarrow \Delta = Z_{\alpha/2} \sqrt{\frac{P_B (1-P_B)}{n}}$$

$$n = \left( \frac{Z_{\alpha/2}}{\Delta} \right)^2 P_B (1-P_B)$$



# Sample Size Calculations

## Attribute Data Case: Estimating A Population Proportion

- *Minimum Sample Size Necessary (95% confidence interval)*

$$n = \left( \frac{1.96}{\Delta} \right)^2 P_B (1-P_B)$$

- *Example*

*We want to estimate the proportion defective (P) within  $\pm 0.02$  (i.e.,  $D = 0.02$ ). We expect P to be approximately 0.05*

$$\begin{aligned} n &= \left( \frac{1.96}{.02} \right)^2 [.05 (1-.05)] \\ &= 456 \end{aligned}$$



## *How To Estimate $P_B$ And $D$*

### ■ *Estimate $P_B$*

- *Take a small pre-sample of data ( $n @ 100$ ) and calculate  $P$*
- *Use  $p$  from an existing control chart*
- *Set  $P_B = .5$  as a worst case (largest  $n$ )*

### ■ *Estimate $D$*

- *See estimation of  $D$  for continuous data*
- *To determine  $D$  for a given sample size*

$$\Delta = Z_{\alpha/2} \sqrt{\frac{P_B (1 - P_B)}{n}}$$



## *Exercises With The Formulas*

### **Assume Sample Size Is Less Than 5% Of The Population Size**

#### **Instructions**

*In your table team, answer the assigned questions and be prepared to report out your answers. Use Minitab when possible. Assume confidence level = 95%*

- 1. Given a sample size of 100, how precisely can we estimate a proportion defective estimated as  $P=0.2$ ?*
- 2. Given an estimated proportion defective guessed to be somewhere in the range of 5% to 15%, how many observations should we take to estimate the proportion defective within 2%?*

#### **Time**

*15 minutes*



# Discrete data sample size spreadsheet



## **Discrete data sample size spreadsheet**

### **The 95% Confidence Interval sheet is:**

Binomial (Pass/Fail) Confidence Intervals (CI) (based on tail areas of the beta distribution) To use this sheet:

1. Type in the desired confidence level
2. Type in the total number of units (good + bad)
3. Type in the total number of opportunities
4. Type the number of failed (defects) units

The CI on the expected failure rate is calculated

### **The Sample Size Required sheet is:**

Sample size required to distinguish between two proportions (based on binomial) ie. how many samples does it take to verify that corrective action has had an impact The sheet uses a starting and ending PPM, requires a CI% and opportunity count to be entered.

### **The Factor Reduction sheet is:**

Given a level of reduction, how many samples are required for many different starting PPM's

**GE Medical Systems**

Revised 2/23/00

Password required to unprotect any sheet is sigma



**ATTRIBUT.XLS is an Excel spreadsheet that handles sample size for discrete data. The Intro Tab is shown here.**



## The 95% Confidence Interval Tab

If we claim to have reduced our defects or missing signatures on invoices etc. down to 1 out of 60, the spreadsheet shows you that with the same level of statistical confidence (95%) we could just as likely have seen 0% defects or 9% defects out of 60.

95% Confidence Intervals for defects						
Confidence -->	0.95					
Units -->	60					
Opportunities -->	1					
TOP's -->	60					
Defects -->	1					
	p(d)	Percent	ppm	Z <sub>ST</sub>		Defects
Upper Limit on Failure Rate	0.0894	8.9%	89399	2.84	<= "worst case" =>	5 95%
Nominal Value	0.01667	1.7%	16667	3.63	<= "best estimate" <->	Confidence
Lower Limit on Failure Rate	0.00042	0.0%	422	4.84	<= "best case" =>	1 Interval

If our project started at 5 defects out of 60 (8.3%) we could not claim to have made a statistical difference.

Another way to read this is if someone claims to have fixed something and they are now running at 1 bad out of their last 60 we could say that we don't buy it. Statistically they could as likely be at 5 out of 60.



## The Sample Size Required Tab

If I started at 5 bad out of 60 (83,333 PPM) and now I think I am at 1 out of 60 (16,666 PPM) how many samples will I need to take to be confident that (with discrete data) I have truly made this claimed improvement?

Sample Size Required To Statistically Confirm PPM Reduction			
	ppm	p(d)	Z <sub>ST</sub>
Process A performance (before change) -->	83333	0.0833	2.88
Process B performance (after change) -->	16666	0.0167	3.63
Opportunities on each unit: -->	1		
Confidence -->	0.95		
Sample size required to distinguish between processes -->	200	200	

**200 Samples!**

200 samples are needed. This is why (see the previous page) any Six Sigma person would not buy the statement that “we made an improvement going from 5/60 to 1/60” . 200 parts need to be counted before we are statistically confident in the change.



## Spreadsheet details . . .

You can change green fields, but not red fields  
Units are the number of items or widgets tested

Confidence -->	0.95
Units -->	10
Opportunities -->	6
TOP's -->	60
Defects -->	5

Opportunities are the number of categories actually looked at on each unit.

Process A performance (before change) -->	83333	0.0833	2.88
Process B performance (after change) -->	16666	0.0167	3.63
Opportunities on each unit: -->	6		
Confidence -->	0.95		
Sample size required to distinguish between processes -->	200	TOP's Units 33	

TOP = units x opportunities

Usually it is meaningful to know how many units need to be run, not how many total opportunities are needed



# *Take Aways—Hypothesis Testing: Discrete Y; Discrete X*

## ■ *Chi-Square Tests*

- *Goodness of Fit Test*
  - *Does our data fit an expected distribution?*
- *Test of Independence*
  - *Does some factor have an affect on the output?*
- *$\chi^2$  statistic*
  - *$\chi^2 > \text{critical value}$  Accept  $H_o$*
  - *$\chi^2 < \text{critical value}$  Reject  $H_o$*
- *p-value*
  - *$p < 0.05$  Accept  $H_o$*
  - *$p > 0.05$  Reject  $H_o$*

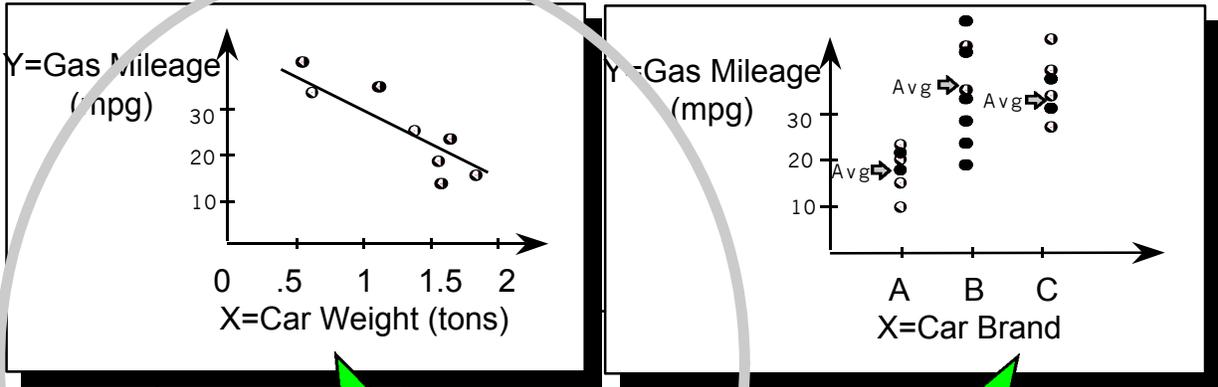


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# *Hypothesis Testing: Continuous $Y$ ; Continuous $X$*



# Continuous Y; Continuous X



X (Factor) treated as:

Continuous

Discrete

Y (Response)

Continuous

- Scatter plot
- Simple Regression

- t-Test
- Homogeneity of Variance
- 1-Way ANOVA
- Mood's

Discrete

- Median
- Contingency Table Anal.
- Chi Square: Independent

Y= Injury Severity

Death	75	60	65
Major	160	115	175
Minor	100	65	135
None	15	10	25

A B C  
X=Car Brand

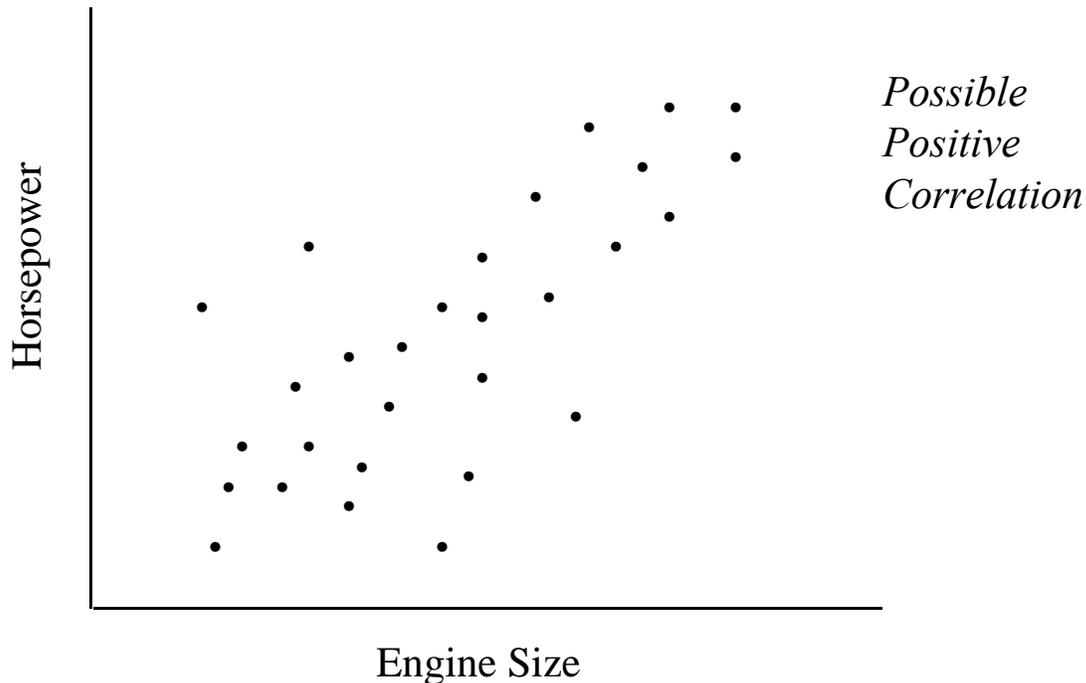
Developed by W. Scott Lasater - GEMIS



## Scatter Plot

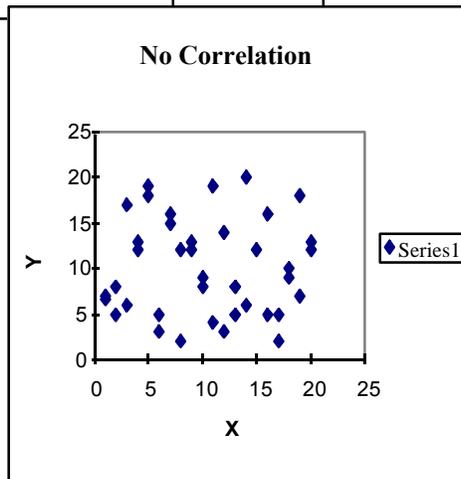
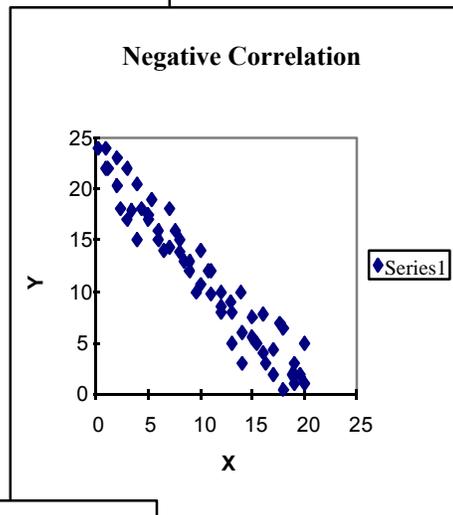
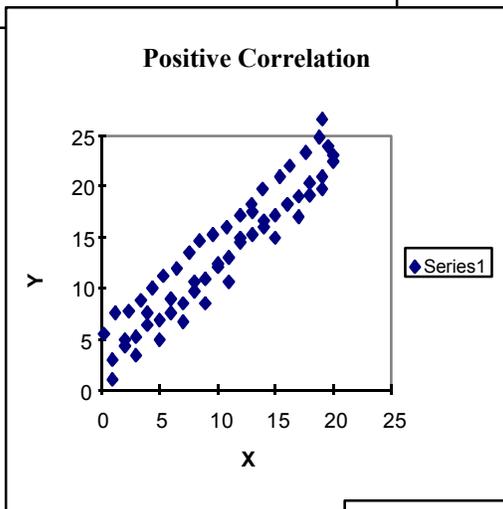
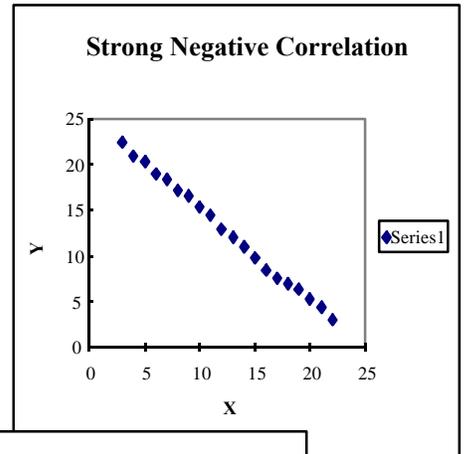
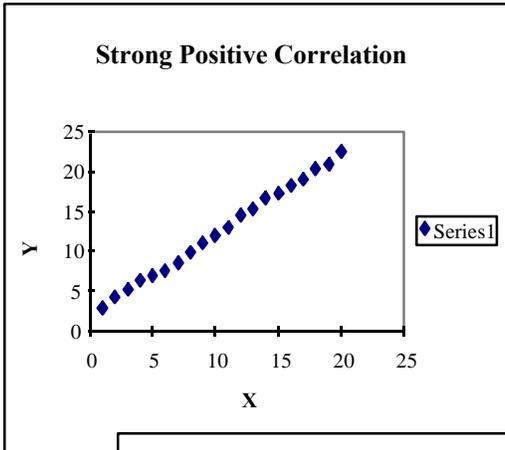
**Purpose:** *To study the possible relationship between one variable and another.*

**When:** *To test a theory that two variables are related.*





# X and Y Data Correlation





## *Building a Scatter Plot*

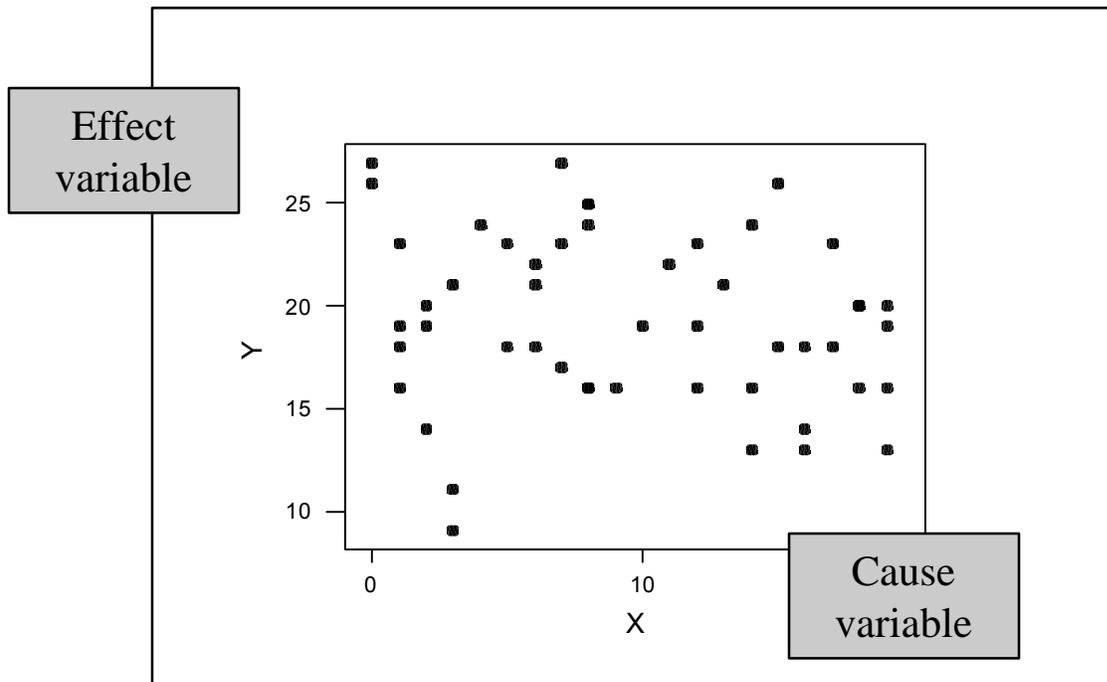
1. *Collect 20 - 100 paired data points*

X	Y
7	17
0	26
1	23
8	16
12	23
19	19
17	18
7	27
3	21
2	20
11	22
0	27
6	22
16	14
3	9
3	11
2	14
6	21
5	18
17	23
.	.
.	.
.	.



# Building a Scatter Plot

## 2. Draw axes



## 3. Plot data

*If a value repeats, circle point for as many times as that value repeats.*

## 4. Analyze data to determine correlation

*Straightness of line and tightness of cluster indicate strength of relationship.*



## Scatter Plot Example

*Is there a relationship between age and years with GE?*

MINITAB FILE: Age\_yrge.mtw

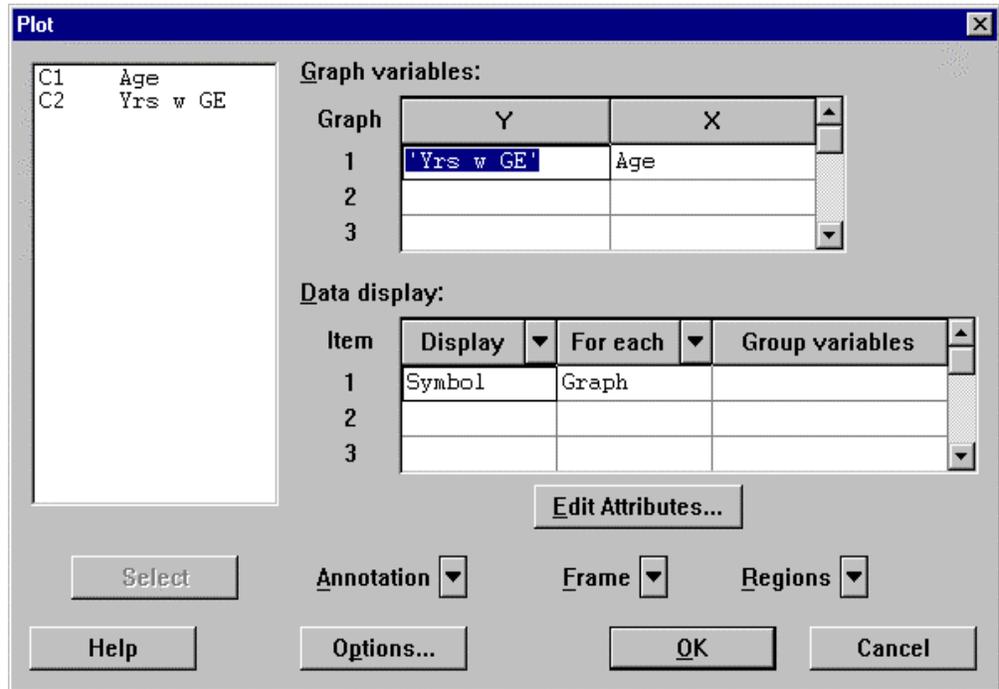
The screenshot shows the Minitab software interface. The 'Graph' menu is open, displaying various plot options. The data table below shows the following information:

	C1	C2	C3	C4	C5	C6	C7
↓	Age	Years with GE					
1	30	27.0000					
2	26						
3	25						
4	29						
5	45	27.0000					



# Scatter Plot Input & Output

1. Double Click →



The Plot dialog box shows the following configuration:

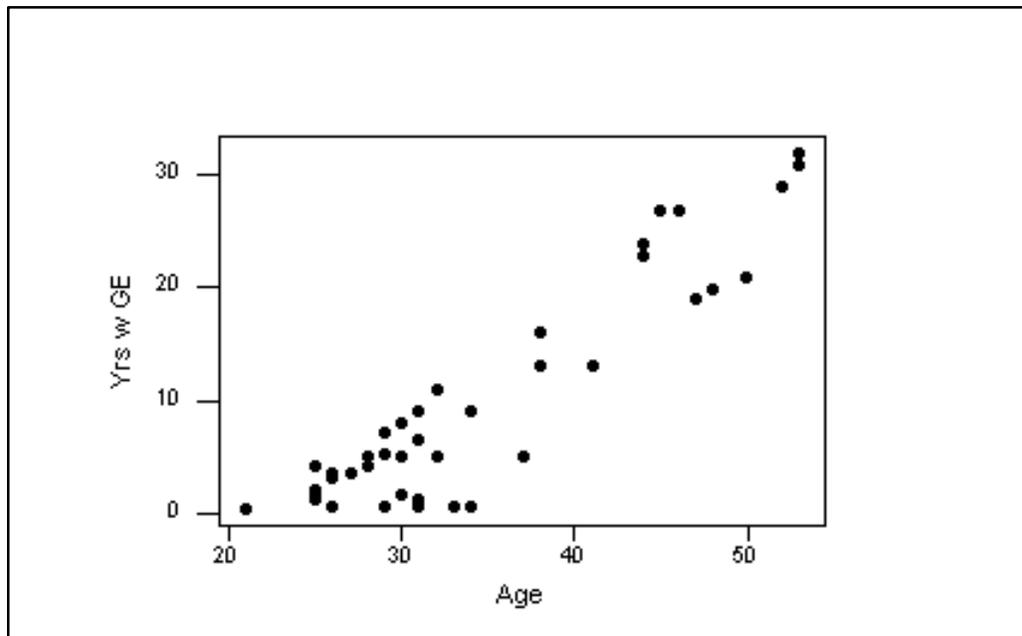
- Graph variables:**

Graph	Y	X
1	'Yrs w GE'	Age
2		
3		
- Data display:**

Item	Display	For each	Group variables
1	Symbol	Graph	
2			
3			

Buttons: Select, Annotation, Frame, Regions, Help, Options..., OK, Cancel. An **Edit Attributes...** button is also present.

2. Click on OK





## *Catapult Scatter Plot Exercise*

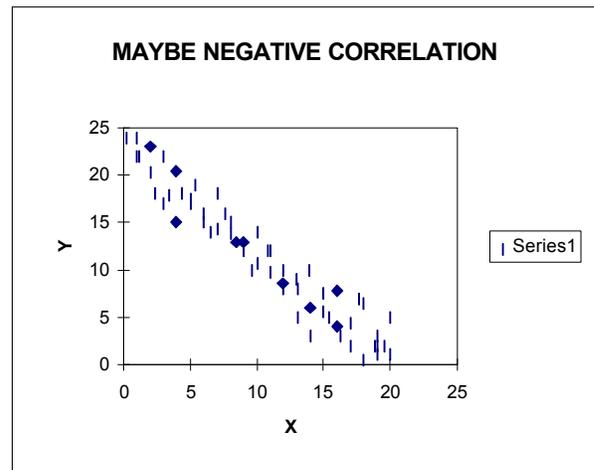
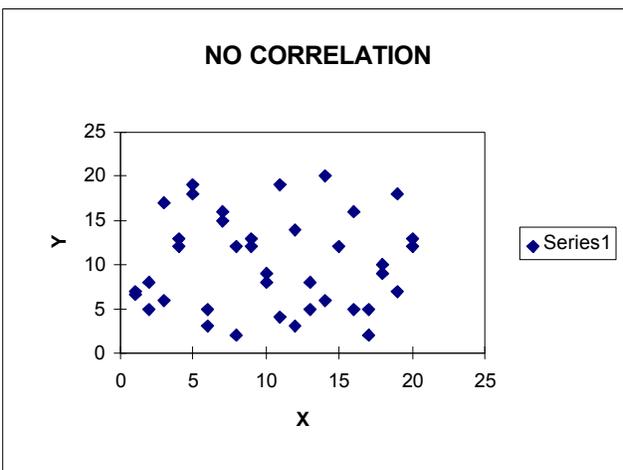
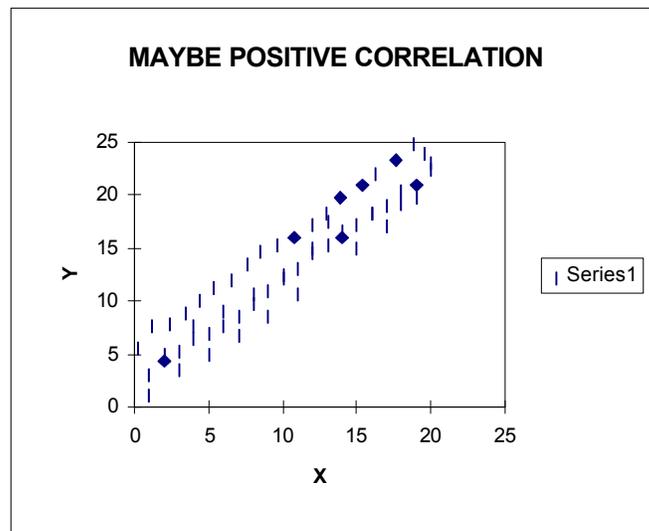
---

- *Use a scatter plot to investigate the possible relationship between the cocking angle and the projectile flight distance.*
- *Use the data in the Minitab file, Catapult.mtw*



# Simple Linear Regression

- *How to fit a line to data*
- *How to quantify correlation*
- *How to predict values*





## *Simple Linear Regression*

- *We have shown how to make scatter plots of data and talked about positive and negative **correlation** of two data sets.*
- *Regression analysis is a statistical technique used to model and investigate the relationship between two or more variables. The model is often used for **prediction**.*
- *Regression is a **hypothesis test**  
 $H_a$ : The model is a significant predictor of the response.*
- *It may be used to analyze relationships between the “Xs,” or between “Y” and “X.”*
- *Regression is a powerful tool, but can never replace engineering or manufacturing process knowledge about trends.*



# Regression Analysis Example

MINITAB FILE: Age\_yrge.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the 'Regression' option is selected, leading to a sub-menu where 'Regression...' is chosen. The worksheet 'Age\_yrge.mtw' is displayed with the following data:

	C1	C2	C3	C4	C5	C6	C7
↓	Age	Yrs w GE					
37	32	11.0000					
38	38	16.0000					
39	31	1.0000					
40	25	1.0000					
41	53	31.0000					

Perform regression using least squares estimation



# Regression Analysis Example

1. Double Click



**Regression**

C1	Age
C2	Yrs w GE

Response: 'Yrs w GE'

Predictors: Age

Select

Help

Graphs... Options... Results... Storage... OK Cancel

2. Click OK

MINITAB - Untitled - [Session]

File Edit Manip Calc Stat Graph Editor Window Help Six Sigma

Regression Analysis

The regression equation is  
Yrs w GE = - 25.1 + 1.01 Age

Predictor	Coef	StDev	T	P
Constant	-25.062	2.483	-10.09	0.000
Age	1.01000	0.07015	14.40	0.000

S = 4.016      R-Sq = 83.5%      R-Sq(adj) = 83.1%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	3342.9	3342.9	207.28	0.000
Residual Error	41	661.2	16.1		
Total	42	4004.2			

Unusual Observations

Obs	Age	Yrs w GE	Fit	StDev Fit	Residual	St Resid
12	34.0	0.500	9.278	0.613	-8.778	-2.21R
15	34.0	0.500	9.278	0.613	-8.778	-2.21R

Current Worksheet: Age\_yrge.mtw      Editable      10:14 PM



## *Minitab Regression Calculation Explanation of Output*

- *The “p-values” for the constant (Y-intercept) and the predictor variables are read exactly as we have explained them in Hypothesis Testing.*
- *$H_a$ : The factor is a significant predictor of the response.*
- *The value  $s$  is the “standard error of the prediction” = standard deviation of the residuals.*
- *R-square is the percent of variation explained by your model.*
- *R-square (adjusted) is the percent of variation explained by your model, adjusted for the number of terms in your model and the number of data points.*
- *The “p-value” for the regression is for whether the entire regression model is significant.*
  - *$H_a$ : The model is a significant predictor of the response.*

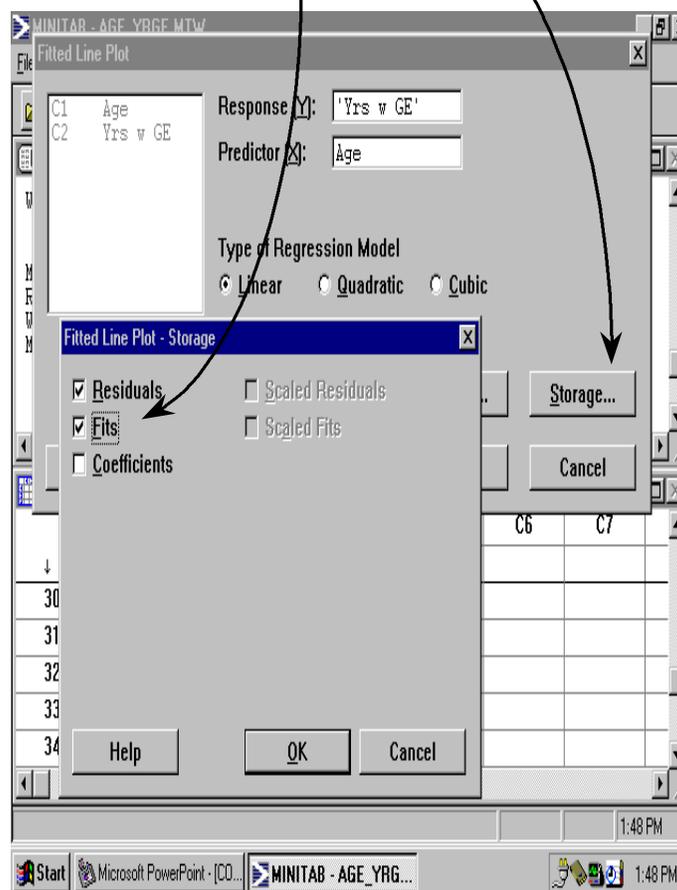


## Correlation & Regression - Residual Plots

Use Residual Plots to test the assumptions of the analysis

When setting up Regression Analysis:

1. Click Storage
2. Select "Residuals" and "Fits"



A **Fit** is the predicted value of response variable for a given value of the predictor variable.

A **Residual** is the difference between an actual observation and the fitted value (the difference between an individual data point and the predicted value).



# Correlation & Regression - Constructing Residual Plots

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and 'Residual Plots' is selected. The 'Residual Plots' dialog box is open, showing the selection of variables C1 (Age), C2 (Yrs w GE), C3 (RESI1), and C4 (FITS1). The dialog box has fields for 'Residuals' and 'Fits', and a 'Title' field. The 'OK' button is highlighted.

	C1	C2	C3	C4	C5	C6	C7
↓	Age	Yrs w GE	RESI1	FITS1			
30	41	12.0	-3.80143	15.8014			
31	44	22.0	3.34074	18.6593			
32	44	22.5	3.84074	18.6593			
33	45	25.0					
34	46	25.0					

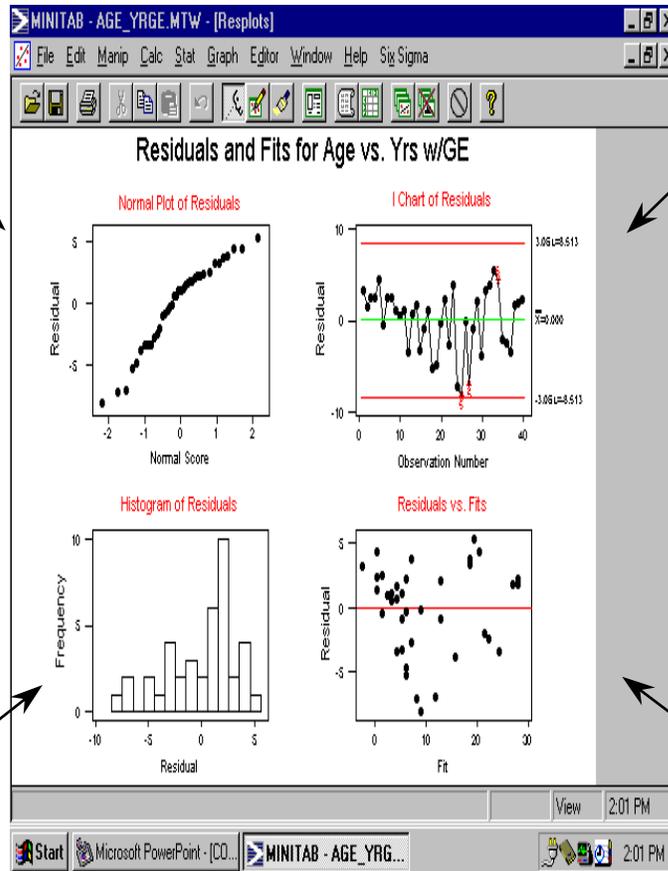
  

	C1	C2	C3	C4	C5	C6	C7
33	45	25.0	5.38813	19.6119			
34	46	25.0	4.43553	20.5645			



# Correlation & Regression - Interpreting the Output

How normal are the residuals?



Trends over time in residuals?

Histogram - bell curve? Outliers? Ignore for small data sets (<30)

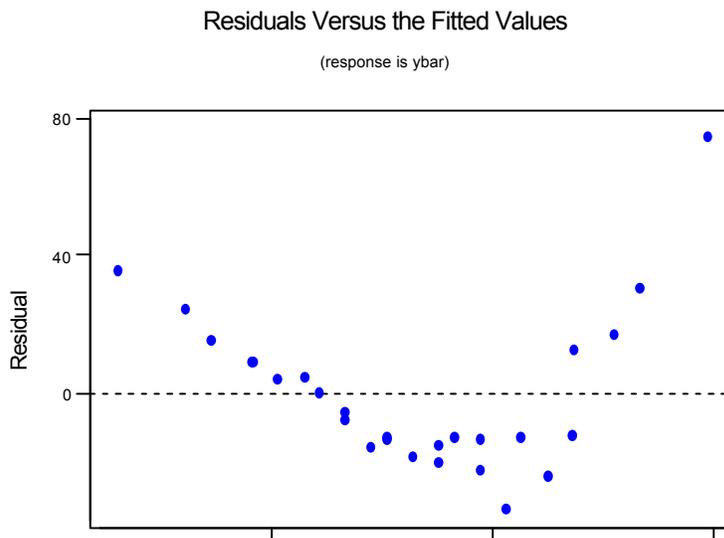
Constant Variance?

**Look for Gross Violations of Assumptions. If You Find Problems, You May Need To Add Additional Terms (e.g. Interactions), Or Transform One Of The Variables**

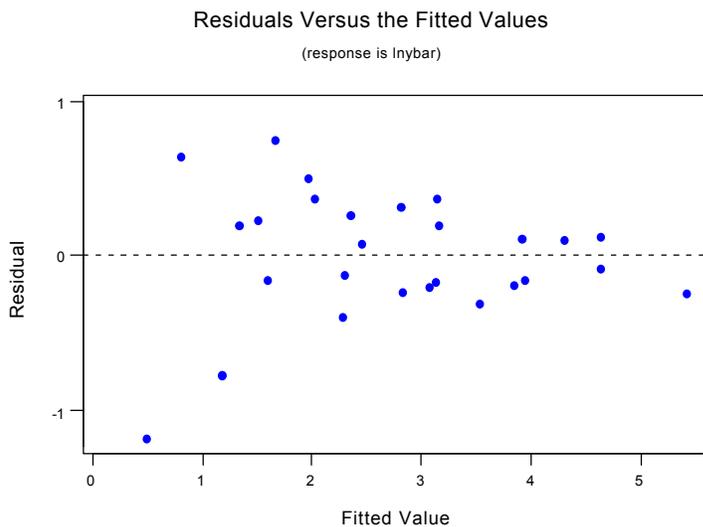


# Correlation & Regression - Example

## Example From GEL Call Center Project (Greg Simpson)



Clear Curvature in Residuals shows non-linear nature of lost calls



After Transforming Data, Residuals look good

*Analyzed DOE Results with Minitab to get Transfer Function*



# Regression Analysis Example: Confidence and Prediction Bands

MINITAB FILE: Age\_yrge.mtw

	C1	C2	C3	C4	C5	C6	C7
↓	Age	Yrs w GE					
1	30	8.0000					
2	26	3.0000					
3	25	1.7000					
4	29	5.2500					
5	45	27.0000					

1. Double Click →

2. Click Options →

**Fitted Line Plot**

Response (Y): 'Yrs w GE'

Predictor (X): Age

Type of Regression Model

Linear     Quadratic     Cubic

Select    Options...    Storage...

Help    OK    Cancel



# Regression Analysis Input & Output

3. Check both boxes



**Fitted Line Plot - Options**

**Transformations**

Logten of Y       Display logscale for Y variable

Logten of X       Display logscale for X variable

**Display Options**

Display confidence bands

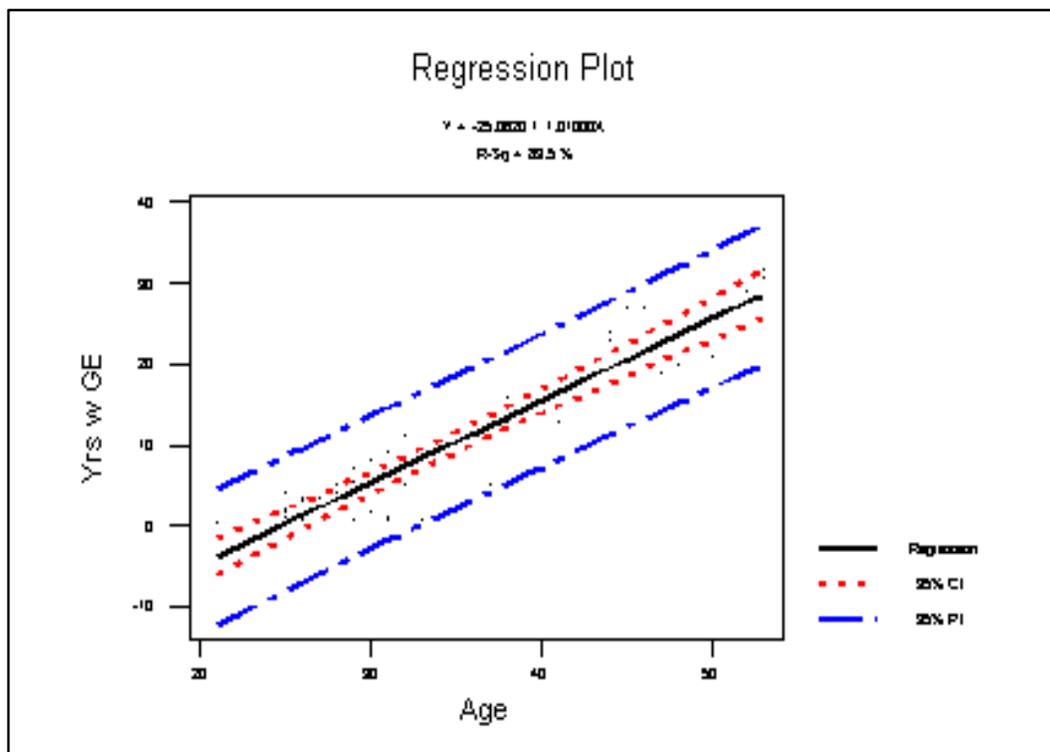
Display prediction bands

**Confidence level:**

**Title:**

4. Click OK





## *Regression Analysis: Confidence and Prediction Bands*

---

- *A confidence band (or interval) is a measure of the certainty of the shape of the fitted regression line. In general, a 95% confidence band implies a 95% chance that the true line lies within the band. [Red lines]*
- *A prediction band (or interval) is a measure of the certainty of the scatter of individual points about the regression line. In general 95% of the individual points (of the population on which the regression line is based) will be contained in the band. [Blue lines]*



## *Regression Analysis: Exercise*

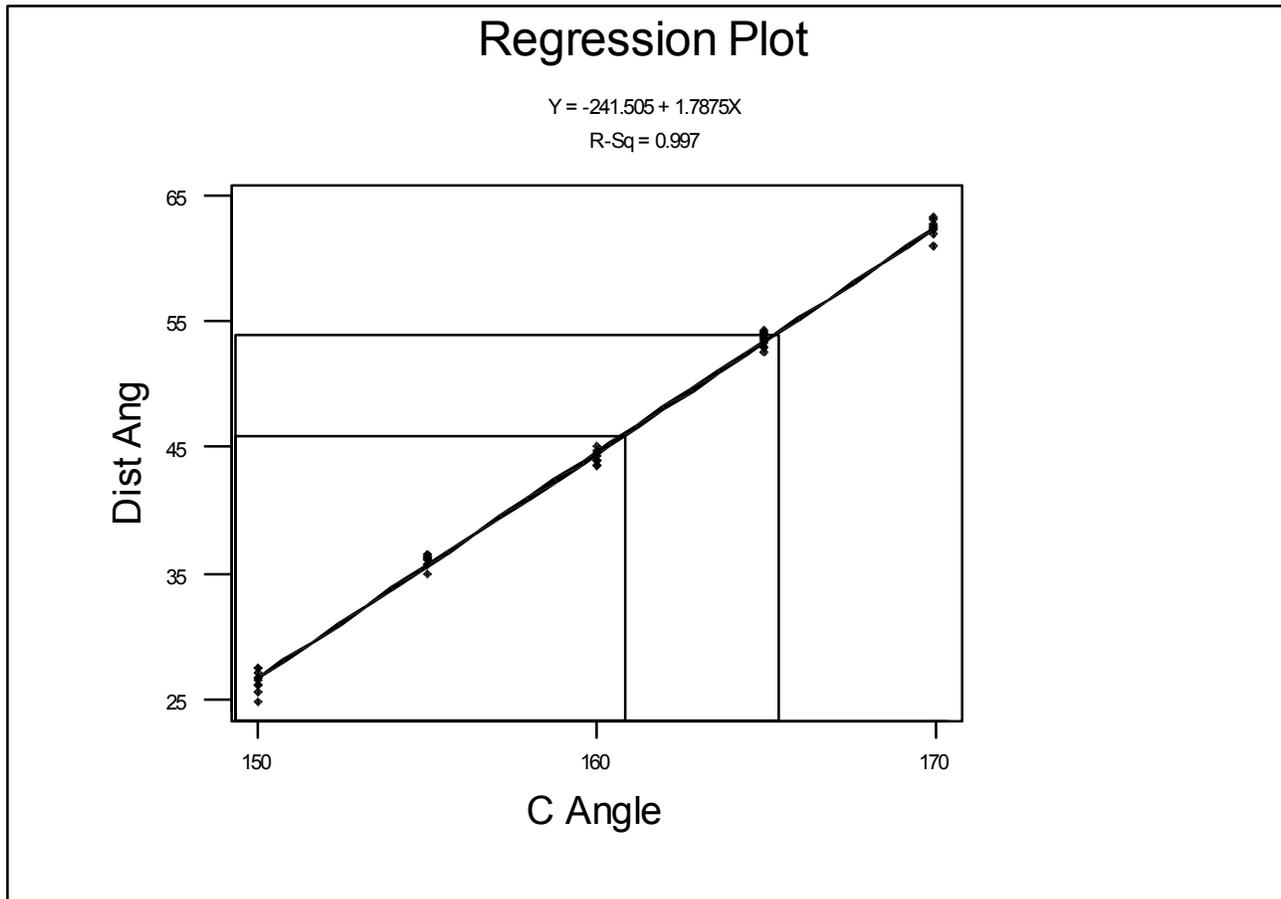
---

- *Perform a regression analysis on the distance versus cocking angle from the file **Catapult.mtw***
- *What can you conclude?*



## Statistical Tolerancing

*If you want to launch the projectile between 46" and 54," what should the tolerance on the cocking angle be?*



Cocking Angle Tolerance:  $161^\circ$ ,  $165^\circ$

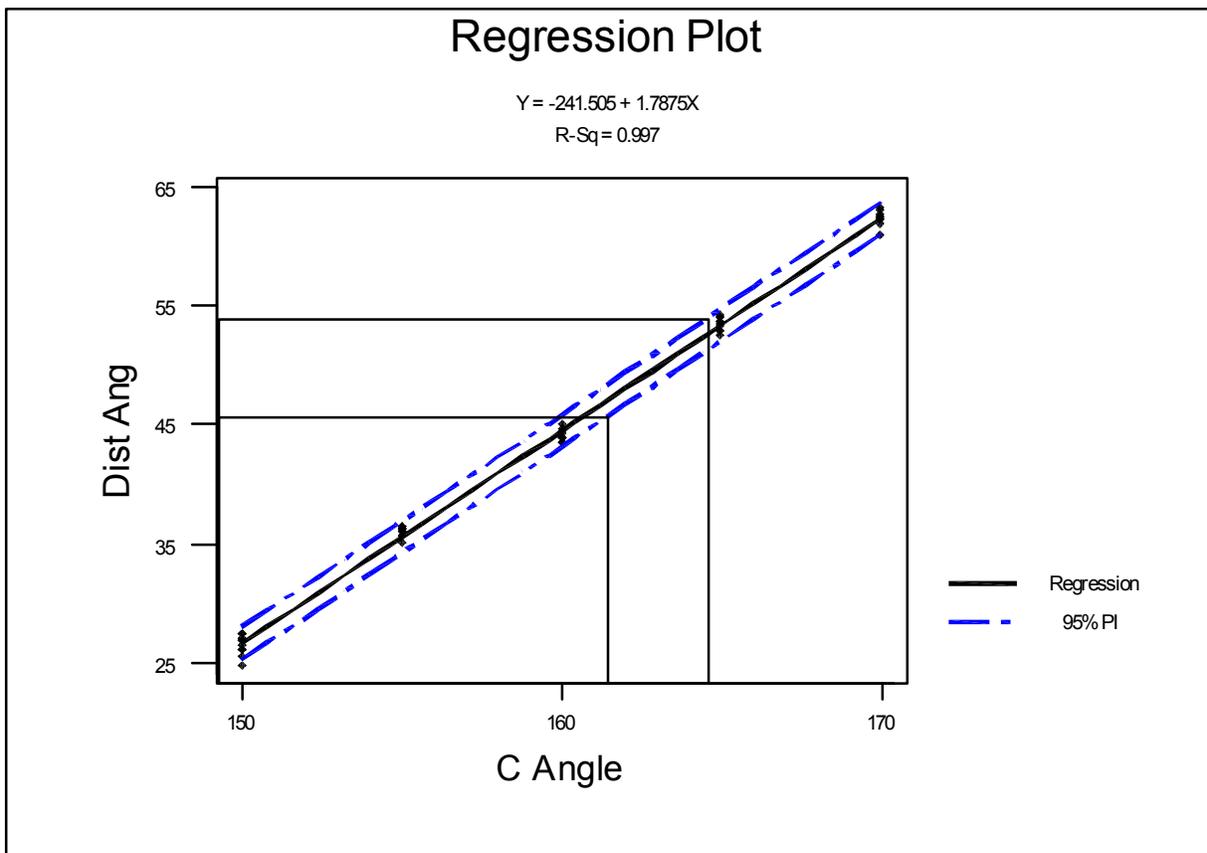
*How does variation around the regression line effect the tolerance?*



# Statistical Tolerancing Including Variation Around Line

Use the prediction interval to include the variation around the regression line.

Draw the USL to the upper line and the LSL to the lower line.



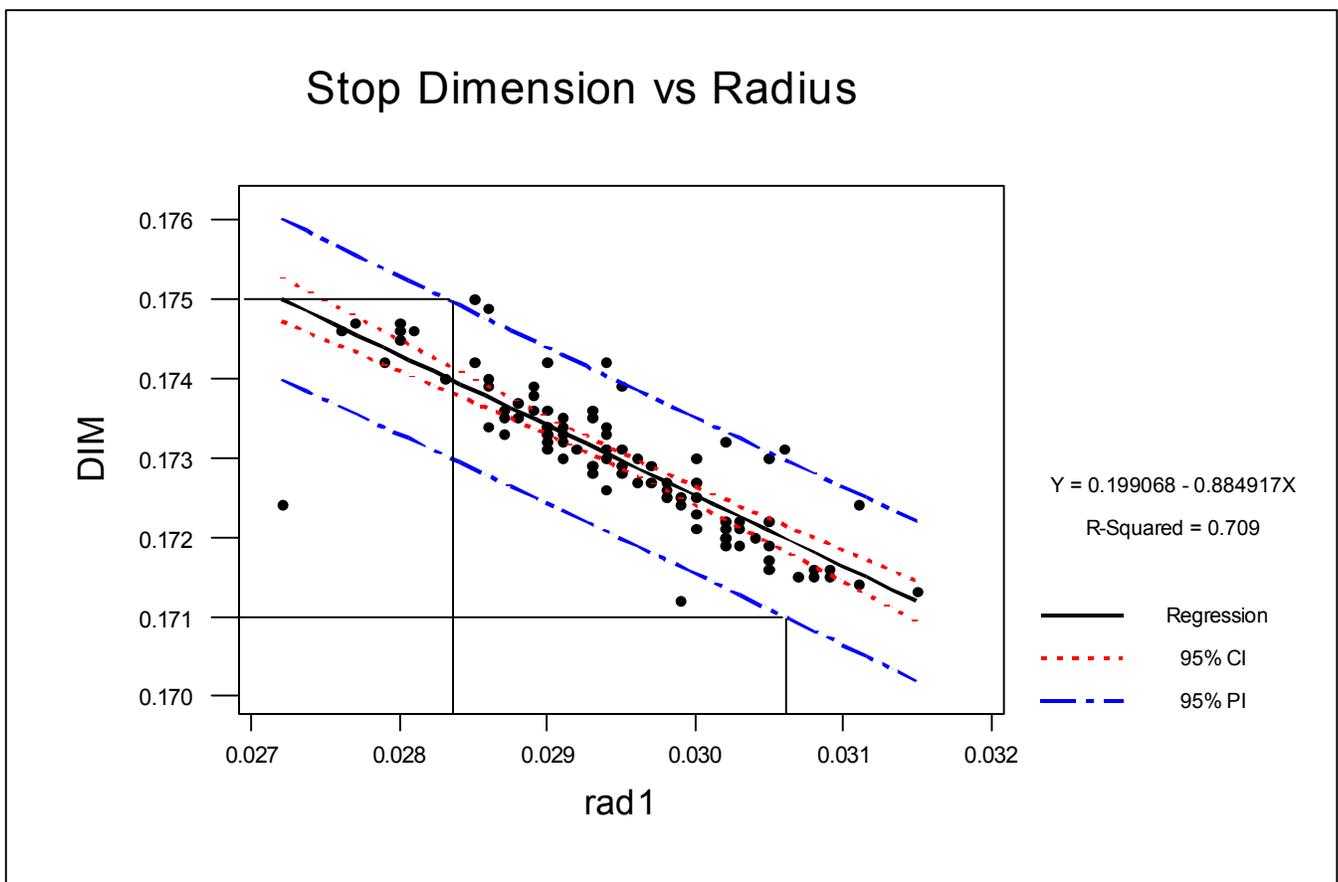
The additional variation reduces the tolerance window to  $162^\circ$ ,  $164^\circ$ .

*How do slope & negative correlation affect the tolerance window?*



## Negative Relationships

*You still draw the USL to the upper line and the LSL to the lower line.*



***Note how the lines cross.***

***The USL for Y defines the LSL for X.***



# Statistical Tolerancing Minitab Example

## MINITAB FILE: Catapult.mtw

**Fitted Line Plot**

Response [Y]: 'Dist Ang'  
Predictor [X]: 'C Angle'

Type of Regression Model  
 Linear    Quadratic    Cubic

Buttons: Select, Options..., Storage..., Help, OK, Cancel

	C14	C15	C16
	150	155	160
1	26.00	36.25	44.00
2	26.00	36.25	44.00
3	26.75	35.75	44.00
4	27.50	36.25	44.25
5	27.00	36.50	43.50

**Fitted Line Plot - Options**

Transformations  
 Logten of Y    Display logscale for Y variable  
 Logten of X    Display logscale for X variable

Display Options  
 Display confidence bands  
 Display prediction bands

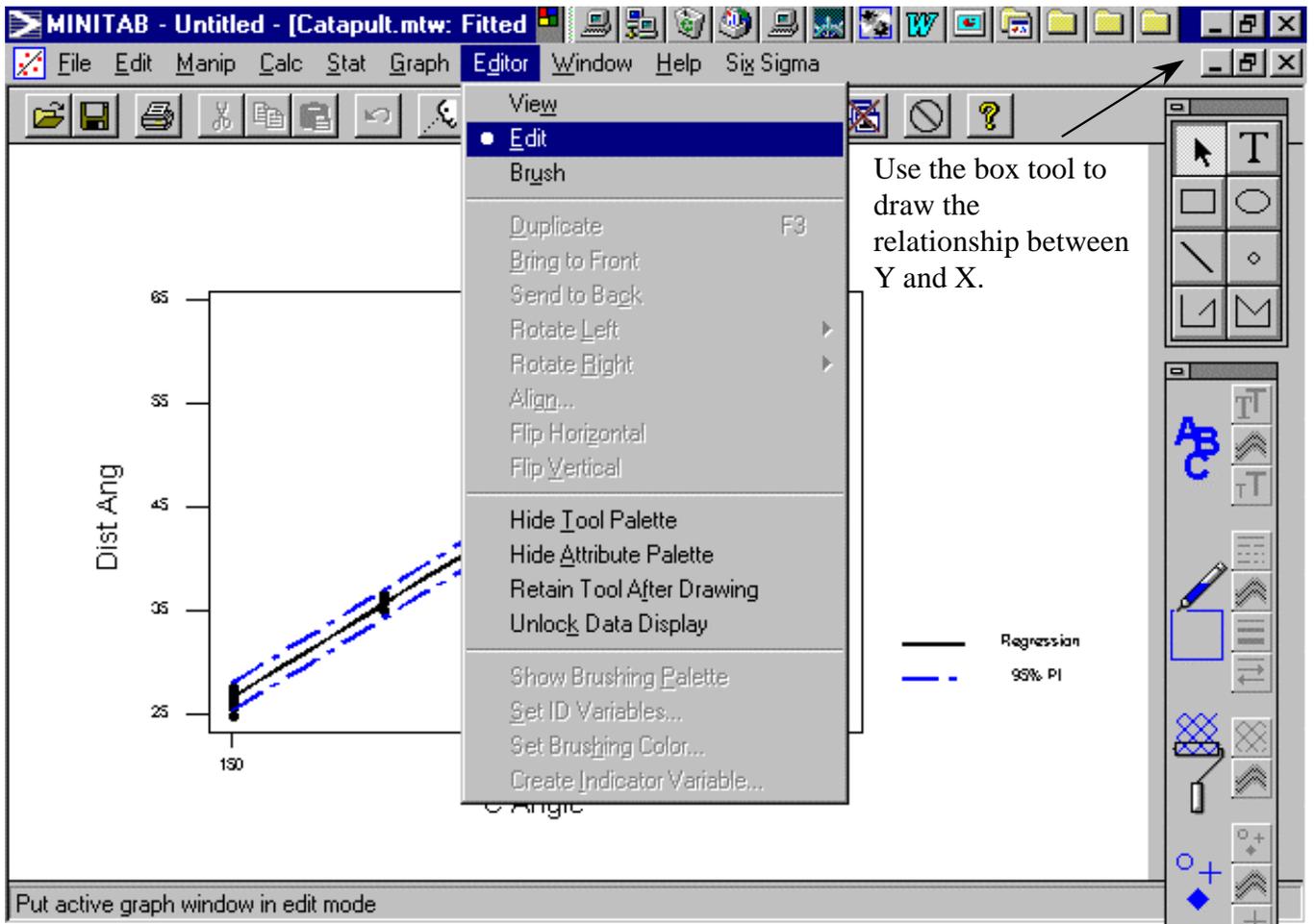
Confidence level: 95.0

Title: \_\_\_\_\_

Buttons: Help, OK, Cancel

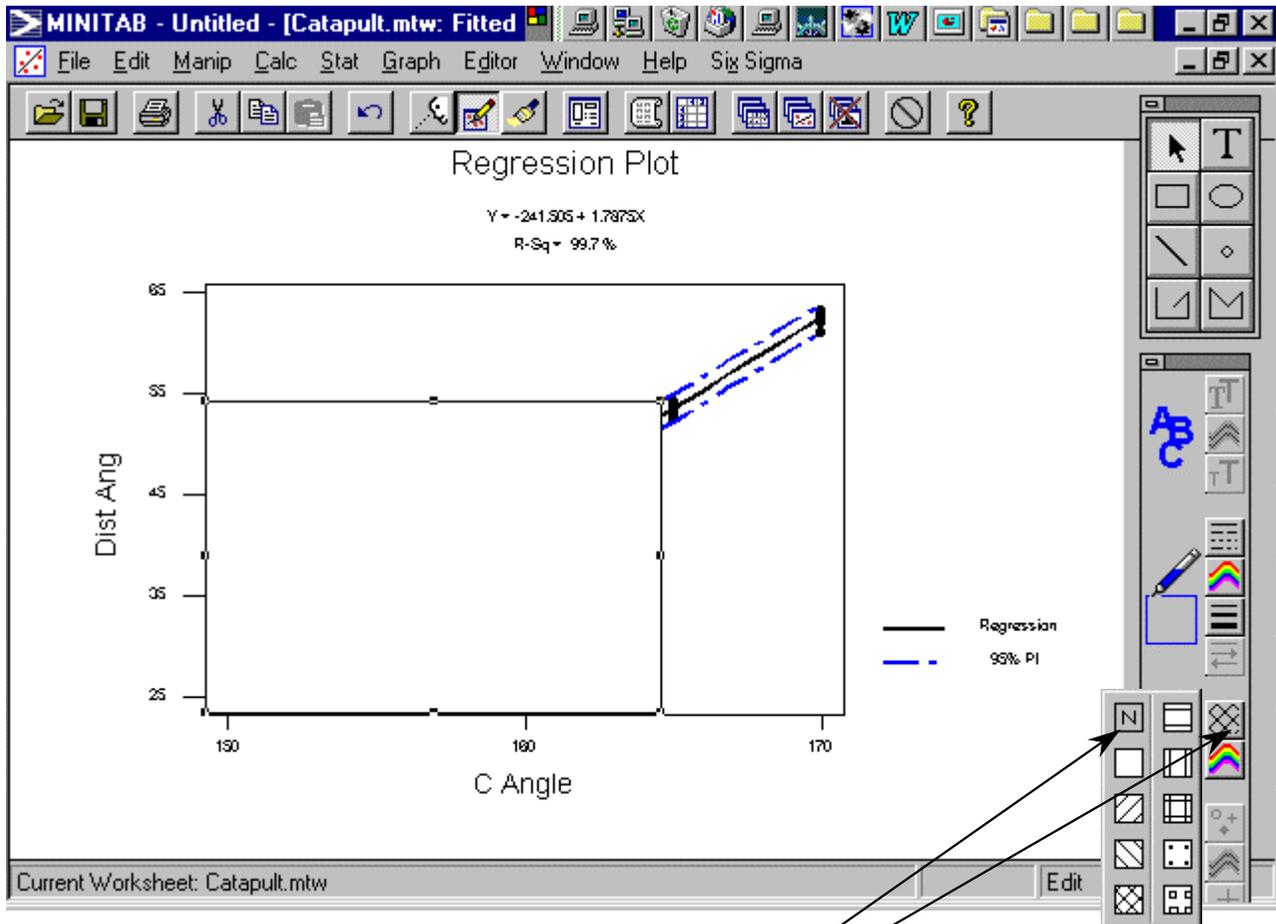


# Drawing the Tolerance Relationship





# Clearing the Boxes



Use the fill tool and the No fill button to make the boxes clear.



# Multiple Regression

**Multiple** regression analysis is a method that enables you to quantify the relationship between a continuous process output (Y) and continuous input factors (Xs), or between a continuous process output (Y) and a combination of continuous and discrete input factors (Xs). Multiple regression is very useful for “mining” historical data to build a model that links the X variables to the Y variable. Such a model completes all *Analyze* phase objectives and may even define the process changes to make in the *Improve* phase. It also allows us to predict the Y output, based on the X values.

After completing this chapter, you will be able to:

- ***Put your data into the correct format for a multiple regression***
- ***Complete a multiple regression analysis and interpret the results***



# *What Questions does Regression Answer?*

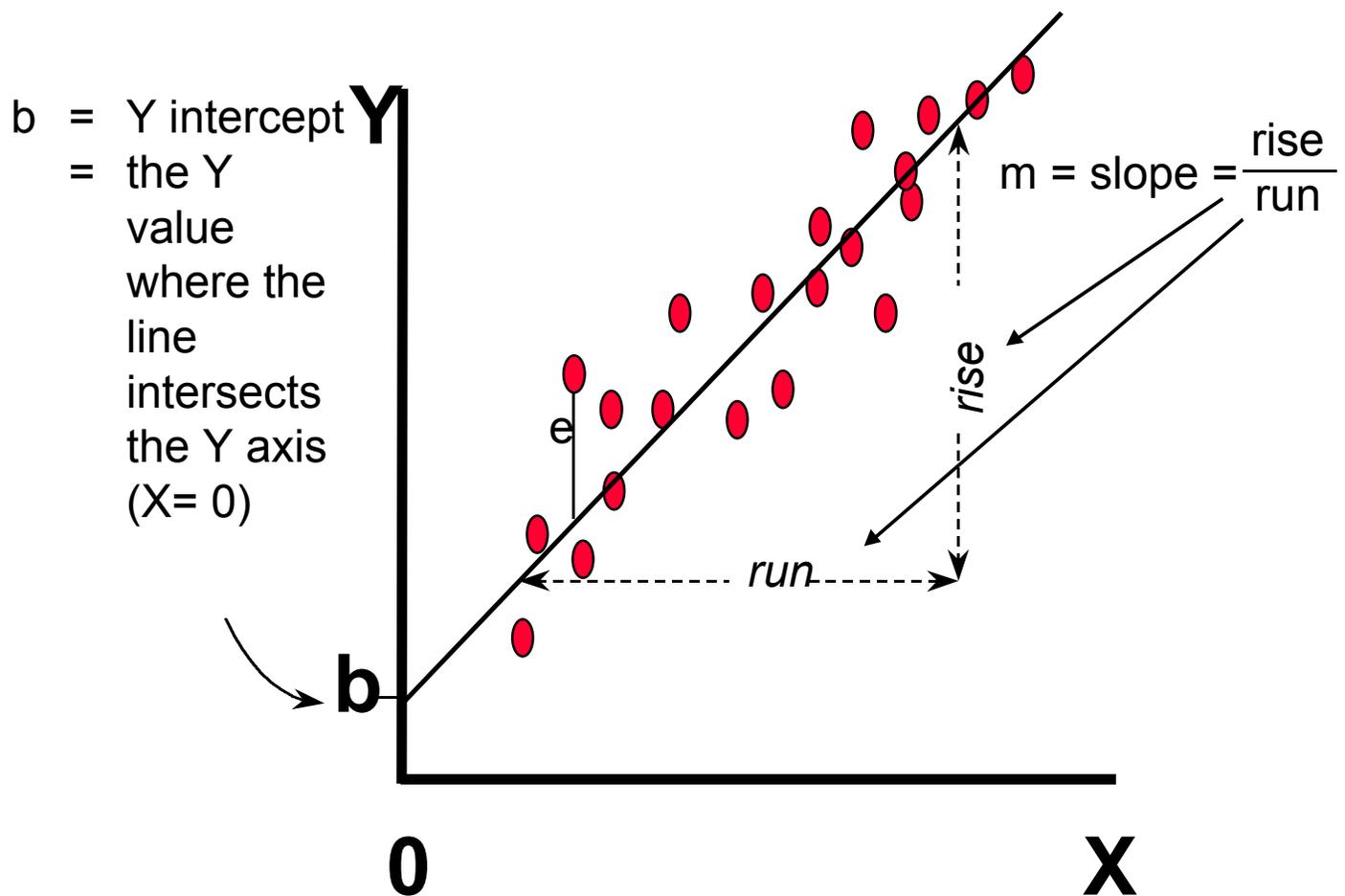
- How good of a job should we be able to do in predicting Y?
- Are there one or more Xs that simultaneously have strong effects on Y?
- Are there non-linear relationships between Y and one or more Xs?
- Are the X variables “clean?” Can the effects of the Xs be separated?
- How do we know that a regression equation is statistically valid?
- Does the model make sense?
- Is the set of Xs under study complete? Are there missing X variables?
- Is the model useful? How much confidence can one have in the model’s prediction?
- Can model predictions be confirmed by testing suggested process changes?



## Key Concepts: Simple Linear Relationship

Recall that a simple linear relationship can be described mathematically by

$$Y = mX + b$$



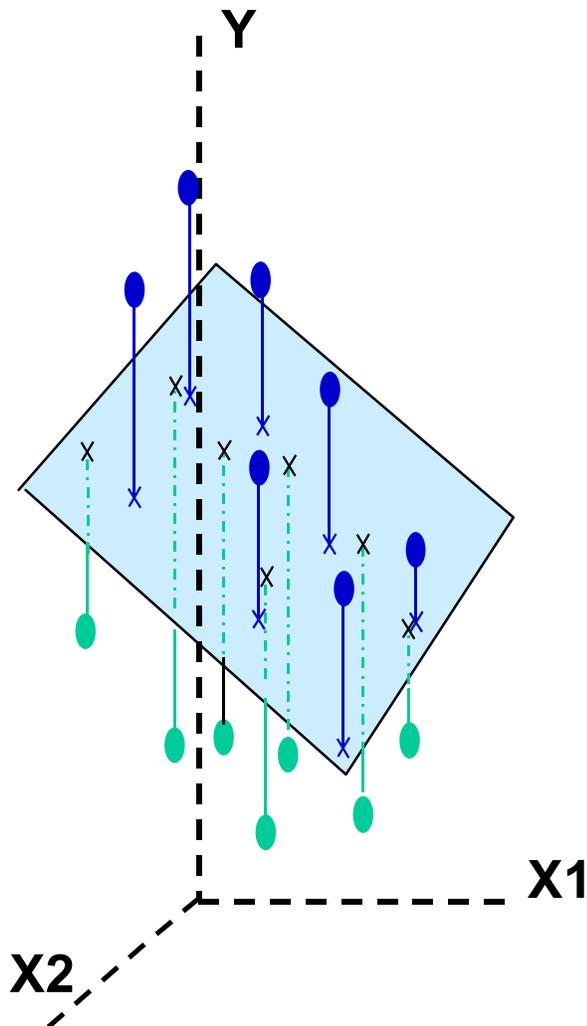
**What if we have two X's?**



## Key Concepts: Multiple Linear Relationship

Now we are fitting points to a plane, rather than a simple line.

The equation of a plane is  $Y = b + m_1(X_1) + m_2(X_2)$



- The plane (or hyperplane) is determined according to the principle of least squares. This means that the sum of the squares of the distances (parallel to the y axis) from the data point to the line (plane or hyperplane) is minimized.
- The best-fit plane is centered in the plotted data with the actual data points dispersed randomly above and below the plane.



## Key Concepts: Multiple Linear Relationship

### Multiple Regression with more than 2 Xs

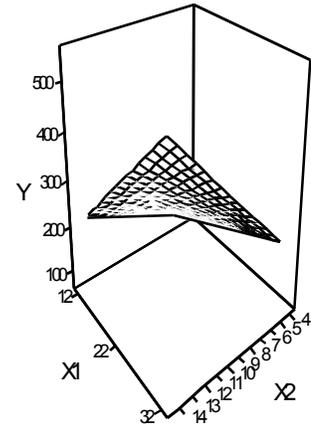
- Beyond two Xs (three dimensions), we cannot visualize the best-fit shape; however, the principles of regression still apply.
- You are fitting the best “hyperplane” with an equation,

$$Y = b + m_1(X_1) + m_2(X_2) + \dots + m_n(X_n)$$

### Multiple Regression with an interaction

- An interaction will change the tilt in the plane and put a kink in the plane.
- You are fitting the best “hyperplane” with an equation,

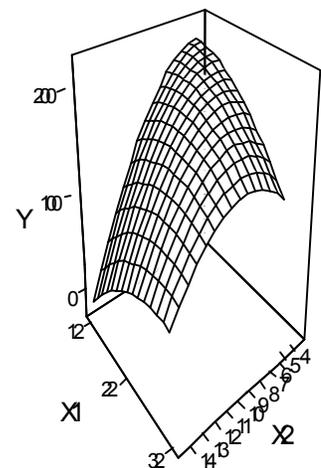
$$Y = b + m_1(X_1) + m_2(X_2) + m_3 (X_1 * X_2)$$



### Multiple Regression with curvature

- Curvature results from higher order terms
- You are fitting a curved plane,

$$Y = b + m_1(X_1) + m_2(X_2) + m_3(X_1 * X_2) + m_4(X_1)^2 + m_5(X_2)^2$$



**Multiple Regression is a powerful tool!**



# *Multiple Regression -- Advantages & Disadvantages*

---

## ***Advantages:***

- Regression extracts a lot of information from historical data.
- Can be used when have both continuous and discrete Xs
- Can gain information on interactions and higher order terms
- At a minimum, regression will reduce your X variable set before designed experimentation.
- Can obtain a predictive model

## ***Disadvantages***

- In some data sets, the X variables are correlated and interactions cannot be determined. Designed experimentation is the way to separate effects and interactions.



## *How does Multiple Regression compare to:*

### **ANOVA**

- ANOVA uses discrete Xs and a continuous Y...Multiple Regression lets us model a combination of discrete and continuous Xs or all continuous Xs with a continuous Y.
- ANOVA answers the percent contribution question...Multiple Regression will provide a model that can be used to predict behavior.

### **ANOVA GLM /Sum of Squares Analysis**

- ANOVA GLM/SS enables percent contributions...Multiple Regression provides information on which variables are significant

### **Design of Experiments (DOE)**

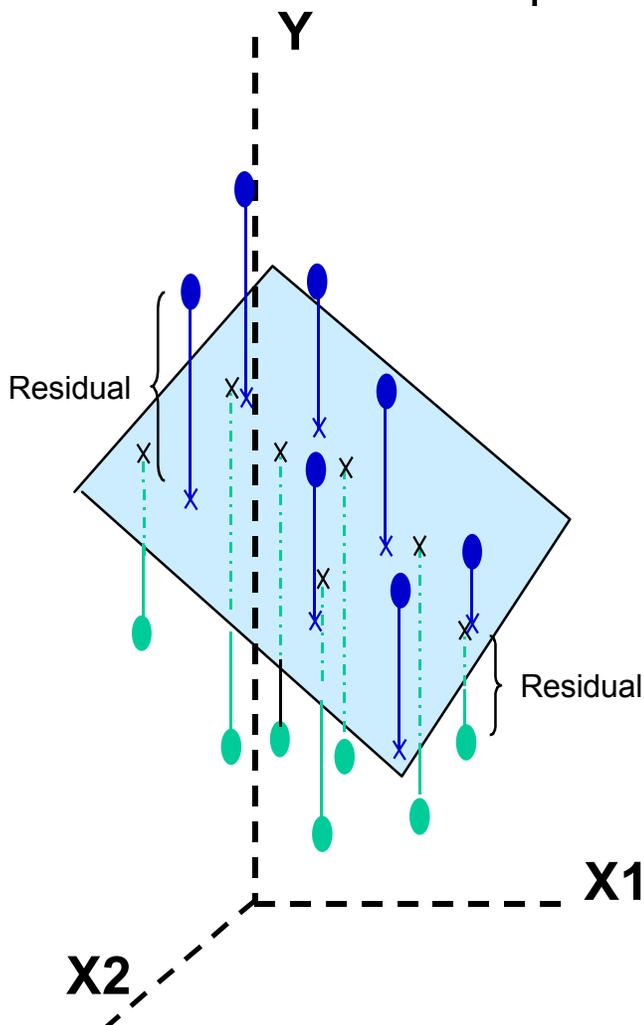
- In designed experiments, the data is collected in a manner that allows the X factors and interaction effects to be separated...Multiple Regression uses historical data in which the X factors and interactions may be correlated with each other. Thus the effects of one factor may be combined with another factor.



## Key Concepts: Residuals

### Residuals --

Are the difference between the actual response value and the fitted value from the equation.



Residuals estimate an inability to predict.

In a valid model, the residuals will be normally distributed about a mean of zero, independent of the X and Y values and randomly distributed over time.



## Regression: Getting Started

### Step

- 1) Review your process map and cause and effect diagram to identify possible Xs. What Xs are controllable versus noise? What things might you change if an X is important?
- 2) Compare the standard deviation of your dataset with the standard deviation from your Measurement Systems Analysis. The larger the standard deviation of your data is compared to the standard deviation from your MSA study, the greater the opportunity to find X's with p-values less than 0.05.
- 3) Draw a sketch of the relationship you believe exists between each X and the process output, Y. This will help you capture your team's theories as well as learnings from other analyses.
- 4) Draw X-Y scatterplots for each X. Do they match your predictions? Do you see any evidence of curvature? How much noise is there?
- 5) **Run a correlation study between the Xs.** If the correlations among Xs are low, the Xs are clean and the effects will separate. If the correlations among Xs are high, the Xs are tied together and you need to check your process knowledge as to which X makes most sense. A correlation between two Xs is high if the correlation coefficient ( $r$ ) is greater than the threshold correlation coefficient ( $r_{\text{threshold}}$ ). This is a rule of thumb for a 95% confidence level.

$$r_{\text{threshold}} = \frac{2}{\sqrt{N + 2}}$$

where  $N$  is the number of data points



## Step 6: Run the Analysis

**Stat>Regression>Regression**

The screenshot shows the Minitab Regression dialog box. The 'Response' field contains 'RespY'. The 'Predictors' field contains 'FactorA FactorB FactorC'. The 'Options...' button is highlighted with a yellow circle and an arrow pointing to it from the text 'Check "Display Variance Inflation Factor" under options'. The 'OK' button is also highlighted with a yellow circle and an arrow pointing to it from the text 'Select the residuals plot options'. Other buttons like 'Select', 'Graphs...', 'Storage...', 'Help', and 'Cancel' are visible.

Enter the response

Enter the X factors

Check "Display Variance Inflation Factor" under options

Select the residuals plot options

Regression will be usually be an iterative process. Select all the X factors believed to be important and enter into the first analysis. Eliminate the insignificant terms one at a time and re-run the analysis. Of course, you'll need to complete the other analysis steps outlined on the next page as well.



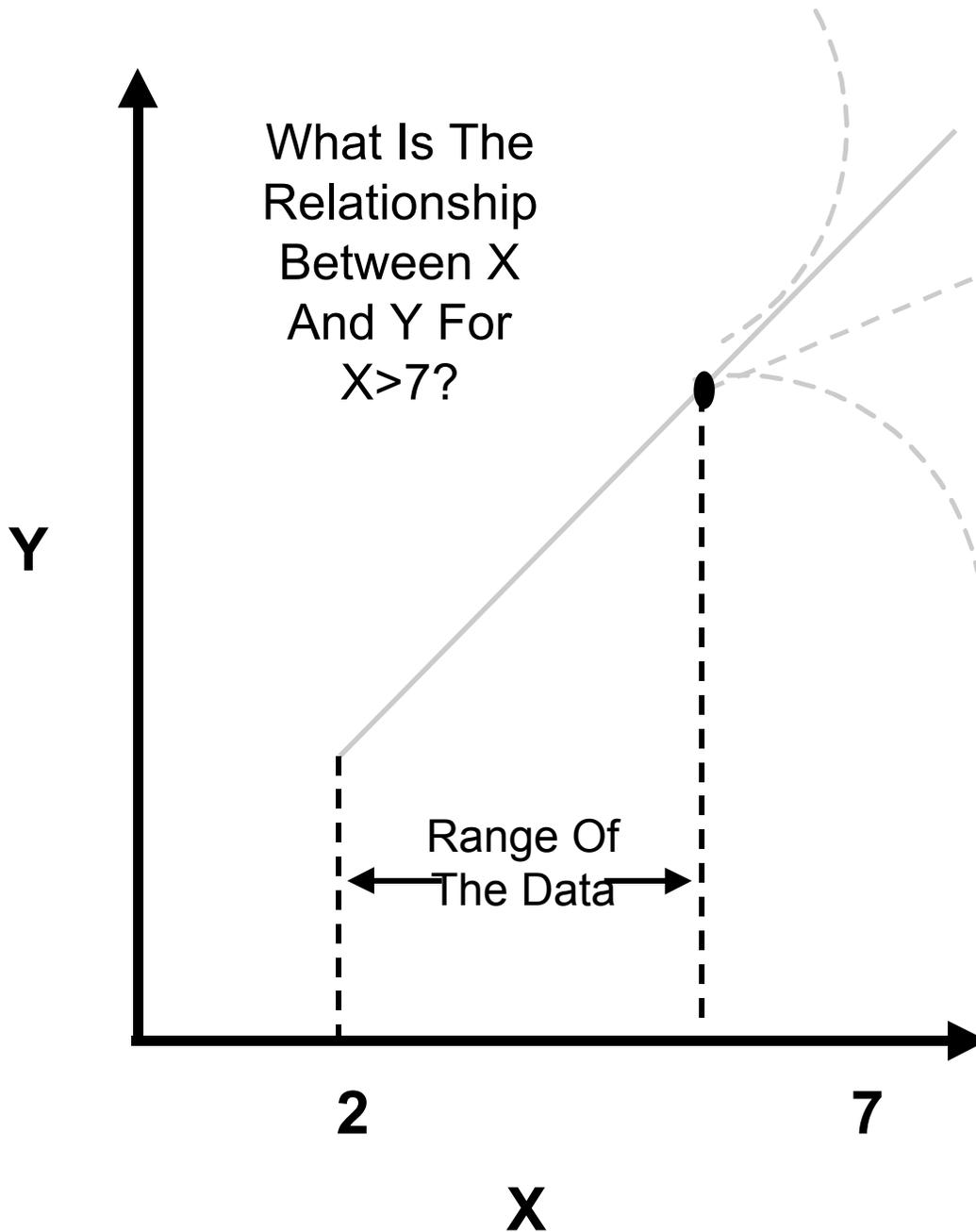
## *Regression: After the Analysis*

### Step

- 7) Is the X significant? Look for p-values  $< 0.05$  or even  $< 0.10$ . Models with insignificant terms overfit the model. Remove the insignificant terms one at a time and systematically. **Note:** if an interaction or squared term for a factor is significant, you must keep the individual factor term.
- 8) Look at the residual plots. Do they indicate any problems with the analysis?
- 9) You are explaining as much variation as can be expected as long as the R-Sq(adj) % + Total %GRR add up between 90 and 100%. Compare the standard deviation of the residuals with the standard deviation of the measurement system (Standard Deviation--Total Gage R&R). The standard deviation of the residuals should not be lower than the standard deviation of the measurement system--check it with an F test, if necessary!
- 10) Do a sanity check -- do the results make sense? Were there surprises? Did the results match your predictions?
- 11) Were there any outlier residuals? If you eliminate them and rerun the regression, do you see any improvement? **Note:** As a rule of thumb, only eliminate the residuals greater than 3 standard deviations from the mean. This should only be about 5% of the data.
- 12) Rerun the analysis as needed for other Xs, or to include non-linear terms.



# Extrapolation Is Risky





---

# *General Linear Model*



## What if you don't have “balanced” data?

- General Linear Model (**GLM**) is the tool to use.
- GLM can handle “unbalanced” data - data sets with **unequal observations per subgroup**.
- Unbalanced data is often seen in historical or baseline data.

### Example:

- ‘Rot1’ (C3) is a continuous response variable that is believed to be a function of Oxygen and Temperature.
- Temperature (C1) is represented by two levels: 10 and 16
- Oxygen (C2) has 3 levels: 2, 6 and 10



# Here's the data set

MINITAB - ROT.MTW - [Data]						
	C1	C2	C3	C4	C5	C6
↓	Temp 1	Oxygen 1	Rot 1			
1	10	2	13			
2	10	2	11			
3	10	2	3			
4	10	6	10			
5	10	6	4			
6	10	6	7			
7	10	10	7			
8	10	10	*			
9	10	10	*			
10	16	2	26			
11	16	2	19			
12	16	2	24			
13	16	6	15			
14	16	6	*			
15	16	6	22			
16	16	10	20			
17	16	10	24			
18	16	10	8			
19						

Missing data

**Why are these data “unbalanced”?**

**Because there are unequal numbers of observations per cell:**

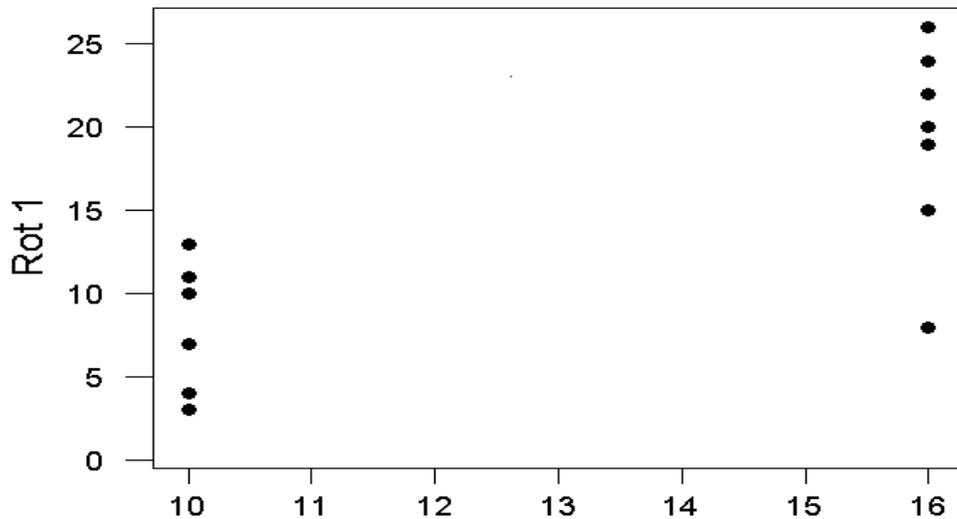
	Temp = 10	Temp = 16
Oxygen = 2	n = 3	n = 3
Oxygen = 6	n = 3	n = 2
Oxygen = 10	n = 1	n = 3



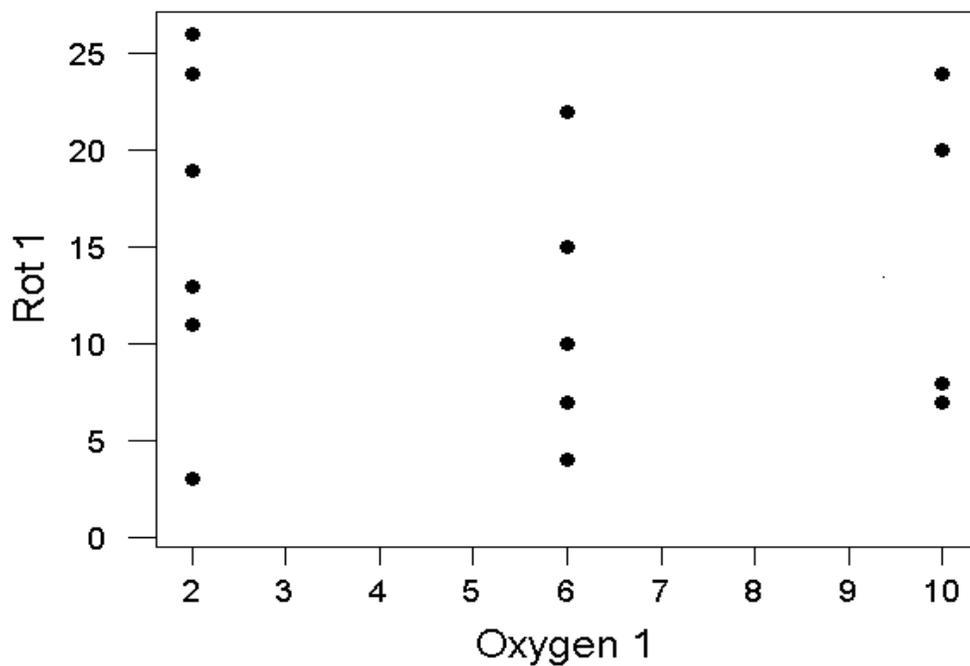
# Graph the data first

Select: **Graph > Plot** and place the appropriate variable in the dialog box for Y and X.

Y = Rot 1  
X = Temp 1



Y = Rot 1  
X = Oxygen 1

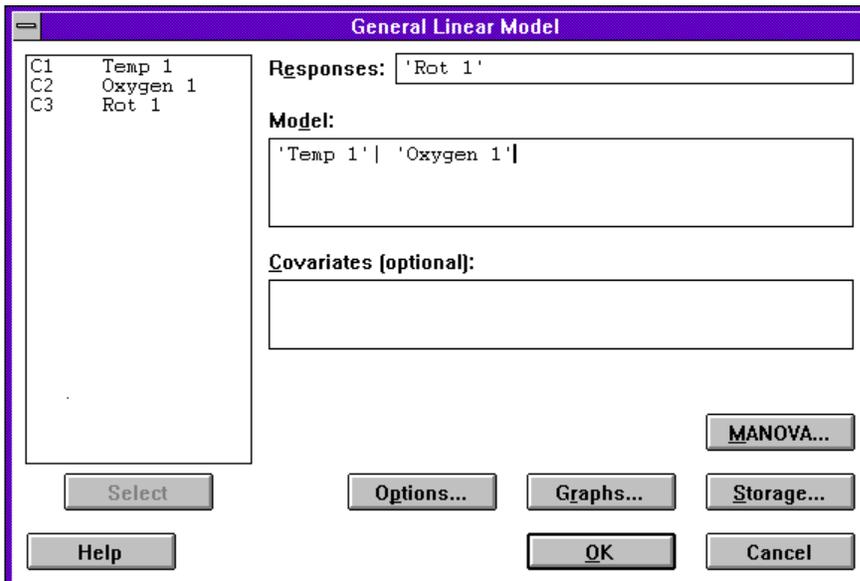
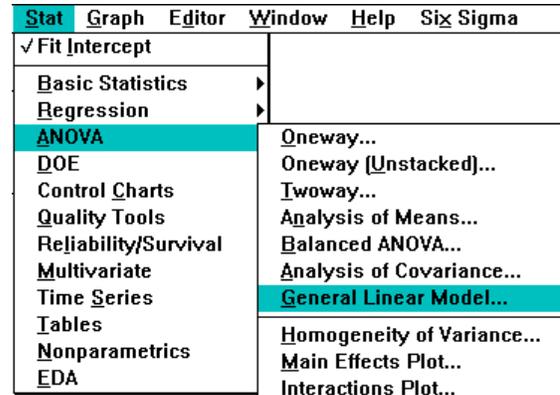




# The GLM analysis...

## Stat>ANOVA>General Linear Model

**Note:** Since we have un-balanced data, we use the GLM analysis option of ANOVA.



Remember to use 'pipes' between 'Temp 1' and 'Oxygen 1' to include interactions

Remember to click on 'Graphs' to create 'Residual vs. fits' graph



## ANOVA Analysis in the Session window

### General Linear Model

Factor	Levels	Values		
Temp1	2	10	16	
Oxygen1	3	2	6	10

### Analysis of Variance for Rot1

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Temp1	1	528.04	453.19	453.19	15.50	0.003
Oxygen1	2	51.19	41.57	20.78	0.71	0.517
Temp1*Oxygen1	2	8.00	8.00	4.00	0.14	0.874
Error	9	263.17	263.17	29.24		
Total	14	850.40				

### Unusual Observations for Rot1

Obs	Rot1	Fit	StDev Fit	Residual	St Resid
7	7.0000	7.0000	5.4075	0.0000	* X
18	8.0000	17.3333	3.1220	-9.3333	-2.11R

R denotes an observation with a large standardized residual  
 X denotes an observation whose X value gives it large influence.

### Interpretation:

(Look at the p-values for the significant factors)

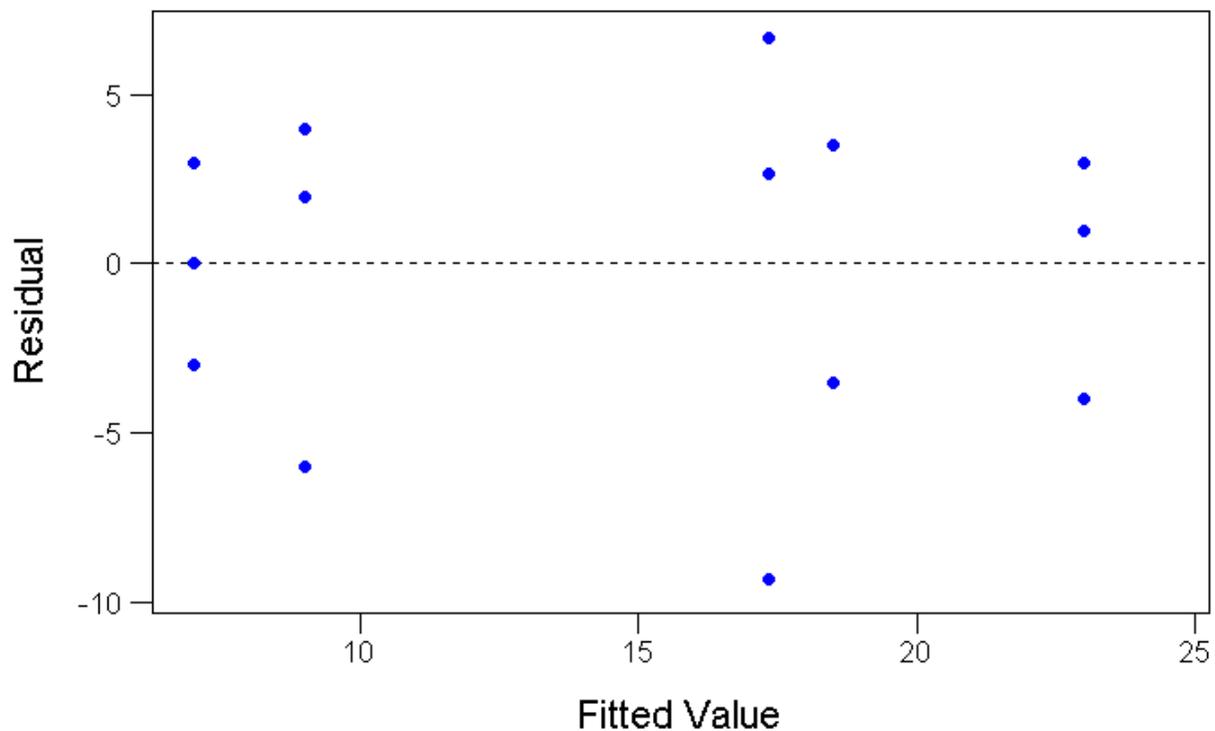
- Temperature is significant,  $p < 0.05$
- Not significant: Oxygen, and the interaction between Oxygen and Temperature,  $p > 0.05$
- The Error term is large relative to the Total SS.  
Possibly search for more “X”s!
- If you get a Minitab error message which says rank deficiency, you do not have enough data to model the terms you have chosen. Try to eliminate terms or see MBB.



## Analyze the Residuals

### Residuals Versus the Fitted Values

(response is Rot 1)



There does not appear to be a pattern in the residuals - we have not gained additional process information from this graph.



---

# *Analyzing Messy Data*

## *Optional Example of GLM*



## *Exercise*

---

**Perform multiple regression and / or GLM using the file messy1.mtw.  
The Y is thrust.**



## *Description Of Messy Data And It's Analysis Problems*

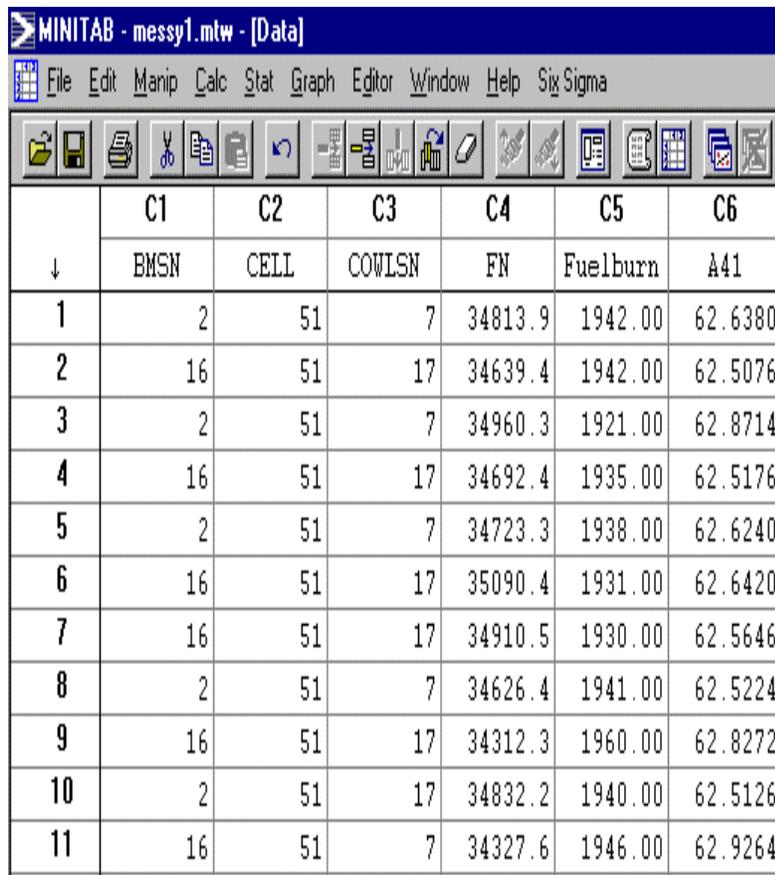
---

- *Messy data is often historical data collected without any design or structure.*
- *Messy data is sometimes called happenstance data.*
- *In many cases, messy data is difficult to analyze because of its lack of structure. For example, it will be unbalanced and often will have missing cells. The X's in the model may be correlated.*
- *A cell is each possible pair of data tag levels in your model.*
  - *Unbalanced means that the number of data points for each cell is unequal.*
  - *A missing cell means no data exists in that cell.*
  - *Multicollinearity means the X's are correlated.*



## Example: Engine Performance Data

File messy1.mtw contains engine performance data collected from past records. It is historical data without any designed structure to it.



	C1	C2	C3	C4	C5	C6
↓	BMSN	CELL	COWLSN	FN	Fuelburn	A41
1	2	51	7	34813.9	1942.00	62.6380
2	16	51	17	34639.4	1942.00	62.5076
3	2	51	7	34960.3	1921.00	62.8714
4	16	51	17	34692.4	1935.00	62.5176
5	2	51	7	34723.3	1938.00	62.6240
6	16	51	17	35090.4	1931.00	62.6420
7	16	51	17	34910.5	1930.00	62.5646
8	2	51	7	34626.4	1941.00	62.5224
9	16	51	17	34312.3	1960.00	62.8272
10	2	51	17	34832.2	1940.00	62.5126
11	16	51	7	34327.6	1946.00	62.9264

BMSN: Bell Mouth  
Serial Number

Cell: Test Cell

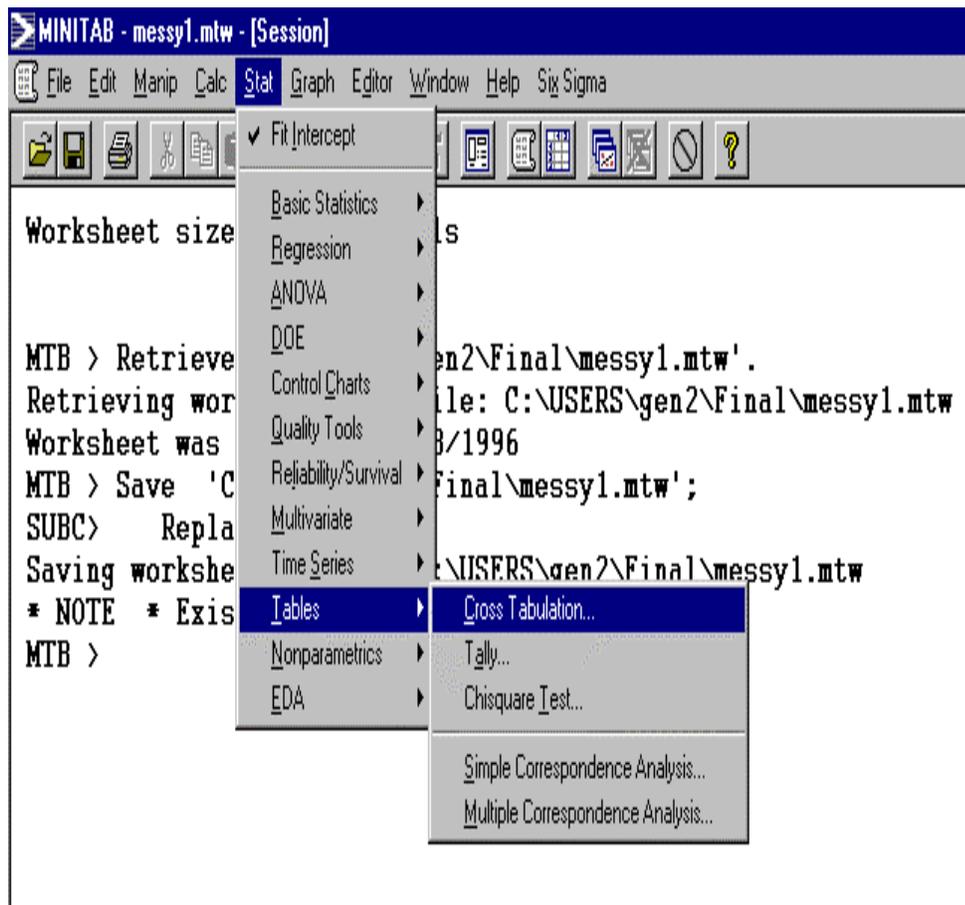
COWLSN: Cowl  
Serial Number

FN: Engine  
Thrust



# Inspection Of The Data

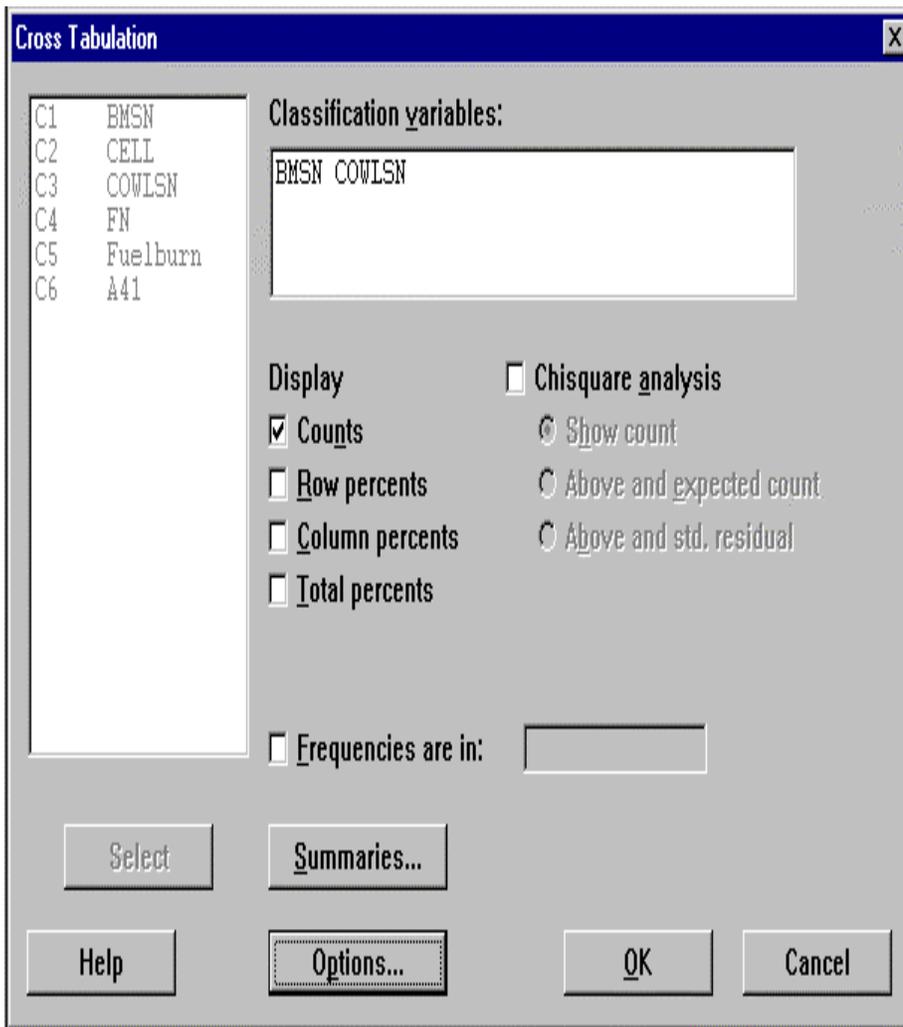
Use the cross tabulation command to view the structure of the data.





## Inspection Of The Data

We will analyze the effects of bell mouth and cowl on fuelburn. Load in the variables for bell mouth and cowl.



For now just check the counts under Display.



## *Inspection Of The Data*

**Rows: BMSN    Columns: COWLSN**

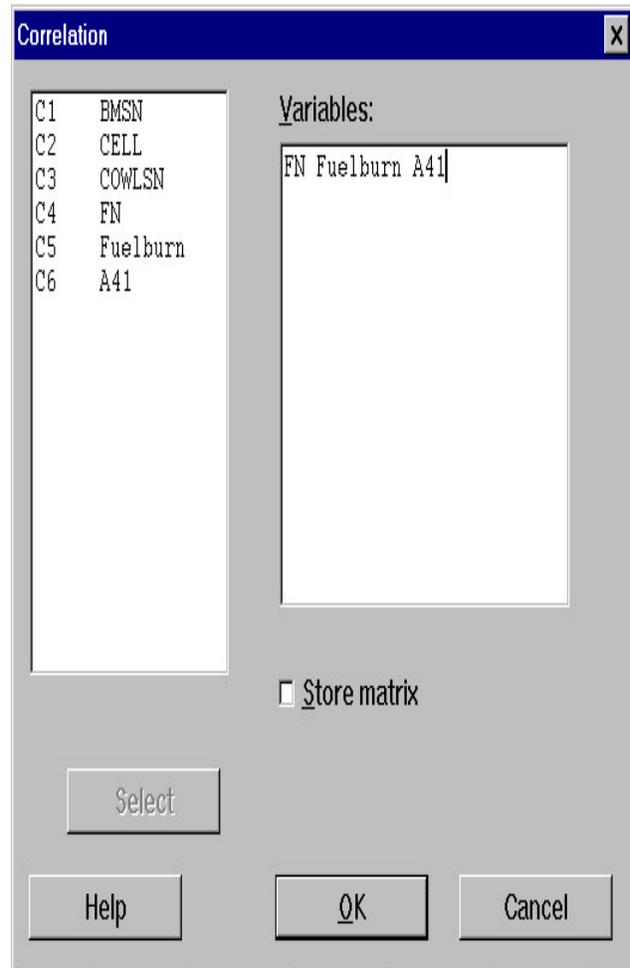
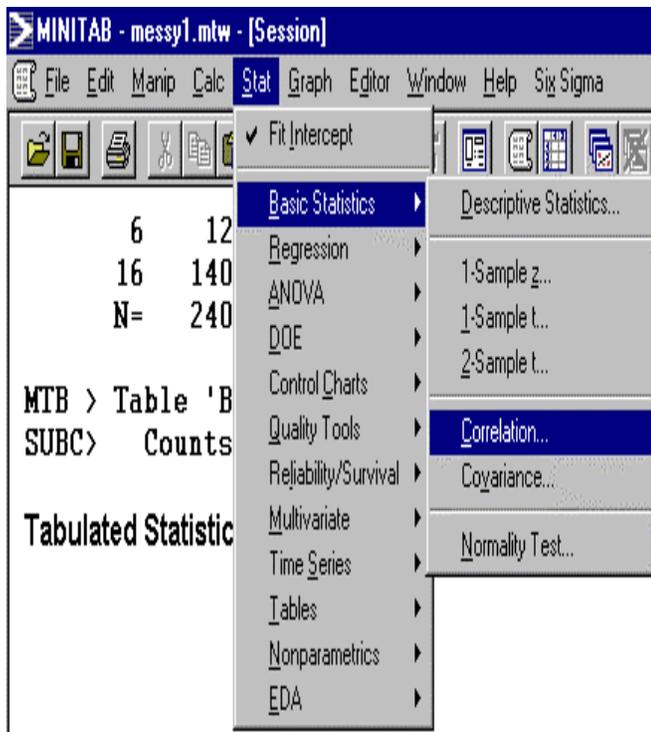
	<b>3</b>	<b>7</b>	<b>12</b>	<b>17</b>	<b>All</b>
<b>1</b>	0	0	3	0	3
<b>2</b>	15	47	3	15	80
<b>4</b>	0	2	3	0	5
<b>6</b>	9	3	0	0	12
<b>16</b>	41	62	29	8	140
<b>All</b>	65	114	38	23	240

Is the data balanced? Are there missing cells?



# Inspection Of The Data

Are the X's correlated? To find out, run a correlation analysis on the continuous X's.





## *Inspection Of The Data*

### **Correlations (Pearson)**

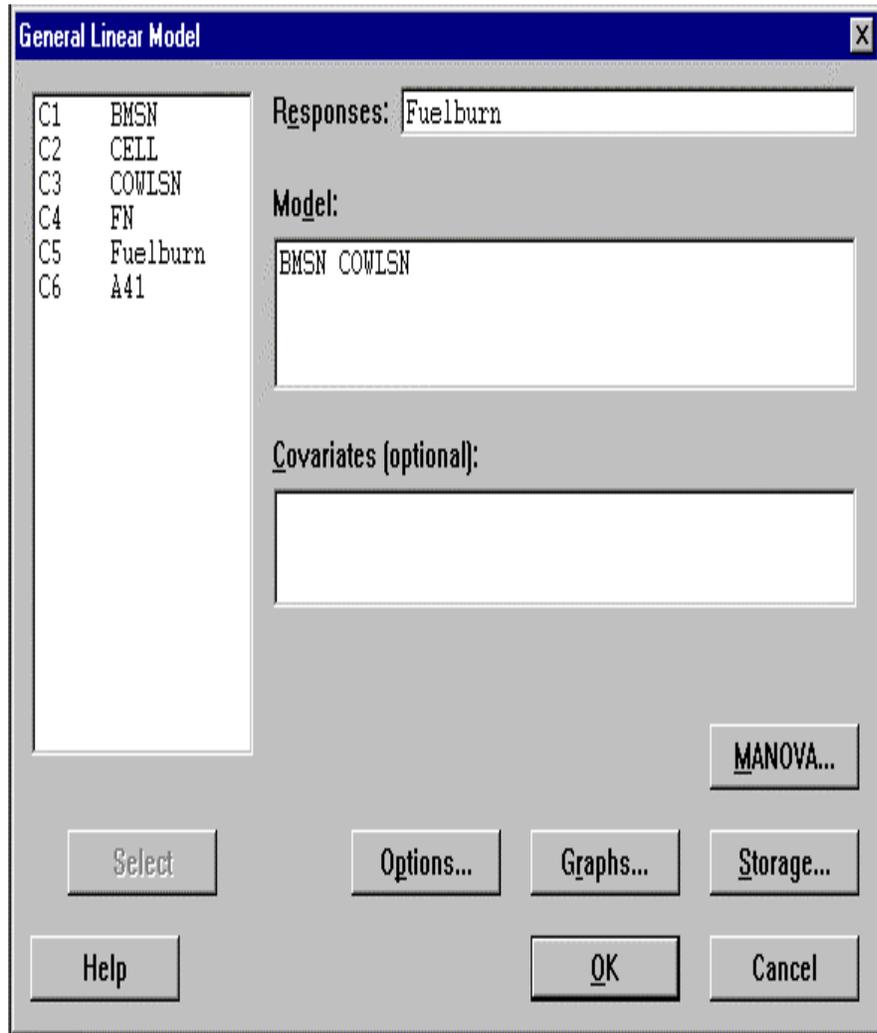
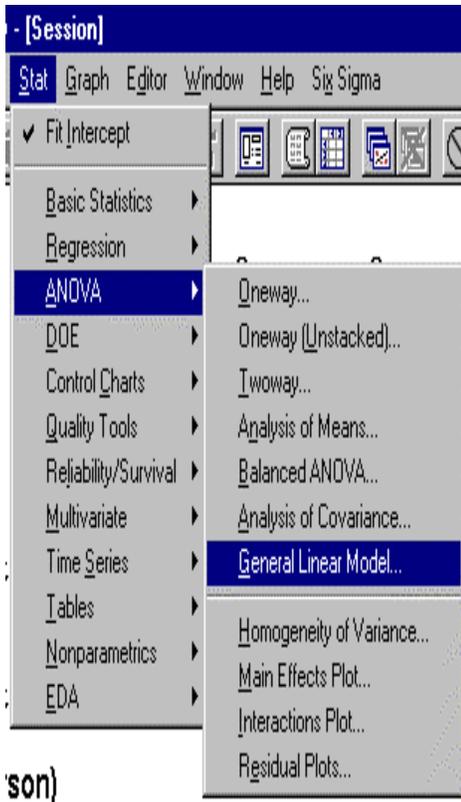
	FN	Fuelburn
Fuelburn	-0.246	
A41	-0.035	-0.108

The correlation coefficient between Fuelburn and A41 is -0.108. This is a low value indicating collinearity is not a problem for this analysis. High values of correlation,  $> 0.9$ , may cause a problem.



# Analysis Using GLM

Using GLM, analyze the effect of bell mouth and cowl on fuel burn.





# Analysis Using GLM

## General Linear Model

Factor	Levels	Values
BMSN	5	1 2 4 6 16
COWLSN	4	3 7 12 17

## Analysis of Variance for Fuelburn

Source	DF	Seq SS	Adj SS	Adj MS	F	P
BMSN	4	425.3	820.6	205.2	1.27	0.282
COWLSN	3	1622.0	1622.0	540.7	3.35	0.020
Error	232	37468.5	37468.5	161.5		
Total	239	39515.8				

## Unusual Observations for Fuelburn

Obs	Fuelburn	Fit	StDev Fit	Residual	St Resid
29	2006.00	1928.98	1.44	77.02	6.10R
150	1906.00	1931.18	1.74	-25.18	-2.00R
.	.	.	.	.	.
.	.	.	.	.	.

What are your conclusions?



## *Analysis Using GLM*

- *Even when the correlation between the  $X$ 's is small, they are still not orthogonal ( independent ).*
- *Therefore, GLM uses the adjusted sum of squares for its hypothesis testing. The  $p$ -values come from using the adjusted mean squares.*
- *The adjusted sum of squares assumes that  $X$  was entered last in the model. Without orthogonality, the order in which the  $X$ 's are entered changes the sequential sum of squares.*
- *This was covered in the Analyze phase.*



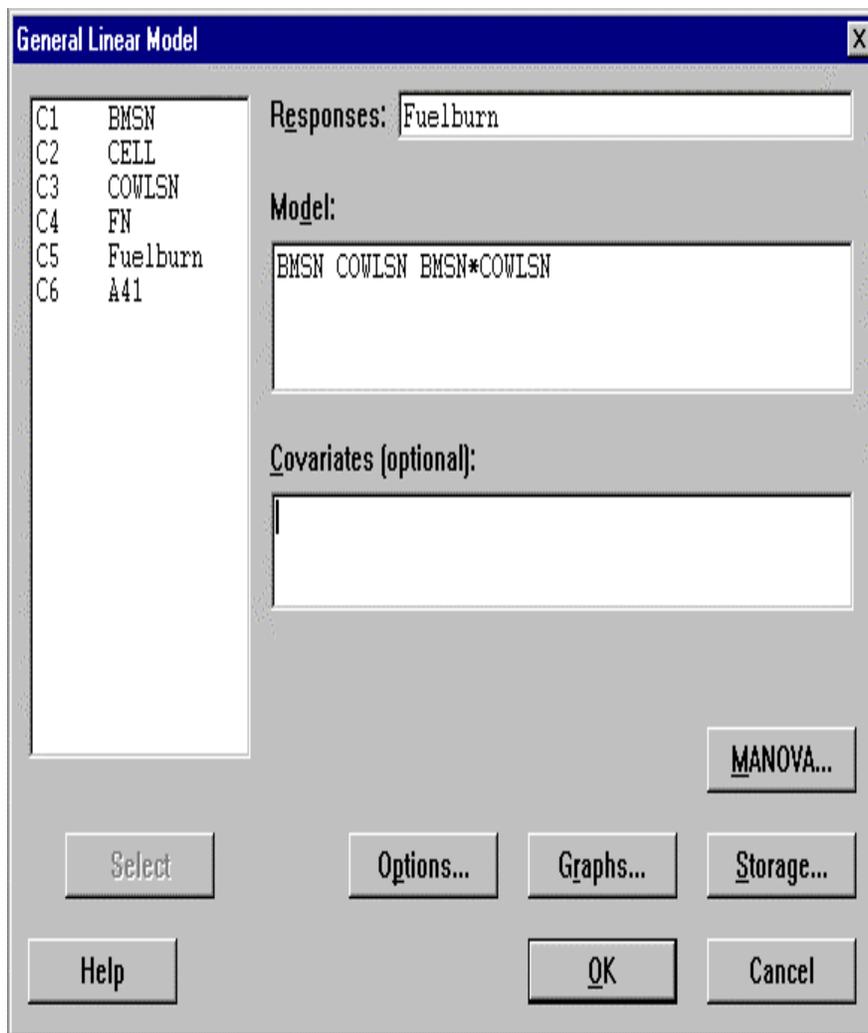
## *Analysis Using GLM*

- *Do the sequential sum of squares add to the total?*
  - *Note that the sequential sum of squares depend on the order in which the X's were entered into the model.*
- *Do the adjusted sum of squares add to the total?*
- *What are the implications of this to estimating the percent contribution of the X's?*



## Analysis Using GLM With Interaction

Suppose you want to investigate the interaction between bell mouth and cowl. Include the interaction in the model window.





# Analysis Using GLM With Interaction

## General Linear Model

### Factor Levels Values

BMSN	5	1	2	4	6	16
COWLSN	4	3	7	12	17	

### Analysis of Variance for Fuelburn

Source	Model DF	Reduced DF	Seq SS
BMSN	4	4	425.33
COWLSN	3	3	1622.00
BMSN*COWLSN		12	5+ 335.64
Error	220	227	37132.83
Total	239	239	39515.80

+ Rank deficiency due to empty cells, unbalanced nesting, collinearity, or an undeclared covariate.  
No storage of results or further analysis will be done.

The rank deficiency means it was unable to estimate the means for all the levels of each X.

Based on our previous investigation of the data, what do you think is the reason for the rank deficiency?



## *Exercise*

---

- *Use file messy2.mtw.*
- *Analyze the effect of bell mouth and cowl on engine thrust.*
- *First inspect the data using the cross tabulation command.*
- *Analyze using GLM with and without the interaction.*



## *Key Learnings*

- *Unbalanced data ensures the  $X$ 's are correlated forcing the use of adjusted sum of squares.*
- *Estimation of the percent contribution of the  $X$ 's to total variability is risky.*
- *Empty cells may make estimation of the level means and interactions impossible.*
  - *This is also true when there is too little data for the number of levels for which the means must be estimated.*



## *Other Concerns In Analyzing Messy Data*

- *You might only have data for a few X's none of which may be a vital X. Much of what is relevant isn't even recorded.*
- *Collected over a long period of time, the important X's could actually change. Standards, procedures, materials, and measurements change over time.*
- *For the given time period over which the data was collected, the range in the X's may be so small that the effect is not detectable.*
- *X's often change together confounding their effects.*
- *A lurking variable, confounded with an X may produce a nonsense correlation. We need X and Y to be causally correlated.*
- *The process may no longer be operating under the conditions that existed when the data was collected.*

All of this should help one appreciate the power of designed experiments.





## *Summary of Hypothesis Tests*

### ■ **Chi-Square Goodness of Fit Test**

- $H_o$ : *The hypothesized distribution is a good fit of the data*
- $H_a$ : *The hypothesized distribution is not a good fit of the data*

### ■ **Chi-Square Test of Independence**

- $H_o$ : *A factor has no effect on the output*
- $H_a$ : *A factor has an effect on the output*

### ■ **Linear Regression**

- $H_o$ : *The model is not a significant predictor of the response*
- $H_a$ : *The model is a significant predictor of the response*



## *Take Aways—Hypothesis Testing: Continuous Y; Continuous X*

- *Scatter Plots: Visual tool to establish a cause and effect relationship between the inputs and the outputs.*
- *Simple Linear Regression*
  - *Statistical technique used to investigate the relationship between 2 variables*
  - *$H_a$ : The factor is a significant predictor of the response*
  - *$R^2$ : percent of variation explained by your model. In general, the closer  $R^2$  is to 1, the better the fit of the model*
  - *Prediction Intervals: 95% of data within the population falls within this band*
  - *Confidence Intervals: There exists a 95% chance that the true line of the population lies within the band*
  - *Prediction Interval: Can be used in statistical tolerancing*
    - *To determine where to set the factor levels to remain within the USL and LSL*



# Summary: Process & Population Sampling

Situation	Purpose/Aim	Considerations	Sample Size	Approaches
Sampling from a process	Take action or predict the future <ul style="list-style-type: none"> <li>■ In control?</li> <li>■ Capable?</li> <li>■ Improve</li> </ul>	<ul style="list-style-type: none"> <li>■ Where you sample</li> <li>■ Frequency</li> <li>■ Grouping</li> <li>■ Representative</li> <li>■ Cost</li> </ul>	<ul style="list-style-type: none"> <li>■ Use guidelines appropriate to analysis tool selected or use population methods on stable processes</li> <li>■ Control charts</li> </ul>	<ul style="list-style-type: none"> <li>■ Subgroup sampling</li> <li>■ Systematic sampling</li> </ul>
Sampling from a large population	Describe or quantify characteristics	<ul style="list-style-type: none"> <li>■ Precision (<math>\pm\Delta</math>)</li> <li>■ Amount of characteristic's variation (s = Standard deviation)</li> <li>■ Confidence level (95%)</li> <li>■ Representative</li> <li>■ Cost</li> </ul>	<ul style="list-style-type: none"> <li>■ Continuous measurement (e.g., Time)               <math display="block">n = \left(\frac{1.96\sigma}{\Delta}\right)^2</math> </li> <li>■ Attribute data (e.g., Proportion defective)               <math display="block">n = \left(\frac{1.96}{\Delta}\right)^2 p(1-p)</math> </li> </ul>	<ul style="list-style-type: none"> <li>■ Random sampling</li> <li>■ Stratified random sampling</li> <li>■ Systematic sampling</li> </ul>

Note: Here,  $\sigma$  is defined as the standard deviation of a population

Note: Some situations are a hybrid of the two situations



# Summary: Sample Size Formulas Values In The Formula

## Sample Size Formulas

Sample size for estimating averages

Sample size for estimating proportions

$$n = \left( \frac{Z_{\alpha/2} \cdot \sigma}{\Delta} \right)^2$$

$$n = \left( Z_{\alpha/2} \right)^2 P_B(1-P_B)$$

Sample Size Depends On	Where You Get It From	Effects On Sample Size (n)	Example
1. $Z_{\alpha/2}$	Where $\alpha$ is 1 – confidence level and $Z_{\alpha/2}$ is the Z-score representing the end point of the interval. For $\alpha = .05$ , $Z_{\alpha/2} = 1.96$ For $\alpha = .01$ , $Z_{\alpha/2} = 2.58$	As $\alpha$ decreases, $Z_{\alpha/2}$ increases and sample size increases.	$\hat{\sigma} = 10$ , $\Delta = 1$ Case 1: $\alpha = .01$ $\Rightarrow 99\%$ , $Z_{\alpha/2} = 2.58$ $n = \left( \frac{2.58 \cdot 10}{1} \right)^2 = 666$ Case 2: $\alpha = .05$ $\Rightarrow 95\%$ precision $n = \left( \frac{1.96 \cdot 10}{1} \right)^2 = 384$
2. $\Delta$ (Delta)	Desired precision of the estimate or half-width of the confidence interval.  Estimate precision based on business considerations, sample size limitations, or scale of scrutiny (measurement resolution).	$\Delta$ decreases as you require more precision (a smaller confidence interval). As $\Delta$ gets smaller, sample size increases.	$\hat{\sigma} = 10$ , $Z_{\alpha/2} = 1.96$ Case 1: $\Delta = 1$ $n = \left( \frac{1.96 \cdot 10}{1} \right)^2 = 384$ Case 2: $\Delta = 1/2$ $n = \quad \quad = 1537$
3. $\sigma$ (Sigma)	The standard deviation of the population you are measuring.  Estimate $\sigma$ by calculating the standard deviation of a small sample of data, using control charts, or taking 1/6 the plausible range of data.	As $\sigma$ increases, sample size increases	$\Delta = 1$ , $Z_{\alpha/2} = 1.96$ Case 1: $\hat{\sigma} = 5$ $n = \left( \frac{1.96 \cdot 5}{1} \right)^2 = 96$ Case 2: $\hat{\sigma} = 10$ $n = \left( \frac{1.96 \cdot 10}{1} \right)^2 = 384$
4. $P_B$ (Proportion defective)	$P_B$ ranges from 0 to 1. It is the proportion of defectives (or alternately, non-defectives) in the population.  Estimate $P_B$ by taking a small sample of data, using control charts or setting $P_B = .5$ as a worst case.	Sample size is highest when $P_B = .5$ . As $P_B$ decreases or increases from $P_B = .5$ sample size decreases.	$\Delta = .05$ , $Z_{\alpha/2} = 1.96$ Case 1: $p = .5$ $n = \left( \frac{1.96}{.05} \right)^2 (.5)(.5) = 384$ Case 2: $p = .25$ $n = \left( \frac{1.96}{.05} \right)^2 (.25)(.75) = 288$



## *Take Aways - Step 6*

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- **A fishbone diagram** is a brainstorming tool
  - identify *Xs* that may impact the *Y* that is important in a project
  - provides a visual display of all possible causes of a specific problem
  
- **A Pareto chart** is used to separate the vital few from the trivial many in a process to determine where to focus improvement efforts.
  - displays the frequency of an output occurring by any category you may choose
  - shows the categories in order of decreasing frequency



## *Take Aways - Step 6*

---

- **Samples** can be used to make inferences about the populations from which they are drawn.
  - In general, it is very difficult to measure the entire population, so data may be collected from a **representative sample**.
  - If the sample data is truly representative of the population, statistical inferences may be made about the entire population.
  
- A **hypothesis** is a conjecture about one or more parameters of a **population**.
  - A Null Hypothesis ( $H_0$ ) states that there is no significant difference between processes or products (status quo).
  - An Alternate Hypothesis ( $H_a$ ) states that there is a significant difference between processes or products.



## *Take Aways - Step 6*

- *There is **risk** involved in hypothesis testing. We associate this risk with the probability of making a wrong decision.*
  - *Type I (alpha) error is rejecting  $H_0$  when  $H_0$  is true.*
    - *accepting that a change has taken place when one has not*
  - *Type II (beta) error is accepting  $H_0$  when  $H_a$  is true.*
    - *accepting that the process has not changed when it really has changed*
  
- *It is not possible to make both a Type I (alpha) error and a Type II (beta) error simultaneously.*
  
- *For the probability of making a Type I error we set the limit Alpha usually at 0.05. For the probability of making a Type II error we set the limit Beta usually at 0.10. The quantity (1-Beta) is called Power of the Test.*



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# *McDonald's Case Study*

## *Optional Example*



## *Measure Select CTQ Characteristic*

*You are the McDonald's Green Belt Project team for this area. In an effort to increase customer satisfaction and capture more market share, you performed customer surveys and a QFD analysis. Through your thorough analysis, you found service time to be a major CTQ. You have also broken service time into two parts:*

- 1. wait time in line*
- 2. order time (time from when you begin to order your food until you are given your food and change)*



## *Measure Data Collection*

- *Your first efforts will focus on order time. You have a hunch that order time may differ for drive-thru versus counter service and for ordering by numbers versus off menu. Two classes of Six Sigma trainees were sent out for lunch, half to the Schenectady (Union St.) McDonald's and half to Clifton Park. About half of the people were asked to order at the drive-thru and half at the counter. The following data was collected:*

*Order Time [seconds]  
Location [1 Schenectady, 2 Clifton Park]  
Service [1 Drive-thru, 2 Counter]  
Order Method [1 By Number, 2 Off Menu]  
Time of Day [hh.mm]*



## *Measure Define Performance Standards*

---

- *To determine what the performance standard should be, you surveyed 100 customers, asking them: “To ensure repeat business, how much time are you willing to spend from when you start to order your food until you are given your food and change?” Based on these results, you determined the performance standard for order time to be 90 seconds.*



## Analyze

---

- *In your analysis of the McDonald's data, you will*
  - *compute the current order time process capability*
  - *determine which factors (location, service, order method) have a significant affect on the output (order time)*



## *Instructions*

---

- ***Minitab file: Mcd.mtw*** contains the McDonald's data.
- *On the following page you will find a list of all the tools we have covered. You will use these tools to perform the analysis.*
- *Before starting the actual analysis of the McDonald's data, take some time as a team to determine the objective, hypotheses (when applicable) and potential conclusions for each of the statistical tools.*
- *At the end of this section, you will find questions about the McDonald's data.*
- *Answer the questions assigned to your team.*
- *Hint for comparing the data subgrouped by 1 factor: you will first have to sort the data by time of day, and then unstack it. For instance, unstacking by location will result in two columns, one for Schenectady and one for Clifton Park. This enables you to do the Run Charts and Histograms.*
- *Present your findings with a Power Point Presentation.*



## Summary of Statistical Tools

Tools	Objective	Hypothesis	Conclusion
Histogram			
Dotplot			
Box Plot			
Run Chart (Sample Size = 1)			
Capability Analysis			
Homogeneity of Variance			
ANOVA			
2-Sample t (Compare Locations)			
2-Sample t (Compare Order Method)			
2-Sample t (Compare Service)			



## Questions for Complete Data Set (All Teams)

MINITAB FILE Mcd.mtw: explanation of subscripts

	Location	Service	Method
1	Schenectady	Drive Thru	By Number
2	Clifton Park	Counter	Off menu

1. *Visualize the distribution of the data. Is the data normally distributed?*

Descriptive Statistics (Graphical Summary) ⇒  
Histogram, Boxplot, Anderson-Darling

2. *What is the order completion process capability, given an upper specification limit of 90 seconds? (leave LSL blank, subgroup rationally)*

Six Sigma Process Report



## *Questions for the Data Subgrouped by 1 factor*

*Teams 1, 4: Schenectady vs. Clifton Park*

*Teams 2, 5: Drive Thru vs. Counter*

*Teams 3, 6: Number vs. Off Menu*

**3.** *Visualize the distribution of the data.*

Histogram

**4.** *Visualize differences between average service time and between service time variation.*

Boxplot

**5.** *Are there any non-random patterns over time?*

Run Chart

**6.** *Is the data normally distributed?*

Normality Test  $\Rightarrow$  Anderson-Darling

**7.** *Do the two groups have the same consistency in speed of service?*

Homogeneity of Variance

**8.** *Is there a statistically significant difference between the average speed of service?*

2-sample t-test



## Questions for Comparing All 8 Subgroups

**9.** *Visualize differences between average service time and between service time variation.*

Boxplot

**10.** *Is the data normally distributed?*

Anderson-Darling

**11.** *Do the eight groups have the same consistency in speed of service?*

Homogeneity of Variance

**12.** *As a corporation, McDonald's has a goal to provide, on average, the same speed of service to customers regardless of location, service type or order method. Based on this sample, does McDonald's reach that goal?*

ANOVA



## *Bonus Question*

**13.** *Given your preference for a combination of service type and order method, would you go to the Schenectady or Clifton Park McDonald's?*

2-sample t-test

<i>Teams 1, 5:</i>	<i>Drive Thru, By Number</i>
<i>Teams 2, 6:</i>	<i>Drive Thru, Off Menu</i>
<i>Teams 3:</i>	<i>Counter, By Number</i>
<i>Teams 4:</i>	<i>Counter, Off Menu</i>



# *Introduction to Improve*

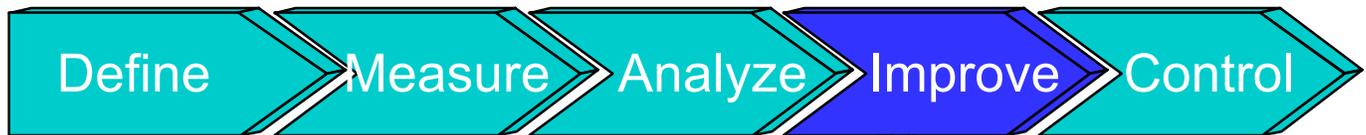


# The 12 Step Process

Step	Description	Focus	Tools	SSQC Deliverables
<b>Define</b>				
A	Identify Project CTQs			Project CTQs (1)
B	Develop Team Charter			Approved Charter (2)
C	Define Process Map			High Level Process Map (3)
<b>Measure</b>				
1	Select CTQ Characteristics	Y	Customer, QFD, FMEA	Project Y (4)
2	Define Performance Standards	Y	Customer, Blueprints	Performance Standard for Project Y (5)
3	Measurement System Analysis	Y	Continuous Gage R&R, Test/Retest, Attribute R&R	Data Collection Plan & MSA (6), Data for Project Y (7)
<b>Analyze</b>				
4	Establish Process Capability	Y	Capability Indices	Process Capability for Project Y (8)
5	Define Performance Objectives	Y	Team, Benchmarking	Improvement Goal for Project Y (9)
6	Identify Variation Sources	X	Process Analysis, Graphical Analysis, Hypothesis Tests	Prioritized List of all Xs (10)
<b>Improve</b>				
7	Screen Potential Causes	X	DOE-Screening	List of Vital Few Xs (11)
8	Discover Variable Relationships	X	Factorial Designs	Proposed Solution (13)
9	Establish Operating Tolerances	Y, X	Simulation	Piloted Solution (14)
<b>Control</b>				
10	Define & Validate Measurement System on X's in Actual Application	Y, X	Continuous Gage R&R, Test/Retest, Attribute R&R	MSA
11	Determine Process Capability	Y, X	Capability Indices	Process Capability Y, X (15)
12	Implement Process Control	X	Control Charts, Mistake Proof, FMEA	Sustained Solution (15), Documentation (16),



# Improve Phase



**7. Screen Potential Causes**

**8. Discover Variable Relationships**

**9. Establish Operating Tolerances**

*Deliverable:*  
*Determine the Vital Few X's That Are Causing Changes in Y.*

*Tools:*  
■ *Screening DOE*

*Deliverables:*  
1) *Establish Transfer Function Between Y and Vital Few X's.*  
2) *Determine Optimal Setting for the Vital Few X's.*  
3) *Perform Confirmation Runs.*

*Tools:*  
■ *Factorial Designs*

*Deliverable: Specify Tolerances on the Vital Few X's.*

*Tools:*  
■ *Simulation*



## *Questions Answered by the Improve Phase*

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- *Why is the improve phase important?*
- *What is the link between improve and my project?*
- *What methods for improvement exist?*
- *Examples of methods used for improvement.*



## *Key Points*

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- *Develop a strategy for improvement.*
- *Characterize and examine the Xs.*
- *For improvement, choose the correct tool.*
- *Consider Improvement methods other than Design of Experiments (DOE).*
- *Pilot the solution.*
- *For more advanced problems use DOE.*



## *Improve Phase Objectives*

- **To develop a *proposed solution*:**
  - *identify an improvement strategy*
  - *experiment to determine a solution*
  - *quantify financial opportunities*
- **To *confirm* that the proposed solution will meet or exceed the quality *improvement goals*:**
  - *a **pilot**: includes one or more small-scale tests of the solution in a real world business environment*
  - *to statistically confirm that an improvement exists (hypothesis tests)*
- **To *identify resources* required for a successful **full-scale** implementation of the solution.**
- **To *plan and execute* full scale implementation including training, support, technology rollout, process and documentation changes.**



## *Improvement Strategy*

---

- *Develop an improvement strategy to provide a framework for developing a solution systematically and efficiently.*
  
- *Strategy will depend upon:*
  - *the nature of your improvement project*
  - *your current level of process knowledge and*
  - *the availability and characterization of the data.*
  
- *Collect data about your process and alternate solutions so you can make informed decisions about how to improve your process.*



## *Improvement Strategy*

---

- *Incorporate different combinations of statistical and quality tools to refine your solution and attain the required process performance.*
  
- *Your improvement strategy may involve:*
  - *Optimizing process performance*
    - *develop a mathematical model for your process by running a design of experiment (DOE) or complete a regression analysis*
    - *determine the best settings for each X*
  
  - *Developing and testing several alternatives by running trial experiments to find the solution that best meets your improvement goals*



## *Improvement Goal*

***The goal of a proposed solution:  
to understand and act on the  
relationship  $Y = f(X)$***

- *Relate the vital Xs to the Y.*
- *Predict the magnitude of the effect of the vital Xs on the Y.*
- *State the direction and magnitude of change in vital Xs to accomplish a change in the Y.*
- *Plan for and implement identified changes in the vital Xs based on the data.*

***Data Driven Analysis  
& Improvement***



## *Characterization of Xs*

### ■ *Operating parameters*

- *Xs that can be set at multiple levels to study how they affect the process Y*
- *changes in their settings impact the Y directly and influence variation*
- *may be continuous and/or discrete*  
*e.g. heat treatment temperature, cycle time, number of people answering phones*

### ■ *Critical Elements*

- *Xs that are independent alternatives*
- *Xs that are not necessarily measurable on a specific scale, but have an affect on the process*  
*e.g. alternative work flow sequences, process standardization, practical solution alternatives*



## *Group Discussion: Identify Vital Xs for a Given Y*

---

*What are key drivers, in both magnitude and direction, of:*

- *RCT sale closure rate?*
- *monthly phone expenses?*
- *turbine efficiency?*
- *drawing accuracy at initial release?*
- *personnel retention rates?*
- *past-due receivables?*
- *training effectiveness?*

*How can we characterize these Xs?*

- *operating parameters*
- *critical elements*

*How would we measure the Xs?*



# *Worksheet: Experimenting to Determine a Solution*

<i><b>If your Xs are ...</b></i>	<i><b>Your improvement strategy is ...</b></i>
<i>Operating parameters (you need to know how they are related to each other and to the Y to develop an appropriate solution)</i>	<ul style="list-style-type: none"><li>• <i>Develop a mathematical model</i></li><li>or</li><li>• <i>Determine the best configuration or combination of Xs</i></li></ul>
<i>Critical elements, (you need to develop and test several practical alternatives to determine which is the best solution)</i>	<ul style="list-style-type: none"><li>• <i>Optimize process flow issues</i></li><li>or</li><li>• <i>Standardize the process</i></li><li>or</li><li>• <i>Develop a practical solution.</i></li></ul>

***Choose the Appropriate Tool***



## Improve Tools

### Basic

- Fishbone
- Box Plot
- Linear Regression
- Hypothesis Tests (z-test, t-test, ANOVA, chi-square, HOV)
- Process Map
- Time Order Plots
- Mistake Proofing
- Multi-vari plot\*
- Force Fields\*
- Kaizen\*

### Intermediate

- DOE (full, fractional DOE)
- Multi-variate Regression\*

### Advanced

- Response Surface\*
- Taguchi (Inner/Outer Array)\*



- complexity
- business impact
- risk
- data availability

**Match Tool to Problem**



## *Selecting the Appropriate Tool*

---

- *For many projects, we may arrive at an acceptable solution using basic tools already developed in the Measure and Analyze phases or by:*
  - *optimizing process flow*
  - *work-outs, benchmarking, best practices, and brainstorming*
    - *generate ideas for alternatives from existing data and process knowledge*
  - *trial experiments or simulation*
  - *standardizing the process*
  - *mistake proofing*
  
- *If more precision is necessary, intermediate or more advanced tools may be used. The majority of this section will focus on the intermediate tools.*



## *Example: Reducing Cycle Time*

---

*A team worked to reduce the time required to provide an answer to a customer's question in the proposal process.*

*Three different people each use a different process to respond to the customer. When all three processes were studied, the following was discovered:*

- *Each process contained a minimum of 5 nonvalue-added steps that made up 70% of the total processing time.*
- *Regardless of risk level or customer, all questions were handled equally.*
- *Incomplete information requests were maintained in the system until the final stages of the proposal process.*



## *Cycle Time Example Improvement Solution*

---

- ✓ *The team worked with the experts to determine alternate processing paths to reduce the nonvalue-added paperwork handoffs and documentation requests.*
- ✓ *They developed an alternate fast-track path for low-risk questions.*
- ✓ *Incomplete information requests were forwarded to the customer for completion.*

***Result: Cycle Time Reduced***



## *Example: Reducing Cost*

- *The telephone expenses have been too high. GE currently pays \$35 per phone line. A project was initiated to identify the key factors that drive up telephone costs. The goal is to reduce the overall telephone expense (Y) to GE.*
- *A fishbone has identified two vital Xs that impact cost (Y)*
  - $X_1 = \text{number of lines in use}$
  - $X_2 = \text{number of lines not in use}$
- *Hence the relation:  $Y = 35 ( X_1 + X_2 )$*
- ***Improvement:*** *Design a new process to identify lines not in use and turn off phone line.*



## *Example: Reducing Delivery Time*

*Problem: There is currently a slow delivery time on small package domestic shipments.*

- *Currently our company is using one vendor, vendor A.*
- *Analysis has shown that vendor, time of year, and distance shipped are vital Xs.*
- *We run an experiment with a potential second vendor, vendor B. We go to a neighboring business who employs Vendor B and collect information on delivery time (in days) under similar conditions of time and distance.*

***How can we test to see if Vendor A is better than Vendor B?***



# Homogeneity of Variance

Perform a homogeneity of variance test on the standard deviations to see if there is a statistically significant difference between the variation in delivery times of Vendor A vs. Vendor B.

$$H_o: s_A^2 = s_B^2$$

$$H_a: s_A^2 \neq s_B^2$$

MINITAB FILE: Vendors.mtw

The screenshot shows the MINITAB software interface. The 'Stat' menu is open, and the path 'Stat > ANOVA > Homogeneity of Variance...' is highlighted. Below the menu, a data table is visible with the following content:

	C1	C2	C3	C4	C5	C6	C7	C8	C9
↓	Vendor A	Vendor B	Time A&B	Vend A & B					
1	5.3	4.2	5.3	1					
2	4.5	3.6	4.5	1					
3	6.7	5.1	6.7	1					
4	6.5	4.8	6.5	1					
5	4.6	5.3	4.6	1					
6	4.6	4.3	4.6	1					
7	6.8	3.8	6.8	1					
8	5.2	4.9	5.2	1					

At the bottom of the window, the text reads: 'Perform Bartlett's and Levene's tests for homogeneity of variance'.



# Homogeneity of Variance

**Homogeneity of Variance Test** [X]

C1	Vendor A
C2	Vendor B
C3	Time A&B
C4	Vend A & B

**Response:** 'Time A&B'

**Factors:** 'Vend A & B'

**Confidence level:** 95.0

**Title:** \_\_\_\_\_

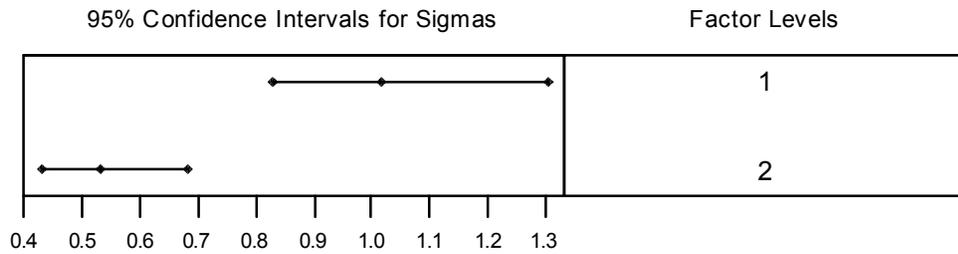
**Select**      **Storage...**

**Help**      **OK**      **Cancel**



# Homogeneity of Variance

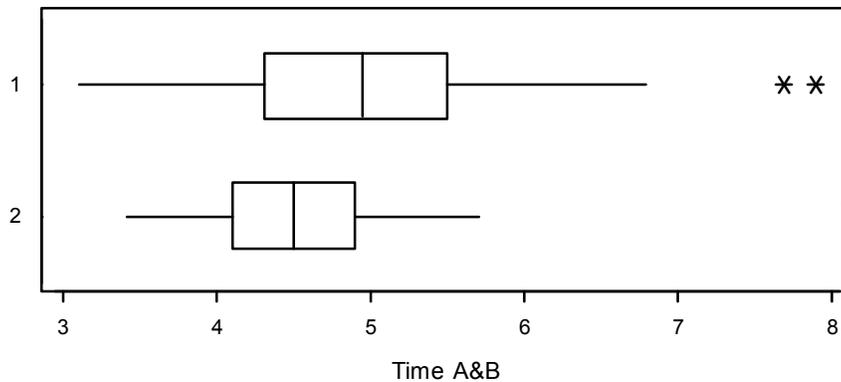
## Homogeneity of Variance Test for Time A&B



Bartlett's-Test

Test Statistic: 3.679

P-Value : 0.000



Levene's Test

Test Statistic: 10.766

P-Value : 0.001

$p < .05$  implies that we accept the alternative hypothesis.  $H_a: \sigma_A^2 \neq \sigma_B^2$

*Vendor A and Vendor B have different variances.*



## Vendor Example: Two sample t-test

Perform a 2 sample t-test on the means to see if there is a statistically significant difference between delivery times of Vendor A vs. Vendor B.

$$H_o: m_A = m_B$$

$$H_a: m_A \neq m_B$$

### MINITAB FILE: Vendors.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and '2-Sample t...' is selected. Below the menu is a data table with columns C1-C9 and rows 1-8. The table contains delivery times for Vendor A and Vendor B, and a column for 'Time A & B'.

	C1	C2	C3	C4	C5	C6	C7	C8	C9
↓	Vendor A	Vendor B	Time A&B	Vend A & B					
1	5.3	4.2	5.3	1					
2	4.5	3.6	4.5	1					
3	6.7	5.1	6.7	1					
4	6.5	4.8	6.5	1					
5	4.6	5.3	4.6	1					
6	4.6	4.3	4.6	1					
7	6.8	3.8	6.8	1					
8	5.2	4.9	5.2	1					

Perform a two-sample t-test and compute a confidence interval for the difference in means



# Two sample t-test

**2-Sample t**

Samples in one column

Samples:

Subscripts:

Samples in different columns

First:

Second:

Alternative:

Confidence level:

Assume equal variances

**2-Sample t - Graphs**

Dotplots of data

Boxplots of data

**2-Sample t**

Samples in one column

Samples:

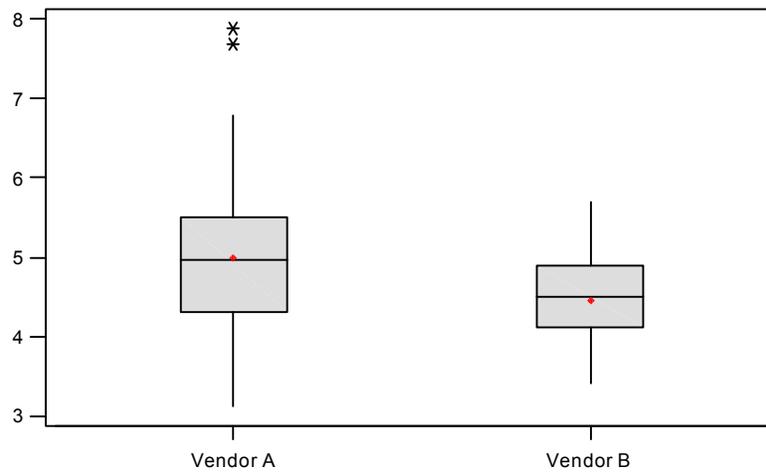
Subscripts:

Assume equal variances



# Data Analysis

Boxplots of Vendor A and Vendor B  
(means are indicated by solid circles)



Two sample T for Vendor A vs Vendor B

	N	Mean	StDev	SE Mean
Vendor A	50	4.99	1.01	0.14
Vendor B	50	4.458	0.529	0.075

95% CI for  $\mu$  Vendor A -  $\mu$  Vendor B: ( 0.21, 0.854)

T-Test  $\mu$  Vendor A =  $\mu$  Vendor B (vs not =): T = 3.29

P = 0.0015 DF = 73

Accept the alternative hypothesis:  $H_a: \mu_A \neq \mu_B$

## Which vendor is better?



# Capability Analysis

MINITAB FILE: Vendors.mtw

The screenshot shows the Minitab software interface. The main window displays a data table with columns C1, C2, C3, C6, C7, C8, and C9. The data is as follows:

	C1	C2	C3	C6	C7	C8	C9
↓	Vendor A	Vendor B	Time A&				
1	5.3	4.2	5.3				
2	4.5	3.6	4.5				
3	6.7	5.1	6.7				
4	6.5	4.8	6.5				
5	4.6	5.3	4.6	1			
6	4.6	4.3	4.6	1			
7	6.8	3.8	6.8	1			
8	5.2	4.9	5.2	1			

The 'Stat' menu is open, showing the path: Stat > Quality Tools > Six Sigma Process Report... The 'Six Sigma Process Report...' option is highlighted. The status bar at the bottom of the window reads: 'Produce six process capability reports'.



# Capability Analysis

**Six Sigma Process Report**

Data are arranged as

Single column: 'Vendor A'

Subgroup size: 1  
(use a constant or an ID column)

Subgroups across rows of:

Lower spec:

Upper spec: 7

Target:  (optional)

Select

Help

Reports

Demographics

OK

Cancel

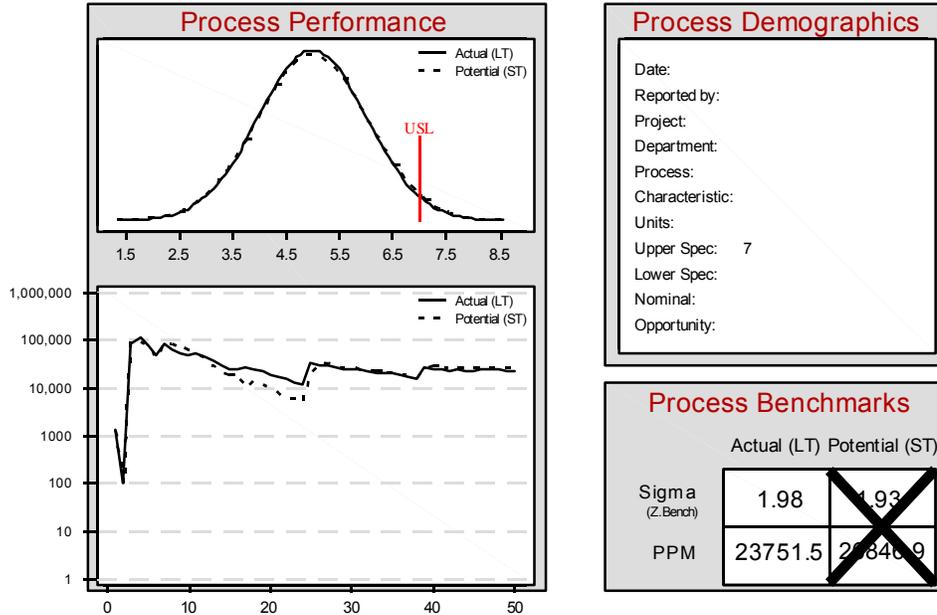
*Note: for a subgroup size of 1, short term values on the process report are invalid since there is not variation within groups.*

*The upper specification limit is 7 days.*

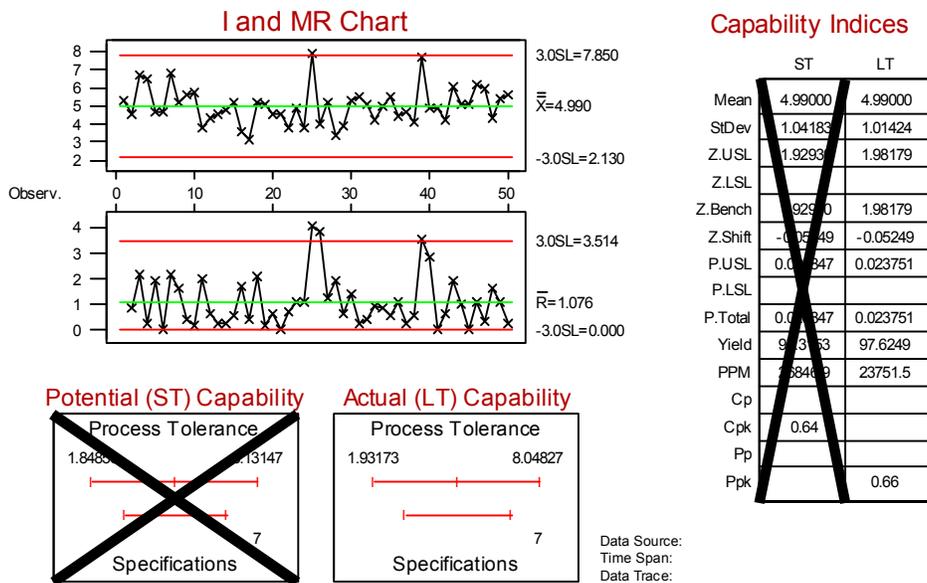


# Capability Analysis

## Report 1: Executive Summary



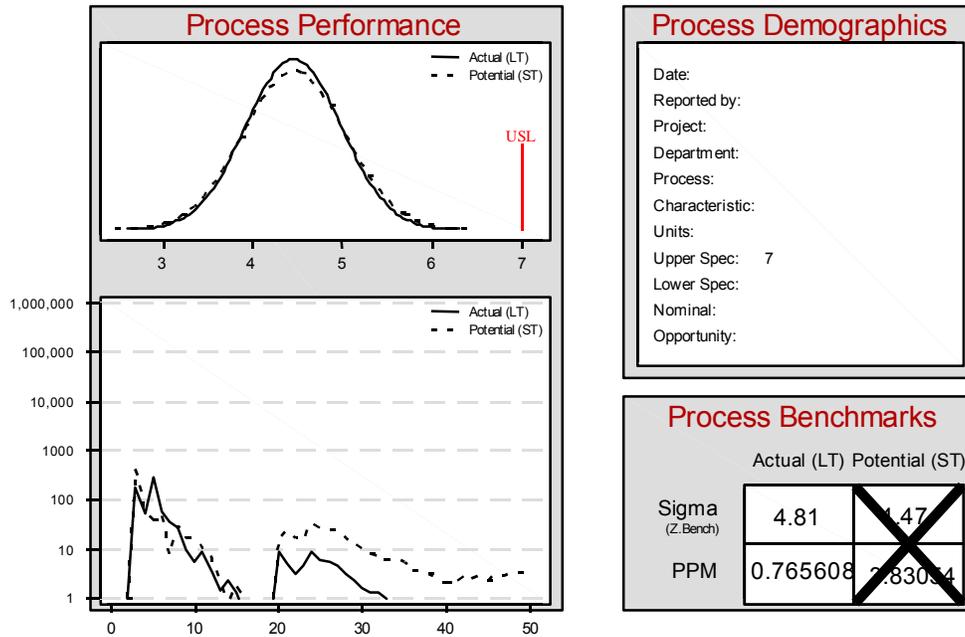
## Report 2: Process Capability for Vendor A



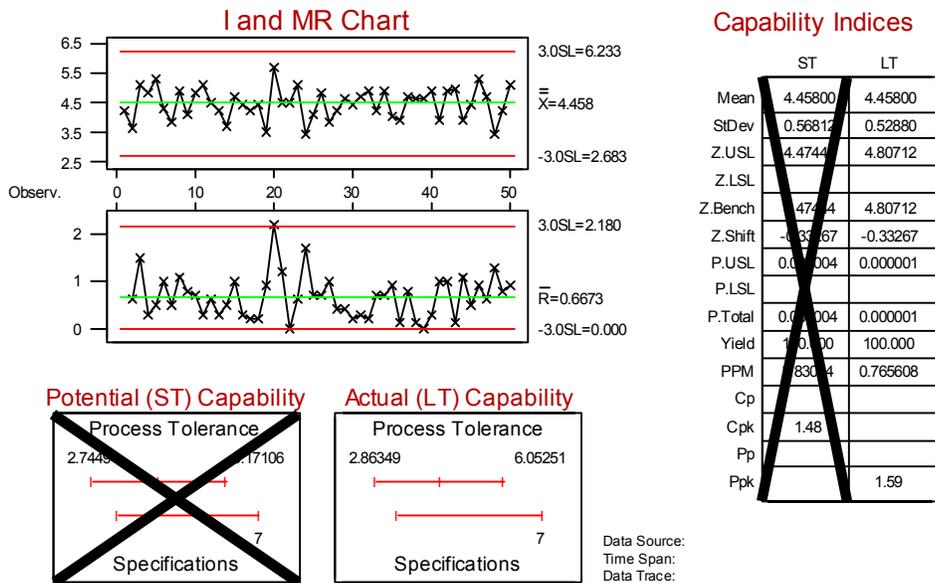


# Capability Analysis

## Report 1: Executive Summary



## Report 2: Process Capability for Vendor B





## *Taking Action*

---

- *Based on data, what is our conclusion?*
- *Switch vendors! Vendor B is better.*
  - *Vendor B has less variation in service time, a lower mean service time, and a higher process capability*
- *Need to consider implications:*
  - *Is the vendor qualified?*
  - *Are there EHS issues?*
  - *Are there ISO issues?*
  - *Impact on other processes?*
  - *Impact on cost?*
- *Pilot to verify improvement.*



## *The Pilot*

---

- *Pilot: a process improvement that you will test on a small scale in a real business environment.*
  
- *The Pilot objective is to collect data from the test site to:*
  - *confirm that your proposed solution will achieve the targeted performance (eg. increasing production or reducing defects)*
  - *to identify any potential implementation problems (technology, training, etc.) prior to full scale implementation*



## *Why Pilot?*

---

- *To better understand the effects of your solution and plan for a successful full-scale implementation.*
- *To release an early version of your solution to a particular market segment that has an urgent need for the change.*
- *To lower your risk of failing to meet improvement goals when the solution is fully implemented.*
- *To more accurately predict monetary savings resulting from your solution.*
- *To justify investments required for full-scale implementation.*
- *To identify potential problems with the solution implementation on a larger scale.*



## *Pilot and Analyze the Results*

---

- *Complete a plan to execute your pilot:*
  - *risk assessment to identify potential unintended consequences of the pilot*
  - *consider issues such as test population, budget, resources, location, and timing*
  - *develop a data collection plan for your pilot*
- *Run the pilot and collect process data.*
- *Analyze the results of the pilot to:*
  - *prove statistically that your solution meets your improvement goals*
  - *identify issues and requirements you need to address to ensure successful full-scale implementation of your solution*
- *Once you have successfully piloted your solution, you can proceed to the Control phase where you will implement the new process on a full scale.*



## *Next Steps*

---

- *Always use data to make improvement decisions.*
- *For simple situations where it is easy to determine the magnitude and direction of the impact of the Xs on the Y, use tools available from Measure and Analyze phases.*
- *For more complex situations, with multiple vital Xs, it may be difficult to determine how the Xs impact the Y. There may be interactions between the Xs and the magnitude and direction of change for the Xs may be difficult to determine. What to do?*
- *On to DOE.....*



## *Take Aways—Introduction to Improve*

- *Develop a strategy for improvement.*
- *Improvements should be based on data driven analysis.*
- *Use the tool that best fits your improvement needs.*
- *When the vital Xs have few interactions use:*
  - *FMEAs*
  - *Process Maps, alternate process flows*
  - *Fishbone*
  - *Pareto*
- *Pilot the solution.*
- *For more advanced problems where the Xs have many interactions, use a Design of Experiments.*

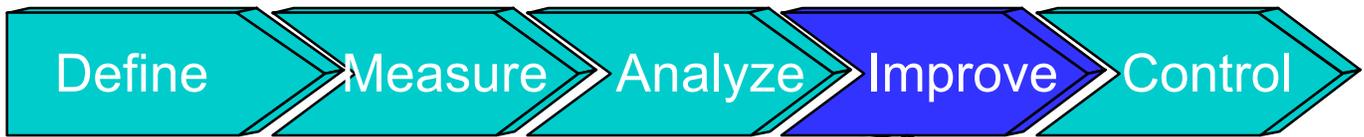


## *Improve Deliverables*

- *Identify key Xs from Paretos & FMEA.*
- *Establish improvement actions for each vital X.*
- *Review applicability of DOE.*
- *Create new process map.*
- *Is there a mistake proof solution?*
- *Establish team “buy-in.”*
- *Does process owner agree on improved process.*
- *Prove process change (Chi square or 2 sample t) Minitab.*
- *Finalize financial savings.*
- *Update Six Sigma Quality Project Tracking database*



# Improve Phase



**7. Screen Potential Causes**

**8. Discover Variable Relationships**

**9. Establish Operating Tolerances**

*Deliverable:*  
*Determine the Vital Few X's That Are Causing Changes in Y.*

*Tools:*  
■ *Screening DOE*

*Deliverables:*  
1) *Establish Transfer Function Between Y and Vital Few X's.*  
2) *Determine Optimal Setting for the Vital Few X's.*  
3) *Perform Confirmation Runs.*

*Tools:*  
■ *Factorial Designs*

*Deliverable: Specify Tolerances on the Vital Few X's.*

*Tools:*  
■ *Simulation*



---

# *Improvement by Design*

## *The Design and Analysis of Experiments*



## *Objective*

- *To decrease the time required to achieve six sigma levels of quality by providing people with tools to characterize and/or improve equipment, processes and products through the application of efficient experimentation and analysis techniques. In particular, to provide understanding of:*
- *Designed experiments using the factorial strategy.*
  - *Techniques for reducing the amount of experimentation*
  - *Graphical methods of analysis of experiments*
  - *Numerical methods of analysis of experiments*



## *Deliverables*

---

- *Identify all “Xs” (independent variables, root causes)*
- *Isolate the “vital few Xs”*
- *Prove statistically that each one is a “vital X”*
- *Quantify the magnitude of the impact of each X*
- *Develop improvement plan (what to do about each X to attain project goals)*
- **IMPROVE THE PROCESS!**



## *Step 7*

---

# *Screen Potential Causes*

*Project deliverable: Determine the Vital Few X's that cause changes in your Y*



## *Step 8*

---

### *Discover Variable Relationships*

*Project deliverable: Determine the Transfer Function between Y and vital few X's*

*Determine optimal settings for the vital few X's*

*Perform the Confirmation Runs*



## *Improvement by Design*

---

*Caution: You have now selected  $x$ 's to begin experimentation, you should ensure the measurement system variability on these  $x$ 's is within guidelines (Step 3).*

*The measurement system used in the experiment may not be the same one used in the actual process.*

*Step 10 will address measurement system variability on the  $x$ 's in the actual application.*



# *Efficient Experimentation*



## *The Problem*

- *Customers of CHI (Cellulose Helicopters Inc.) have been complaining about the limited flight time of CHI helicopters.*
- *Management wants to increase flight time to improve customer satisfaction.*
- *You are put in charge of this improvement project.*

***How would you approach this problem?***

(This problem is adapted from *Designing Industrial Experiments: The Engineer's Key to Quality*, by Box, Bisgaard and Fung, Madison, Wisconsin.)



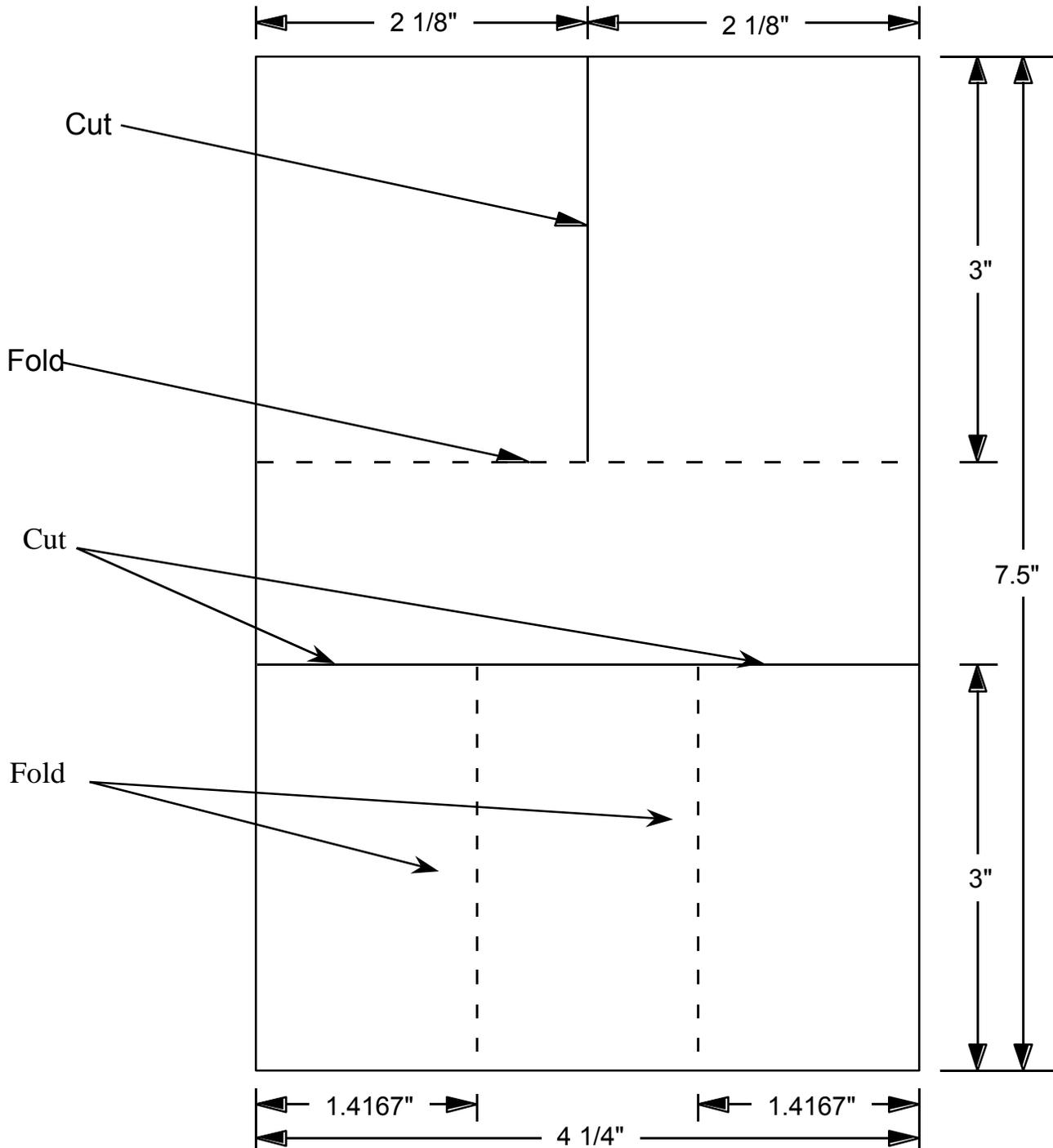
## *The Approach*

---

- *Baseline the current situation.*
- *Identify key factors affecting flight time.*
- *Identify potential improvements to be evaluated.*
- *Develop an experimentation plan.*
- *Conduct the experiment.*
- *Evaluate the results.*
- *Recommend improvements.*
- *Plan and implement the improvements.*
- *Provide a mechanism for control.*



# The Standard Design





# *What is the Baseline Performance of the Standard Design?*

---

*Trial 1*      *Trial 2*      *Trial 3*      *Average*

*Team 1*

*Team 2*

*Team 3*

*Team 4*

*Team 5*

*Team 6*

*Team 7*

*Team 8*



## *Possible Helicopter Modifications*

---

*To Increase Flight Time:*

—

—

—

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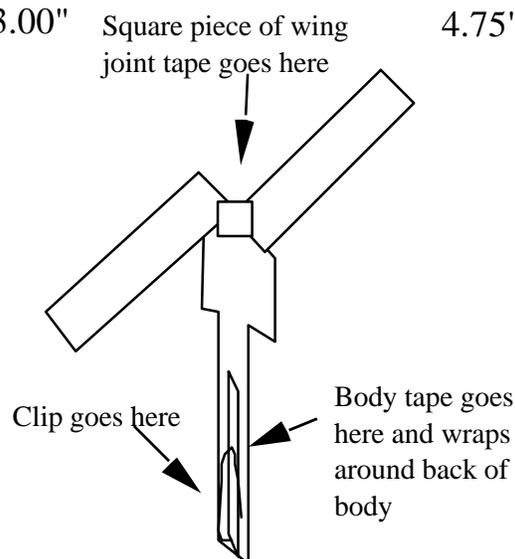
—



# Factors That May Affect Flight Time of CHI Helicopters

- *The Cellulose Helicopter Association has authorized for flight testing certain modifications to the standard design. Allowable settings for the factors that may vary are shown below.*

Factor	Suggested Levels	
	<u>Standard</u>	<u>Allowable Changes</u>
Paper Type	Recycled (yellow)	Copier (white)
Paper Clip	No	Yes
Taped Body	No	3 inches of adhesive tape
Taped Wing Joint	No	Yes
Body Width	1.42"	2.00"
Body Length	3.00"	4.75"
Wing Length	3.00"	4.75"





## *Project Description: Phase I*

### ■ *Project Mission*

- *Find the combination of factors that most consistently maximizes the flight time of a \_\_\_\_\_ foot drop.*

### ■ *Project Constraints*

- *Your team is authorized to conduct a Phase I investigation into the plausibility of the above mission. Your materials and test budget for this phase is limited to \$1,000,000*
- *Each prototype costs \$100,000 to build*
  - *Additionally, it costs \$10,000 to conduct each flight test*

***Your team must issue a report in 45 minutes.***



## *Roles and Responsibilities*

<b>Role</b>	<b>Responsibility</b>	<b>Who</b>
Lead Engineer	Lead the team in deciding which prototypes to build. Has final say on which prototypes are built and tested.	_____
Test Engineer	Leads the team in conducting the flight tests of all prototypes. Has final say on how tests are conducted.	_____
Assembly Engineer	Leads the team in building the prototypes. Has final say on all building issues.	_____
Finance Manager	Leads the team in tracking expenses. Has responsibility for keeping the team on budget.	_____
Recorder	Leads the team in recording data from the trials.	_____



## *Project Report*

---

- *Prepare a Phase I report on your recommendations for increasing flight time. Include:*
  - *Recommendations for an improved helicopter design*
  - *Predicted flight time at improved setting*
  - *How much money did you use?*
  - *What experimental strategy did you use to arrive at the above?*
  - *How did you analyze your data?*
  - *Recommendations for future prototypes to construct and test*



## *Exercise*

---

- *List the things your team did to hold conditions similar across tests:*

—

—

—

—

—

—

—



## Stick-With-A-Winner Strategy

Description	PT	PC	TB	TW	BW	BL	WL	Result
1. Standard	-	-	-	-	-	-	-	2.1
2. Paper Trial	+	-	-	-	-	-	-	2.6
3. Clip Trial	+	+	-	-	-	-	-	2.4
4. Taped Body Trial	+	-	+	-	-	-	-	2.5
5. Taped Wing Trial	+	-	-	+	-	-	-	2.8
6. Wide Body Trial	+	-	-	+	+	-	-	2.9
7. Long Body Trial	+	-	-	+	+	+	-	2.7
8. Long Wing Trial	+	-	-	+	+	-	+	3.2

### **Interpretation:**

*Since Trial 2 out-performed Trial 1, the paper is changed in the rest of the trials. Since Trial 3 was inferior to Trial 2, the rest of the trials are conducted without the clip (the standard level).*

*Note: Trial 4 is compared to Trial 2 in order to determine whether or not to proceed with a taped body.*

### **Key:**

PT = Paper Type  
PC = Paper Clip  
TB = Taped Body  
TW = Taped Wing Joint  
BW = Body Width  
BL = Body Length  
WL = Wing Length

#### Factor Levels

standard    changed  
-            +



## One-Factor-At-A-Time Strategy

Description	PT	PC	TB	TW	BW	BL	WL	Result
1. Standard	-	-	-	-	-	-	-	2.0
2. Paper Trial	+	-	-	-	-	-	-	2.5
3. Clip Trial	-	+	-	-	-	-	-	1.9
4. Taped Body Trial	-	-	+	-	-	-	-	1.9
5. Taped Wing Trial	-	-	-	+	-	-	-	2.2
6. Wide Body Trial	-	-	-	-	+	-	-	2.3
7. Long Body Trial	-	-	-	-	-	+	-	2.5
8. Long Wing Trial	-	-	-	-	-	-	+	2.3

**Key:**

PT = Paper Type  
PC = Paper Clip  
TB = Taped Body  
TW = Taped Wing  
BW = Body Width  
BL = Body Length  
WL = Wing Length

Factor Levels

standard      changed  
-                    +



## *Key Learnings*

---

- *The intuitive approach to multi-factor experimentation involves varying only one factor at a time.*
- *One-factor-at-a-time and stick-with-a-winner strategies can fail to determine which factors are important and be inefficient with respect to the amount of information provided in each trial.*
- *In one-factor-at-a-time experimentation, the presence of variation in model construction, test, and measurement can make it difficult to see the effect of the factors under study.*
- *Use care when running experiments. Pay attention to the measurement process. Plan how to keep other factors not under study constant.*



# One-at-a-Time Strategy

## 3 Factors, 2 Levels

Trial	Factor 1	Factor 2	Factor 3
1	-	-	-
2	+	-	-
3	-	+	-
4	-	-	+

*What combinations of factor settings are missing?*

5	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
6	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
7	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
8	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

“-” represents low settings.  
“+” represents high settings.



## *Full Factorial Layout 3 Factors, 2 Levels*

<b>Std. Order</b>	<b>Factor 1</b>	<b>Factor 2</b>	<b>Factor 3</b>
1	–	–	–
2	+	–	–
3	–	+	–
4	+	+	–
5	–	–	+
6	+	–	+
7	–	+	+
8	+	+	+

*For 3 factors, each at 2 levels, there are  $2^3 = 2 \times 2 \times 2 = 8$  combinations of factor settings.*

*Notice the pattern of factor settings in the standard order.*



## Example: A 2<sup>3</sup> Factorial Layout

- For Three Factors That May Affect Helicopter Flight Time

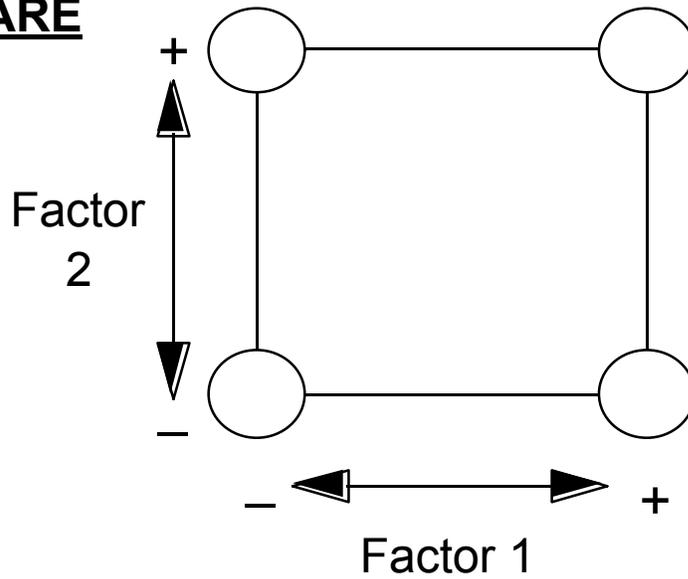
<b>Std. Order</b>	<b>Paper Type</b>	<b>Body Width</b>	<b>Body Length</b>
1	Recycled	1.42	3.00
2	Copier	1.42	3.00
3	Recycled	2.00	3.00
4	Copier	2.00	3.00
5	Recycled	1.42	4.75
6	Copier	1.42	4.75
7	Recycled	2.00	4.75
8	Copier	2.00	4.75

	<b>Std. -</b>	<b>New +</b>
Paper Type	Recycled	Copier
Body Width	1.42"	2.00"
Body Length	3.00"	4.75"

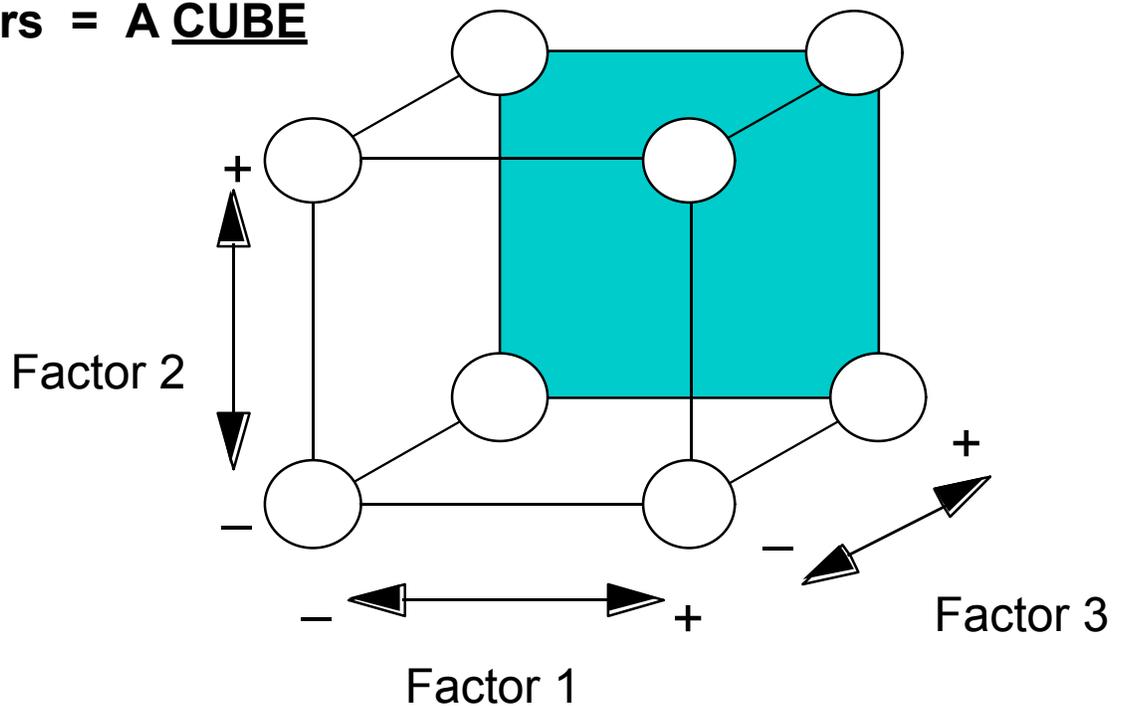


# Visualizing the Experimental Space

2 Factors = A SQUARE

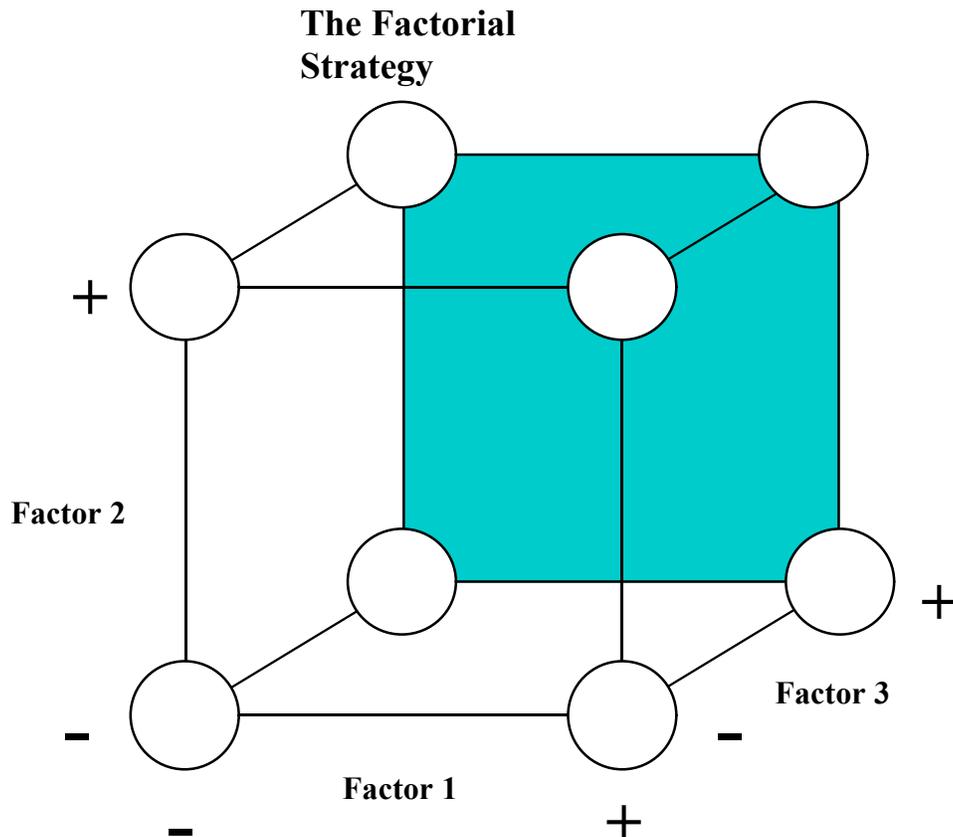


3 Factors = A CUBE





## 3 Factors: Cube Layout

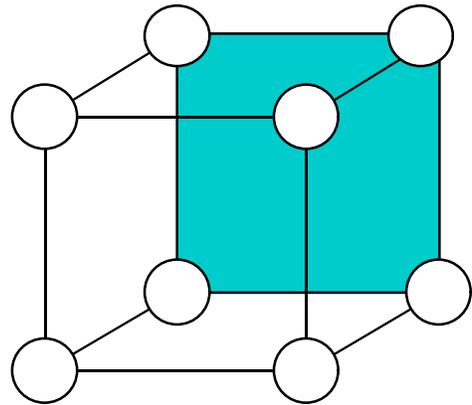
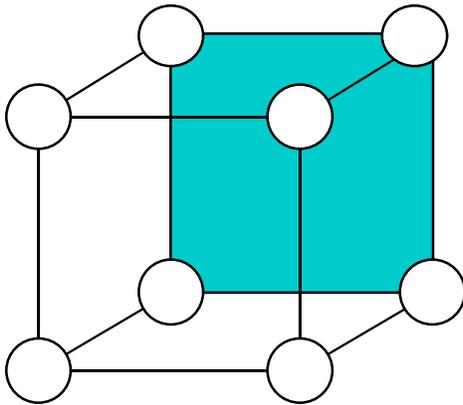


- A cube helps us visualize the experimental space covered by the 3 factors.
- Each corner represents 1 set of experimental conditions.
- $2^3 = (\text{Two Levels})^{(\text{Three Factors})} = 8$  experimental conditions.



## 4 Factors: *Cube Layout*

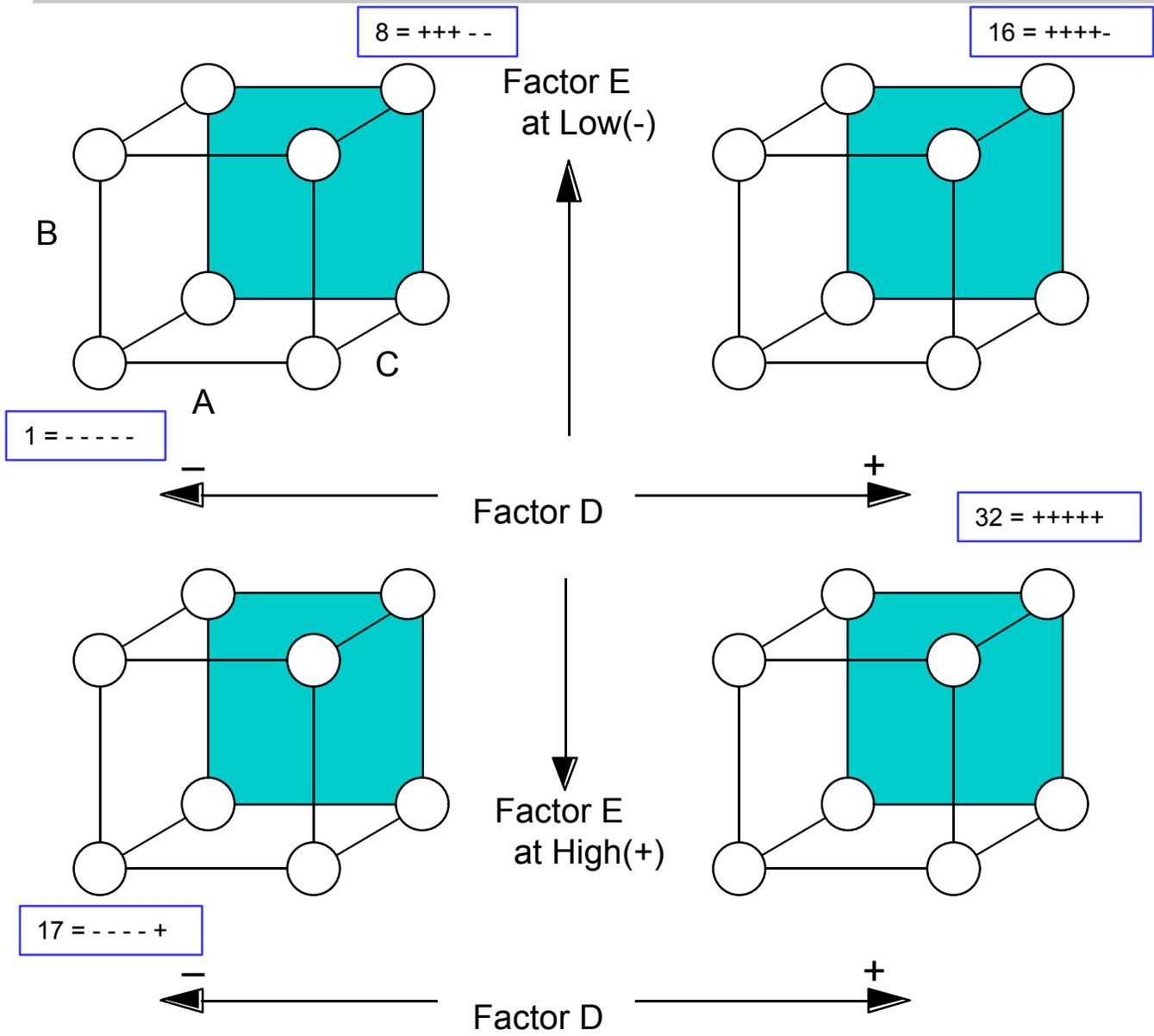
4 Factors = Two **CUBES**



**5 factors = four cubes? What does it look like?**



# 5 Factors: Cube Layout



- **Review: 5 factors = A, B, C, D, E**
- **$2^5 = 32$  Runs [note: 4 Cubes x 8 = 32 “corners”]**
- **What do shaded faces represent?**





## *Exercise*

---

- *How many experimental conditions are there for 7 factors, each at 2 levels?*
- *Write the full factorial design, using standard order, for an experiment with 3 factors, each at 2 levels.*



## *Summary: Factorial Strategy ( $2^k$ )*

- *One-at-a-time designs explore a potentially misleading portion of the design space.*
- *Full factorial designs cover the entire design space.*
- *Full factorial designs are easy to lay out because of the repeating pattern in the standard order.*
- *The number of experimental conditions for a 2-level experiment with  $k$  factors is  $2^k = 2 \times 2 \times 2 \times \dots \times 2$  ( $k$  times).*



# *Process of Experimentation*



# *Process of Experimentation*

---

1. *Define Project*
  - *Identify responses*
2. *Establish Current Situation*
3. *Perform Analysis*
  - *Identify factors*
  - *Choose factor levels*
  - *Select design*
  - *Randomize runs*
  - *Collect data*
  - *Analyze data*
  - *Draw conclusions*
  - *Verify results*
4. *Determine Solutions*
5. *Record Results*
6. *Standardization*
7. *Determine Future Plans*



## Replication

- **Definition:** *Multiple execution of all or part of the experimental process with the same factor settings.*
  - *It is not the same as multiple measurements or tests on a single piece, lot, or model*
- **Why?**
  - *To measure experimental variability.*
    - So we can decide whether the difference between responses is due to the change in factor levels (an induced special cause) or to common cause variability*
    - To see more clearly whether or not a factor is important*
  - *To obtain two responses for each set of experimental conditions*
    - Location*
    - Spread*
  - *Replication provides the opportunity for factors that are unknown or uncontrollable to balance out. Along with randomization, replication acts as a bias decreasing effect*



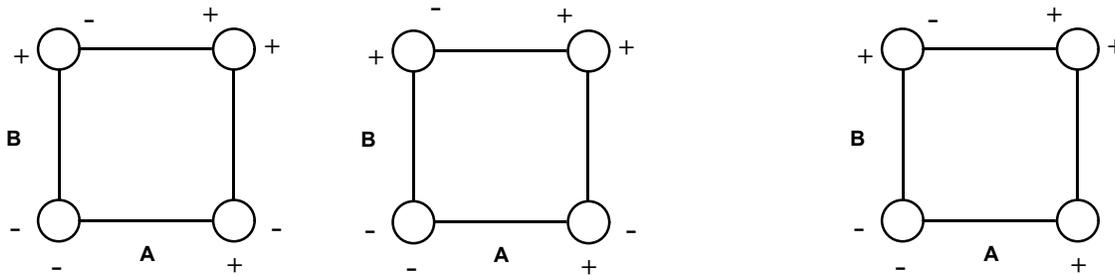
## Replication vs. Repetition

**Replication:** Multiple execution of all or part of the experimental process with the same factor settings.

—build different experimental units

**Repeat tests:** 2 or more observations that have the same levels for all factors

—performed on the same experimental unit



DESIGN LAYOUT EXAMPLE

$2^2$  design - 1 replication

$2^2$  design with 1 repetition

Std. Order	Response		Std. Order	Obs 1	Obs 2	Avg
1	-	-	1	$y_{11}$	$y_{21}$	$\bar{V}_1$
2	+	-	2	$y_{12}$	$y_{22}$	$\bar{V}_2$
3	-	+	3	$y_{13}$	$y_{23}$	$\bar{V}_3$
4	+	+	4	$y_{14}$	$y_{24}$	$\bar{V}_4$
1	-	-				
2	+	-				
3	-	+				
4	+	+				



## *Randomization: The Experimenter's Insurance*

---

- **Definition:** *To assign the order in which the experimental trials will be run using a random mechanism.*
  - *It is not the standard order*
  - *It is not running in an order that is convenient*
  - *Minitab will randomize for us*
  
- **Why?**
  - *Averages the effect of any lurking variables over all of the factors in the experiment*
  - *Helps validate statistical conclusions made from the experiment*



# Lurking Variables

## ■ Definition\*:

- *A variable that has an important effect and yet is not included among the factors under consideration because:*
  - *Its existence is unknown*
  - *Its influence is thought to be negligible*
  - *Data on it are unavailable*

## ■ Safeguard:

- *Randomize the order of the experimental trials to protect against the effect of lurking variables*

## ■ Action:

- *If the lurking variable creates a trend it can be compensated for in the numerical analysis*
- *Conclusions can then be drawn from the original factors that are not affected by such lurking variables*

\*Source: Joiner, Brian L. "Lurking Variables: Some Examples," *The American Statistician*, November 1981, Volume 35, No. 4, pp. 227-233.



## *An Example of a Lurking Variable*

*The Agricultural Sciences Department at North Carolina State University developed a new and improved chicken feed that would supposedly promote plumper and meatier chickens. The school contracted with a local poultry provider (Holly Farms) and conducted a series of studies testing the new product. The NC State Mathematics Department was asked to develop a DOE to support the above tests.*

*Preliminary calculations were made and two populations of chickens were identified, tagged (this becomes very important later), and segregated. One population was fed the standard feed and the other fed the new and improved feed. After feeding the two populations of chickens, statistically significant samples from each population were slaughtered and weighed (what we would refer to as 'Destructive Testing'). The outcome of the experiment was obviously trying to prove that chickens on the new feed weighed more than those on the old feed. In this case the  $Y = \text{Weight}$  and  $X = \text{Type of feed}$ .*

*After reviewing the data, the scientists were surprised to learn that there was no statistical difference between the two populations. The average weight (from the samples) was actually slightly higher (although not statistically higher -  $p\text{-value} > 0.05$ ) for those chickens fed the standard feed. Obviously, this baffled the scientists involved in the experiment.*

*After a few weeks of evaluating the experiment and the data, one of the grad students asked Holly Farms for a map of their facility.*



## *Lurking Variable Example (cont.)*

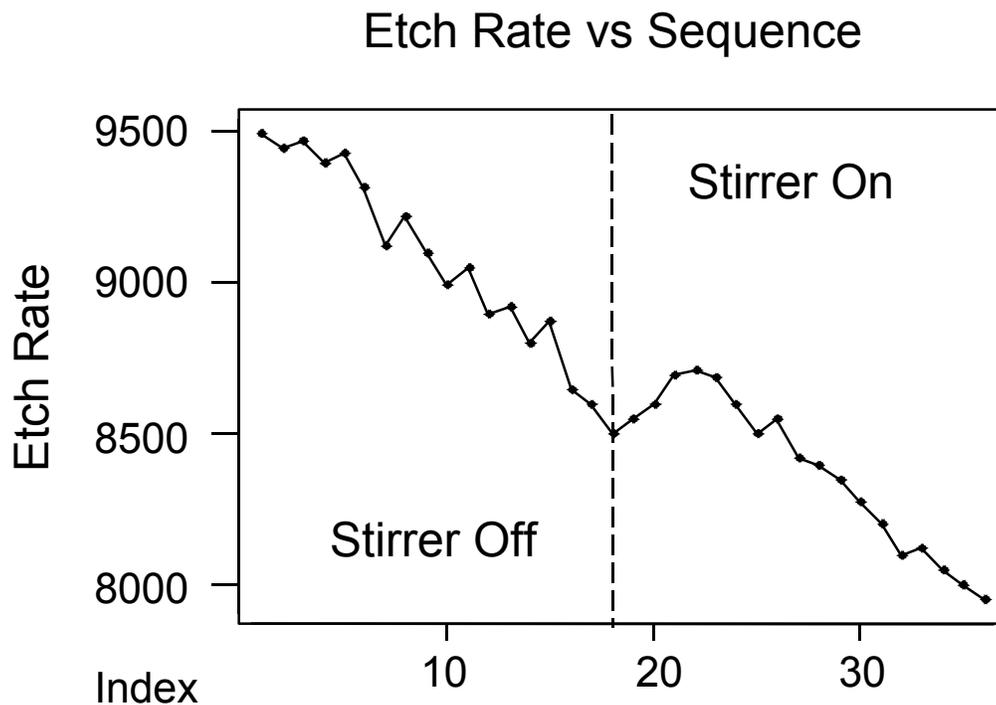
*After reviewing the map, the student noticed that some of the chicken houses were located immediately next to the slaughter house. This raised a question in the student's mind and he decided to drive out to the farm for some first-hand observations. He was escorted to the slaughter house area and immediately noticed that the chickens located in the houses next to the slaughter house demonstrated significantly higher levels of activity - i.e. clucking, pecking, and running around like... well... like chickens. After another review of the experimental data (by tag number), it was discovered that all of the chickens on the new feed had been located in the house immediately adjacent to the slaughter house - a lurking variable had been identified. (NOTE: without the tag numbers being recorded, this lurking variable may have never been discovered once again illustrating the importance of proper, thorough data collection). After reviewing his findings with the team, it was decided to introduce a new variable into the experiment - chicken house location.*

*The experiment was re-run with the new variable included (i.e. - chicken locations were randomized) and the results analyzed. On the second attempt, the results validated the scientists original hypothesis - the new feed produced plumper, meatier chickens. Evidently, those chickens located next to the slaughter house experienced higher stress levels and subsequent weight loss. As a result, Holly Farms opted to use the new feed AND also relocated all chicken houses AWAY from their slaughter houses.*

*Moral of the Story: Don't keep your chickens too close to the slaughter house.*



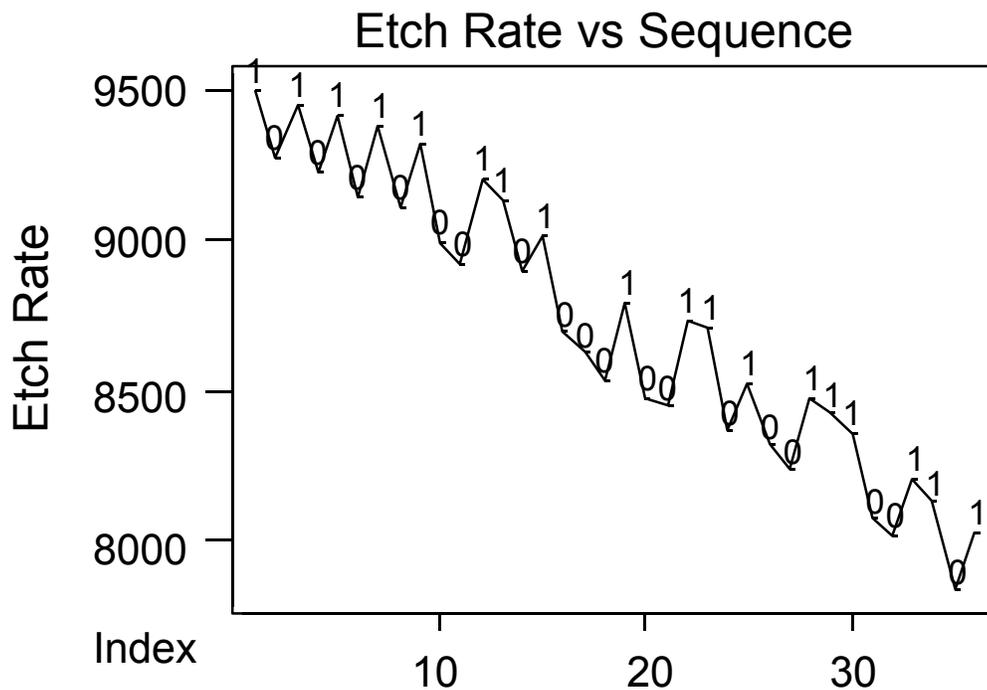
## Why Randomize?



*Bath degradation obscures factor effects in a design run in standard order.*



## Example: Why Randomize? (cont.)



0=stirrer off, 1=stirrer on

*In the randomized experiment the stirrer effect is visible despite the effect of bath degradation.*



## Useful Terminology

- **Experiment:** A test under defined conditions to determine an unknown effect, to illustrate or verify a known law, or to establish a hypothesis. See design of experiment.
- **Experimental Error:** Variation in observations made under identical test conditions. The amount of variation that cannot be attributed to the variables included in the experiment.
- **Factor:** Independent variables.
- **Interaction:** Factors are said to interact if the effect that one factor has on the response is dependent on the level of another factor(s).
- **Level:** The settings of a factor.
- **Randomization:** Assigning the order in which to run the experimental conditions using a random mechanism.
- **Repetition:** Multiple measurements or tests on a single piece, lot, or model.
- **Replication:** Multiple execution of all or part of the experimental process with the same factor settings on different pieces, lots or models.



# *The Process of Experimentation*

- ① *Define Project*
  - *Identify responses*
- ② *Establish Current Situation*
- ③ *Perform Analysis*
  - *Identify factors*
  - *Choose factor levels*
  - *Select design*
  - *Randomize runs*
  - *Collect data*
  - *Analyze data*
  - *Draw conclusions*
  - *Verify results*
- ④ *Determine Solutions*
- ⑤ *Record Results*
- ⑥ *Standardization*
- ⑦ *Determine Future Plans*



## *Data Collection Objectives*

---

- *What is your goal, or expected outcome, for collecting data?*
- *In general terms, what data do you need to collect to satisfy the objective?*
- *What process or product will you monitor to collect the data?*



## *Decide What to Measure*

---

- *What data is needed?*
- *Determine the specific measures that must be gathered to meet the objectives.*
- *Identify each Y or X that you need to measure.*
- *What is the operational definition for each measurement?*
- *Write each definition to ensure that the members of your team have a common understanding of the data to be collected.*



## *Decide How to Measure*

- *Determine the measurement tool that is required.*
  - *Does a tool already exist?*
  - *Do we need a new tool?*
  - *Examples of tools: stopwatches, gages, eyes, rulers, micrometers, computers, surveys, thermometers, scales, questionnaires, X-ray machines, etc.*
  
- *Determine the appropriate sample size*
  - *Random samples are important*
  - *Capture the behavior of a process or population accurately while minimizing the expense of data collection*



# *The Process of Experimentation*

- ① *Define Project*
  - *Identify responses*
- ② *Establish Current Situation*
- ③ *Perform Analysis*
  - *Identify factors*
  - *Choose factor levels*
  - *Select design*
  - *Randomize runs*
  - *Collect data*
  - *Analyze data*
  - *Draw conclusions*
  - *Verify results*
- ④ *Determine Solutions*
- ⑤ *Record Results*
- ⑥ *Standardization*
- ⑦ *Determine Future Plans*



## *Steps in Analysis: Full Factorial, Replicated Designs*

---

**Diagnostics:**  
*Is data OK?*

1. *Plot the raw data.*
2. *Plot the residuals.*

**Analysis:**  
*Make inferences*

3. *Examine factor effects.*
4. *Confirm impressions with statistical procedures.*
5. *Summarize conclusions.*



## *Plot the Raw Data*

---

### *First Step in Analysis*

#### ■ *Why?*

- *To find “defects” in the data set: typos, mistakes, missing values, problems*
- *To get a feel for the actual data, to become familiar with it*
- *To get impressions of what the data is going to tell us, to guide our analysis*
- *To look for trends that might have interfered with the experiment*
- *To reduce the chances that misleading or erroneous conclusions are drawn*

#### ■ *How?*

- *Time Order Plots*
- *Box Plots*



## *Time Order Plot*

---

- *Plot the data in the order it was produced.*
- *If there is more than one way to order the data, make a plot for each:*
  - *Order for running experiment*
  - *Order for taking measurements*
  - *Spatial arrangements of measurements*
- *Look for:*
  - *Outliers*
    - *Indicate typos, mistakes, or extreme special causes*
  - *Trends and non-random patterns*  
*indicate lurking variables*
  - *Obvious factor effects*

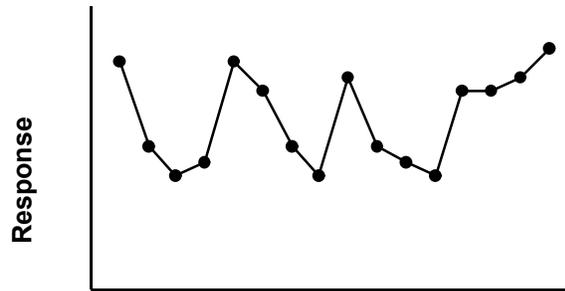


# Time Order Plot Examples



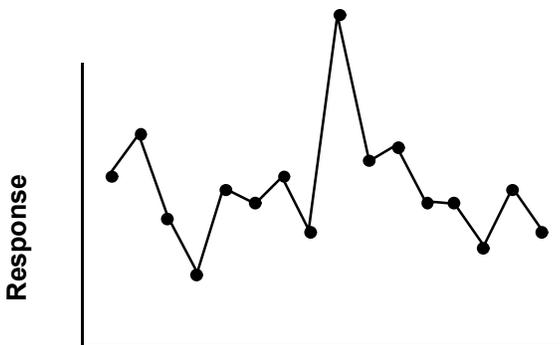
Order of Experiment

a) Trend: Response is increasing over time.



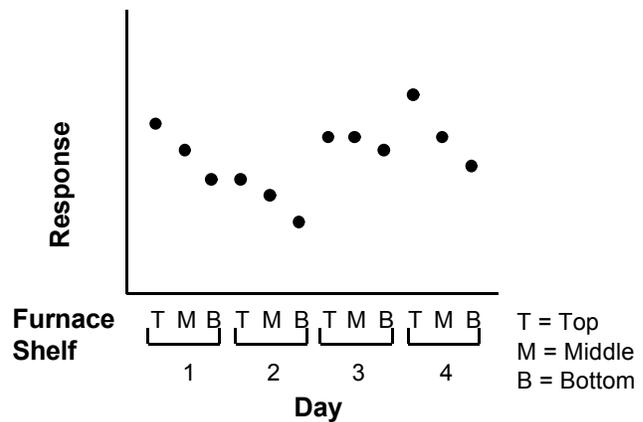
Order of Experiment

c) Possible factor effect:  
Points are separated into two levels.  
Label them.



Order of Taking Measurements

b) There is no trend associated with the measurement process  
Note the outlier.



d) Spatial arrangement: Ordering the data by shelf position reveals that the response decreases with the shelf height.



# Outlier

## ■ Definition

- A data point that is far enough away from the rest of the data that you question whether it “belongs” to the data set
- It sometimes indicates a mistake or some other special cause

## ■ Action

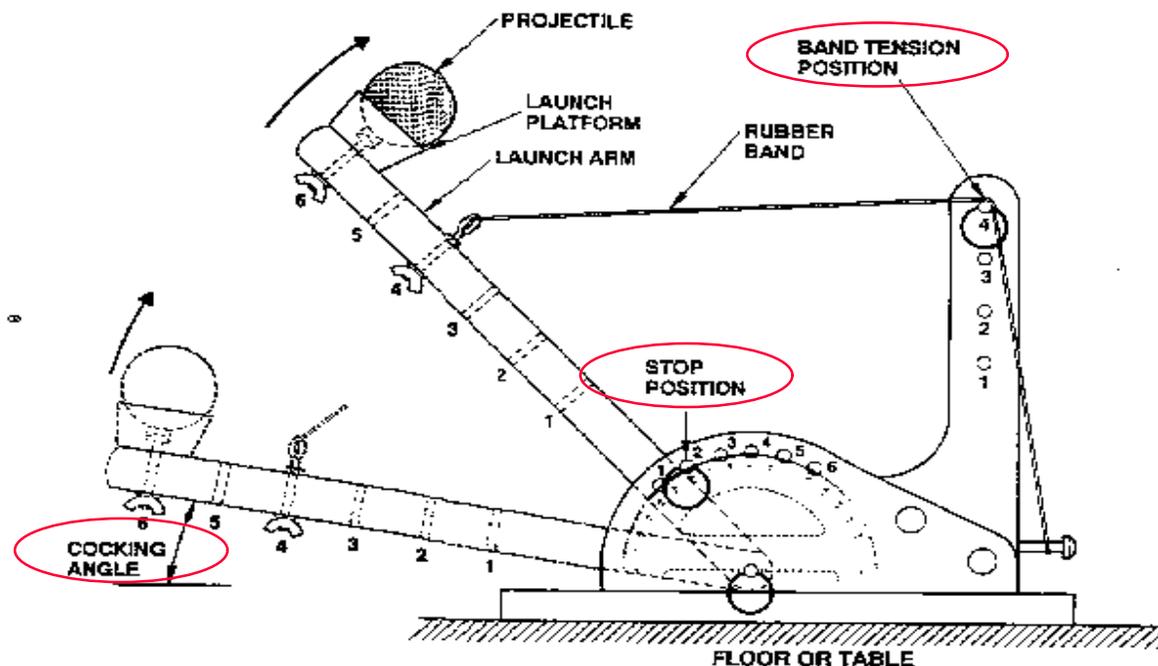
- Correct the data, if it is a mistake
  - Correct the cause, if possible — learn from it
  - If no explanation can be found, **leave the outlier in the data set** for now and continue with the analysis (do not “throw it out”)
- ## ■ Focus quick improvement efforts on the outliers and reduce variation



## Catapult Example

An experiment was conducted with the catapult introduced in Step 4. The experiment consisted of three factors set at 2 levels. Three replicates were conducted and the runs were randomized. The operator, projectile fired, and rubber band were all held constant during the experiment. The data from the experiment can be found in the Minitab worksheet: Catapult\_V2.MTW

<u>Factor</u>	<u>-</u>	<u>+</u>
Band Position	1	4
Stop Position	2	6
Cocking Angle	150 <sup>0</sup>	180 <sup>0</sup>





# Example

## Time Order Plot of Catapult Distances

MINITAB FILE: Catapult\_V2.MTW

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and 'Quality Tools' is selected. The 'Run Chart...' option is highlighted. The background data table is as follows:

	C1	C2	C3			C7	C8	C9	
	StdOrder	RunOrder	CenterPt	E	6	Cocking Angle	Distance	Exp Con	
1	20	1	1		6	150	48.75	4	
2	23	2	1		6	180	49.50	7	
3	11	3	1	1	1	6	150	49.75	3
4	9	4	1	1	1	2	150	49.50	1
5	2	5	1	1	4	2	150	49.50	2
6	4	6	1	1	4	6	150	48.25	4
7	1	7	1	1	1	2	150	50.75	1
8	19	8	1	1	1	6	150	49.25	3
9	24	9	1	1	4	6	180	50.50	8
10	3	10	1	1	1	6	150	48.50	3



# Time Order Plot Results

**Run Chart**

C1	StdOrder
C2	RunOrder
C3	CenterPt
C4	Blocks
C5	Band Positic
C6	Stop Positic
C7	Cocking Angl
C8	Distance
C9	Exp Con

Data are arranged as

Single column: Distance

Subgroup size: 1

(use a constant or an ID column)

Subgroups across rows of:

For data in subgroups

Plot subgroup means

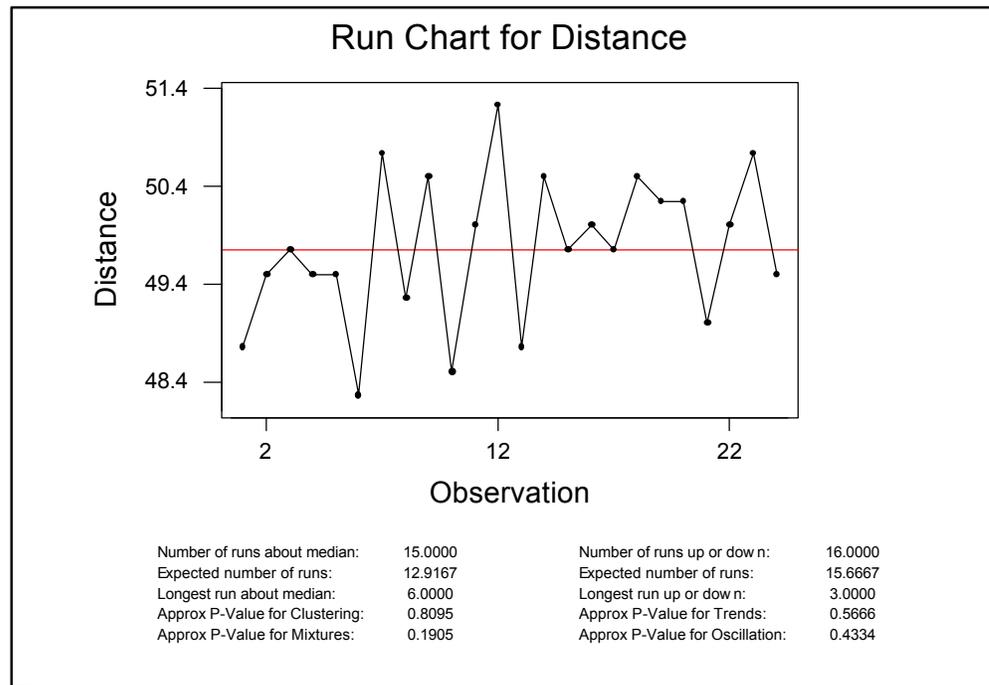
Plot subgroup medians

Select Help Options... OK Cancel

*Do you see any trends, non-random patterns or outliers?*

Visually this data looks OK.

p-values > .05 mean no problems with Clustering, Mixtures, Trends or Oscillation - this is good!





## *Box Plots*

- *Make a box plot of all the data.*
- *Make a box plot for each factor and stratify it by the high and low levels.*
- *Make a box plot for each set of experimental conditions.*
- *Why?*
  - *To get a feel for the variability in all the data*
  - *To look for outliers*
  - *To get impressions of the effects of factors*

*Look for:*

  - Shifts in the average*
  - Shifts in the variability*



# Example

## Box Plots of Catapult Distance

MINITAB FILE: Catapult\_V2.MTW

The screenshot shows the MINITAB software interface. The 'Graph' menu is open, and 'Boxplot...' is selected. The data table below shows the following information:

	C1	C2	C3	C4	C5	C6	C7	C8	C9	
	StdOrder	Run	Order	Rep	ks	Band Position	Stop Position	Cocking Angle	Distance	Exp Con
1	20	1	1	1	1	4	6	150	48.75	4
2	23	2	1	1	1	1	6	180	49.50	7
3	11	3	1	1	1	1	6	150	49.75	3
4	9	4	1	1	1	1	2	150	49.50	1
5	2	5	1	1	1	4	2	150	49.50	2
6	4	6	1	1	1	4	6	150	48.25	4
7	1	7	1	1	1	1	2	150	50.75	1
8	19	8	1	1	1	1	6	150	49.25	3
9	24	9	1	1	1	4	6	180	50.50	8
10	3	10	1	1	1	1	6	150	48.50	3



# Box Plot Results

1. Double clicking chooses the variables to graph.

**Boxplot**

Graph variables: Y (measurement) vs X (category)

Graph	Y	X
1	Distance	
2	Distance	'Band Positi>>
3	Distance	'Stop Positi>>

Data display:

Item	Display	For each	Group variable
1	IQRRange Box	Graph	
2	Outlier S>>	Graph	
3			

Buttons: Select, Annotation, Frame, Regions, Help, Options..., OK, Cancel

2. Click on OK

**Boxplot Options**

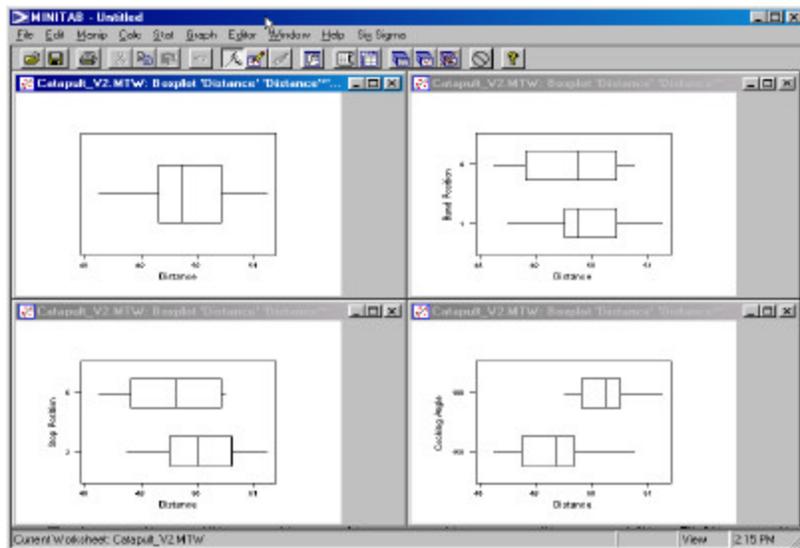
Transpose X and Y

Buttons: Help, OK, Cancel

*Do you see any obvious factor effects or outliers?*

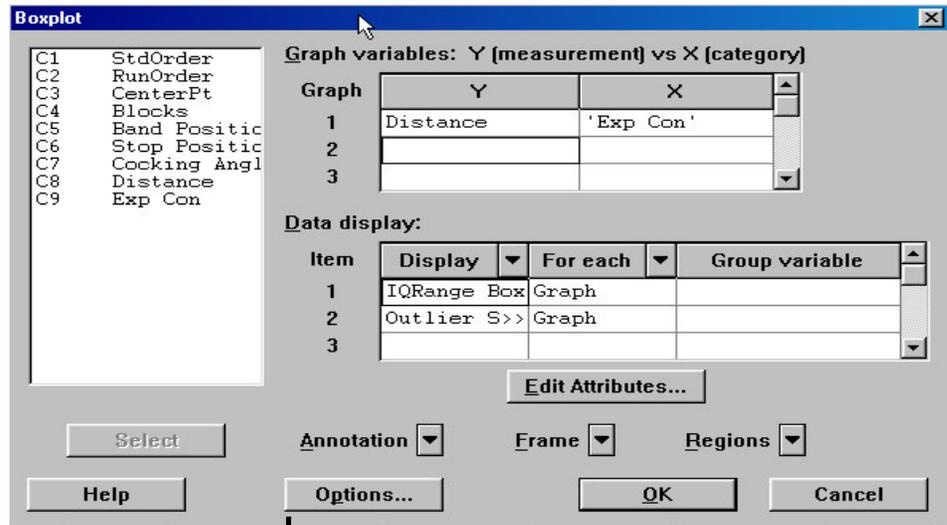
Cocking angle looks like it affects the distance the ball is fired.

No outliers.



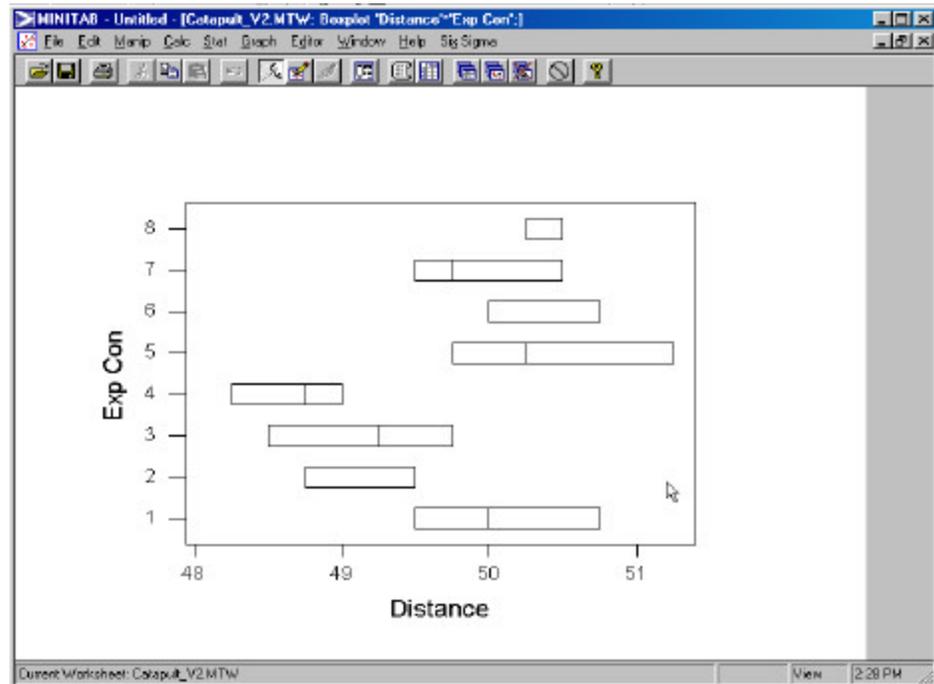


# Box Plot Results



*How consistent is the variability across experimental conditions?*

Experimental condition 5 seems to have the most variability. May want to examine condition 5 for typos, testing problems, etc. *What does this tell us about the experimental variability? (see next pages)*





## Variation Review

### Components of Variation

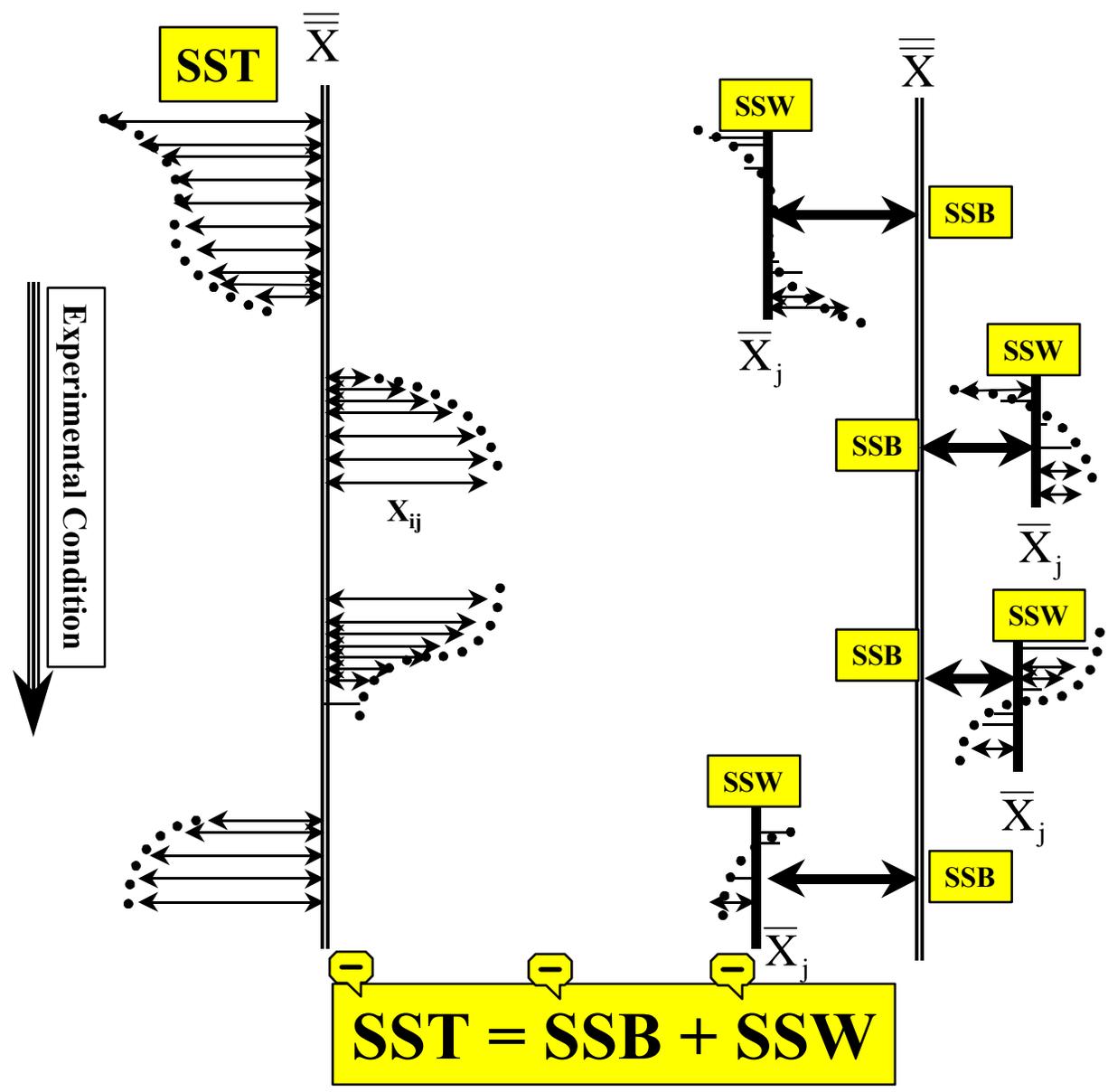
$$\text{SST} \qquad \text{SSB} \quad + \quad \text{SSW}$$
$$\sum_{j=1}^g \sum_{i=1}^n (x_{ij} - \bar{x})^2 = n \sum_{j=1}^g (\bar{x}_j - \bar{x})^2 + \sum_{j=1}^g \sum_{i=1}^n (x_{ij} - x_{j\cdot})^2$$

Residual

*N* = Total # of observations  
*n* = # of observations in each subgroup  
*g* = # of subgroups  
*n* x *g* = total number of observations



# Components of Variation



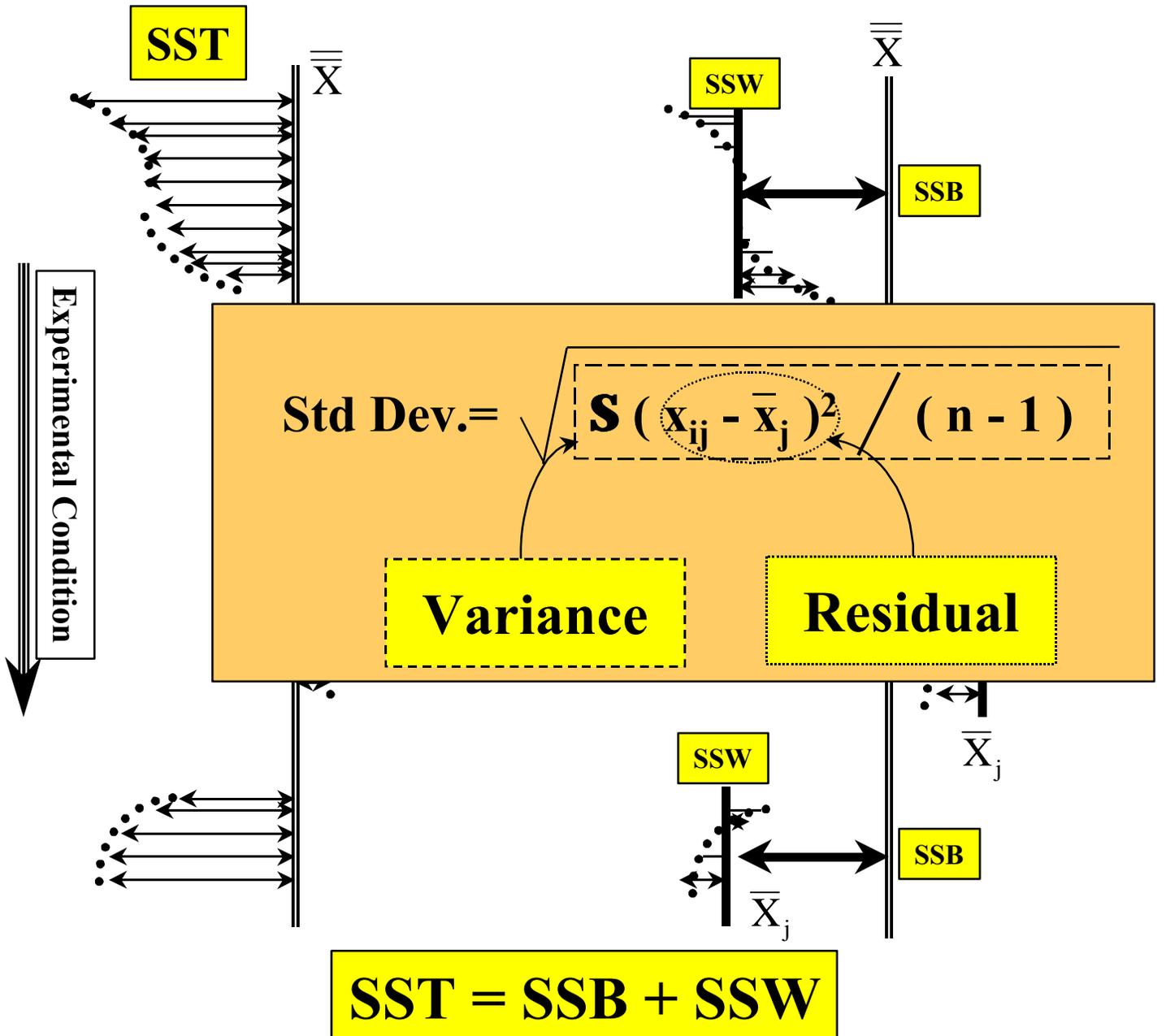
**SST = Total Sum of the Squares**  
**SSW = Sum of the Squares Within**  
**SSB = Sum of the Squares Between**

$$\begin{aligned}
 SST &= \sum \sum (x_{ij} - \bar{x})^2 \\
 SSW &= \sum \sum (x_{ij} - \bar{x}_j)^2 \\
 SSB &= n \sum (\bar{x}_j - \bar{x})^2
 \end{aligned}$$

Residual  
 $x_{ij} - \bar{x}_j$



# Components of Variation



SST = Total Sum of the Squares  
 SSW = Sum of the Squares Within  
 SSB = Sum of the Squares Between

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$SST = \mathbf{S} \mathbf{S} (x_{ij} - \bar{x})^2$   
 $SSW = \mathbf{S} \mathbf{S} (x_{ij} - \bar{x}_j)^2$   
 $SSB = \mathbf{n} \mathbf{S} (\bar{x}_j - \bar{x})^2$

Residual  
 $x_{ij} - \bar{x}_j$



## *Estimate of Experimental Variability*

- *The Pooled Standard Deviation is an estimate of the size of experimental variability or common cause variation associated with the experiment.*
  - *Is the amount of variability that is not explained by the factors*
  - *Indicates the average amount of variability among runs performed at the same experimental conditions*
  - *Is **not** the standard deviation of all the **data** combined*
  - *“Pools” the standard deviation calculated from each condition across all experimental conditions*

$$s_p = \sqrt{\text{Average of Variances}}$$

- *Assumes the variability is the same at each experimental condition*
- *Cannot be calculated without replication*
- ***Purpose:** To help us judge whether factors in the experiment had an effect or not.*

\* This formula holds if there is an equal number of replicates at each condition, otherwise use a weighted average.



# Minitab Calculation of $s_p$

## MINITAB FILE: Catapult\_V2.MTW

The screenshot shows the Minitab software interface. The main window displays the 'Catapult\_V2.MTW' worksheet with the following data:

	C1	C2	C3	C4	C5	C6	C7	C8	C9
	StdOrder	RunOrder	CenterPt	Blocks	Band Position	Stop Position	Cocking Angle	Distance	Exp Con
3	11	3							3
4	9	4							1
5	2	5							2
6	4	6							4
7	1	7							1
8	19	8							3

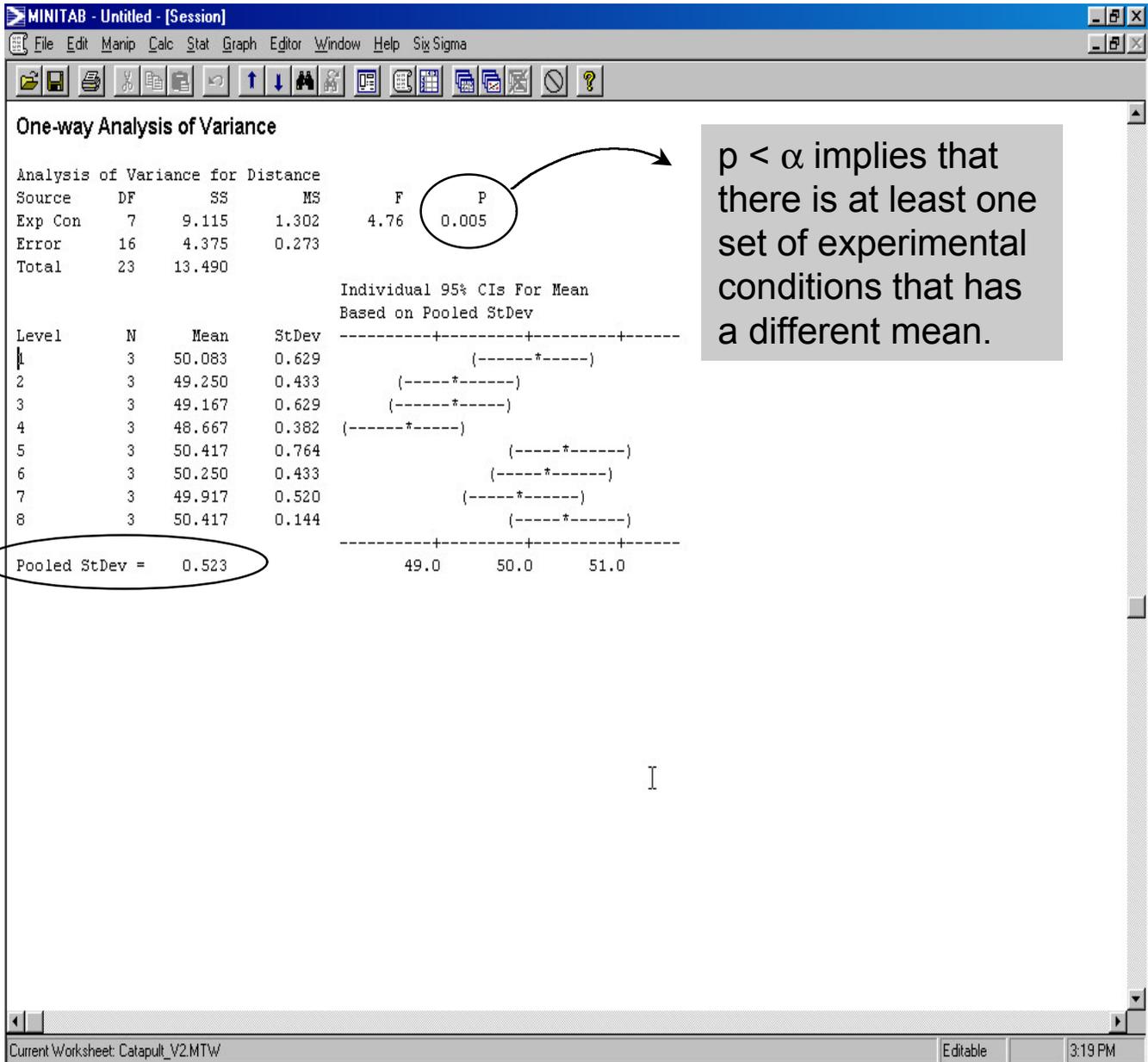
The 'One-way Analysis of Variance' dialog box is open, showing the following settings:

- Response: Distance
- Factor: 'Exp Con'
- Comparisons... (highlighted)
- Store residuals
- Store fits

Buttons at the bottom of the dialog box include: Select, Graphs..., Help, OK, and Cancel.



# Minitab Calculation (cont.)





## *Steps in Analysis: Full Factorial, Replicated Designs*

**Diagnostics:**

**Is data OK?**

1. *Plot the raw data.*

2. ***Plot the residuals.***

**Analysis:**  
**Make inferences**

3. *Examine factor effects.*

4. *Confirm impressions with  
statistical procedures.*

5. *Summarize conclusions.*



## *Plot the Residuals*

### *Second Step in Analysis*

- *Residuals are the “leftover” variation in the data after accounting for the main cause of variation: different experimental conditions.*
- *Like the pooled standard deviation, residuals tell us about common cause variation.*
- *Notice the formula for the standard deviation is based on the residuals.*

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n - 1}}$$

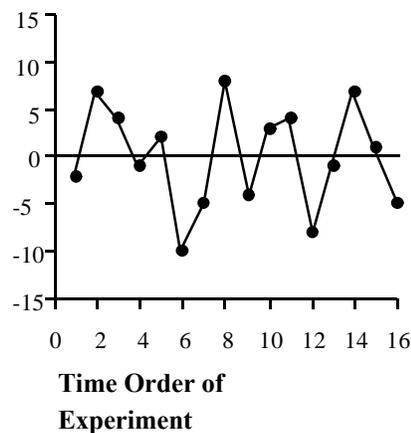
- *Residuals usually follow a normal distribution.*
- *Examining residuals carefully and taking appropriate action will increase the accuracy of our conclusions.*



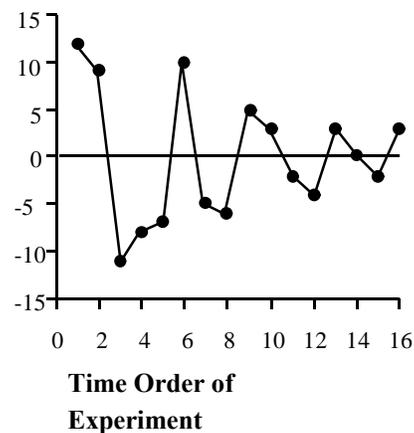
## Residuals Plot 1

*Plot the residuals against time order*

a) Residuals



b) Residuals



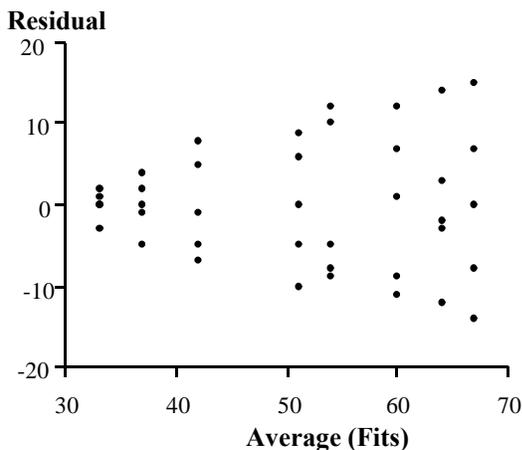
### ■ Why?

- *To ensure that the experimental variability has only common causes associated with it*
- *To look for lurking variables (trends, outliers, or non-random patterns) that might influence our conclusions. They may have been hidden in other plots*

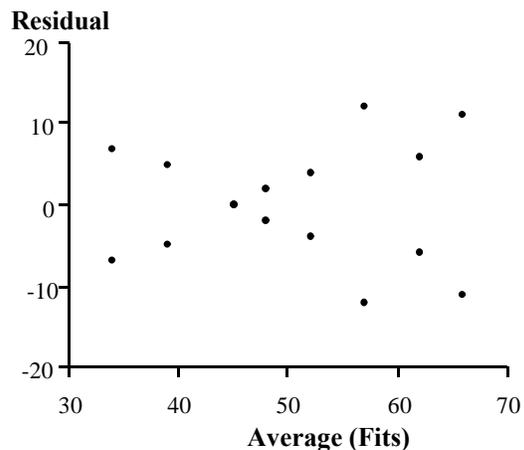


## Residuals Plot 2

*Plot the residuals against the average (“Fits” in Minitab) for each experimental condition.*



a) 5 replicates of 8 experimental conditions



b) 2 replicates of 8 experimental conditions

### ■ Why?

- *To look for a non-random pattern, such as a megaphone shape*
  - *The megaphone shape indicates that variation increases as the average increases*
  - *Ignore the pattern indicated by the symmetry of dots around 0. It is not a special cause. Two replicates will always appear perfectly matched.*



## *Residual Plot 3*

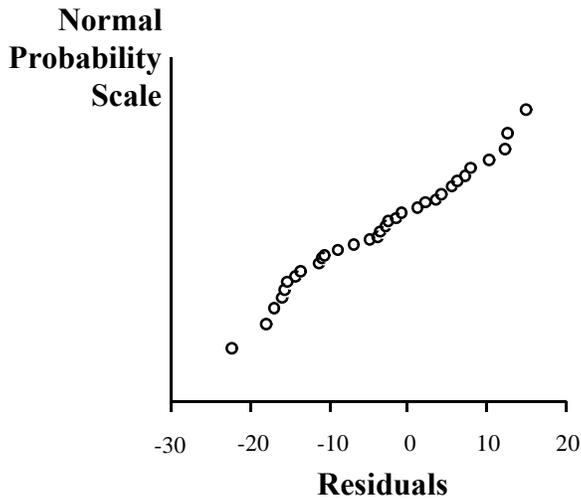
*Plot the residuals on a normal probability scale.*

### ■ *Why?*

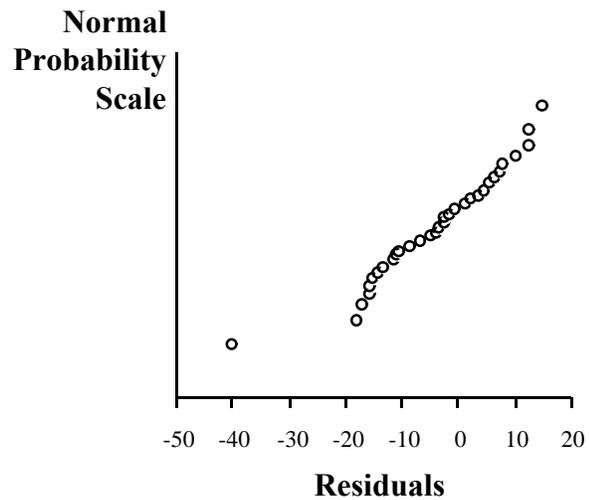
- *To look for major deviations from non - “straight line” relationships. This implies the residuals are not normally distributed. The variation in the experiment is not random.*
- *To look for outliers*



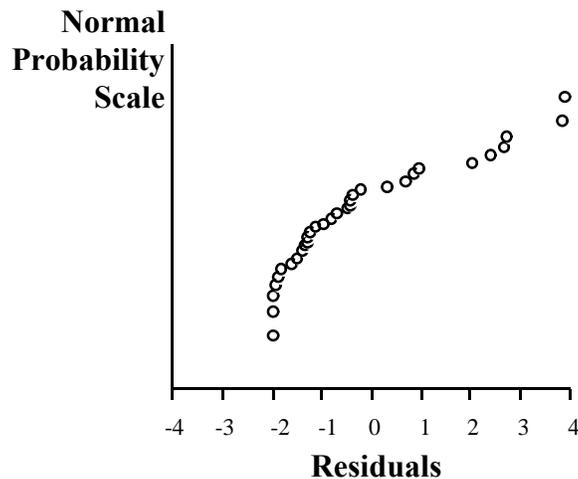
# Normal Probability Plots



a) A straight-line relationship indicates the data follow a normal distribution.



b) These values follow a normal distribution except for the outlier. Check it.



c) This S-shape indicates these values are not normally distributed.



# Creating Residual Plots with Minitab

MINITAB FILE: Catapult\_V2.MTW

**One-way Analysis of Variance**

Source	DF	SS	MS
Exp Con	7		
Error	16		
Total	23		

Level	N	Mean	StDev
1	3	50.083	0.629
2	3	49.250	0.433
3	3	49.167	0.629
4	3	48.667	0.382
5	3	50.417	0.764

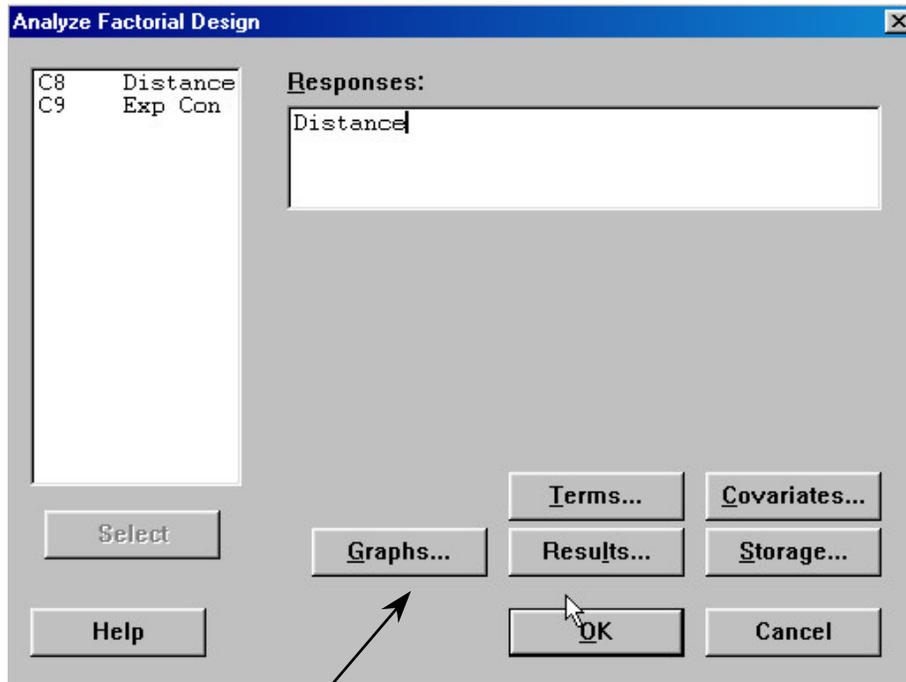
  

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
↓	StdOrder	RunOrder	CenterPt	Blocks	Band Position	Stop Position	Cocking Angle	Distance	Exp Con	
1	20	1	1	1	4	6	150	48.75	4	
2	23	2	1	1	1	6	180	49.50	7	
3	11	3	1	1	1	6	150	49.75	3	
4	9	4	1	1	1	2	150	49.50	1	
5	2	5	1	1	4	2	150	49.50	2	
6	4	6	1	1	4	6	150	48.25	4	

Fit a model to the experimental data



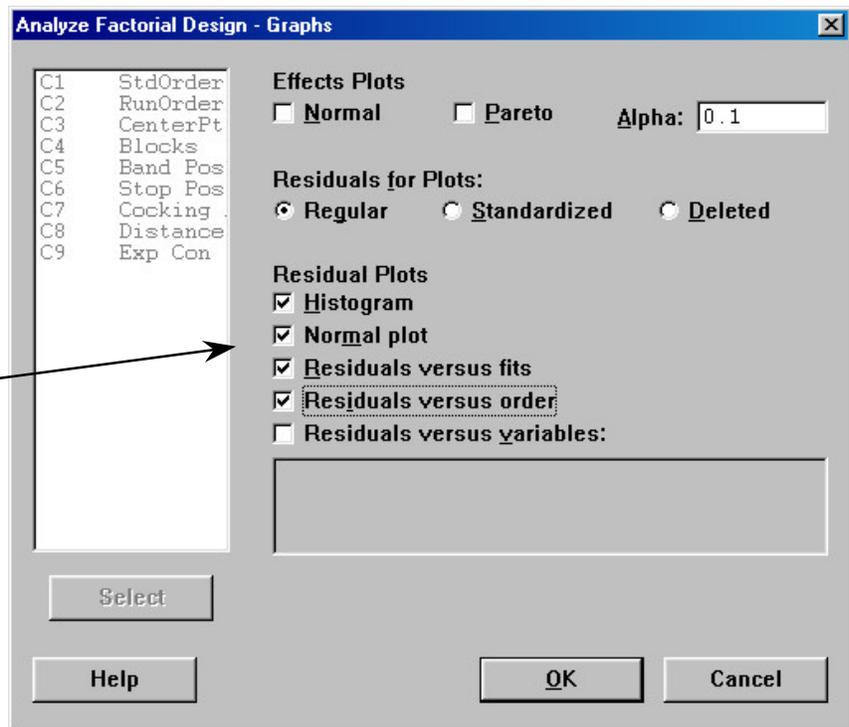
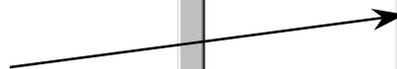
# Creating Residual Plots with Minitab (cont.)



Click on:

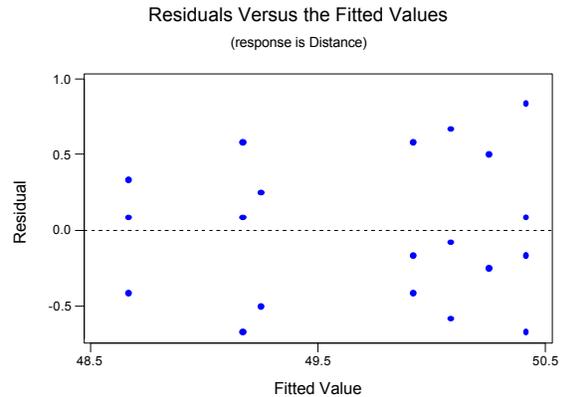
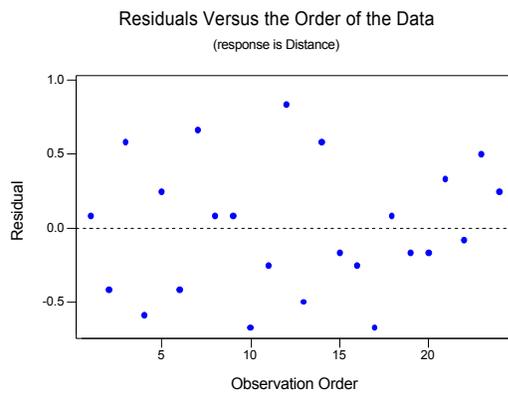


Check top four residual plots.





# Residual Plot Results

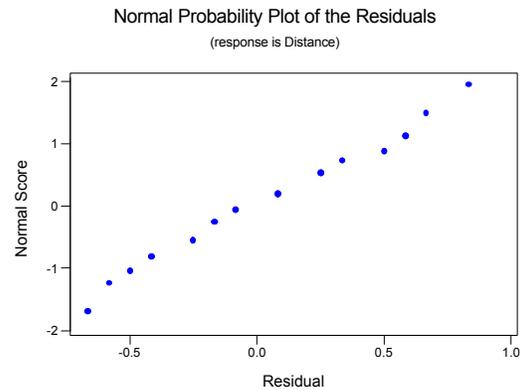
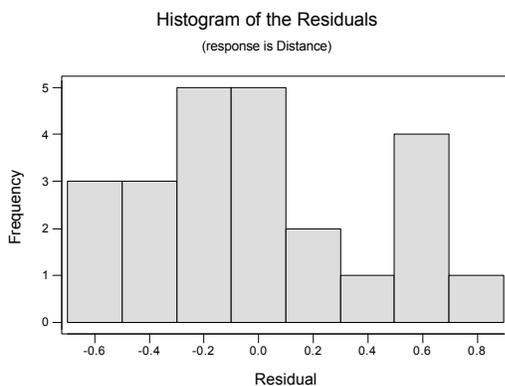


*Do you see any trends, non-random patterns or outliers?*

**No, this data looks OK.**

*Do you see any correlation between residuals and fits (means)? Megaphones?*

**No, this data looks OK.**



*Are the residuals normally distributed?*

**Yes, this data looks OK.**



## *Actions from Residual Plots*

- *If you have an **outlier**, try to find an explanation and correct the data. If you cannot find an explanation, leave it in your data as it is.*
- *If you see a **trend** in the residuals over time, you may have a lurking variable. Look for the cause and correct the situation.*
- *If you have a **correlation** between the residuals and fits, try to understand from where this relationship is coming. Is it something you can control? Is it inherent in your process? Control your experiment better.*
- *If your residuals are **not normally distributed**, you may have a significant factor for which you are not accounting. Look for the factor and include it in your next experiment.*
- *If you need more help, call your MBB or local statistician.*



## *Steps in Analysis: Full Factorial, Replicated Designs*

---

**Diagnostics:** 1. *Plot the raw data.*  
**Is data OK?** 2. *Plot the residuals.*

**Analysis:** 3. ***Examine factor effects.***  
**Make inferences** 4. *Confirm impressions with statistical procedures.*  
5. *Summarize conclusions.*



## *Examine Factor Effects*

### ■ *The Third Step in Analysis*

$$Y = f(X_1, X_2, X_3, \dots, X_n)$$

Y (Response) = Distance

$X_i$  (Factors) = Band Position, Stop Position,  
Cocking Angle

- *How do the factors affect the response?*
- *How do the combinations (interactions) of factors affect the response?*
  - *We can write the equation that answers these questions*

*Distance = Constant + Band Position Effect + Stop Position Effect + Cocking Angle Effect + Band Position\*Stop Position Interaction Effect + Band Position\*Cocking Angle Interaction Effect + Stop Position\*Cocking Angle Interaction Effect + Band Position\*Stop Position\* Cocking Angle Interaction Effect*



## The “Average” Distance Launched

*How far is the “average” distance launched?*

Std. Order	Band Position	Stop Position	Cocking Angle	Data			Average
				(Rep 1)	(Rep 2)	(Rep 3)	
1	–	–	–	49.5	50.75	50.00	
2	+	–	–	49.5	48.75	49.50	
3	–	+	–	49.75	49.25	48.50	
4	+	+	–	48.75	48.25	49.00	
5	–	–	+	51.25	49.75	50.25	
6	+	–	+	50.00	50.00	50.75	
7	–	+	+	49.50	50.50	49.75	
8	+	+	+	50.50	50.50	50.25	
				Sum			<u>1194.50</u>
				Overall Average			<u>49.77</u>

*Distance = 49.77 + Band Position Effect + Stop Position Effect + Cocking Angle Effect + Band Position\*Stop Position Interaction Effect + Band Position\*Cocking Angle Interaction Effect + Stop Position\*Cocking Angle Interaction Effect + Band Position\*Stop Position\* Cocking Angle Interaction Effect*



## Main Effect - Cocking Angle

### How does Cocking Angle affect Distance?

Std. Order	Band Position	Stop Position	Cocking Angle	Average	Cocking Angle “-”	Cocking Angle “+”
1	-	-	-	50.10	50.10	
2	+	-	-	49.25	49.25	
3	-	+	-	49.17	49.17	
4	+	+	-	48.67	48.67	
5	-	-	+	50.42		50.42
6	+	-	+	50.25		50.25
7	-	+	+	49.92		49.92
8	+	+	+	50.42		50.42

Cocking Angle “-” Total 197.19  
Cocking Angle “-” Average 49.30

Cocking Angle “+” Total 201.01  
Cocking Angle “+” Average 50.25

Cocking Angle Effect = [Cocking Angle “+” Average] - [Cocking Angle “-” Average] = 0.95

**As Cocking Angle goes from “-” to “+”  
distance *increases* by 0.95 inches.**



# Main Effects - Band & Stop Positions

## How does Band Position affect Distance?

	Band Position “-”	Band Position “+”
	—	—
	—	—
	—	—
	—	—
<b>Total:</b>	—	—
<b>Average:</b>	—	—
	[Band Position “+” Average] - [Band Position “-” Average] = ___	

## How does Stop Position affect Distance?

	Stop Postition “-”	Stop Position “+”
	—	—
	—	—
	—	—
	—	—
<b>Total:</b>	—	—
<b>Average:</b>	—	—
	[Stop Position “+” Average] - [Stop Position “-” Average] = ___	



## *Main Effects - Band & Stop Positions*

### **How does Band Position affect Distance?**

	Band Position "-"	Band Position "+"
	50.10	49.25
	49.17	48.67
	50.42	50.25
	49.92	50.42
Total:	199.61	198.59
Average:	49.90	49.65

$$[\text{Band Position "+" Average}] - [\text{Band Position "-" Average}] = -.25$$

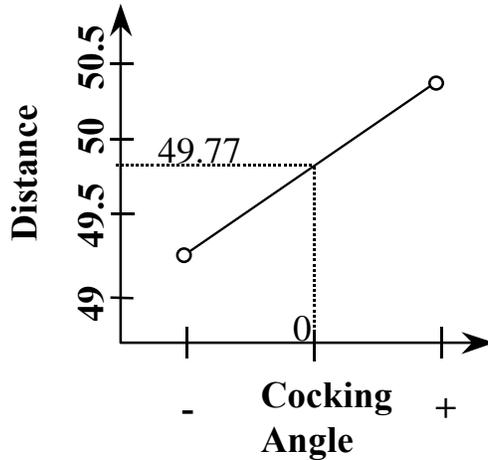
### **How does Stop Position affect Distance?**

	Stop Position "-"	Stop Position "+"
	50.10	49.17
	49.25	48.67
	50.42	49.92
	50.25	50.42
Total:	200.02	198.18
Average:	50.00	49.55

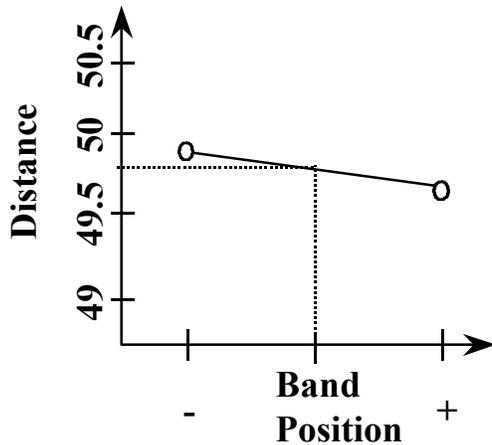
$$[\text{Stop Position "+" Average}] - [\text{Stop Position "-" Average}] = -.45$$



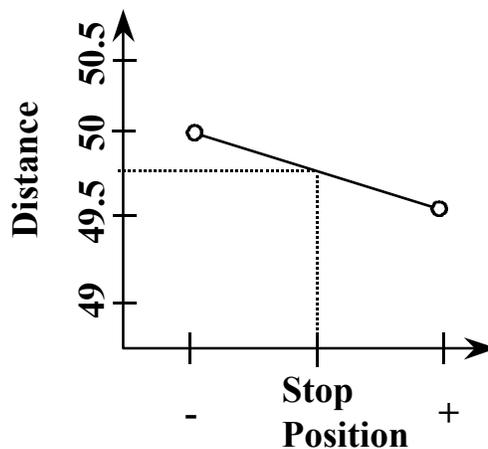
## Main Effect Plots



As Cocking Angle goes from “-” to “+” distance *increases* by 0.95 inches.



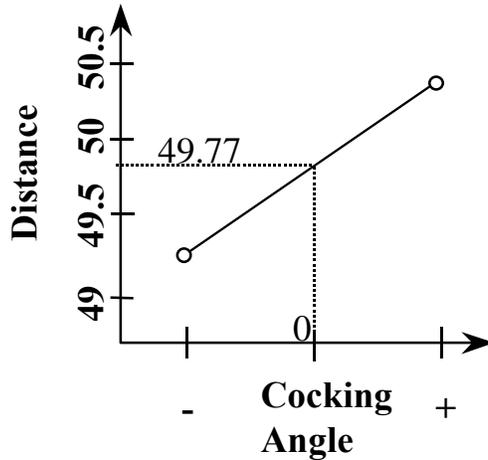
As Band Position goes from “-” to “+” distance *decreases* by 0.25 inches.



As Stop Position goes from “-” to “+” distance *decreases* by 0.45 inches.



# Factor Coefficients



As Cocking Angle goes from “-” to “+” distance *increases* by 0.95 inches.

A **Factor Effect** is the change in response due to a “two-unit” (-1 to +1) change in the factor.

A **Factor Coefficient** is the change in response due to a “one-unit” (-1 to 0 or 0 to +1) change in the factor.

**Factor Coefficient = Factor Effect / 2**

When Cocking Angle moved from -1 to 0, distance increased by 0.48 inches (0.95 / 2).

**The Coefficient for Cocking Angle = 0.48 inches**

$Distance = 49.77 - 0.13 * Band\ Position - 0.23 * Stop\ Position + 0.48 * Cocking\ Angle + Band\ Position * Stop\ Position\ Interaction\ Effect + Band\ Position * Cocking\ Angle\ Interaction\ Effect + Stop\ Position * Cocking\ Angle\ Interaction\ Effect + Band\ Position * Stop\ Position * Cocking\ Angle\ Interaction\ Effect$



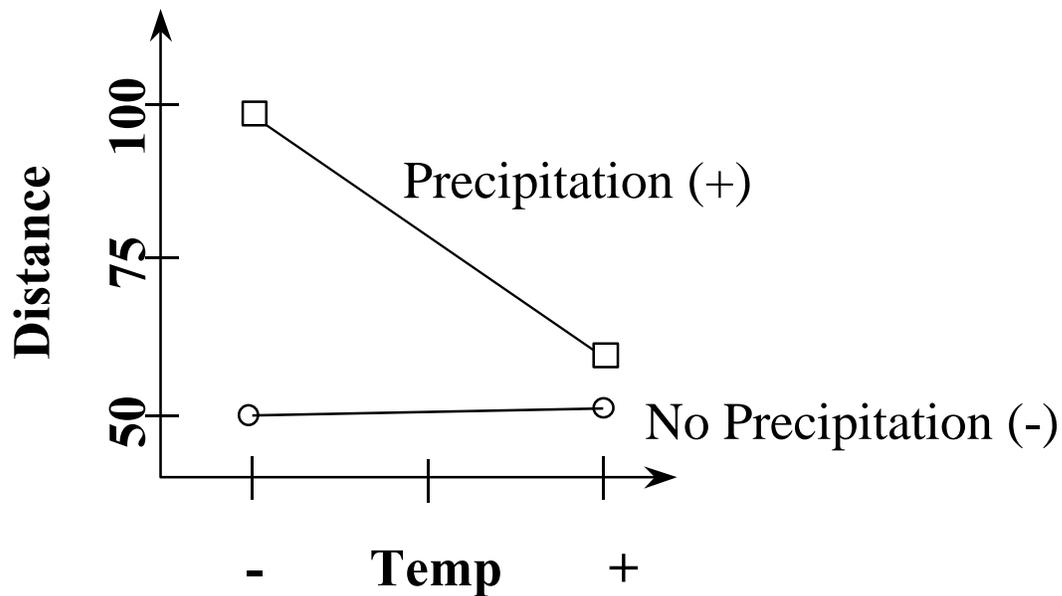
## *Factor Interactions*

- **Definition:** *Factors are said to interact if the effect that one factor has on the response is dependent on the level of another factor(s).*
- **Example:** *The effect that ambient temperature has on stopping distance of a car is dependent on whether or not there is precipitation.*
  - *A car will stop in the same distance for temperatures above or below freezing, if there is no precipitation*
  - *A car will take much longer to stop when the temperature is below freezing than above freezing, if there is precipitation*
  - **Response: Stopping Distance (feet)**
  - **Factor A: Ambient Temperature**
    - *Below Freezing*
    - + *Above Freezing*
  - **Factor B: Precipitation**
    - *No Precipitation*
    - + *Precipitation*



## *Example: Stopping Distance Interaction*

<u>Temp.</u>	<u>Precip.</u>	<u>Distance</u>
-	-	50
+	-	54
-	+	100
+	+	60

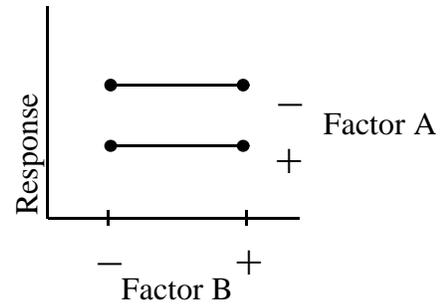
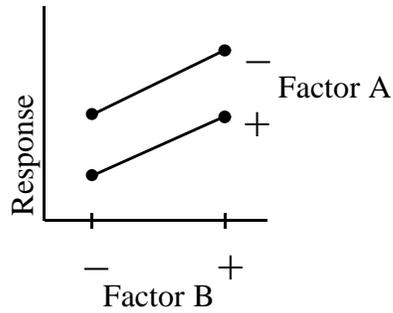


***There is an Interaction Between Temperature and Precipitation***

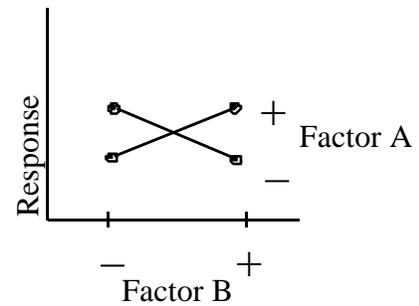
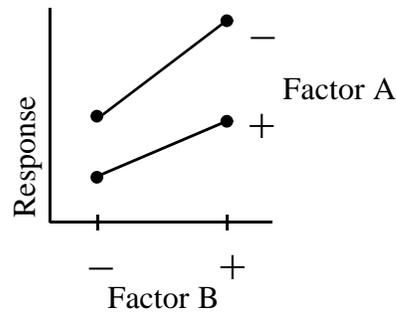


# Interpreting Interaction Plots

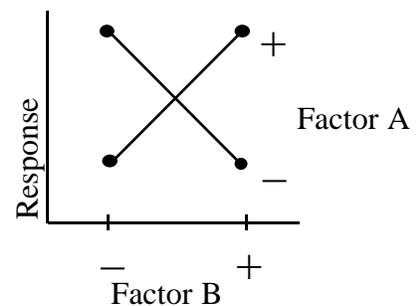
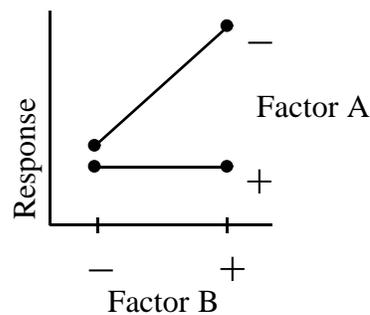
No Interaction



Mild Interaction



High Interaction



Adapted from Lawson, John and John Erjavac, *Basic Experimental Strategies and Data Analysis* Provo, UT: Brigham Young University, p. 104.



## Interaction Effect

- To calculate the Interaction Effect, it is necessary to make an Interaction Column in our design matrix by multiplying the Temp. and Precip. factor level columns together.

$$(-)(-) = + \quad (+)(-) = - \quad (+)(+) = +$$

<u>Temp.</u>	<u>Precip.</u>	<u>T*P</u>	<u>Distance</u>
-	-	+	50
+	-	-	54
-	+	-	100
+	+	+	60

$$\text{T*P Effect} = (\text{T*P “+” Average}) - (\text{T*P “-” Average})$$

$$\text{T*P Effect} = (50+60)/2 - (54+100)/2$$

$$\text{T*P Effect} = - 22 \text{ feet}$$

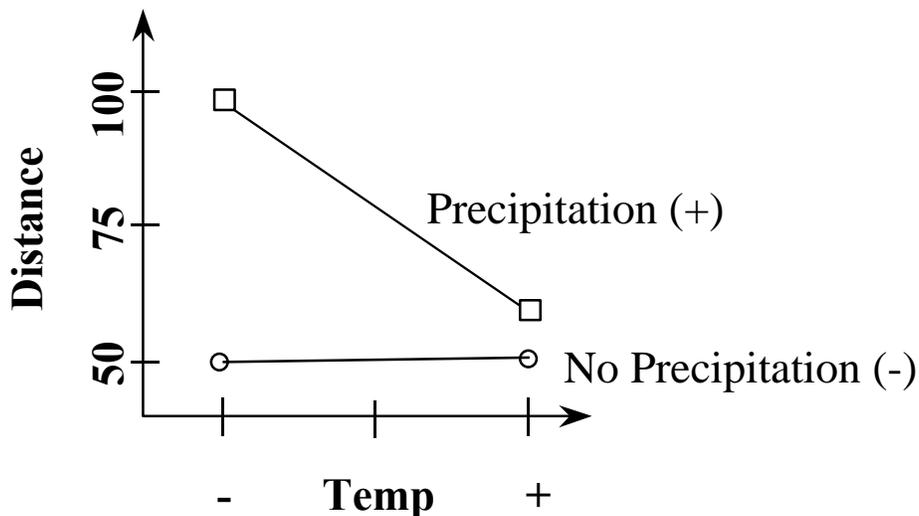
$$\text{T*P Coefficient} = - 11 \text{ feet}$$

$$\text{Stopping Distance} = \text{Average} + \text{Main Effects} - 11(\text{T*P})$$



## Interaction Interpretation

- *Stopping Distance = Average + Main Effects - 11(T\*P)*
  - *To stop 11' shorter, set T\*P to "+" (+,+ or -,-)*
  - *Which one should you pick? +,+ or -,-?*



- *You could use the graph to help you decide. (-,- stops at 50' and +,+ stops at 60') Pick -,-.*
- *Or you could use the Main Effect information to help you decide.*



---

## *Stopping Distance Exercise*

---

- 1. Calculate the Average and the Main Effects for the Stopping Distance data.*
- 2. Complete the equation from the previous page.*
- 3. What levels would you set the factors at to calculate the minimum stopping distance?*



## *Factor Names and Interaction Symbols*

- *It is useful to abbreviate factor names. There are several ways to do this:*
  - *Choose the first letter or first two letters of each factor*
  - *Assign factors a letter: A, B, C, D, . . .*
- *Denote the interaction between Factor A and Factor B as:*
  - *A x B or AB*
- *For two factors (A and B) there is one interaction: AB*
- *For three factors there are*
  - *Three two-way interactions: AB, AC, BC*
  - *One three-way interaction: ABC*
- *How many interactions are there for four factors (A,B,C,D)?*



## *Interaction Columns for a 2<sup>3</sup> Design*

<b>Std. Order</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>AB</b>	<b>AC</b>	<b>BC</b>	<b>ABC</b>	<b>Average Response</b>
1	-	-	-	+	+	+	-	_____
2	+	-	-	-	-	+	+	_____
3	-	+	-	-	+	-	+	_____
4	+	+	-	+	-	-	-	_____
5	-	-	+	+	-	-	+	_____
6	+	-	+	-	+	-	-	_____
7	-	+	+	-	-	+	-	_____
8	+	+	+	+	+	+	+	_____



## Example

### Worksheet for Computing Catapult Effects

Std. Order	BP	SP	CA	BPSP	BPCA	SPCA	BPSPCA	Average Distance
1	-	-	-	+	+	+	-	<u>50.10</u>
2	+	-	-	-	-	+	+	<u>49.25</u>
3	-	+	-	-	+	-	+	<u>49.17</u>
4	+	+	-	+	-	-	-	<u>48.67</u>
5	-	-	+	+	-	-	+	<u>50.42</u>
6	+	-	+	-	+	-	-	<u>50.25</u>
7	-	+	+	-	-	+	-	<u>49.92</u>
8	+	+	+	+	+	+	+	<u>50.42</u>
$\Sigma (+)$	<u>198.59</u>	—	—	<u>199.61</u>	—	—	<u>199.26</u>	Sum the Ave. Distance corresponding to all the pluses.
$\Sigma (-)$	<u>199.61</u>	—	—	<u>198.59</u>	—	—	<u>198.94</u>	Sum the Ave. Distance corresponding to all the minuses.
$\Sigma (+) + \Sigma (-)$	<u>398.20</u>	—	—	<u>398.20</u>	—	—	<u>398.20</u>	Arithmetic Check: Should equal the same number in each column.
$\Sigma (+) - \Sigma (-)$	<u>-1.02</u>	—	—	<u>1.02</u>	—	—	<u>0.32</u>	
$[\Sigma (+) - \Sigma (-)] \div 4$	<u>-0.25</u>	<u>-0.46</u>	<u>0.95</u>	<u>0.25</u>	<u>0.42</u>	<u>0.29</u>	<u>0.08</u>	These are the "EFFECTS" or the signal from the experiment.

$$\begin{aligned}
 \text{Distance} = & 49.77 - 0.13 * \text{Band Position} - 0.23 * \text{Stop Position} + 0.48 * \\
 & \text{Cocking Angle} + 0.13(\text{Band Position} * \text{Stop Position}) + \\
 & 0.21(\text{Band Position} * \text{Cocking Angle}) + 0.15(\text{Stop} \\
 & \text{Position} * \text{Cocking Angle}) + 0.04(\text{Band Position} * \text{Stop} \\
 & \text{Position} * \text{Cocking Angle})
 \end{aligned}$$



# Catapult Example Using Minitab

MINITAB FILE: Catapult\_V2.MTW

**One-way Analysis of Variance**

Source	DF	SS	MS
Exp Con	7		
Error	16		
Total	23		

Level	N	Mean	StDev
1	3	50.083	0.629
2	3	49.250	0.433
3	3	49.167	0.629
4	3	48.667	0.382
5	3	50.417	0.764

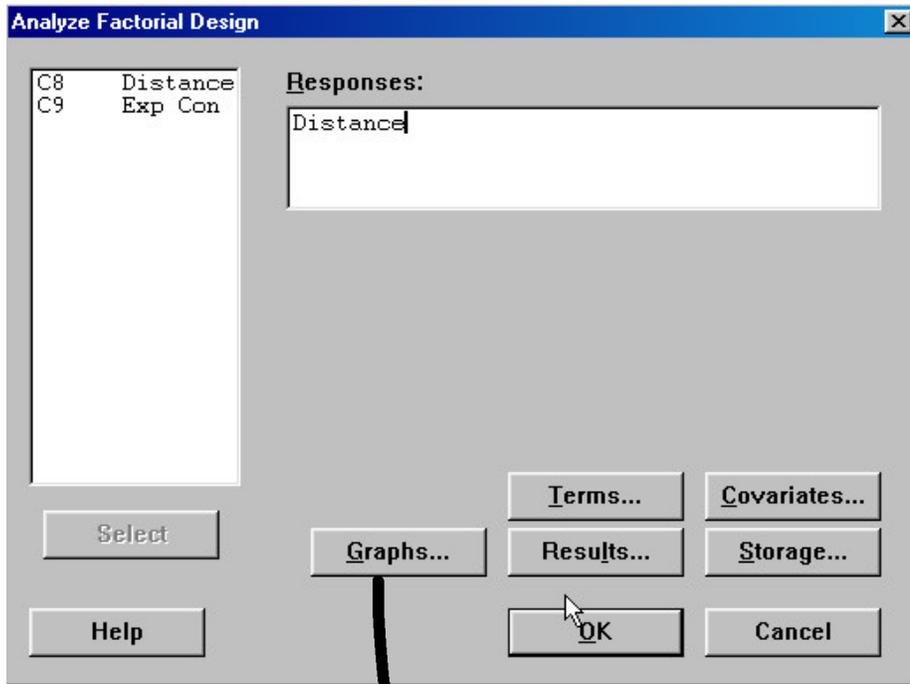
  

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
↓	StdOrder	RunOrder	CenterPt	Blocks	Band Position	Stop Position	Cocking Angle	Distance	Exp Con	
1	20	1	1	1	4	6	150	48.75	4	
2	23	2	1	1	1	6	180	49.50	7	
3	11	3	1	1	1	6	150	49.75	3	
4	9	4	1	1	1	2	150	49.50	1	
5	2	5	1	1	4	2	150	49.50	2	
6	4	6	1	1	4	6	150	48.25	4	

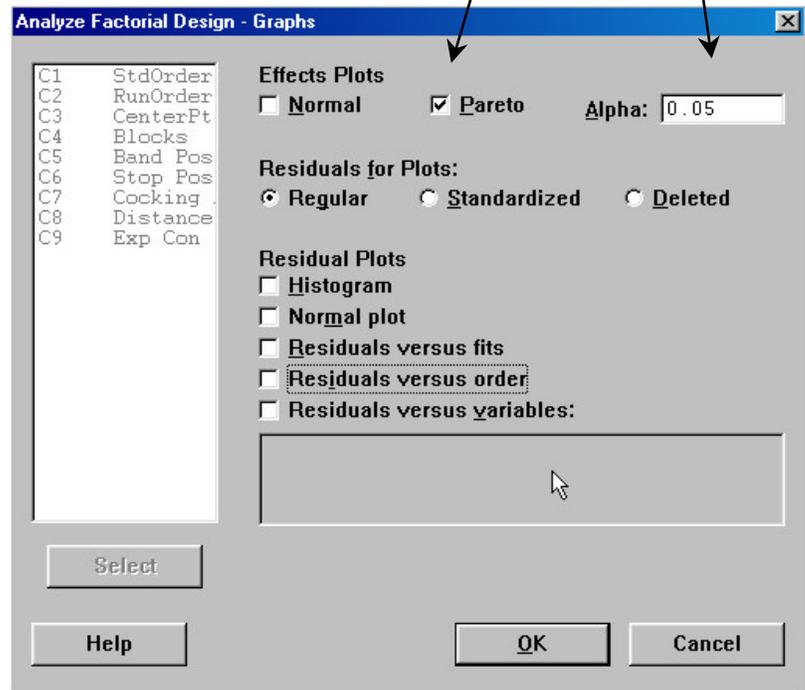
Fit a model to the experimental data



# Minitab Input

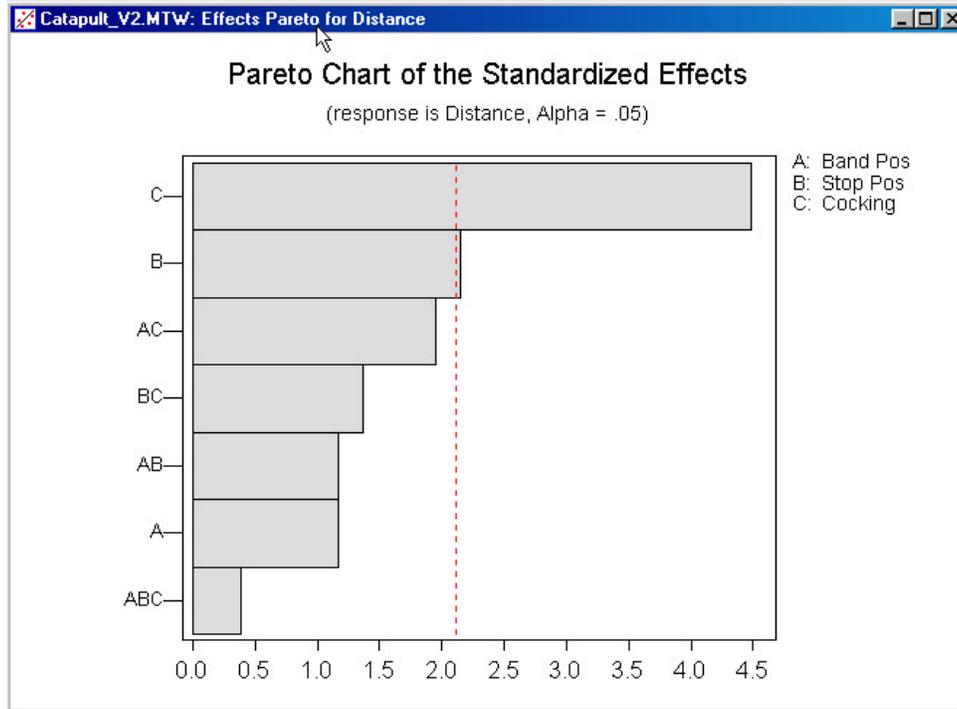


Change alpha to .05  
Choose Pareto





# Minitab Results



Session

### Fractional Factorial Fit

Estimated Effects and Coefficients for Distance (coded units)

Term	Effect	Coef	StDev Coef	T	P
Constant		49.7708	0.1067	466.29	0.000
Band Pos	-0.2500	-0.1250	0.1067	-1.17	0.259
Stop Pos	-0.4583	-0.2292	0.1067	-2.15	0.047
Cocking	0.9583	0.4792	0.1067	4.49	0.000
Band Pos*Stop Pos	0.2500	0.1250	0.1067	1.17	0.259
Band Pos*Cocking	0.4167	0.2083	0.1067	1.95	0.069
Stop Pos*Cocking	0.2917	0.1458	0.1067	1.37	0.191
Band Pos*Stop Pos*Cocking	0.0833	0.0417	0.1067	0.39	0.701

Analysis of Variance for Distance (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	7.1458	7.14583	2.38194	8.71	0.001
2-Way Interactions	3	1.9271	1.92708	0.64236	2.35	0.111
3-Way Interactions	1	0.0417	0.04167	0.04167	0.15	0.701
Residual Error	16	4.3750	4.37500	0.27344		
Pure Error	16	4.3750	4.37500	0.27344		
Total	23	13.4896				



# Factorial Plots Setup

MINITAB FILE: Catapult\_V2.MTW

Analysis of Variance for Distance

Source	DF	Seq	F	P
Main Effects	3	7.34	3.71	0.001
2-Way Interactions	3	1.8271	1.82708	0.64236
3-Way Interactions	1	0.0417	0.04167	0.04167
Residual Error	16	4.3750	4.37500	0.27344
Pure Error	16	4.3750	4.37500	0.27344
Total	23	13.4896		

7	1	7	1	1	1	2	150	50.75	1		
8	19	8	1	1	1	8	150	49.25	3		
9	24	9	1	1	4	6	180	50.50	8		
10	3	10	1	1	1	6	150	48.50	3		
11	22	11	1	1	4	2	180	50.00	6		
12	5	12	1	1	1	2	180	51.25	5		
13	18	13	1	1	4	2	150	48.75	2		
14	14	14	1	1	1	6	180	50.50	7		

Check Main and Interaction

**Factorial Plots**

**Main effects (response versus levels of 1 factor)** Setup...

**Interaction (response versus levels of 2 factors)** Setup...

**Cube (response versus levels of 2 to 8 factors)** Setup...

Type of Means to Use in Plots

**Data Means**

**Fitted Means**

Help OK Cancel

Click Setup for both



# Factorial Plots

**Factorial Plots - Main**

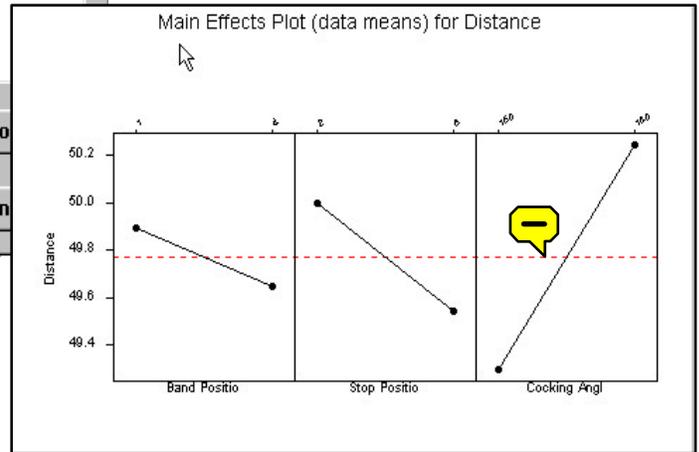
Responses: Distance

Factors to Include in Plots

Available:

Selected: A: Band Positio, B: Stop Positio, C: Cocking Angl

Buttons: Select, Help, OK, Cancel, Options



**Factorial Plots - Interact**

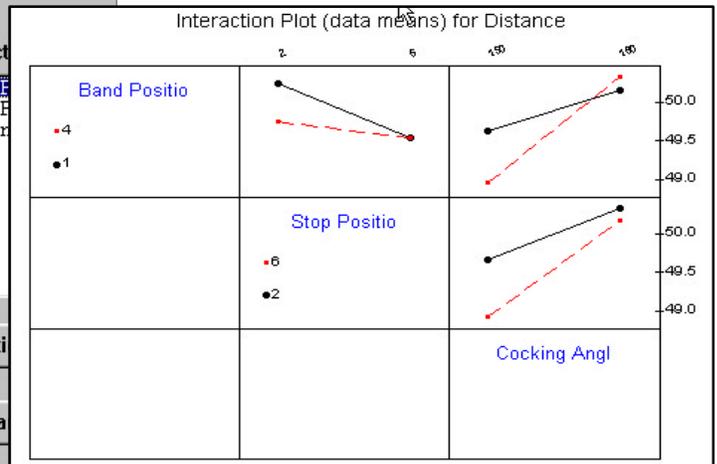
Responses: Distance

Factors to Include in Plots

Available:

Selected: A: Band Positio, B: Stop Positio, C: Cocking Angl

Buttons: Select, Help, OK, Cancel, Options





## *Steps in Analysis: Full Factorial, Replicated Designs*

**Diagnostics:** 1. *Plot the raw data.*  
**Is data OK?** 2. *Plot the residuals.*

**Analysis:** 3. *Examine factor effects.*  
**Make inferences** 4. **Confirm impressions with statistical procedures.**  
5. *Summarize conclusions.*



# Confirm Impressions

## The Third Step in Analysis

- By now you have a good idea about the influence of the factors in the experiment. You can confirm these impressions statistically by performing a Hypothesis Test.
- $H_o$ : Factor has no effect on the results
- $H_a$ : Factor has an effect on the results
- $p > \alpha$ : Reject  $H_a$
- $p < \alpha$ : Accept  $H_a$

Session

Fractional Factorial Fit

Estimated Effects and Coefficients for Distance (coded units)

Term	Effect	Coef	StDev Coef	T	P
Constant		49.7708	0.1067	466.29	0.000
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Band Pos*Stop Pos	0.2500	0.1250	0.1067	1.17	0.259
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3-Way Interactions	1	0.0417	0.04167	0.04167	0.15	0.701
Residual Error	16	4.3750	4.37500	0.27344		
Pure Error	16	4.3750	4.37500	0.27344		
Total	23	13.4896				

**Only Cocking Angle has a significant affect on the distance.**



## 4. Practical Significance

### MINITAB FILE: Catapult\_V2.MTW

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the 'ANOVA' option is selected, leading to a sub-menu where 'General Linear Model...' is highlighted. Below the menu, there are several lines of text indicating macro execution:

```
* NOTE * There is partial confounding, no alias table was printed.  
Executing from file: C:\PROGRAM FILES\MTBWIN\MACROS\FFMain.MAC  
  
Macro is running ... please wait  
Executing from file: C:\PROGRAM FILES\MTBWIN\MACROS\FFInt.MAC  
  
Macro is running ... please wait
```

The data table below shows 12 rows of experimental data with 12 columns (C1-C12). The columns are labeled as follows: C1 (StdOrder), C2 (RunOrder), C3 (CenterPt), C4 (Blocks), C5 (Band Position), C6 (Stop Position), C7 (Cocking Angle), C8 (Distance), C9 (Exp Con), C10, C11, and C12.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
↓	StdOrder	RunOrder	CenterPt	Blocks	Band Position	Stop Position	Cocking Angle	Distance	Exp Con			
1	20	1	1	1	4	6	150	48.75	4			
2	23	2	1	1	1	6	180	49.50	7			
3	11	3	1	1	1	6	150	49.75	3			
4	9	4	1	1	1	2	150	49.50	1			
5	2	5	1	1	4	2	150	49.50	2			
6	4	6	1	1	4	6	150	48.25	4			
7	1	7	1	1	1	2	150	50.75	1			
8	19	8	1	1	1	6	150	49.25	3			
9	24	9	1	1	4	6	180	50.50	8			
10	3	10	1	1	1	6	150	48.50	3			
11	22	11	1	1	4	2	180	50.00	6			
12	5	12	1	1	1	2	180	51.25	5			

Perform multivariate and univariate analysis of variance on balanced or unbalanced data



## Practical Significance (cont.)

**General Linear Model**

C1 StdOrder  
C2 RunOrder  
C3 CenterPt  
C4 Blocks  
C5 Band Positic  
C6 Stop Positic  
C7 Cocking Angl  
C8 Distance  
C9 Exp Con

Responses: Distance

Model:  
'Band Position' 'Stop Position' 'Cocking Angle'

Random factors:

Covariates... Options... Comparisons...  
Graphs... Results... Storage...  
Select Help OK Cancel

Session

General Linear Model

Factor	Type	Levels	Values
Band Pos	fixed	2	1 4
Stop Pos	fixed	2	2 6
Cocking	fixed	2	150 180

Analysis of Variance for Distance, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Band Pos	1	0.3750	0.3750	0.3750	1.18	0.290
Stop Pos	1	1.2604	1.2604	1.2604	3.97	0.060
Cocking	1	5.5104	5.5104	5.5104	17.37	0.000
Error	20	6.3437	6.3437	0.3172		
Total	23	13.4896				

Unusual Observations for Distance

Obs	Distance	Fit	StDev Fit	Residual	St Resid
7	50.7500	49.6458	0.2299	1.1042	2.15R

R denotes an observation with a large standardized residual.



## *Steps in Analysis: Full Factorial, Replicated Designs*

**Diagnostics:** 1. *Plot the raw data.*  
**Is data OK?** 2. *Plot the residuals.*

**Analysis:** 3. *Examine factor effects.*  
**Make inferences** 4. *Confirm impressions with statistical procedures.*  
5. **Summarize conclusions.**



## *Summarize Conclusions*

### *The Fifth Step in Analysis*

- *List all the conclusions you have made during the analysis.*
- *Interpret the meaning of these results. For example, relate them to known physical properties, engineering theories, or your own personal knowledge.*
- *Make recommendations.*
- *Formulate and write conclusions in simple language.*

*We have completed Step 3 in the 7 step method for Improvement. Your recommendation (solution) should be confirmed, operationalized, and standardized in the remaining four steps (Confirm Solutions, Operationalize Results, Standardization, and Develop Future Plans).*



## *Compute Prediction Model*

- *The important effects from the analysis of a designed experiment can be used to develop a model to predict conditions not included in the experiment.*
- *Include all statistically significant effects.*
  - *Effects that are not significant do little to improve prediction and will add to the complexity of the model*
- *When using models for prediction, remember:*
  - *Interpolation within the region of experimentation is reasonably safe*
  - *Extrapolation beyond the region of experimentation is unwise unless verified by experimentation*
  - *Model coefficients are based on data that is variable. Predictions from models will be approximations subject to uncertainty.*



## *Catapult Prediction Model*

- *Since the only statistically significant factor was Cocking Angle, only this term is included in the prediction model.*

$$*Distance = 49.77 + 0.48 * Cocking Angle*$$

- *To maximize Distance set:*
  - *Cocking Angle = 180<sup>0</sup> (+)*

$$*Distance = 49.77 + 0.48(+)* = 50.25 inches$$



## *Prediction Model for Standard Deviation:*

**Step 1:** Prepare a “**reduced design matrix**” — in standard order with the Mean and **Standard Deviation** of the response at each Experimental Condition.

[Catapult Case:  $2^3 = 8$  Exp. Conditions with 3 replications]

BP = Band Position, SP = Stop Position, CA = Cocking Angle

**Example:**

<b>Exp Cond</b>	<b>BP</b>	<b>SP</b>	<b>CA</b>	<b>Mean</b>	<b>StDev</b>
1	-1	-1	-1	50.083	0.629
2	1	-1	-1	49.250	0.433
3	-1	1	-1	49.167	0.629
4	1	1	-1	48.667	0.382
5	-1	-1	1	50.417	0.764
6	1	-1	1	50.250	0.433
7	-1	1	1	49.917	0.520
8	1	1	1	50.417	0.144

*Enter the reduced matrix in the file Catapult\_V2.mtw in columns C10-C15. In order for Minitab to recognize the new design, we must define it in Minitab. We do this by using the command Define Custom Factorial Design.*



# Define Custom Factorial Design

MINITAB FILE: Catapult\_V2.MTW

MINITAB - Untitled

File Edit Manip Calc Stat Graph Editor Window Help Six Sigma

Basic Statistics  
Regression  
ANOVA  
DOE  
Control Charts  
Quality Tools  
Reliability/Survival  
Multivariate  
Time Series  
Tables  
Nonparametrics  
EDA  
Power and Sample Size

Level N  
1 3  
2 3  
3 3  
4 3  
5 3  
6 3  
7 3  
8 3

Pooled StDev = 0.523

Stat > DOE > Define Custom Factorial Design...

Catapult\_V2.MTW \*\*\*

	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21	C22	C2
↓	Exp Con	BP	SP	CA	Mean	StDev								
1	1	-1	-1	-1	50.083	0.629								
2	2	1	-1	-1	49.250	0.433								
3	3	-1	1	-1	49.167	0.629								
4	4	1	1	-1	48.667	0.382								
5	5	-1	-1	1	50.417	0.764								
6	6	1	-1	1	50.250	0.433								
7	7	-1	1	1	49.917	0.520								
8	8	1	1	1	50.417	0.144								
9														
10														
11														
12														

Create a factorial design from data that is already in the worksheet

Step 2: Define the custom design.



# Minitab Input

1. Select factors  
“BP”, “SP”,  
and “CA.”

The dialog box 'Define Custom Factorial Design' has a list of factors on the left: C1 StdOrder, C2 RunOrder, C3 CenterPt, C4 Blocks, C5 Band Pos, C6 Stop Pos, C7 Cocking, C8 Distance, C9 E C, C10 Exp Con, C11 BP, C12 SP, C13 CA, C14 Mean, C15 StDev. The 'Factors:' field contains 'BP SP CA'. The '2-level factorial' radio button is selected. Buttons include 'Select', 'Low/High...', 'Designs...', 'Help', 'OK', and 'Cancel'.

2. Select Designs.

3. Select these  
options for  
standard  
order, run  
order, center  
points, and  
blocks.

The dialog box 'Define Custom 2-Level Factorial - Design' has a list of factors on the left: C10 Exp Con, C14 Mean, C15 StDev. The 'Standard Order Column' section has 'Order of the data' selected. The 'Run Order Column' section has 'Order of the data' selected. The 'Center Points' section has 'No Center Points' selected. The 'Blocks' section has 'No blocks' selected. Buttons include 'Select', 'Help', 'OK', and 'Cancel'.

4. Select OK.



# Prediction Model for Standard Deviation

MINITAB FILE: Catapult\_V2.MTW

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and 'Analyze Factorial Design...' is selected. The 'Catapult\_V2.MTW' data table is visible at the bottom. The table has 12 rows and 14 columns. The first column is labeled 'Exp Con' and contains values 1 through 12. The second column is labeled 'BP' and contains values -1, 1, -1, 1, -1, 1, -1, 1, and is empty for rows 9-12. The third column is labeled 'SP' and contains values -1, -1, 1, 1, -1, 1, 1, 1, and is empty for rows 9-12. The fourth column is labeled 'CA' and contains values -1, -1, -1, 1, 1, 1, 1, 1, and is empty for rows 9-12. The fifth column is labeled 'Mean' and contains values 50.083, 49.250, 49.167, 48.667, 50.417, 50.250, 49.917, 50.417, and is empty for rows 9-12. The sixth column is labeled 'StDev' and contains values 0.629, 0.433, 0.629, 0.382, 0.764, 0.433, 0.520, 0.144, and is empty for rows 9-12. The seventh column is labeled 'StdOrder1' and contains values 1, 2, 3, 4, 5, 6, 7, 8, and is empty for rows 9-12. The eighth column is labeled 'RunOrder1' and contains values 1, 2, 3, 4, 5, 6, 7, 8, and is empty for rows 9-12. The ninth column is labeled 'Blocks1' and contains values 1, 1, 1, 1, 1, 1, 1, 1, and is empty for rows 9-12. The tenth column is labeled 'CenterPt1' and contains values 1, 1, 1, 1, 1, 1, 1, 1, and is empty for rows 9-12. The eleventh and twelfth columns are empty. The status bar at the bottom says 'Fit a model to the experimental data'.

	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21	C22
↓	Exp Con	BP	SP	CA	Mean	StDev	StdOrder1	RunOrder1	Blocks1	CenterPt1			
1	1	-1	-1	-1	50.083	0.629	1	1	1	1			
2	2	1	-1	-1	49.250	0.433	2	2	1	1			
3	3	-1	1	-1	49.167	0.629	3	3	1	1			
4	4	1	1	-1	48.667	0.382	4	4	1	1			
5	5	-1	-1	1	50.417	0.764	5	5	1	1			
6	6	1	-1	1	50.250	0.433	6	6	1	1			
7	7	-1	1	1	49.917	0.520	7	7	1	1			
8	8	1	1	1	50.417	0.144	8	8	1	1			
9													
10													
11													
12													

**Step 3: Run Factorial Analysis using StDev as response.**



# Minitab Input

1. Select Responses: StDev.

2. Select Terms.

The 'Analyze Factorial Design' dialog box shows a list of variables on the left: C10 (Exp Con), C14 (Mean), and C15 (StDev). The 'Responses:' field contains 'StDev'. At the bottom, there are buttons for 'Select', 'Terms...', 'Covariates...', 'Graphs...', 'Results...', 'Storage...', 'Help', 'OK', and 'Cancel'. An arrow points from the '2. Select Terms.' instruction to the 'Terms...' button.

3. Include terms in the model up through order 3.

4. Select OK.

The 'Analyze Factorial Model - Terms' dialog box has a dropdown menu set to '3' for 'Include terms in the model up through order:'. Under 'Available Terms:', there is a list: A:BP, B:SP, C:CA. Under 'Selected Terms:', there is a list: A:BP, B:SP, C:CA, AB, AC, BC, ABC. Between the lists are buttons for '>', '>>', '<', '<<', 'Cross', and 'Default'. At the bottom, there are checkboxes for 'Include blocks in model' and 'Include center point column as a term in the model', and buttons for 'Help', 'OK', and 'Cancel'.



# Minitab Output

**Fractional Factorial Fit**

Estimated Effects and Coefficients for StDev (coded units)

Term	Effect	Coef
Constant		0.4918
BP	-0.2875	-0.1437
SP	-0.1460	-0.0730
CA	-0.0530	-0.0265
BP*SP	-0.0240	-0.0120
BP*CA	-0.0660	-0.0330
SP*CA	-0.1205	-0.0603
BP*SP*CA	0.0015	0.0007

Analysis of Variance for StDev (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	0.213562	0.213562	0.0711875	*	*
2-Way Interactions	3	0.038905	0.038905	0.0129682	*	*
3-Way Interactions	1	0.000004	0.000004	0.0000045	*	*
Residual Error	0	0.000000	0.000000	0.0000000		
Total	7	0.252471				

Current Worksheet: Catapult\_V2.MTW | Editable | 5:28 PM

**None of the factors has a significant *dispersion* effect**



## *Prediction Model for Standard Deviation*

**Step 4:** Build **Prediction Model** using “significant” factors — remember to include first order “parents” of significant higher-order, “child” interactions.

### **Model**

**StDev = Constant**

### **Model with coefficient values plugged in:**

**StDev = 0.4918**

***Six Sigma Goal = reduce variation!!***

**What factor settings,  $X_i$  values = low vs. high, minimize StDev of the response variable Y?**



## *Prediction Model for Standard Deviation*

### **Step 5: Explore Prediction Models:**

- optimize **Center** (Distance) and **Spread** (StDev)
- explore “trade-offs” vs. goals (CTQs)
- implement “settings” — **Improve** (as in MAIC)
- track results — establish **Control**

#### **Models:**

$$\text{Distance} = 49.77 + 0.48 \cdot \text{CA}$$

$$\text{StDev} = 0.4918$$

*Are there any trade-offs? Can we “win” both ways?*



## *Trade-offs? Can we “win” both ways?*

To help us see the trade-offs, we can take a graphical look at:

Mean = location effects vs.  
StDev = dispersion effects

We have two options for setting this up in Minitab:

STAT >ANOVA  
Main Effects Plot  
Response = Mean (and then StDev)  
Factors = BP, SP, CA

or:

STAT >DOE >Factorial Plots  
> Main Effects  
Responses = Mean StDev  
(i.e., can select both at same time)  
Factors = BP, SP, CA



## *Prediction Model for Standard Deviation*

**Step 6:** *Determine Factor Settings — Six-Sigma solution = on Target (Mean) & minimal variation (StDev).*

**EXAMPLE:** *Goal = CTQs — Catapult Case:*

1. *“launch reasonable distance — say 50 inches or better [CTQ1 = “Quality: Performance”]*
2. *“minimize variation in distance” — customer wants to see very little difference in distance projectiles are launched [CTQ2 = “Consistency”] plus  $CTQ2 > CTQ1$  [from QFD]*

**Solution:**

Minimize StDev:

$$StDev = 0.4918$$

Distance > 50 will also give minimum StDev above:

$$Distance = 49.77 + 0.48 \cdot CA$$

$$CA = +1$$

$$Distance = 49.77 + 0.48 = 50.25$$



# Another Example

**DOE Example** A 2-level factorial experiment was performed to determine the effects that three factors had on the force applied to the quill of a numerically controlled mill. The three factors are:

- 1) the direction of cut with respect to the direction the metal was rolled
- 2) clearance angle of cutting tool, and
- 3) feed rate

There were three replications made at each of the eight conditions. The order of the twenty-four runs was randomized to make the effects of any unknown or uncontrollable factors appear as random variability in the results. Strain gauges were mounted to the quill, leads connected to a strain gauge amplifier, and the amplifier output connected to an eight bit A/D converter in a PC computer.” \*

\*This experiment was performed by Michael B. Seamons, Brigham Young University, April 1985.

Factor Levels:	Low (-),	High (+)
Direction of cut with respect to the direction the metal was rolled (D):	15 degrees,	75 degrees
Clearance Angle (A):	5 degrees,	10 degrees
Feed Rate (R):	4 in./min.,	12 in./min.

Run Order	Experimental Conditions	R	A	D	Force
1	8	+	+	+	57.9
2	6	+	-	+	61.4
3	8	+	+	+	57.1
4	6	+	-	+	62.5
5	4	+	+	-	49.9
6	1	-	-	-	42.1
7	7	-	+	+	57.8
8	5	-	-	+	61.2
9	2	+	-	-	40.7
10	3	-	+	-	50.9
11	5	-	-	+	60.9
12	4	+	+	-	53.0
13	8	+	+	+	57.0
14	1	-	-	-	38.9
15	2	+	-	-	43.2
16	5	-	-	+	59.8
17	2	+	-	-	43.8
18	1	-	-	-	41.4
19	4	+	+	-	53.5
20	3	-	+	-	47.3
21	7	-	+	+	55.3
22	6	+	-	+	59.1
23	3	-	+	-	51.6
24	7	-	+	+	56.4

## MINITAB FILE: Quill.mtw

This DOE material prepared for GE and copyright © 1990 by Joiner Associates Inc. and Improvement Professionals



## *Analyzing the Data*

---

1. *Plot and analyze the raw data.*  
—check for stability, shape, center, spread, outliers, obvious factor effects
2. *Develop the prediction model for force:*
  - a. *What values should you use for R, A, and D to maximize force?*
  - b. *What values would you use to minimize force?*
3. *What is the prediction model for “dispersion effects,” for StdDev?*
4. *Use a condensed matrix to obtain Main Effects Plots for both force and StdDev.*
5. *Any comments about the DOE residual analysis?*

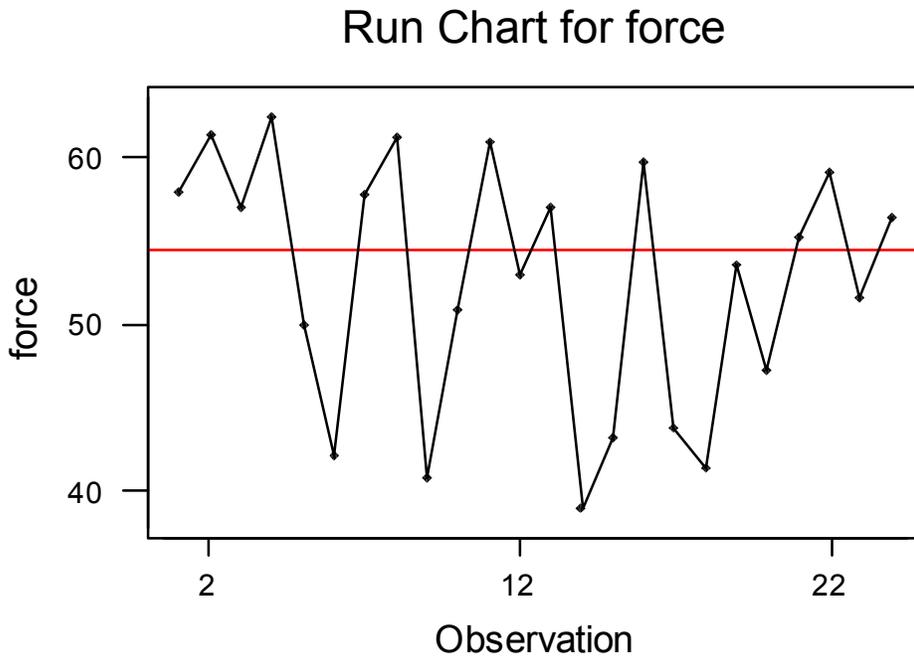
*Use Minitab file Quill.mtw*



# Stability

## ■ STAT -> QUALITY TOOLS -> RUN CHART

- SINGLE COLUMN: force
- SUBGROUP SIZE: 1



Number of runs about median: 13.0000  
Expected number of runs: 13.0000  
Longest run about median: 4.0000  
Approx P-Value for Clustering: 0.5000  
Approx P-Value for Mixtures: 0.5000

Number of runs up or down: 17.0000  
Expected number of runs: 15.6667  
Longest run up or down: 2.0000  
Approx P-Value for Trends: 0.7490  
Approx P-Value for Oscillation: 0.2510

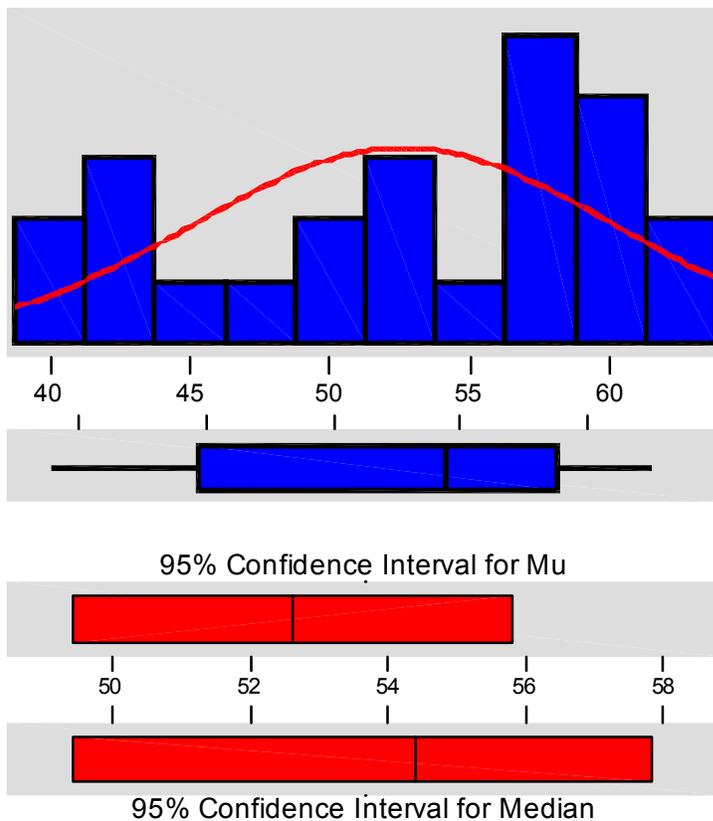


# Descriptive Statistics

## ■ STAT -> BASIC STATISTICS -> DISPLAY DESCRIPTIVE STATISTICS

- VARIABLE: force
- GRAPHS: graphical summary

### Descriptive Statistics



Variable: force

Anderson-Darling Normality Test

A-Squared: 0.743

P-Value: 0.046

Mean 52.6125

StDev 7.5178

Variance 56.5168

Skewness -5.0E-01

Kurtosis -1.12088

N 24

Minimum 38.9000

1st Quartile 44.6750

Median 54.4000

3rd Quartile 58.8000

Maximum 62.5000

95% Confidence Interval for Mu

49.4380 55.7870

95% Confidence Interval for Sigma

5.8429 10.5456

95% Confidence Interval for Median

49.4496 57.8173

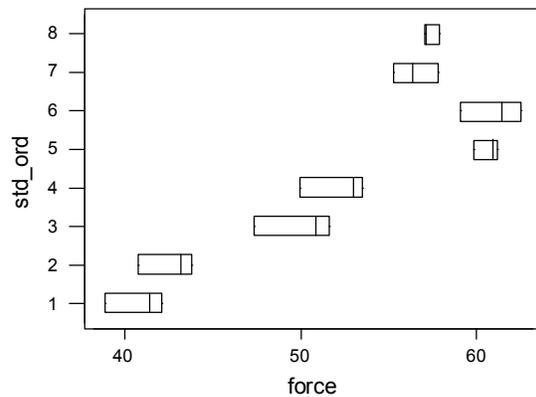
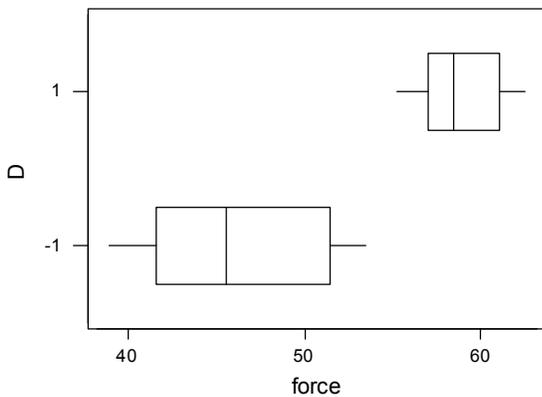
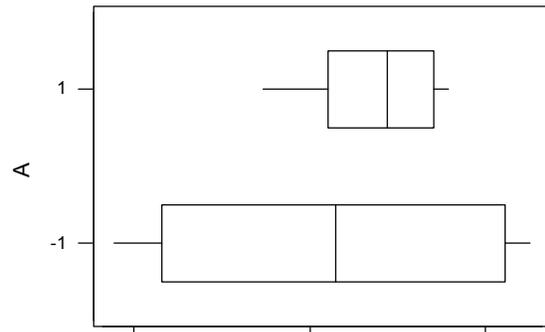
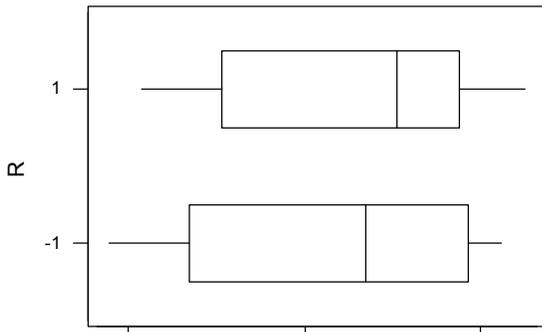


# Box Plots

## ■ GRAPH -> BOXPLOT

— GRAPH: Y: force      X: R  
                          force            A  
                          force            D  
                          force            std\_order

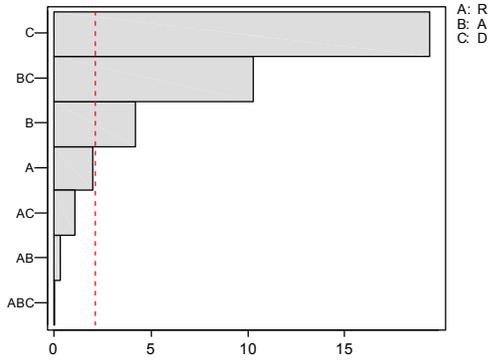
— OPTIONS: transpose X & Y



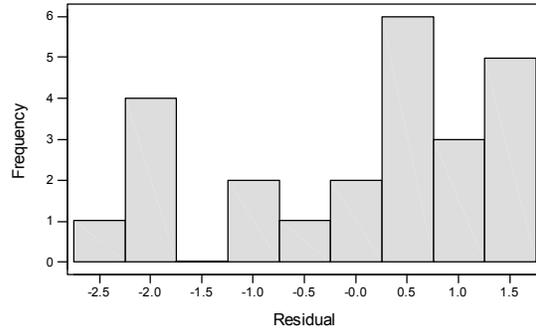


# Analyzing the DOE

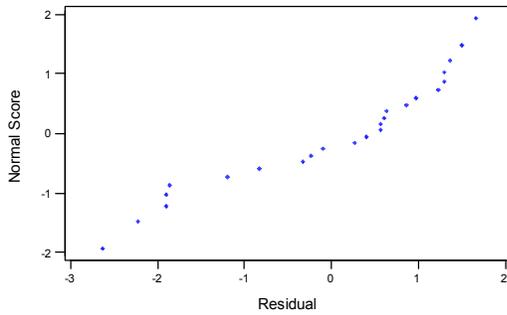
Pareto Chart of the Standardized Effects  
(response is force, Alpha = .05)



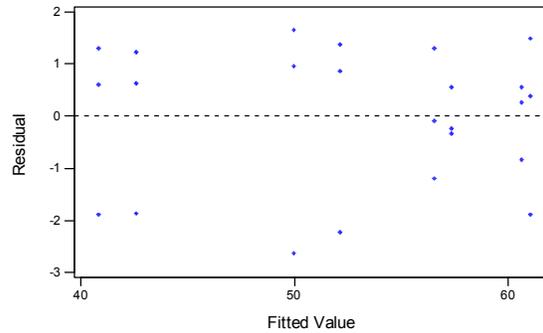
Histogram of the Residuals  
(response is force)



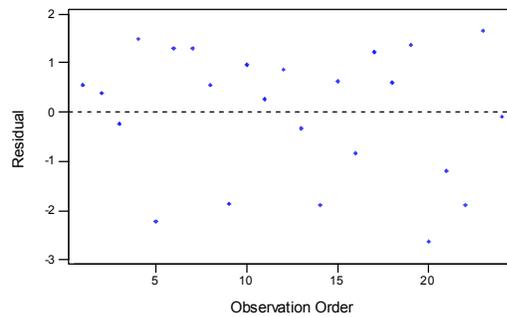
Normal Probability Plot of the Residuals  
(response is force)



Residuals Versus the Fitted Values  
(response is force)



Residuals Versus the Order of the Data  
(response is force)





# DOE Solution for Force

## Fractional Factorial Fit

### Estimated Effects and Coefficients for force (coded units)

Term	Effect	Coef	St Dev	Coef	T	P
Constant		<b>52.613</b>	0.3230	162.91	<b>0.000</b>	
R	1.292	0.646	0.3230	2.00	0.063	
A	2.725	1.362	0.3230	4.22	<b>0.001</b>	
D	12.508	6.254	0.3230	19.36	<b>0.000</b>	
R*A	0.225	0.113	0.3230	0.35	0.732	
R*D	-0.692	-0.346	0.3230	-1.07	0.300	
A*D	-6.625	-3.312	0.3230	-10.26	<b>0.000</b>	
R*A*D	0.008	0.004	0.3230	0.01	0.990	

### Analysis of Variance for force (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	993.31	993.315	331.105	132.27	0.000
2-Way Interactions	3	266.52	266.518	88.839	35.49	0.000
3-Way Interactions	1	0.00	0.000	0.000	0.00	0.990
Residual Error	16	40.05	40.053	2.503		
Pure Error	16	40.05	40.053	2.503		
Total	23	1299.89				

$$\text{Force} = 52.613 + 1.362 A + 6.254 D - 3.312 AD$$



Note the Constant Term = 52.6 =  $\bar{X}$

(as discussed in Descriptive Stats)



Also  $SST = 1300$  .... with  $SSW$  (Error) = 40 and  $SSB = 1260$

**What is the Prediction Model for Force ?**



## *Prediction Model for Standard Deviation*

### **Review = “Condensed Matrix” and Main Effects Plots for Mean & StDev**

The condensed matrix:

<b>ExpCon</b>	<b>R1</b>	<b>A1</b>	<b>D1</b>	<b>Mean</b>	<b>StDev</b>
1	-1	-1	-1	40.800	1.682
2	1	-1	-1	42.567	1.644
3	-1	1	-1	49.933	2.307
4	1	1	-1	52.133	1.950
5	-1	-1	1	60.633	0.737
6	1	-1	1	61.000	1.735
7	-1	1	1	56.500	1.253
8	1	1	1	57.333	0.493

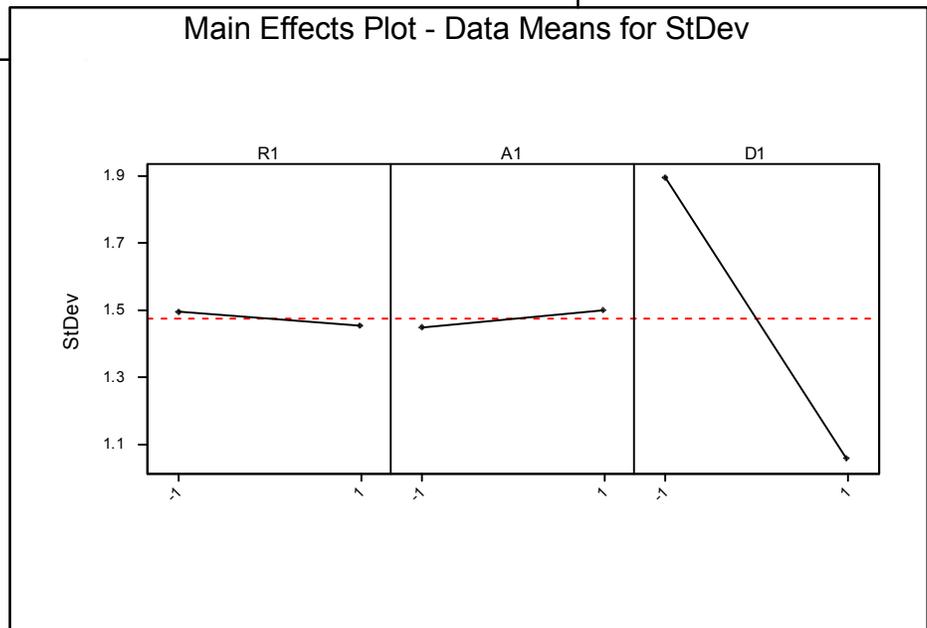
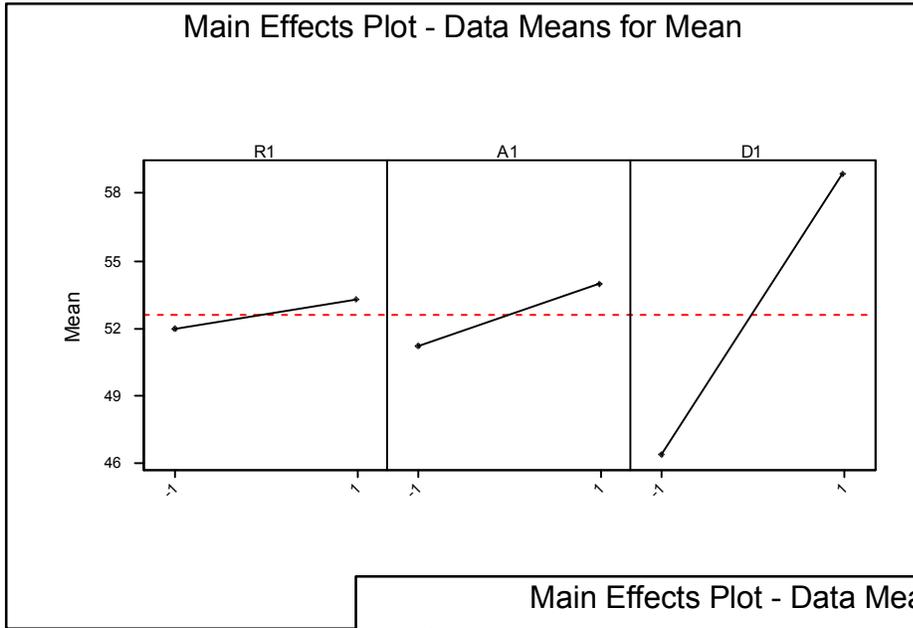
#### **Used to obtain:**

- 1. the Prediction Model for the StdDev and**
- 2. the Main Effects Plots of Mean and StdDev.**

**StDev = 1.475**



# Main Effects Plots



**What are the “trade-offs” ?**



## *5 Steps: Analyzing Full Factorial Designs with Replicates*

<b>Step</b>	<b>Description</b>	<b>Type of Tool</b>	<b>What to Look For</b>
1	<i>Plot raw data</i>	<i>Time Order Plot</i>	<i>Outliers, trends, non-random patterns, obvious factor effects</i>
		<i>Graphical Summary</i>	<i>Normality, mean, standard deviation</i>
		<i>Box Plots, Multi-vari Plot</i>	<i>Outliers, obvious factor effects</i>
2	<i>Plot residuals</i>	<i>Time Order Plot</i>	<i>Non-random patterns, trends, outliers</i>
		<i>Scatter Plot of Residuals vs. X</i>	<i>Megaphone shape or non-random appearance</i>
		<i>Normal Probability Plot of Residuals</i>	<i>Non-“straight line” relationship, outliers</i>
		<i>Dot Plot of Residuals</i>	<i>Bell-shape (normal distribution), outliers</i>



## *5 Steps: Analyzing Full Factorial Designs with Replicates*

<b>Step</b>	<b>Description</b>	<b>Type of Tool</b>	<b>What to Look For</b>
3	<i>Examine</i>	<i>Calculate Effects</i>	<i>Magnitude and sign</i>
	<i>Factor Effects</i>	<i>Effects Plots</i>	<i>Which effects are the biggest</i>
		<i>Interaction Plots</i>	<i>Non-parallel lines</i>
4	<i>Confirm Impressions</i>	<i>Hypothesis Test (p-value)</i>	<i>Identify important effects</i>
5	<i>Summarize Conclusions</i>		<i>Interpret important effects in terms of physical situation</i>



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# *Reducing the Size of Experiments*



## *Reducing the Size of Factorial Experiments*

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- *Many factors potentially impact the quality of any process/product.*
- *The factorial strategy is an efficient approach to experimentation.*
- *When factors are investigated at two levels, the number of experimental runs is  $2^k$ , where  $k$  denotes the number of factors.*
- *This can result in a large number of runs, even with a relatively small number of factors.*



## *Number of Runs Required...*

*... for a 2 Level Factorial with k Factors*

<b>Number of Factors</b>	<b>Number of Runs</b>
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
•	•
•	•
•	•
15	32,768
•	•
•	•
•	•
20	1,048,576



## *Information Available from 2-level Factorial Designs*

<b>Number of Factors</b>	<b>Main Effects</b>	<b>2-way Interactions</b>	<b>Higher Order Interactions</b>
1	1	—	—
2	2	1	—
3	3	3	1
4	4	6	5
5	5	10	16
6	6	15	42
7	7	21	99
8	8	28	219
9	9	36	466
10	10	45	968
•	•	•	•
•	•	•	•
•	•	•	•
15	15	105	32,647
•	•	•	•
•	•	•	•
•	•	•	•
20	20	190	1,048,365



## *Example of Information Available...*

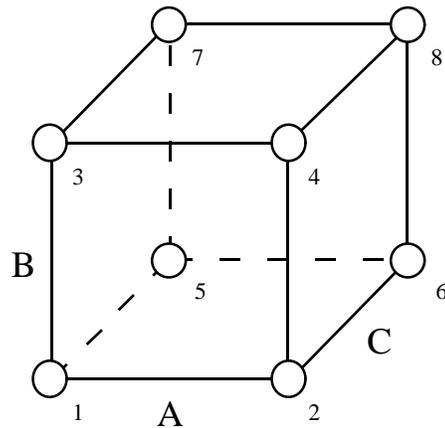
*...from a Full Factorial (4 Factors)*

	<b>Number</b>
Overall Average	1
Main effects:        A   B   C   D	4
2-way interactions: AB   AC   AD   BC   BD   CD	6
3-way interactions: ABC   ABD   ACD   BCD	4
4-way interactions: ABCD	1



# Reducing the Size of a Factorial Experiment

<u>Std. Order</u>	<u>A</u>	<u>B</u>	<u>C</u>
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+



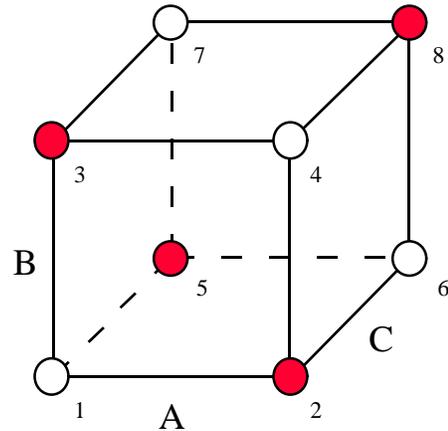
*Equipment delays allow you time to run only 4 trials  
in the allotted time.*

*Which 4 trials will you choose?*



# Choosing the Half Fraction

<u>Std. Order</u>	<u>A</u>	<u>B</u>	<u>C</u>
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+



We can select:

The 4 unshaded trials,  
**Std.**

<u>Order</u>	<u>A</u>	<u>B</u>	<u>C</u>
1	-	-	-
4	+	+	-
6	+	-	+
7	-	+	+

or

The 4 shaded trials,  
**Std.**

<u>Order</u>	<u>A</u>	<u>B</u>	<u>C</u>
2	+	-	-
3	-	+	-
5	-	-	+
8	+	+	+

<b>Design</b>	<b>Number of runs</b>
Full Factorial	$2^k = 2^3 = 8$
Half Fraction	$2^{k-1} = 2^{3-1} = 2^2 = 4$

Half Fraction designs use *Half* the runs of Full Factorial designs.



## *Properties of a Properly Selected Half Fraction*

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- *The design is nicely balanced, that is, each factor is studied the same number of times at each level. (equal number of + and -)*
- *The design collapses into a full factorial. Should any factor turn out not to matter, the result is a full factorial in the other two factors.*
- *The design covers much of the region of interest.*



## Constructing a Half Fraction for Four Factors

<u>A</u>	<u>B</u>	<u>C</u>	<u>D = ABC</u>
-	-	-	-
+	-	-	+
-	+	-	+
+	+	-	-
-	-	+	+
+	-	+	-
-	+	+	-
+	+	+	+

Full factorial  
for 3 factors

- List the full factorial for three factors. This is called the base design.
- The fourth factor is assigned to the 3-factor interaction for the other three factors.
- Recall: a 3-factor interaction column is obtained by multiplying the three main effect columns. So  $D = ABC$ . Note:  $+ \times + = +$

$$+ \times - = -$$

$$\text{so: } - \times - \times - = -$$

$$+ \times - \times - = +$$



## *General Rule for Constructing a Half Fraction for k Factors*

- *Define the base design as a full factorial for the first  $k-1$  factors.*
- *Assign the  $k^{\text{th}}$  factor to the interaction of the first  $k-1$  factors from the base design.*
- *This interaction is found by multiplying the factor level settings together for the first  $k-1$  factors from the base design.*
- *Number of runs =  $N = 2^{k-1}$*



## *Exercise*

### *Half Fraction of a $2^5$ Factorial*

- *Generate a half fraction experiment for  $k = 5$  factors, A, B, C, D, and E.*

**Factors**

**A**

**B**

**C**

**D**

**E**



# Answers

## Half Fraction of a $2^5$ Factorial

<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>E</u>
-	-	-	-	+
+	-	-	-	-
-	+	-	-	-
+	+	-	-	+
-	-	+	-	-
+	-	+	-	+
-	+	+	-	+
+	+	+	-	-
-	-	-	+	-
+	-	-	+	+
-	+	-	+	+
+	+	-	+	-
-	-	+	+	+
+	-	+	+	-
-	+	+	+	-
+	+	+	+	+

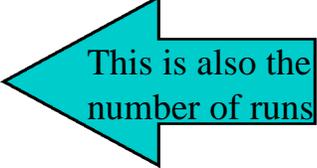


## Trade-Offs

### Between Full Factorial and Half Fraction Designs

#### Number of Effects Computed

<u>Effects</u>	<u>Full Factorial</u>	<u>Half Fraction</u>
Mean	1	1
Main Factors	5	5
2 Factor Int.	10	10
3 Factor Int.	10	—
4 Factor Int.	5	—
5 Factor Int.	1	—
<i>Total Effects Computed</i>	32	16



This is also the number of runs

- ***Are the additional runs worth it?***
- ***What happens to the higher order interactions?***



## *A 2-Factor Experiment with Confounded Effects*

<u>Run</u>	<u>Factor A</u>	<u>Factor B</u>	<u>Response</u>
1	–	–	130
2	–	–	125
3	–	–	133
4	–	–	130
5	+	+	50
6	+	+	85
7	+	+	79
8	+	+	93

*The effects of Factors A and B are **confounded**.*

**Confounding** is the combining of the effects of two or more factors into one resulting number such that the magnitude of the effects of the individual factors cannot be separated.



## Confounding in the Half Fraction

<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>AB</u>	<u>CD</u>
-	-	-	-	+	+
+	-	-	+	-	-
-	+	-	+	-	-
+	+	-	-	+	+
-	-	+	+	+	+
+	-	+	-	-	-
-	+	+	-	-	-
+	+	+	+	+	+

$$A = BCD$$

$$B = ACD$$

$$C = ABD$$

$$D = ABC$$

$$AB = CD$$

$$AC = BD$$

$$AD = BC$$

$$\text{mean} = ABCD$$

***For the half fraction, each letter must be present on one side of the equal sign or the other.***



## Exercise

### Confounding in the Half Fraction of a $2^5$

<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>E = ABCD</u>
-	-	-	-	+
+	-	-	-	-
-	+	-	-	-
+	+	-	-	+
-	-	+	-	-
+	-	+	-	+
-	+	+	-	+
+	+	+	-	-
-	-	-	+	-
+	-	-	+	+
-	+	-	+	+
+	+	-	+	-
-	-	+	+	+
+	-	+	+	-
-	+	+	+	-
+	+	+	+	+

#### Questions

1. What main effect is confounded with ABCD?
2. What main effect is confounded with ABCE?
3. Prove that AB is confounded with CDE.
4. Guess what AC is confounded with.



## Answer

### Confounding in the Half Fraction of a $2^5$

1. E is confounded with ABCD.
2. D is confounded with ABCE.

3.

<u>AB</u>	<u>CDE</u>
+	+
-	-
-	-
+	+
+	+
-	-
-	-
+	+
+	+
-	-
-	-
+	+
+	+
-	-
-	-
+	+

4. BDE is confounded with AC.



## Example

- Analyzing the Full Factorial and the Half Fraction for an Electrochemical Hole Drilling Experiment

	Factors	Levels	
		-	+
A	Volts	94	150
B	Electrolyte Concentration	60	120
C	Tool Angle	8	10
D	Pressure	17	23

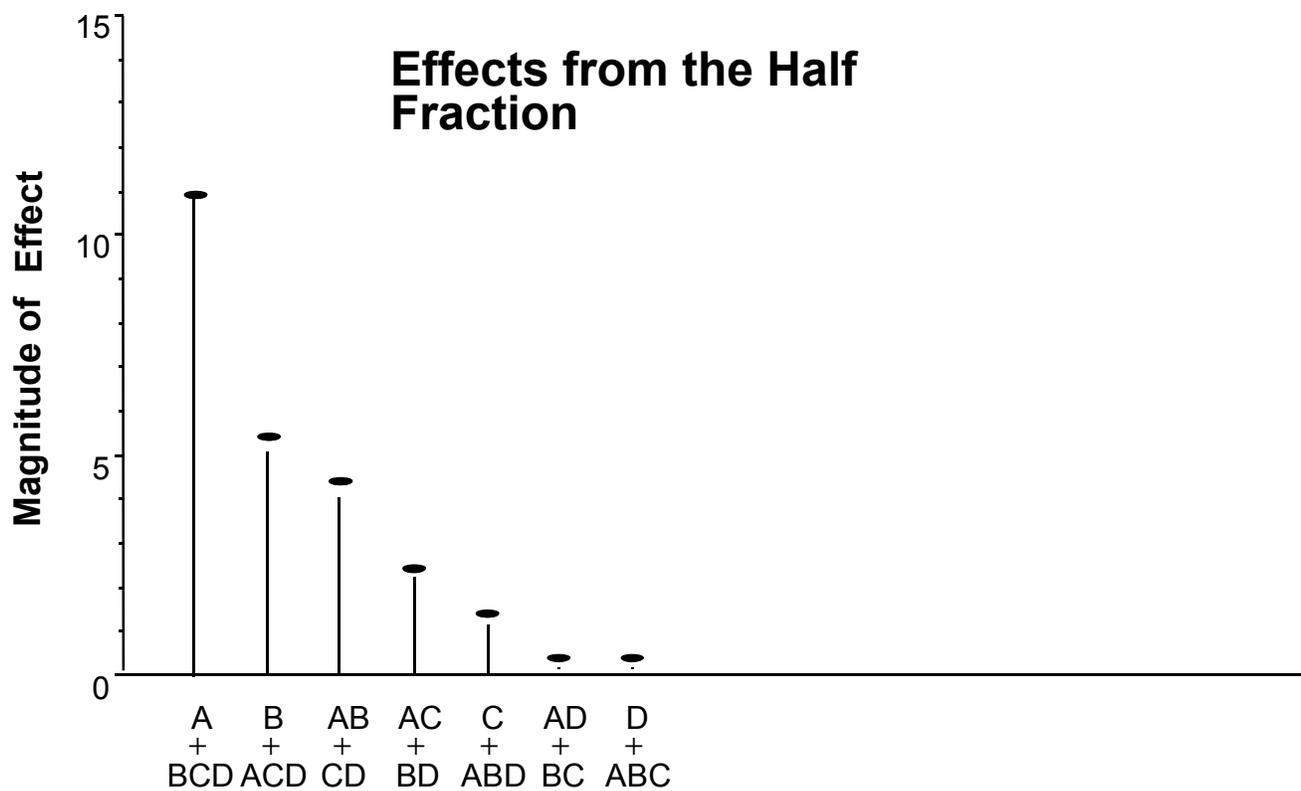
Test No.	A	B	C	D	Diameter (in thousandths)
1	-	-	-	-	54
2	+	-	-	-	46
3	-	+	-	-	62
4	+	+	-	-	46
5	-	-	+	-	53
6	+	-	+	-	46
7	-	+	+	-	60
8	+	+	+	-	47
9	-	-	-	+	56
10	+	-	-	+	45
11	-	+	-	+	63
12	+	+	-	+	46
13	-	-	+	+	50
14	+	-	+	+	45
15	-	+	+	+	66
16	+	+	+	+	47

 = Half fraction runs

Source: J. Bemesderfer, pp. 195-202.

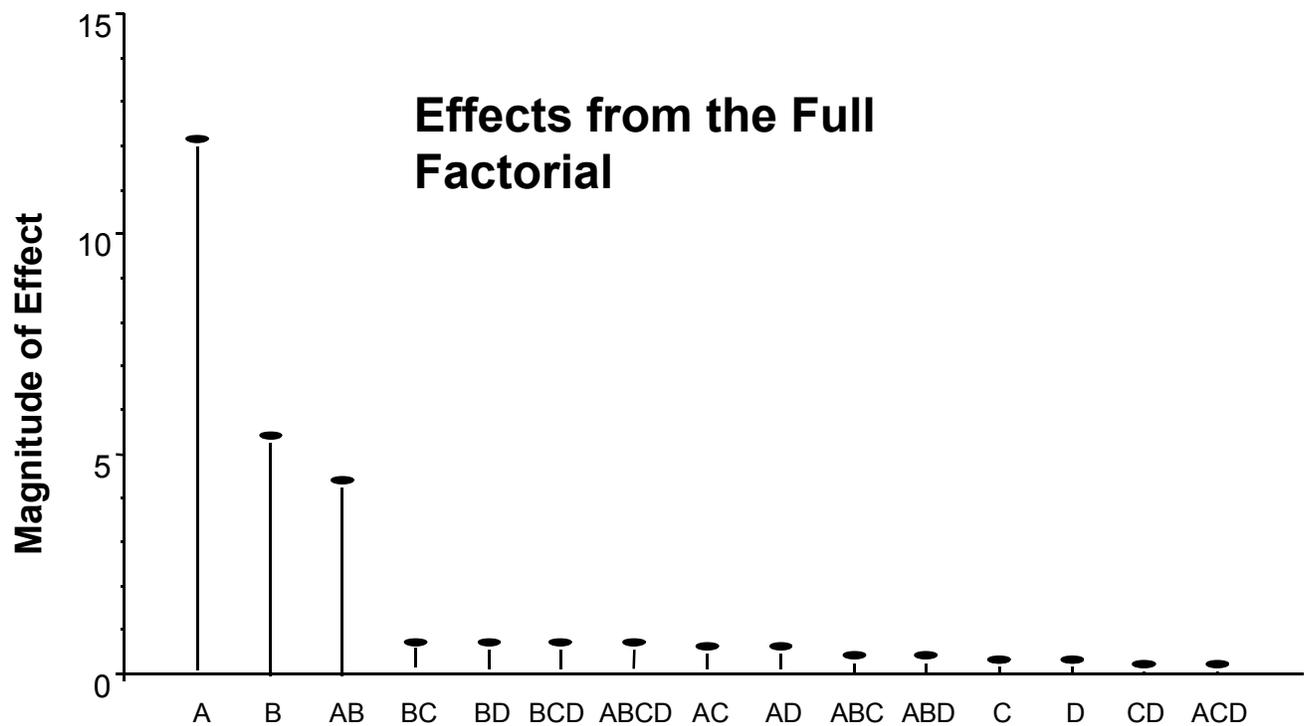


# Effects Plot for the Half Fraction





# Effects Plot for the Full Factorial





## *What is Meant by the Expression “AB + CD”*

- *In this experiment AB and CD are confounded.*
- *We have been referring to this confounding as  $AB = CD$ . We can prove this equivalence by multiplying columns from the design together.*
- *We estimate the effect of the AB interaction by averaging all the responses performed at the +AB level and subtracting the average of all the responses performed at the -AB level. When we do this, we are simultaneously estimating the effect of the interaction CD.*
- *The estimate that results is neither the AB interaction nor the CD interaction exclusively, but instead, is the sum of these interactions.*
- *Thus, **when we refer to confounding in the design**, we use = since the patterns of + and - signs are identical.*
- *But **when we compute effects**, we use “+” to remind us that the effect we have calculated may be a combination of all the confounded effects.*



## *Summary of the Half Fraction*

*The half fraction of a full factorial can often provide the same information as the full factorial, with only half the number of runs.*

<b>Benefits</b>	<b>Costs</b>
<ul style="list-style-type: none"><li><i>• Fewer runs saves time and money.</i></li></ul>	<ul style="list-style-type: none"><li><i>• More complicated to analyze (must understand confounding).</i></li><li><i>• In designs with few runs, important effects (such as 2-way interactions) are confounded.</i></li></ul>

*Note: Some texts (and Minitab) call confounding - aliasing.*



# A Strategy for Choosing the Appropriate Design

## The Knowledge Line

	Current State of Process Knowledge			
	Low			High
<b>Type of Design</b>	Main Effects	Fractional Factorial	Full Factorial	Response Surface
<b>Usual # of Factors</b>	>5	4–10	1–5	2-3
<b>Purpose:</b>				
• <b>Identify</b>	Most important factors	Some interactions	Relationships among factors	Optimal factor settings
• <b>Estimate</b>	Crude direction for improvement	Some interpolation	All main effects and interactions	Curvature in response, empirical models

**Step 7**      **Screening** →

**Step 8**      → **Optimization**



## *What are Screening Designs?*

- *They study the main effects of a large number of factors.*
- *They contain at least the same number of runs plus one as factors.*
- *They are useful in the early stages of investigation where it is desirable to go from a large list of factors that **may** affect the response to a small list of factors that **do** affect the response.*



## Example:

### Design Constructing a 7-Factor Screening

How many runs do you need for a 7-Factor Screening Design?

What design is this similar to?

Factors	Runs
2	$2^2 = 4$
3	$2^3 = 8$
4	$2^4 = 16$

Screening Design for 7 Factors	A	B	C	D	E	F	G
Full Factorial in A, B, & C	A	B	C	AB	AC	BC	ABC
	-	-	-	+	+	+	-
	+	-	-	-	-	+	+
	-	+	-	-	+	-	+
	+	+	-	+	-	-	-
	-	-	+	+	-	-	+
	+	-	+	-	+	-	-
	-	+	+	-	-	+	-
	+	+	+	+	+	+	+

1. Write out the full factorial design in 3 factors.

2. Create the columns used to calculate all interactions.

3. Assign 4 factors to the columns created above.



## Example (cont.)

# Confounding of Main Effects and 2-Factor Interactions in the 7-Factor Screening Design

Starting with:

$$\begin{aligned}
 D &= AB \\
 E &= AC \\
 F &= BC \\
 G &= ABC
 \end{aligned}$$

Results in the following confounding of main effects and 2 factor interactions:

$$\begin{aligned}
 A &= BD = CE = FG \\
 B &= AD = CF = EG \\
 C &= AE = BF = DG \} \\
 D &= AB = CG = EF \\
 E &= AC = BG = DF \\
 F &= AG = BC = DE \\
 G &= AF = BE = CD
 \end{aligned}$$

C	A	E	AE	B	F	BE	D	G	DG
-	-	+		-	+		+	-	
-	+	-		-	+		-	+	
-	-	+		+	-		-	+	
-	+	-		+	-		+	-	
+	-	-		-	-		+	+	
+	+	+		-	-		-	-	
+	-	-		+	+		-	-	
+	+	+		+	+		+	+	



# Example: 15-Factor, 16-Run Screening Design

Screening Design for 15 Factors	A	B	C	D	K	L	M	N	O	P	E	F	G	H	J
Full Factorial in A, B, C, & D	A	B	C	D	AB	AC	AD	BC	BD	CD	ABC	ABD	ACD	BCD	ABCD
	-	-	-	-	+	+	+	+	+	+	-	-	-	-	+
	+	-	-	-	-	-	-	+	+	+	+	+	+	-	-
	-	+	-	-	-	+	+	-	-	+	+	+	-	+	-
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	-	+	+	+	-	-	-	+	+	+	-	-	-	+	-
	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

1. Write out the full factorial design in 4 factors.
2. Create the columns used to calculate all interactions. They are all possible products of the first four columns.
3. Assign 11 factors to the columns created above.



## Another Useful Screening Design

### The 2-level Plackett-Burman Designs to Study 11 Factors in 12 Runs

	<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>E</u>	<u>F</u>	<u>G</u>	<u>H</u>	<u>I</u>	<u>J</u>	<u>K</u>
1	+	+	-	+	+	+	-	-	-	+	-
2	-	+	+	-	+	+	+	-	-	-	+
3	+	-	+	+	-	+	+	+	-	-	-
4	-	+	-	+	+	-	+	+	+	-	-
5	-	-	+	-	+	+	-	+	+	+	-
6	-	-	-	+	-	+	+	-	+	+	+
7	+	-	-	-	+	-	+	+	-	+	+
8	+	+	-	-	-	+	-	+	+	-	+
9	+	+	+	-	-	-	+	-	+	+	-
10	-	+	+	+	-	-	-	+	-	+	+
11	+	-	+	+	+	-	-	-	+	-	+
12	-	-	-	-	-	-	-	-	-	-	-



## *When to Use Plackett-Burman Designs*

- *Use them when it is too costly to run the  $2^k$  (8, 16, or 32-run) screening design.*
  - *Example: 10 factors but 16-run design is too costly*
- *Use them only in these circumstances since the “cost” of this design is the loss of information about where the two factor interactions are confounded.*

*Note: There are Plackett-Burman designs available for  $4(i)$  runs where “i” is an integer.*



## *Screening Design Selection*

---

### **Number of Factors**

### **Design**

5	8-run $2^{5-1}$
6-11	12-run Plackett-Burman
6-8	Resolution IV fractional factorial
16-19	20-run Plackett-Burman
20-23	24-run Plackett-Burman
24-27	28-run Plackett-Burman
16-31	32-run design based on a $2^5$



## *Exercise*

---

### *Describe Possible Designs for 6 Factors*

- *Suppose you have six factors that you want to investigate.*
  - *What designs might you consider?*
  - *Under what conditions would you favor each of the above designs?*



## Answer

### Describe Possible Designs for 6 Factors

- Three possible designs covered thus far are:
  - The 64-run full factorial
  - The 32-run half fraction
  - The 8-run screening design
- The table below explains the conditions when each design is favored. The full and half fractions have similar properties.

**Designs Available for 6 Factors**

	<b>Full factorial or half fraction</b>	<b>Screening design</b>
<b>State of knowledge</b>	Reasonably high	Low
<b>Cost of runs</b>	Cheap	Expensive
<b>Presence of interaction</b>	Expect interactions	Expect few interactions
<b>Number of factors expected to influence the response</b>	Many	Only a few
<b>Size of the effects you wish to detect</b>	About the same size as the experimental error, $s_p$	About three times the experimental error, $s_p$



## *What About Intermediate Conditions?*

- *For conditions which are not as extreme as the ones that favor the full factorial or the screening design, a 16-run design is available for a 6-factor experiment. This quarter fraction is the design of choice under the conditions described below.*

**Designs Available for 6 Factors**

	<b>Full factorial or half fraction</b>	<b>Smaller fraction (Quarter fraction)</b>	<b>Screening design</b>
<b>State of knowledge</b>	Reasonably high	Moderate	Low
<b>Cost of runs</b>	Cheap	Moderate	Expensive
<b>Presence of interaction</b>	Expect interactions	May be 2-factor interactions; higher ones are unlikely	Expect few interactions
<b>Number of factors expected to influence the response</b>	Many	Unsure, few or all	Only a few
<b>Size of the effects you wish to detect</b>	About the same size as the experimental error, $s_p$	About two times the experimental error, $s_p$	About three times the experimental error, $s_p$



# Bridging the Gap

## ■ Fractional Factorials Bridge the Gap Between Full Factorials and Screening Designs.

- Fractional factorials are the design of choice when you have already determined which factors are likely to be important and want to learn more about the effect of these factors.

### Current State of Process Knowledge

	Low			High
<b>Type of Design</b>	Main Effects	Fractional Factorial	Full Factorial	Response Surface
<b>Usual # of Factors</b>	>5	4–10	1–5	2-3
<b>Purpose:</b>				
• <b>Identify</b>	Most important factors	Some interactions	Relationships among factors	Optimal factor settings
• <b>Estimate</b>	Crude direction for improvement	Some interpolation	All main effects and interactions	Curvature in response, empirical models
<b>Step 7</b>	Screening →			
<b>Step 8</b>	Optimization →			



# Example: A Fractional Factorial

Quarter Fraction of a  $2^6 = 2^{6-2}$

Run	A	B	C	D	E	F
1	-	-	-	-	-	-
2	+	-	-	-	+	-
3	-	+	-	-	+	+
4	+	+	-	-	-	+
5	-	-	+	-	+	+
6	+	-	+	-	-	+
7	-	+	+	-	-	-
8	+	+	+	-	+	-
9	-	-	-	+	-	+
10	+	-	-	+	+	+
11	-	+	-	+	+	-
12	+	+	-	+	-	-
13	-	-	+	+	+	-
14	+	-	+	+	-	-
15	-	+	+	+	-	+
16	+	+	+	+	+	+

1. List the full factorial for four factors.

This is called the base design. ( $2^{6-2} = 2^4$ )

2. The fifth factor, E, is assigned to the 3-factor interaction of A, B, and C. ( $E = ABC$ )

3. Likewise, the sixth factor, F, is assigned to the 3-factor interaction of B, C, and D. ( $F = BCD$ )



## Summary of Creating Fractional Factorials

- *The half fraction is derived by starting with a base design of a full factorial in one less than the desired number of factors, and using an interaction column (a product of columns in the base design) to create the extra column.*
- *Similarly, the quarter fraction is derived by starting with a base design of a full factorial in **two** less than the desired number of factors and using **two** interaction columns to create the extra columns.*
- *Likewise, a  $\frac{1}{2^p}$  fraction is derived by starting with a base design of a full factorial in **p** less than the desired number of factors, and using **p** interaction columns to create the extra columns.*

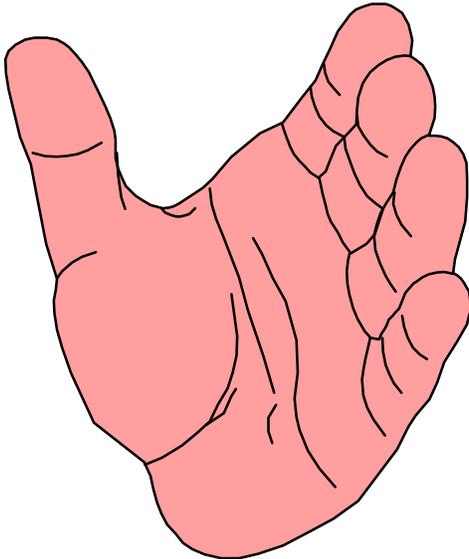


## Resolution

- *Understanding the Degree of Confounding in a Fractional Factorial*
  - *The “cost” of running fractional factorials is that effects and interactions will be confounded.*
  - *We use a single Roman numeral, called resolution, to describe the degree of confounding.*
  - *Resolution tells us the size of the effects we expect to see confounded.*
  - **Resolution III designs** *have main effects confounded with 2-factor and higher order interactions, but not with other main effects.*
  - **Resolution IV designs** *have main effects confounded with 3-factor and higher order interactions, but not with other main effects or 2-factor interactions. In other words, main effects are clear of 2-factor interaction.*
  - **Resolution V designs** *have main effects confounded with 4-factor and higher order interactions, but not other main effects, 2- or 3-factor interactions. Two-factor interactions are confounded not with each other, but with 3-factor or higher order interactions. In other words, 2-factor interactions are clear of each other.*



## Resolution — “Hand” Method



1. Hold up number of fingers equal to design Resolution—for Resolution V = 5 fingers.
2. Use other hand to grab number of fingers equal to the Main/Interaction Effects you wish to investigate for confounding—i.e., to determine what Main Effects are confounded with, grab one finger.

3. The remaining number of fingers is the lowest level of interaction effects which are confounded.

=====

[In the example above, the result is “*4 - factor interactions and higher*” are confounded with Main Effects—since there are four (4) fingers remaining]

=====

Review: Given a Resolution IV design, what are the second order interactions confounded with?



## *Exercise Resolution*

---

- *If you expect several 2-factor interactions to be important, what resolution design should you select?*
- *What is the advantage of a resolution V design over a resolution IV?*
- *What is the resolution of screening designs?*



## *Answer Resolution*

---

- *Select a design of either resolution IV or V.*
- *A resolution V design has all 2-factor interactions clear of each other. Thus, you will always be able to tell which 2-factor interactions are important. In a resolution IV design, 2-factor interactions are confounded together. If you get a large interaction effect, it will not always be clear which particular 2-factor interaction is the one that is important. Sometimes you can figure it out by what you know about the experiment, but sometimes additional runs are required to resolve the ambiguity.*
- *Screening designs are resolution III.*



# Selecting the Appropriate Design

Number of Factors

	3	4	5	6	7	8	9	10	11	
4	$2_{III}^{3-1}$									
8	$2^3$	$2_{IV}^{4-1}$	$2_{III}^{5-2}$	$2_{III}^{6-3}$	$2_{III}^{7-4}$					
16	$2^3$ 2 times	$2^4$	$2_{V}^{5-1}$	$2_{IV}^{6-2}$	$2_{IV}^{7-3}$	$2_{IV}^{8-4}$	$2_{III}^{9-5}$	$2_{III}^{10-6}$	$2_{III}^{11-7}$	
32	$2^3$ 4 times	$2^4$ 2 times	$2^5$	$2_{VI}^{6-1}$	$2_{IV}^{7-2}$	$2_{IV}^{8-3}$	$2_{IV}^{9-4}$	$2_{IV}^{10-5}$	$2_{IV}^{11-6}$	↖ (1/128)
64	$2^3$ 8 times	$2^4$ 4 times	$2^5$ 2 times	$2^6$	$2_{VII}^{7-1}$	$2_{V}^{8-2}$	$2_{IV}^{9-3}$	$2_{IV}^{10-4}$	$2_{IV}^{11-5}$	↖ (1/64)
128	$2^3$ 16 times	$2^4$ 8 times	$2^5$ 4 times	$2^6$ 2 times	$2^7$	$2_{VIII}^{8-1}$	$2_{VI}^{9-2}$	$2_{V}^{10-3}$	$2_{V}^{11-4}$	↖ (1/32)
		↖ (16)	↖ (8)	↖ (4)	↖ (2)	↖ (1)	↖ (1/2)	↖ (1/4)	↖ (1/8)	↖ (1/16)

Legend for  $2_R^{K-P}$

- 2 = Number of levels of each factor
- K = Number of factors
- P = Number of factors assigned to interactions
- K-P = Factors required to generate the basic design
- $2^{K-P}$  = Number of runs
- R = Design Resolution



## Using the Look-up Tables

1. An engineer has 7 factors that she wants to study at two levels. She believes that all of these factors will affect the responses of interest and that some two-way interactions are likely.

- Which design should she select?
- How many runs does this design have?
- What size fraction is this?
- What are 2-factor interactions confounded with in this design?

2. Complete the following table:

Number of Runs	Maximum # of Factors that can be Studied at Resolution III	Maximum # of Factors that can be Studied at Resolution IV	Maximum # of Factors that can be Studied at Resolution V
4		–	–
8			–
16			
32			
64			
128			

### Bonus Question

3. What design would you use to study 5 factors when you can easily afford to make 32 runs?



## *Using the Look-up Tables (cont.)*

1. a.  $2_{IV}^{7-3}$

b.  $2^{7-3} = 2^4 = 16$

c. *This is a  $2^{-3} = 1/2^3 =$  one-eighth fraction.*

d. *Since this design is resolution IV, 2-factor interactions are confounded with each other.*



## Using the Look-up Tables (cont.)

2.

Number of Runs	Maximum # of Factors that can be Studied at Resolution III	Maximum # of Factors that can be Studied at Resolution IV	Maximum # of Factors that can be Studied at Resolution V
4	3	—	—
8	7	4	—
16	15	8	5
32	31		* 6
64	63		8
128	127		

\* This design is actually resolution VI.

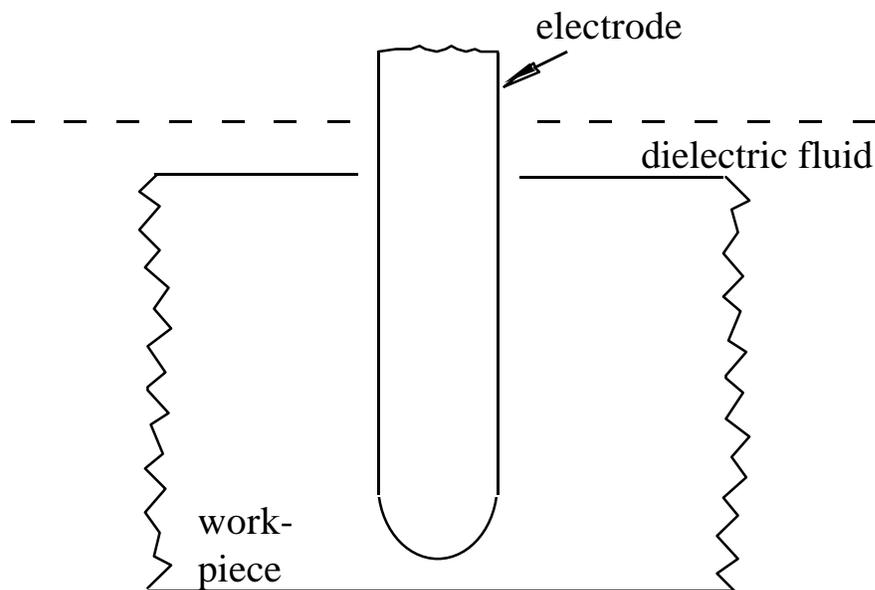
3. Consider running the half fraction of the 5-factor design. With the additional runs that you can easily make, replicate this half fraction.

*The half fraction is of resolution V. Resolution V designs are generally adequate (unless you expect 3-factor interactions), and with replication, you can additionally investigate if the factors affect the variability of the response.*



## *Example: Raycon EDM Machine*

Electrical discharge machining (EDM) removes material from a work piece by a controlled “sparkout” method, with the electrode and workpiece both submerged in a dielectric fluid. Eight controls on the panel are factors of interest.



**Illustration of Electrical Discharge Machining  
EDM Process**

Adapted from Bemserfer, John L., "The Use of Designed Experiments to Improve Productivity," General Electric Aircraft Engines, 25 September 1979, p. 56.



## Example (cont.)

# Raycon EDM Machine, a $2_{IV}^{8-4}$ Design

Key Factor	Low (-)	High (+)
A-Pulse	5	11
B-Interval	250	350
C-Current	10	16
D-Arc Level	7	9
E-Arc Suppress	7	9
F-Short Delay	1	3
G-Servo Feed	14	22
H-Servo Response	30	60

Run	A	B	C	D	(ABC) (ABD) (ACD) (BCD)				Responses		
					E	F	G	H	Wear	Volts	Width
1	-	-	-	-	-	-	-	-	.075	72	15.7
2	+	-	-	-	+	+	+	-	.030	65	16.7
3	-	+	-	-	+	+	-	+	.065	66	16.0
4	+	+	-	-	-	-	+	+	.065	63	16.7
5	-	-	+	-	+	-	+	+	.135	60	17.0
6	+	-	+	-	-	+	-	+	.200	66	17.7
7	-	+	+	-	-	+	+	-	.107	64	17.1
8	+	+	+	-	+	-	-	-	.180	72	17.7
9	-	-	-	+	-	+	+	+	.065	60	15.7
10	+	-	-	+	+	-	-	+	.015	72	15.7
11	-	+	-	+	+	-	+	-	.040	65	15.7
12	+	+	-	+	-	+	-	-	.065	72	16.3
13	-	-	+	+	+	+	-	-	.140	71	17.1
14	+	-	+	+	-	-	+	-	.180	64	19.0
15	-	+	+	+	-	-	-	+	.240	66	18.4
16	+	+	+	+	+	+	+	+	.185	60	19.4

MINITAB FILE: Raycon1.mtw



## Example (cont.)

### Raycon EDM Machine Estimates for Effects

<u>Wear</u>	<u>Volts</u>	<u>Width</u>	
.1117	66.125	16.994	= Average
.0066	1.25	0.813	= A
.0134	-0.25	0.338	= B
.1184	-1.50	1.863	= C
.0091	0.25	0.337	= D
-.0259	0.50	-0.163	= E
-.0091	-1.25	0.013	= F
-.0216	-7.00	0.338	= G
.0191	-4.00	0.162	= H
.0041	0.25	-0.088	= AB + CE + DF + GH
.0241	-1.00	0.238	= AC + BE + DG + FH
-.0166	0.25	0.062	= AD + BF + CG + EH
.0009	0.50	0.113	= AE + BC + DH + FG
.0191	-0.75	0.237	= AF + BD + CH + EG
.0216	-0.50	0.763	= AG + BH + CD + EF
-.0166	1.00	-0.213	= AH + BG + CF + DE



## Example (cont.)

### Fractional Factorial Fit

Estimated Effects and Coefficients for Wear (coded units)

Term	Effect	Coef
Constant		0.11169
A	0.00663	0.00331
B	0.01337	0.00669
C	0.11838	0.05919
D	0.00912	0.00456
E	-0.02587	-0.01294
F	-0.00913	-0.00456
G	-0.02162	-0.01081
H	0.01912	0.00956
A*B	0.00413	0.00206
A*C	0.02413	0.01206
A*D	-0.01662	-0.00831
A*E	0.00087	0.00044
A*F	0.01912	0.00956
A*G	0.02162	0.01081
A*H	-0.01663	-0.00831

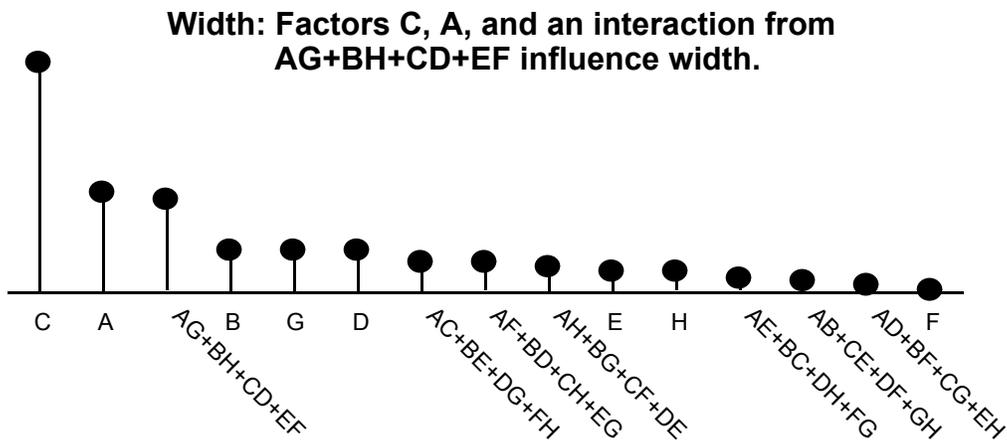
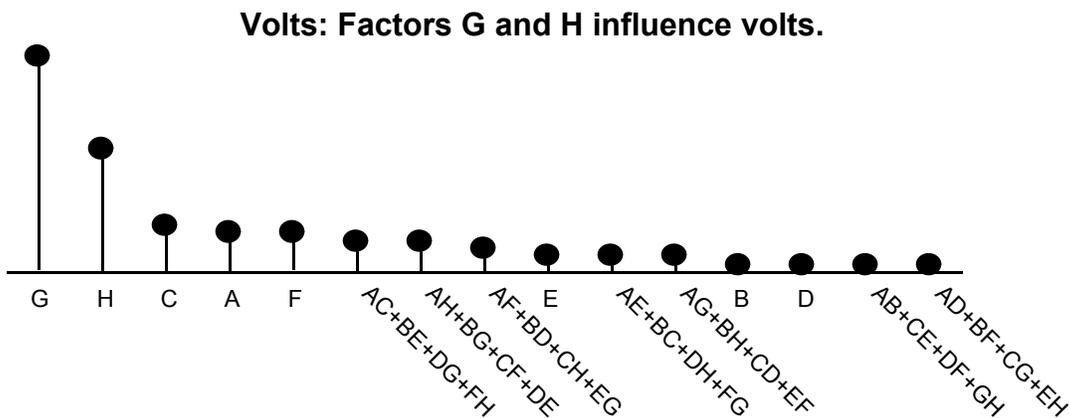
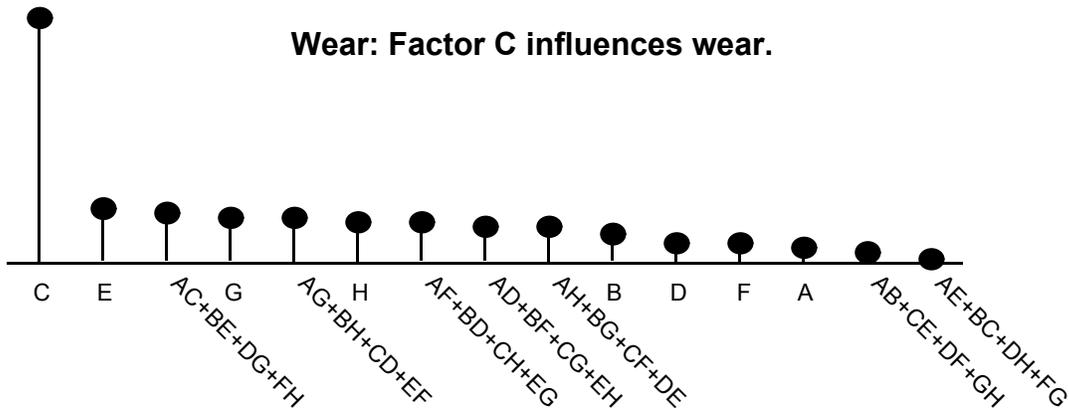
Analysis of Variance for Wear (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	8	0.063620	0.063620	0.007952	*	*
2-Way Interactions	7	0.007944	0.007944	0.001135	*	*
Residual Error	0	0.000000	0.000000	0.000000		
Total	15	0.071563				



# Effects Plots

## for the Responses in the Raycon EDM Machine





# Analyzing Volts

## Fractional Factorial Fit

Estimated Effects and Coefficients for Volts (coded units)

Term	Effect	Coef
Constant		66.125
A	1.250	0.625
B	-0.250	-0.125
C	-1.500	-0.750
D	0.250	0.125
E	0.500	0.250
F	-1.250	-0.625
G	-7.000	-3.500
H	-4.000	-2.000
A*B	0.250	0.125
A*C	-1.000	-0.500
A*D	0.250	0.125
A*E	0.500	0.250
A*F	-0.750	-0.375
A*G	-0.500	-0.250
A*H	1.000	0.500

Analysis of Variance for Volts (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	8	283.00	283.00	35.375	*	*
2-Way Interactions	7	12.75	12.75	1.821	*	*
Residual Error	0	0.00	0.00	0.000		
Total	15	295.75				



# Analyzing Width

## Fractional Factorial Fit

Estimated Effects and Coefficients for Width (coded units)

Term	Effect	Coef
Constant		16.9938
A	0.8125	0.4063
B	0.3375	0.1687
C	1.8625	0.9313
D	0.3375	0.1687
E	-0.1625	-0.0812
F	0.0125	0.0063
G	0.3375	0.1688
H	0.1625	0.0812
A*B	-0.0875	-0.0438
A*C	0.2375	0.1187
A*D	0.0625	0.0312
A*E	0.1125	0.0562
A*F	0.2375	0.1187
A*G	0.7625	0.3813
A*H	-0.2125	-0.1062

Analysis of Variance for Width (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	8	18.095	18.095	2.2619	*	*
2-Way Interactions	7	3.054	3.054	0.4363	*	*
Residual Error	0	0.000	0.000	0.0000		
Total	15	21.149				



## *Determining Which Interaction Affects the Response*

- *There are two choices:*
  - *Foldover*
    - *We can repeat a similar design with all or some of the signs changed. The resulting design combined with the existing one will have fewer confounded interactions.*
  - *Perform a few additional runs*
    - *Run a small number of runs to figure out which interaction is affecting the response*
- *Both of these techniques are not too complicated, but slightly beyond the scope of this course, and we suggest you get assistance should such a situation arise in your experiment.*



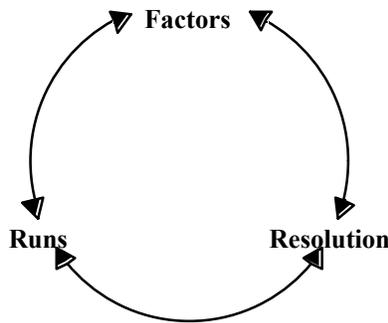
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# *Selecting an Experimental Design*



## Selecting an Experimental Design

Experimenters would generally like to study many factors, at high resolution, in a few runs. But often it is not possible to meet these desires simultaneously and trade-offs are necessary.



	<b><u>Who/What Defines</u></b>	<b><u>How Defined</u></b>
<b>Factors</b>	Subject matter specialists. (Engineers, Operators, etc.)	Brainstorming, process flowchart, cause-and-effect diagrams, consensus
<b>Resolution</b>	Potential for interactions	Statistical theory
<b>Runs</b>	Budget, priorities System under study	Affordability Effect size, variability size



## Detectable Effect Size

- *What is the smallest effect we can consistently detect with the current number of experimental runs?*
- *First we must identify the current level of variability (standard deviation) for each response variable.*
- *Information on the standard deviation of experimental variability can be obtained from:*
  - *Prior experiments*
  - *Statistical process control charts*
  - *Pilot runs*
  - *Similar processes*
  - *Educated guess*
  - *Estimate  $s_p$  using  $s_{st}$*
- *We can then compute the smallest effect size which can be detected 19 out of 20 times (power = 95%) for any 2-level design with the following formula:*

$$\text{Detectable Effect Size (DES)*} \cong 8 \times (\text{Std. Dev.}) \div \sqrt{\text{Number of Runs}}$$

\*Source: Wheeler, R.E. "Portable Power". *Technometrics*. Vol. 16. 1974. p. 193.



## Examples

### Using the Detectable Effect Size Formula

#### ■ How small of an effect will a design detect?

##### — In the Catapult Experiment

– Standard deviation = 0.63 bends ( $s_p$  calculated from 30 groups)

– Number of runs = 24

$$\text{Detectable Effect} \cong 8 \times 0.63 \div \sqrt{24} = 1.03 \text{ Inches}$$

– Thus, the smallest effect that will consistently be detected is 1.03 inches.

#### ■ How many runs are required to consistently detect an effect of a given size?

##### — In the Catapult Experiment

– Standard deviation = 0.63 bends

– Desired detectable effect size = 0.25 inches

$$0.25 \cong 8 \times 0.63 \div \sqrt{\text{Number of Runs}}$$

– Number of runs =  $\left(\frac{(8)(0.63)}{0.25}\right)^2 = 407$

– Since the Catapult Experiment has 3 factors, the full factorial, 8-run design will need to be replicated. The nearest multiple of a  $2^3$  design has 408 runs. Thus, 51 reps of the 8-run design are necessary ( $8 * 51 = 408$ ).



## Examples

Using the Detectable Effect Size Formula cont.

- How does the detectable effect size relate to the number of runs when the standard deviation is unknown?
  - Plugging  $s_p$  into the formula and solving for the detectable effect size in  $s_p$  units yields:

<u>No. of Runs</u>	<u>Detectable Effect</u>
8	$2.8 s_p$
16	$2 s_p$
32	$1.4 s_p$
64	$1 s_p$

$$DES = \frac{8}{\sqrt{N}} s_p$$

- Thus, a 64-run design will consistently detect an effect only if it is larger than or equal to the standard deviation of the response.



## *How to Select an Experimental Design*

- ① *Select the number of factors ( $k$ ) to be studied.*
- ② *Decide on the minimum required Resolution ( $R$ ).*
  - *If you are trying to screen the list of factors to determine which show large effects, a Resolution III design should be sufficient*
  - *If instead, you need to understand the interactions between factors, a Resolution IV or V design will be necessary*
    - *Use a Resolution V design over a Resolution IV when you expect a lot of important 2-factor interactions.*
- ③ *Using  $k$  and  $R$ , select a design from page 41 of the previous tabbed section (Reducing the Size of Experiments) or the “Display Available Designs” option under DOE  $\mathcal{P}$  Create Factorial Design in Minitab. This gives the minimum number of runs ( $N$ ) required to satisfy  $k$  and  $R$ .*



## *How to Select an Experimental Design (cont.)*

- 4 Evaluate the selected minimum number of runs ( $N$ ).
  - Consider budget and priorities
  - Evaluate detectable effect size (DES):
$$DES = 8 \times (\text{Std. Dev.}) / \sqrt{N}$$
    - Use  $s_{st}$  for  $s_p$  if  $s_p$  can not be estimated
  - If DES is too large, increase  $N$  by evaluating the number of runs necessary to achieve desired DES:
    - $N = [8 \times (\text{Std. Dev.} / \text{Desired DES})]^2$
  - Achieve this increased  $N$  either by choosing a larger fraction or replicating your design
- 5 Check for compatibility among  $k$ ,  $R$ , and  $N$ .
  - If minimum  $R$  is not achievable, decrease  $k$  or  $R$ , or increase  $N$
  - If more than the minimum  $N$  is required to detect the desired effect size, replicate the design or increase  $k$  or  $R$
  - If  $k$ ,  $R$ , and  $N$  are compatible but there is no replication, add center points. (However, center points do not affect the detectable effect size)



## *How to Select an Experimental Design (cont.)*

- ⑥ *See if the neighboring designs in the table offer a significant advantage.*
  - *Example 1: If you have selected the  $2^{6-2}_{IV}$  design, you can add up to two more factors and still have a design which is resolution IV. You may want to try to consider adding additional factors, since there is little cost in terms of what you can estimate from the design. Here you can increase  $k$  with no change in  $R$  or  $N$ .*
  - *Example 2: If you have selected the  $2^5$  design you can, for the same number of runs, instead do the  $2^{5-1}$  design, replicated. Here you can sacrifice a little in  $R$  and get the benefits of a replicated design.*

**Don't Run The Design If It Won't Give You What You Want!**



## Example: Design Selection I

- Dana has identified 11 factors that she would like to investigate. She is concerned that a few 2-factor interactions may be important, so she wants a Resolution IV design. Due to budget constraints she can afford about 21 runs. She wants to detect effects which are 1.5 times the pooled standard deviation.

- 1 Number of factors,  $k = 11$ .
- 2 Minimum required Resolution,  $R = IV$ .
- 3 The design is \_\_\_\_\_ with  $N =$  \_\_\_\_\_
- 4 What is the Detectable Effect Size for 21 runs?  
 $8s_p \div \sqrt{21} = 1.75s_p$  (doesn't make 1.5 goal)  
 What is the Detectable Effect Size for 32 runs  
 $8s_p \div \sqrt{32} = 1.41s_p$  (makes 1.5 goal)  
 What is the fewest number of runs required to achieve  
 $DES = 1.5s_p$ ?  
 $N = (8 \div 1.5)^2 = 29$  runs

- 5 Compatible?  
No!

k	R	N	Design	Detectable Effect
11	IV	32	$2_{IV}^{11-6}$	$1.4s_p$
8	IV	16 + 5 c.p.	$2_{IV}^{8-4}$	$2.0s_p$
11	III	16 + 5 c.p.	$2_{III}^{11-7}$	$2.0s_p$
11	III	20	Plackett-Burman 20 run	$1.8s_p$



## Example: Design Selection II

David is experimenting with a process where runs are cheap. The pooled standard deviation is about 2.5 and he wants to detect an effect of 2.5 or greater. In a previous experiment he has identified 5 factors that he wants to investigate along with all 2-factor interactions.

1. Number of factors,  $k = \underline{\hspace{2cm}}$ .
2. Minimum required resolution,  $R = \underline{\hspace{2cm}}$ .
3. The design is            with  $N = \underline{\hspace{2cm}}$  runs.
4. Number of runs
  - a. The budget is not a concern.
  - b. Detectable Effect Size =  $8 \times 2.5 \div \sqrt{16} = 5$ .
  - c.  $N$  for desired DES =  $[8 \times 2.5 \div 2.5]^2 = 64$  runs.
5. Compatible?

<u>k</u>	<u>R</u>	<u>N</u>	<u>Design</u>	<u>Detectable Effect</u>
5	V	16	$2^{5-1}_V$	5
5	V	64	$2^{5-1}_V$ (4 reps)	2.5
5	full	64	$2^5$ (2 reps)	2.5



# *Experimenter's Checklist*



# Experimenter's Checklist

## 1 Preliminaries:

- a Identify the budgetary restrictions for the project.
- b Examine the literature and past investigations in this area.

## 2 Identify Responses, Factors, and Factor Levels:

- c Select one or more measurable responses.
- d Operationally define the measurement procedure and estimate the variability of each response.
- e Identify all of the factors that may impact the response of interest.
- f Examine all pairs of factors for those that may be coupled (interact).
- g Decide where this project fits on the Knowledge Line and the minimum resolution required to move it up.
- h Set the high and low levels for each factor.
- i Review combinations of factor levels for potential problems.





## *Experimenter's Checklist (cont.)*

### **3** *Select Design:*

- j** *Select a design that allows you to examine the desired number of factors with the required resolution for the current state of knowledge.*
- k** *Decide on the number of experimental trials allowed by the budget using the 25% rule or other constraints.*
- l** *Calculate a detectable effect size for each response variable.*
- m** *If it is desirable to detect a smaller effect, calculate a new number of experimental trials and re-examine the budget.*
- n** *Build some replication into the final design, if possible.*
- o** *Consider whether center points should be added to the design.*

### **4** *Randomize the Run Order:*

- p** *Examine the factors and the physical layout of the experiment to see if restrictions on the randomization are necessary.*
- q** *Control any factors not in the experiment that can be controlled (held constant).*
- r** *Randomize over any factors not in the experiment that cannot be controlled.*
- s** *Select the materials for the experimental samples at random from the available inventory.*
- t** *Randomize the experimental trials consistent with any restrictions that may be necessary.*



## *Experimenter's Checklist (cont.)*

### **5** *Collect the Data:*

- u** *Prepare a written procedure and data collection form with room for all pertinent information, including written comments.*
- v** *Schedule the needed machines, technicians, materials, etc.*
- w** *If necessary, provide training to anyone involved in doing the experiment, including those who randomize and run the tests, take measurements, etc.*
- x** *Label and save all samples and results if possible.*
- y** *Monitor the performance of the experiment carefully (be there). Keep a log book of events, especially deviations from the plan.*
- z** *Review the raw data as it is collected and correct any mistakes immediately.*

### **6** *Analyze the Data:*

- a** *Plot the raw data in various ways.*
- b** *If the experiment includes replications, compute averages, standard deviations, and residuals for each experimental condition and plot them in various ways.*
- c** *Compute the factor effects and interactions and plot them in various ways.*
- d** *Where useful, develop a prediction model to relate factors to responses.*
- e** *When possible and appropriate, confirm impressions from plots with appropriate statistical analyses.*



## *Experimenter's Checklist (cont.)*

### **7** *Draw, Verify, and Report Conclusions:*

- f** *Interpret the results of the experiment using all known information (physical and statistical).*
- g** *Formulate and write the conclusions in simple, non-statistical language intelligible to peers.*
- h** *Verify the conclusions with additional runs.*
- i** *If appropriate, go on to the next iteration of study.*
- j** *Prepare a written report of the conclusions with additional runs.*
- k** *If appropriate, go on to the next iteration of study.*
- l** *Prepare a written report of the conclusions and recommendations.*
- m** *Review progress and make recommendations to your team.*





# 1 *Preliminaries*

---

- a *Identify the budgetary restrictions for the project.*
- b *Examine the literature and past investigations in this area.*



## *Is a Designed Experiment Appropriate?*

- *Not if preliminary work reveals an obvious cause and a solution exists. DON'T DO IT!*
- *Experiment if:*
  - *A root cause of the problem cannot be found*
  - *Root causes have been found and removed, but further improvement is desired*
  - *Many potential factors affect the response*
  - *You want to know (quantify) the relationship between the factors and the response*



## **2** *Identify Responses, Factors, and Factor Levels*

- c Select one or more measurable responses.*
- d Operationally define the measurement procedure and estimate the variability of each response.*
- e Identify all of the factors that may impact the response of interest.*
- f Examine all pairs of factors for those that may be coupled (interact).*
- g Decide where this project fits on the Knowledge Line and the minimum resolution required to move it up.*
- h Set the high and low levels for each factor.*
- i Review combinations of factor levels for potential problems.*



## *Selecting Response Variables*

- *What is the most important response or key quality characteristic?*
- *Is it measurable? If not:*
  - *Select a substitute response that measures a property that is related to the desired response*
  - *Several substitutes may be needed to adequately describe the desired response*
- *Collect data on all responses of interest to maximize information return on the experiment.*
- *Where possible, consider variability as a response variable. This requires replications!*
- *It is often helpful to define the direction for improvement for each response:*
  - *Smaller is better*
  - *Larger is better*
  - *On target is better*



## *Response Measurement Systems*

- *There are two types of response variables.*
  - *Qualitative: Opinion, ratings , rankings, etc.*
  - *Quantitative: Measurement instrument*
- *Effective measurement processes must be:*
  - *Well defined: Clear operational definition*
  - *Stable: Regular maintenance and calibration schedule*
  - *Accurate: “Agree” with an acceptable calibration standard*
  - *Repeatable: Same under same conditions*
  - *Reproducible: Same under different conditions*
- *Estimate the variability of the response.*
- *An experiment can be run to evaluate the measurement process and to understand which factors affect it most.*
  - *Called a gage reproducibility and repeatability study*
  - *Identifies sources of appraiser and instrument variation*
- *For details on how to evaluate your measurement processes, see ASQC’s Measurement Systems Analysis: Reference Manual in the reference list.*



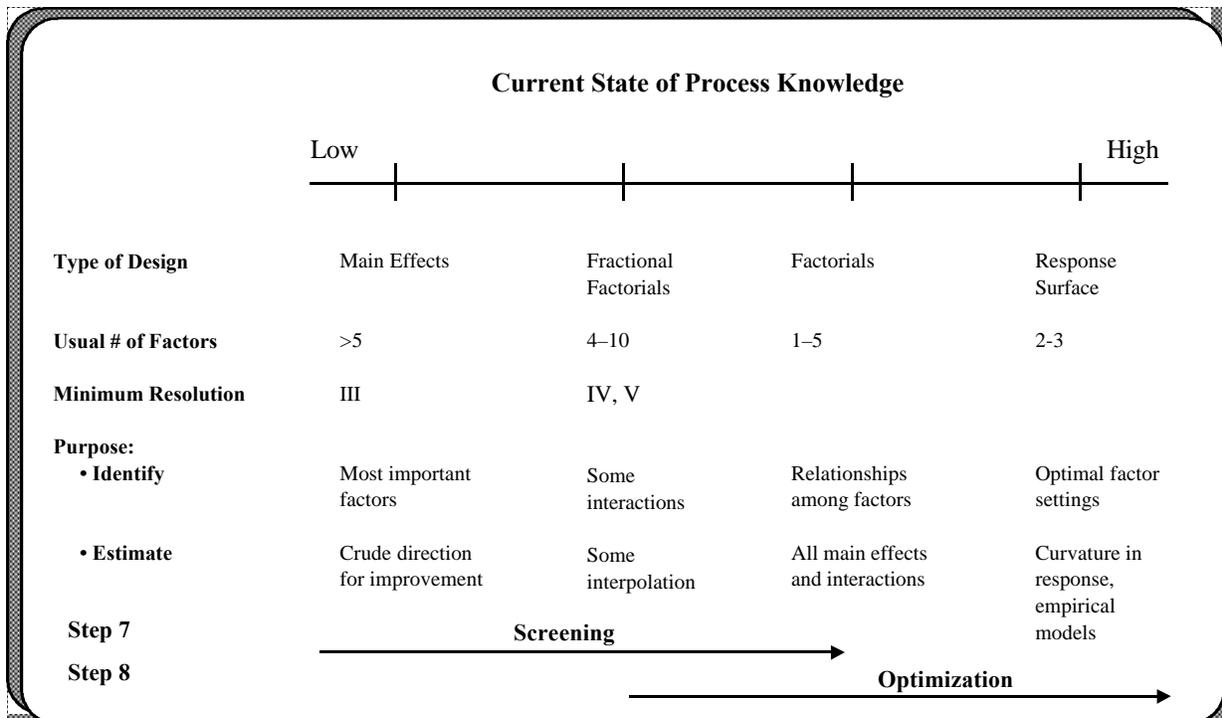
## *Choosing Factors for the Experiment*

- *Visit the work area and observe the process or examine a process flow diagram to generate ideas on what factors might influence the experiment.*
- *Arrange a formal brainstorming meeting with all the people who might have ideas on which factors might have an influence.*
- *Use a cause-and-effect diagram to organize the list of potential factors, to prevent you from overlooking some, and to help select the ones to experiment with.*
- *Categorize the selected factors as controllable or uncontrollable.*
- *Identify pairs of factors that might interact.*



# Assessing the Current State of Knowledge

- *Have all factors that affect the process/product/equipment been identified?*
- *Is the effect of the factors on the response understood?*
- *Is there a coupling or interaction between some of the factors?*
- *Is the process/product/equipment performing at its peak or optimum level?*





## *Strategies for Moving Up the Knowledge Line to Optimization*

- *Spend 25% of budget on the first experiment.*
- *Plan on several experiments.*
- *Experimentation is sequential:*
  - *Start with many factors to find the few important ones*
  - *Look for interactions between important factors. Look for curvature in the response*
  - *Find “best” settings of each factor. Establish the relationship between variables.*

*“Real life problem solving using statistical methods is a dynamic process that often is like detective work... ‘Quality detectives’ follow hunches and get inspired by evidence collected in conjunction with their subject matter expertise. Data inspire them to follow new leads in pursuit of never-ending quality and productivity improvement. It is with a sequential use of experimental design and analysis that the real power of statistics applied to industrial problems is released.”\**

\*Source: Bisgaard, S. “The Quality Detective: A Case Study,” Center for Quality and Productivity Improvement. Report No. 32. June 1988.



## *Setting Levels for Each Factor*

*“To find out what happens to a system when you interfere with it, you have to interfere with it (not just passively observe it).”*

*- George E.P. Box*

- *Setting quantitative factor levels requires subject matter knowledge and engineering intuition.*
  - *Set them far enough apart so that if the factor is active it will have a fair chance of being detected*
  - *Set them further apart than you would normally feel comfortable with*
  - *Don't set them so far apart that the resulting product or response variable will be of no value (e.g., significantly non-linear)*
  - *Consider purpose of experiment (e.g., optimization will require broader ranges than simply determining which X's have what effect on the current process)*
  - *Setting qualitative factor levels is simply a matter of deciding, for example, which vendor is “low” and which is “high”*
- *Sometimes the low (“-”) level is used to represent the standard settings. However, it is better to make sure quantitative settings are logical, i.e., higher values are “+” and lower values are “-”, regardless of which setting is standard.*



## *Reviewing Factor Level Combinations for Potential Problems*

- *Are any combinations potentially hazardous (maybe all factors set at higher levels)?*
- *Will any combinations produce useless results (maybe all factors set at low levels)?*
- *If there is a questionable factor combination you can:*
  - *Run it first to check it,*
  - *Adjust the levels, or*
  - *Reassign the factors so that the problem combination does not appear in the design.*
- *Do any combinations result in scrapped parts? (consider economics).*



## *Other Considerations for Good Experimental Procedure*

- *Try to incorporate the range of operating conditions into your experiment in a controlled way. If you use different machines, materials, or operators in production, introduce these variables as factors in your experiment.*
- *If you suspect uncontrolled factors to be affecting the response in your experiment, at least measure these variables. For example, if you suspect the temperature of the environment affects the response, but you decide not to control it, you can record the temperature for each run. In the analysis of your experiment, you can look for a relationship between the response and temperature.*
- *If your initial experiments are performed under carefully controlled conditions, be prepared to do some later experiments under conditions more like those over which you hope the results to hold.*

**Note:** *Paying attention to these items will increase the likelihood that conclusions will verify.*



### 3 *Select Design*

- j Select a design that allows you to examine the desired number of factors with the required resolution for the current state of knowledge.*
- k Decide on the number of experimental trials allowed by the budget using the 25% rule or other constraints.*
- l Calculate a detectable effect size for each response variable.*
- m If it is desirable to detect a smaller effect, calculate a new number of experimental trials and re-examine the budget.*
- n Build some replication into the final design, if possible.*
- o Consider whether center points should be added to the design.*



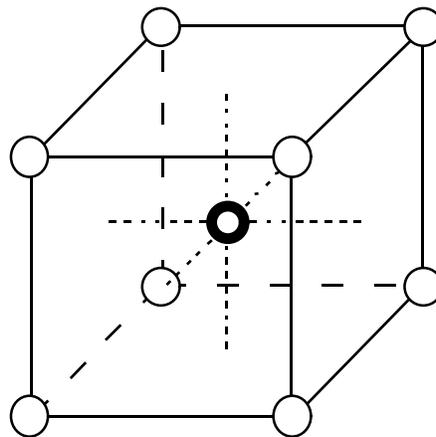
## *Replication*

- *Definition: Multiple execution of all or part of the experimental process with the same factor settings.*
  - *It is not the same as multiple measurements on a single piece or lot*
- *Why?*
  - *To measure experimental variability so we can decide whether the difference between responses is due to the change in factor levels (an induced special cause) or to common cause variability*
  - *To see more clearly whether or not a factor is important*
  - *To obtain two responses for each set of experimental conditions*
    - *Location*
    - *Spread*



## What are Center Points?

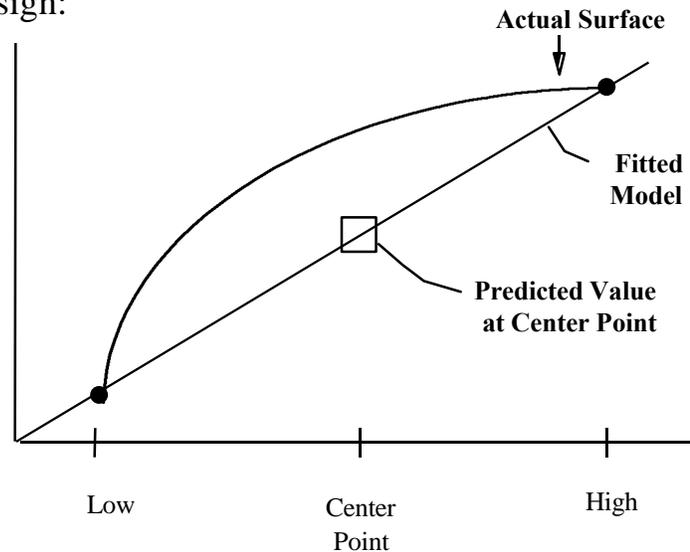
- **Center Points:** *Runs performed with all factors set halfway between their high and low level.*
  - *Used to obtain genuine replication without repeating all the experimental conditions*
  - *Usually replicated several times to estimate experimental variability*
  - *Valid for quantitative factors only.*  
*Represented by “0” (halfway between “-” and “+”)*
  - *Used also to check for curvature or a non-linear response*





## Check for Curvature in Designs with Center Points

Consider a  $2^1$   
Design:



- *The predicted value at the center point is the average of the observed value at the low and high points.*
- *If some experimental runs were actually made at the center point, the difference between the average of the observations at the center point and the average of the factorial points could be used to estimate how much curvature was present.*



## *Two Ways to Order Center Points*

- *Option 1: Add the number of center points to the total number of conditions and completely randomize all the runs.*
- *Option 2: Run the center point first and insert the others evenly spaced throughout the remaining runs.*

*Option 2 is preferred because it provides a better:  
Estimate of experimental variability across the entire experiment.  
Detection of lurking variables.*



## *Example of Ordering Center Points*

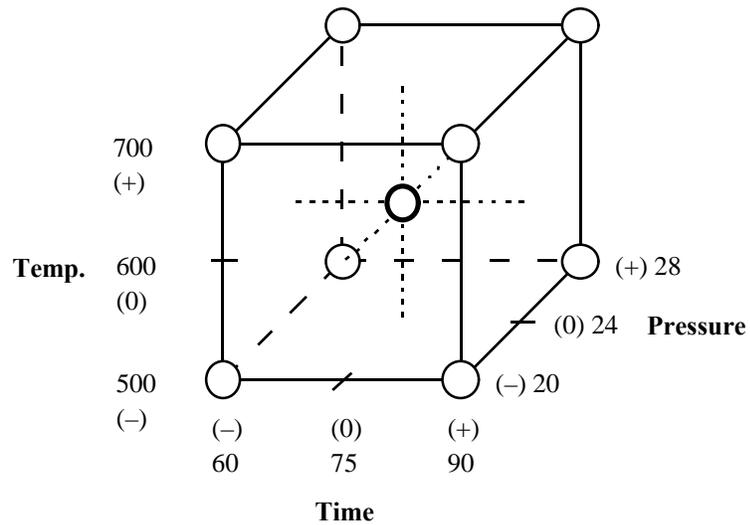
- *Given a 16-Run Experiment with four additional center points*
  - *Option 1:*
    - *Label the center points as conditions 17, 18, 19, and 20.*
    - *Completely randomize 20 runs.*
  - *Option 2:*
    - *Randomize the order of the 16 runs.*
    - *Determine at what interval to insert the center points:*
      - *20 runs divided by 3 equals 6.7*
    - *Insert a center point every 6 runs, i.e., at run order 1, 7, 13, and 19.*
    - *Re-number the order of all runs accordingly.*



# Example:

## Center Points for a 2<sup>3</sup> Design

Std. Order	Coded Settings			Actual Settings		
	A	B	C	A Time	B Temp	C Pressure
Base Design	1	-	-	60	500	20
	2	+	-	90	500	20
	3	-	+	60	700	20
	4	+	+	90	700	20
	5	-	-	60	500	28
	6	+	-	90	500	28
	7	-	+	60	700	28
	8	+	+	90	700	28
Center Points	9	0	0	75	600	24
	10	0	0	75	600	24
	11	0	0	75	600	24
	12	0	0	75	600	24





## Curvature Check

- For the Raycon EDM Machine Experiment, is there curvature with Width?

MINITAB FILE: Raycon2.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the 'DOE' (Design of Experiments) sub-menu is selected. The 'Define Custom Factorial Design...' option is highlighted. Below the menu, a data table is visible with columns C9 through C17. The table contains experimental data for 'Terms', 'Wear', 'Volts', 'Order', 'Blocks', 'CenterPt', and 'Blocks'.

	C9	C10	C11		C15	C16	C17
	Terms	Wear	Volts	Order	Blocks	CenterPt	Blocks
14	15	0.180000	60	14	1	1	
15	16	0.240000	60	15	1	1	
16	17	0.185000	60	16	1	1	
17	18	0.114000	60	17	1	0	
18		0.183000	61	18	1	0	
19		0.165000	58	19	1	0	
20							
21							

Create a factorial design from data that is already in the worksheet



# Minitab Input

1. Select Factors  
A-H.

The dialog box 'Define Custom Factorial Design' has a list of factors on the left: C1 A, C2 B, C3 C, C4 D, C5 E, C6 F, C7 G, C8 H, C9 Terms, C10 Wear, C11 Volts, C12 Width, C13 StdOrder, C14 RunOrder, C15 Blocks, C16 CenterPt, C17 Blocks1. The 'Factors:' field contains 'A-H'. The '2-level factorial' radio button is selected. Buttons include 'Select', 'Low/High...', 'Designs...', 'Help', 'OK', and 'Cancel'. An arrow points to the 'Designs...' button.

2. Select Designs.

3. Select standard  
order, run  
order, center  
points, and  
blocks.

The dialog box 'Define Custom 2-Level Factorial - Design' has a list of factors on the left: C10 Wear, C11 Volts, C12 Width, C13 StdOrder, C14 RunOrder, C15 Blocks, C16 CenterPt, C17 Blocks1. The 'Standard Order Column' section has 'Specify by column:' selected with 'StdOrder' in the text box. The 'Run Order Column' section has 'Specify by column:' selected with 'RunOrder' in the text box. The 'Center Points' section has 'Specify by column:' selected with 'CenterPt' in the text box. The 'Blocks' section has 'Specify by column:' selected with 'Blocks1' in the text box. Buttons include 'Select', 'Help', 'OK', and 'Cancel'. Arrows point from the text '3. Select standard order, run order, center points, and blocks.' to the 'Specify by column:' options for Standard Order Column, Run Order Column, Center Points, and Blocks.

4. Select OK.



# Analyze the Factorial Design

MINITAB FILE: Raycon2.mtw

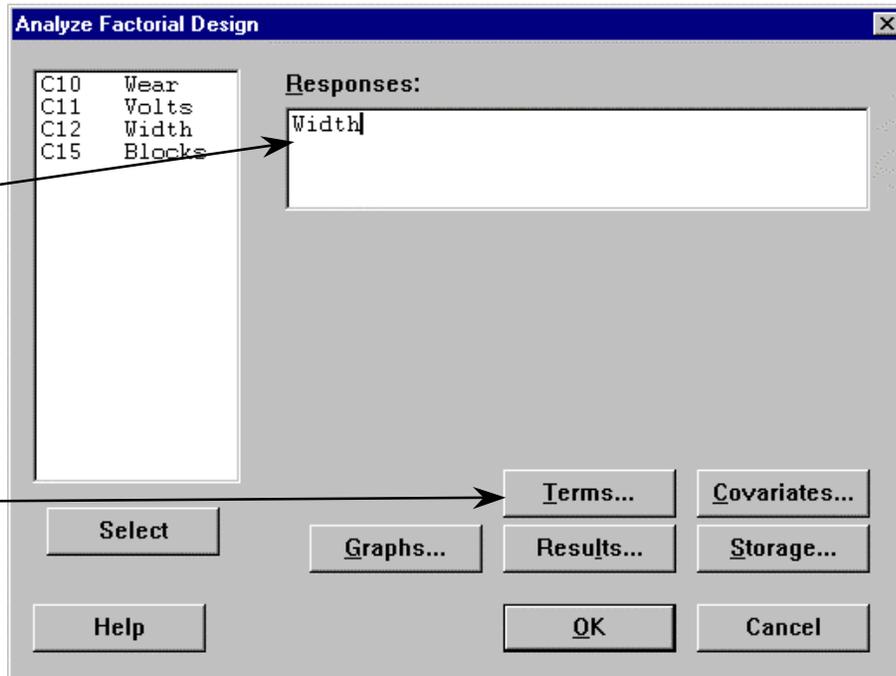
The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the 'DOE' (Design of Experiments) sub-menu is selected. Within the DOE sub-menu, the 'Analyze Factorial Design...' option is highlighted. The background shows a data table with the following columns and rows:

	C9	C10	C11		C15	C16	C17
	Terms	Wear	Volts	der	Blocks	CenterPt	Blocks
14	15	0.180000	6	14	1	1	
15	16	0.240000	6	15	1	1	
16	17	0.185000	60	16	1	1	
17	18	0.114000	60	17	1	0	
18		0.183000	61	18	1	0	
19		0.165000	58	19	1	0	
20							
21							

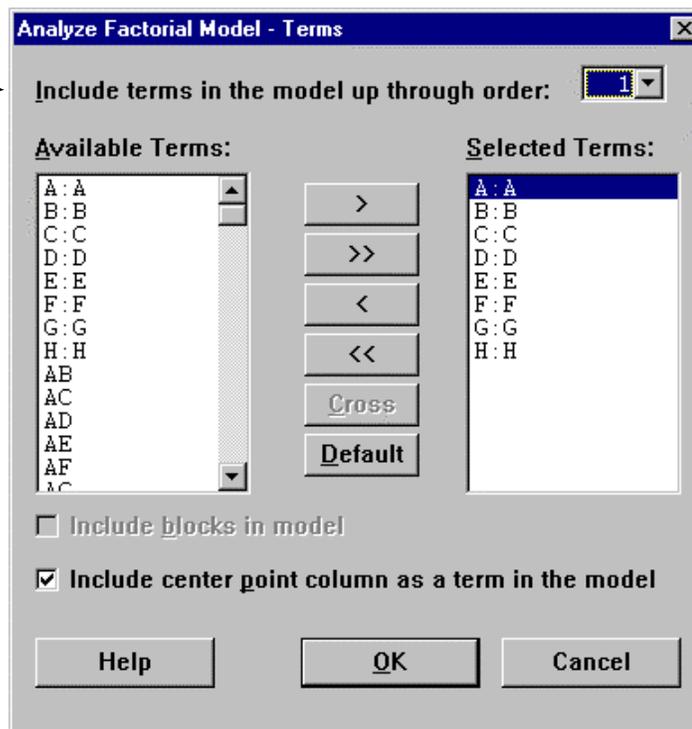


# Analyze the Factorial Design

1. Select response width.



3. Include Terms up through order 1.



4. Select OK.



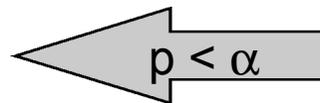
## Curvature Check

Analysis of Variance for Width (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	8	18.0950	18.0950	2.2619	5.93	0.008
Curvature	1	3.0917	3.0917	3.0917	8.10	0.019
Residual Error	9	3.4344	3.4344	0.3816		
Lack of Fit	7	3.0544	3.0544	0.4363	2.30	0.337
Pure Error	2	0.3800	0.3800	0.1900		
Total	18	24.6211				

$H_0$ : No Curvature

$H_a$ : Curvature is Significant



Note that the  $p$ -value for curvature is 0.019.

Therefore, we conclude that there is significant curvature.



## *Curvature: Now What?*

- *If we conclude that curvature is unimportant, then*
  - *we are in a relatively flat region of the response surface*
  - *we can move in the direction of improvement. (Extrapolation with prediction model)*
  - *we can verify predictions with experimentation*
  
- *If we conclude that curvature is important, then*
  - *even interpolation with the prediction model is suspect*
  - *further experimentation in the current region should be done to build a better model*
  - *a response surface design is required (three or more levels), but is not covered here*



## *Curvature Check Exercise*

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- *Using the file called Raycon2.mtw, conduct a check for curvature on both Volts and Wear.*



# DOE for Variances

Use the std deviation as the response to determine s-hat equation

## Fractional Factorial Fit

Estimated Effects and Coefficients for std dev

Term	Effect	Coef
Constant		7.500
Pin	5.000	2.500
Start	-3.000	-1.500
Pin*Start	-1.000	-0.500

$$\hat{S} = 7.5 + (2.5)A - (1.5)B - (0.5)AB$$

Analysis of Variance for std dev

Source	DF	Seq SS	Adj SS	Adj MS
Main Effects	2	34.000	34.000	17.000
2-Way Interactions	1	1.000	1.000	1.000
Residual Error	0	0.000	0.000	0.000
Total	3	35.000		



## *Analyzing the Results of the DOE:*

### **Determine the optimal settings**

*We can use the prediction model for the mean ( $\hat{Y}$ ) to determine the average  $Y$  for given values of  $X$  within our experimental range. Using the  $\hat{s}$  equation, we can determine how the factors should be set to minimize the variation.*



## *For the catapult . . .*

$$\hat{Y} = 195 + 15(\text{Start Angle}) + 10(\text{Start Angle} \times \text{Pin Position})$$

$$\hat{S} = 7.5 + 2.5(\text{Pin Position}) - 1.5(\text{Start Angle}) - 0.5(\text{Start Angle} \times \text{Pin Position})$$

*Our initial goal was to determine which variables have the biggest effect on how far the ball travels. From the analysis, we determined that start angle had the biggest effect on the average distance. Pin Position had no effect on the mean, except as an interaction. Pin Position does affect the standard deviation. Less variation is observed at the low level (-1) for pin position. If we wanted to launch the ball consistently to an average distance (Y-hat) of 197 inches, we would fix pin position at the -1 setting and solve the equation for start angle. Note that these equations must be solved using the coded (-1, or +1) units, then converted to actual units!!*



## *For the catapult . . .*

$$\hat{Y} = 195 + 15(\text{Start Angle}) + 10(\text{Start Angle} \times \text{Pin Position})$$

$$\hat{S} = 7.5 + 2.5(\text{Pin Position}) - 1.5(\text{Start Angle}) - 0.5(\text{Start Angle} \times \text{Pin Position})$$

$$197 = 195 + 15(\text{Start Angle}) + 10(\text{Start Angle} \times (-1))$$

$$= 195 + 5(\text{Start Angle})$$

$$2 = 5(\text{Start Angle})$$

$$0.4 = \text{Start Angle}$$

*Converting from the coded units:*

*At these settings (-1 pin, 0.4 SA), we would predict a standard deviation of:*

$$S\text{-hat} = 7.5 - 2.5 - 1.5(.4) + 0.5(0.4) = 4.6$$



## **4** *Randomize the Run Order*

- p Examine the factors and the physical layout of the experiment to see if restrictions on the randomization are necessary.*
- q Control any factors not in the experiment that can be controlled (held constant).*
- r Randomize over any factors not in the experiment that cannot be controlled.*
- s Select the materials for the experimental samples at random from the available inventory.*
- t Randomize the experimental trials consistent with any restrictions that may be necessary.*



## *Randomization - The Experimenter's Insurance*

- *Randomization: Assign the order in which to run the experimental conditions using a random mechanism.*
- *Benefits: Prevents the effect of a lurking variable from being mistakenly attributed to another factor. Therefore, valid statistical conclusions can be drawn in spite of lurking variables.*
- *Restricted Randomization: Sometimes it is impossible to completely randomize all the runs. Do the best you can. The trade-off is losing the insurance of valid conclusions.*
- *To guard against known lurking variables, you can*
  - *Hold them constant*
  - *Randomize*



## *Situations Where Randomization is Critical*

- *Changes over time*
  - *Etch bath degradation*
  - *Tool wear*
  - *Instrument drift*
  - *Operator learning curve*
  - *Shifts (day, night, graveyard)*
- *Changes in experimental material*
  - *Sick chickens*
  - *Sample aging*
  - *Different vendors or batches*
- *Change in plans*
  - *Equipment availability*
  - *Premature stopping of experiment*

***Control the factors you can and randomize for protection against the rest.***



## Using a Random Number Table

- *Select an appropriate table for N, the number of runs in the experiment.*
- *Close your eyes to pick a column.*
- *The column of numbers represents the order in which to run the experimental conditions. For example, if you choose the first column, the first run of your experiment will be condition 8. The second run will be condition 12. The last run will be condition 2.*
- *Cross off the column you used and select a different one next time.*

Experimental Conditions	Std. Order #	Random Orders																		
	1	8	2	7	7	4	1	14	9	5	9	15	8	16	4	9	12	12	1	9
2	12	7	15	13	15	14	12	10	6	8	13	4	11	14	4	6	11	4	1	6
3	1	12	16	12	14	15	6	6	8	6	5	3	7	6	3	16	7	12	8	8
4	11	16	5	6	7	4	3	11	2	5	9	11	5	15	12	11	2	10	3	10
5	13	5	10	11	16	12	4	15	11	3	1	12	14	8	1	7	4	8	11	2
6	10	11	6	4	9	2	2	5	3	13	3	7	2	2	13	9	13	11	15	4
7	15	9	9	16	11	10	1	8	7	12	7	1	6	3	8	15	16	9	6	12
8	5	3	1	5	13	3	5	12	10	10	12	6	9	13	5	1	3	2	5	16
9	14	6	2	1	1	8	13	4	13	7	10	15	8	7	16	2	1	15	7	9
10	3	8	8	14	10	11	7	1	9	2	11	13	15	12	11	5	14	5	12	5
11	4	15	3	2	6	6	9	3	12	1	2	10	4	9	2	14	8	7	10	13
12	9	14	4	15	8	16	10	2	15	15	14	2	1	11	10	4	5	6	13	11
13	7	10	11	10	3	7	16	16	1	11	6	5	12	10	7	8	6	3	14	7
14	16	13	14	9	5	9	11	7	14	14	16	9	13	5	6	3	10	13	4	15
15	6	4	12	8	2	13	15	13	4	16	4	16	10	16	14	13	15	16	16	14
16	2	1	13	3	12	5	8	14	16	4	8	14	3	1	15	10	9	14	2	1



## **5** *Collect the Data*

- u** *Prepare a data collection form with room for all pertinent information, including written comments.*
- v** *Schedule the needed machines, technicians, materials, etc.*
- w** *If necessary, provide training to anyone involved in doing the experiment, including those who randomize and run the tests, take measurements, etc.*
- x** *Label and save all samples and results if possible.*
- y** *Monitor the performance of the experiment carefully (be there). Keep a log book of events, especially deviations from the plan.*
- z** *Review the raw data as it is collected and correct any mistakes immediately.*



## **6** *Analyze the Data*

- a *Plot the raw data in various ways.*
- b *If the experiment includes replications, compute averages, standard deviations, and residuals for each experimental condition and plot them in various ways.*
- c *Compute the factor effects and interactions and plot them in various ways.*
- d *Where useful, develop a prediction model to relate factors to responses.*
- e *When possible and appropriate, confirm impressions from plots with appropriate statistical analyses.*



## **7** *Draw, Verify and Report Conclusions*

- f** *Interpret the results of the experiment using all known information (physical and statistical).*
- g** *Formulate and write the conclusions in simple, non-statistical language intelligible to peers.*
- h** *Verify the conclusions with additional runs.*
- i** *If appropriate, go on to the next iteration of study.*
- j** *Prepare a written report of the conclusions with additional runs.*
- k** *If appropriate, go on to the next iteration of study.*
- l** *Prepare a written report of the conclusions and recommendations.*
- m** *Review progress and make recommendations to your team.*



## Verify Conclusions

- *Definition: Performing additional runs to confirm that the conclusions drawn from the experiment are correct. There are several reasons why improvement we see in an experiment may not prove to be attainable in the future:*
  - *We may not understand the response correctly. We may be misled by the design's confounding or the response may be too complex to represent with a simple factorial model.*
  - *Conclusions may depend on unknown conditions present during experimentation.*
  - *Differences between laboratory and production environment or equipment may affect the results. This is commonly referred to as scale-up differences.*
  - *Before spending time and money to implement the conclusions from an experiment, it is wise to verify them. This verification is done before a solution is implemented in the Control phase.*



## *Why Don't Experiments Verify Conclusions?*

- **Reason #1: Response Not Correctly Understood**
  - *It is possible that an effect we attributed to a particular main effect or interaction may be in fact due to some other term. If there was an ambiguity in your analysis, you may want to go back and consider other options in interpreting confounded effects.*
  - *It is also possible (but uncommon) that the simple factorial analysis has not done a good job at representing the behavior of the response in the range we have studied. If the response variable moves up and down repeatedly over the range under study, we will not be able to approximate it very well with our simple analysis.*



## *Why Don't Tests Verify Conclusions?*

- **Reason #2: Unknown Conditions Present During Experimentation**
  - *There are an infinite number of things that can change between when we run the experiment and when we verify the results. One or more of these changes, that we may not even be aware of, may have an important affect on the response. Some examples are given below:*
    - *Ambient temperature changes*
    - *Different raw material uses*
    - *Slightly modified procedure*
    - *Measurement equipment drifts*
    - *Deterioration of chemical baths*
  - *If your results fail to verify, investigate to see if any changes of the type mentioned above have occurred. Be on the look out for differences between then and now that may be having some effect on the response. If you find a likely culprit, it may be useful to run a small experiment to see if it is, in fact, affecting the response.*



# *Why Don't Tests Verify Conclusions?*

---

## ■ *Reason #3: Scale-up Differences*

- *Laboratory results sometimes fail to verify in production due to scale-up differences. Often in the lab, we set up conditions that are quite different than the conditions under which we hope the results will apply. We are very careful to hold all unstudied factors constant, to use the purest of materials, the most careful procedures, the best machinery, small batch sizes, etc.*
- *When we “scale-up” and try to apply the results to less controlled conditions, often the results do not verify.*
- *Be on guard for the kinds of differences mentioned above and investigate if your production results fail to verify.*



## *How to Verify an Experiment*

*There are two primary ways to verify an experiment*

- *Repeat the entire experiment or run a similar one. For example, a similar experiment might be to select only the factors with the largest effects and run a larger fraction in these factors.*
- *Identify the “best” operating conditions from the experimental results. (These may or may not be a run we have made.) Make several runs at these conditions to see if they perform as expected. If feasible, use a control chart on the response and alternate between stretches at the original and “best” conditions and look for a signal of a special cause on the chart.*

*Or, you can run another designed experiment with the “best” settings of the important factors as one corner of the experiment. Make relatively small changes from the “best” settings for the factor levels, always moving in the direction of improvement for each factor. This would allow for verification of the experimental results as well as examination of the sensitivity of the response to small changes in the important factors.*



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# *Other Designs for Experiments*



# *Design for Experiments*

## **Type of Design**

## **Purpose**

### *Full Factorial*

*All factors at 2 levels  
All factors at 3 (or  
more) levels  
Factors at mixed levels*

*Supplies information on all  
factors and interactions. No  
confounding.*

### *Fractional Factorial*

*2 levels  
3 or more levels  
Mixed levels*

*Reduces the number of runs  
by confounding information  
on interactions.*

### *Screening*

*Fractional factorials  
Plackett-Burman  
“Taguchi Methods”*

*Finds which of many factors  
are most important.*

### *Response Surface*

*Central Composite  
designs  
Box Behnken  
designs*

*Optimizes the settings of the  
factors when curvature exists  
in the response.*



## *Design for Experiments (cont.)*

### **Type of Design**

### **Purpose**

#### *Blocked Designs*

*One blocking factor  
with 2, 4, or 8 levels*

*One or more  
blocking factors  
(complete, incomplete,  
split plot, Latin Squares, etc.)*

*Controls known sources of  
variation that are not  
intrinsically of interest.*

*Handles difficult randomization:  
some factors cannot be  
changed as easily as others.*

#### *Components of Variation*

*Identifies sources of variation in  
measurements. Example: between  
engines, between test cells, within  
test cells.*

#### *EVOP (Evolutionary Operation)*

*Applies DOE in manufacturing when  
it is not possible to shut down for  
experimental runs.*

#### *Mixture*

*Finds the optimum proportion of  
elements in a mixture. Example:  
from fruit juice to rocket fuel.*



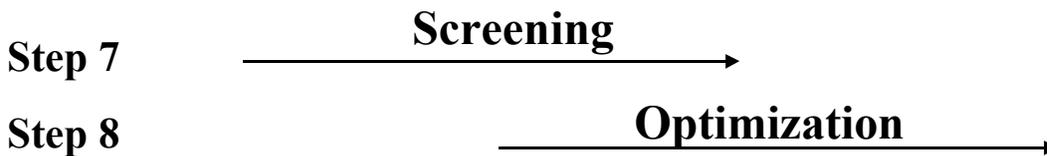
# Strategy for Choosing the Appropriate Design

## The Knowledge Line

### Current State of Process Knowledge



Type of Design	Screening	Fractional Factorials	Factorials	Response Surface
Usual # of Factors	>5	4-10	1-5	2-3
<b>Purpose:</b>				
•Identify	Most important factors	Some interactions	Relationships among factor	Optimal factor settings
•Estimate	Crude direction for improvement	Some interpolation	All main effects and interactions	Curvature in response, empirical models





# *Demonstration II*



## *Demonstration II*

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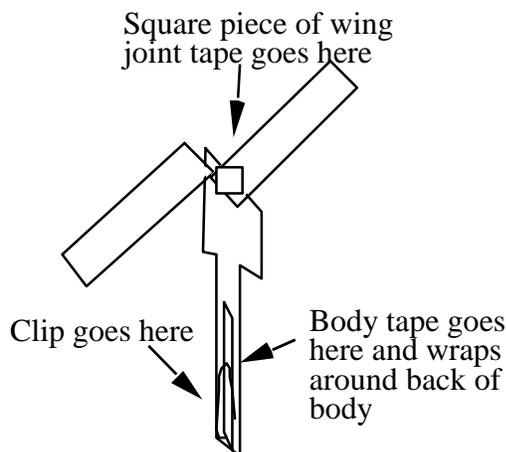
- *Using what you now know about factorial experiments, repeat the Cellulose Helicopter project to maximize flight time.*
- *Create prediction models for both the mean and standard deviation of flight time.*
- *Estimate your best  $Z_{st}$  given a lower specification limit of \_\_\_\_\_ sec.*



## *Factors That May Affect Flight Time of CHI Helicopters*

- *The Cellulose Helicopter Association has authorized for flight testing certain modifications to the standard design. Allowable ranges for the factors that may vary are shown below:*

Factor	Suggested Levels	
	Standard	Allowable Changes
Paper Type	Recycled (yellow)	Copier (white)
Paper Clip	No	Yes
Taped Body	No	3 inches of adhesive tape
Taped Wing Joint	No	Yes
Body Width	1.42"	2.00"
Body Length	3.00"	4.75"
Wing Length	3.00"	4.75"





## *Phase II: Project Description*

---

### ■ *Project Mission:*

- *Find the combination of factors that most consistently maximizes the flight time of a \_\_\_\_\_ foot drop*

### ■ *Project Constraints:*

- *Each prototype costs: \$100,000 to build*
- *Additionally it costs: \$10,000 to conduct a single flight time test*
- *You must do both a screening design (Part A) and a main test (Part B)*
- *You must run a final test (Part C) to verify the improvements you have identified*
- *Your materials and test budget for this phase is limited to \$5,000,000*
- *Your team must issue a report*



## *Part A: Screening Design*

---

- *A 7-factor screening design must be run to verify any and all assumptions the team has made as a result of Phase I.*
- *Choose your screening design, set it up in Minitab and collect your data.*
- *Analyze your data:*
  - *Box Plots*
  - *Run Charts*
  - *Pareto of Effects*
  - *Factorial Plots (main effects)*
- *Determine the factors you want to carry over to Part B of your experiment and give statistical results to support your decision.*



## *Part B: Main Test*

- Choose your main experiment design. It **must** include **at least two** replicates. Set it up in Minitab and collect your data.
- Analyze your data:
  - Box Plots
  - Run Charts
  - Residuals Plots
  - Pareto of Effects
  - Factorial Plots (main effects and interactions)
- Develop a prediction equation for flight time of your recommended model.
- Develop a prediction equation for variation.
- Calculate the detectable effect size for your experimental design given your  $s_p$ .
- Define the features of your recommended model and give statistical results to support your recommendation.



## *Part C: Verification Test*

- *Determine how many flights you want or can afford to make to verify the design improvements you have identified through your experiment.*
- *Collect your data.*
- *Analyze your data for randomness and normality.*
- *Determine your new flight time process capability against a flight time lower specification of \_\_\_\_\_ seconds.*
- *Demonstrate whether or not you have achieved a statistically significant difference in average flight time from your baseline model.*
- *Summarize your costs against the budget.*



## *Tips & Rules*

- *Good designs include REPLICATION, RANDOMIZATION, and verification.*
- *Design Selection: factors, resolution (possibility of interactions), runs (DES, budget).*
- *In Minitab: name factors to make table readable.*
- *Set up production line to share the manufacturing work.*
- *Label helicopters clearly.*
- *Establish a good measuring process.*
- *Make notes on unusual flight observations, i.e. stability.*
- *Less experimental variation means more conclusive results.*
- *Hints: consistent wing angle, fold stability, body folding, release method, storage of helicopters, avoid air currents.*



## *Typical Time Requirements*

---

### *Part A:*

- *½ hr: screening design selection and setup in Minitab*
- *½ hr: production and flight tests*
- *1 hr: analysis of screening design/selection of second design*

### *Part B:*

- *½ hr: experimental design selection and setup in Minitab*
- *1 ¼ hr: production and flight tests*
- *1 hr: analysis of second design*

### *Part C:*

- *¼ hr: verification tests*
- *1 hr: preparation of report*



## *Roles and Responsibilities*

<b>Role</b>	<b>Responsibility</b>	<b>Who</b>
<i>Lead Engineer</i>	<i>Leads the team in deciding which prototypes to build. Has final say on which prototypes are built and tested.</i>	_____
<i>Test Engineer</i>	<i>Leads the team in conducting the flight tests of all prototypes. Has final say on how tests are conducted.</i>	_____
<i>Assembly Engineer</i>	<i>Leads the team in building the prototypes. Has final say on all building issues.</i>	_____
<i>Finance Manager</i>	<i>Leads the team in tracking expenses. Has responsibility for keeping the team on budget.</i>	_____
<i>Recorder</i>	<i>Leads the team in recording data from the trials.</i>	_____



## *Project Report*

- *Prepare a Phase II report on your recommendations for increasing flight time. Include:*
  - *Recommendations for an improved helicopter design*
  - *Predicted flight time at improved setting*
  - *How much money did you use?*
  - *What experimental strategy did you use to arrive at the above?*
  - *How did you analyze your data?*
  - *Your two prediction models*
  - *Your  $Z_{st}$*
  - *Your DES*



## *Exercise*

---

- *What differences did you notice between Demo I and Demo II?*



## *Take Aways—Steps 7 and 8*

- *One-Factor-At-A-Time and Stick-With-A-Winner approaches are limited with regard to the amount of information they provide about a process.*
- *Full Factorial design is an approach that tests every possible combination of the factors.*
- *Full Factorial designs have a pattern that is easy to lay out and will result in  $2^k$  different experimental conditions for an experiment with  $k$  factors at two levels.*
- *A well-designed experiment will include replication, as opposed to repetition.*
- *Randomizing the order in which the experiment is conducted can reduce the bias that a lurking variable might introduce.*



## *Take Aways—Steps 7 and 8*

- *Before analyzing the data from a designed experiment, diagnostic steps should be undertaken with both the raw data and residuals.*
- *Any outliers that are identified should be investigated. Do not throw away the outliers unless there exists very extreme circumstances to which you can associate the point.*
- *Focus on the outliers to reduce the variation within your process.*
- *Replicated experimental designs will allow the analysis of residuals and the calculation of an estimate of experimental variability, the pooled standard deviation.*



## *Take Aways—Steps 7 and 8*

- *There is an interaction when the effect that one factor has on the response of interest depends on the level of another factor.*
  
- *After calculating the effects of the factors and their interactions, a prediction equation for the mean and standard deviation of the response may be developed using the terms that are statistically significant.*
  - $Y = f(X_1, X_2, \dots, X_n)$
  
- *The prediction equation may be used to determine the desired settings of the factors to achieve the desired response.*



## *Take Aways—Steps 7 and 8*

- *For a full factorial design, the number of experimental runs grows exponentially with the number of factors, which frequently makes it too costly and time consuming to perform an experiment.*
- *Fractional designs may allow investigation with a smaller number of runs.*
- *Much of the information obtained in a full factorial can be obtained using only a fraction of the full factorial.*
- *There are methods for creating fractional factorial designs.*
  - *half fraction*
  - *quarter fraction*
  - *etc.*



## *Take Aways—Steps 7 and 8*

- *Fractional designs result in fewer runs but introduce confounding, which makes the interpretation of results from an experiment more difficult.*
- *Screening designs are useful as a first iteration in a series of experiments.*
  - *only provide information about main factors*
  - *can be used to take a large list of factors and reduce it to a smaller number of factors to be investigated in another experiment*
  - *main effects are confounded with two-factor interactions*
- *Resolution describes the degree of confounding present in a fractional factorial design.*



## *Take Aways—Steps 7 and 8*

- *Detectable Effect Size describes the smallest effect an experiment will consistently detect.*
  - *When deciding on the number of runs, consider the detectable effect size*
  
- *Several things are important when selecting an experimental design:*
  - *Number of factors to be studied*
  - *Potential for interactions*
  - *Budget/Time constraints*
  - *Need for replication*
  
- *Selecting the proper design often involves a trade-off among number of factors, resolution, and number of runs.*



## *Take Aways—Steps 7 and 8*

- *Designed experiments are one of many tools that may be used when improving a process.*
- *Our level of knowledge concerning the process/product/equipment is a key determinant in the selection of a design strategy.*
- *When experimenting, what is learned from the initial experiment will shape the next experiment. This is the sequential nature of experimentation.*
- *Stable processes and well defined, stable measurement systems help assure successful experimentation.*

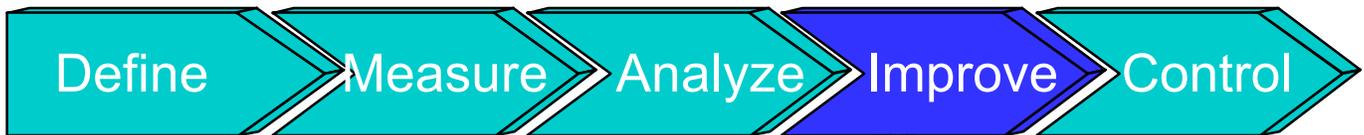


## *Take Aways—Steps 7 and 8*

- *The 25% rule is a helpful guideline when performing a series of experiments. The 25% rule states that 25% of a budget should be reserved for an initial screening study, or other first phase experiments.*
- *Center points may be used to estimate experimental variability in lieu of replication.*
- *Center points may also be used to check for curvature in the response of interest.*
- *Selection of factor levels and identification of response variables require subject matter knowledge and experience.*
- *Well designed experiments include both replication and randomization.*
- *Conclusions should be verified with additional experimentation.*



# Improve Phase



**7. Screen Potential Causes**

*Deliverable: Determine the Vital Few X's That Are Causing Changes in Y.*

*Tools:*  
■ *Screening DOE*

**8. Discover Variable Relationships**

*Deliverables:*  
1) *Establish Transfer Function Between Y and Vital Few X's.*  
2) *Determine Optimal Setting for the Vital Few X's.*  
3) *Perform Confirmation Runs.*

*Tools:*  
■ *Factorial Designs*

**9. Establish Operating Tolerances**

*Deliverable: Specify Tolerances on the Vital Few X's.*

*Tools:*  
■ *Simulation*



## Step 9

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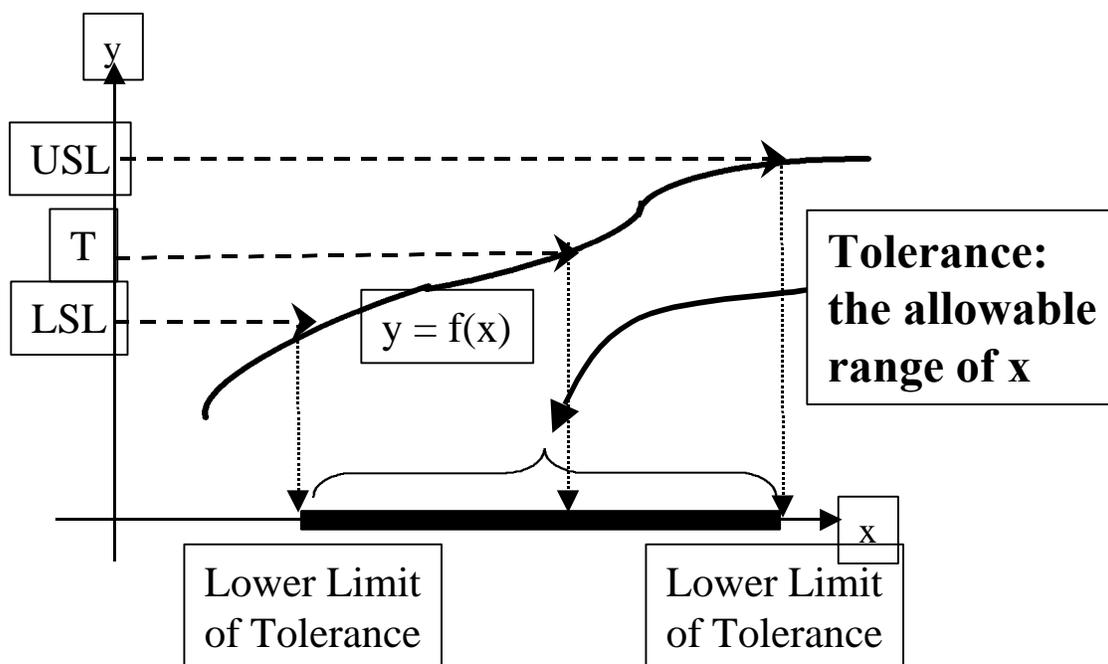
*Step 8 provided the experimental techniques to establish the relationship between the measurable  $y$  characteristics and the controlling  $x$  factors. In step 9, those relationships, so-called transfer functions, will be used to define the key operating parameters and tolerance to achieve the desired performance of the CTQ's. The concept is very straightforward, one should be able to set the tolerance of  $x$  factors if the  $x$ - $y$  relationship and the specifications of  $y$  are given. A quick example is following.*



# Principle of Tolerancing

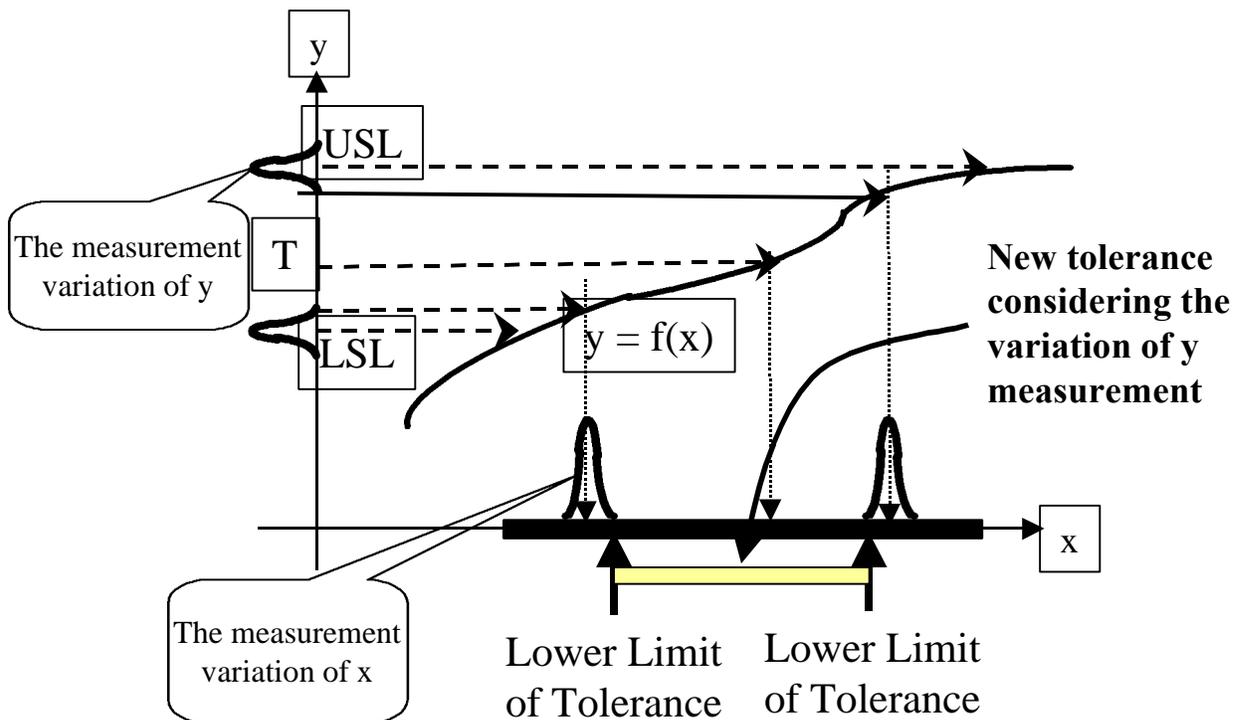
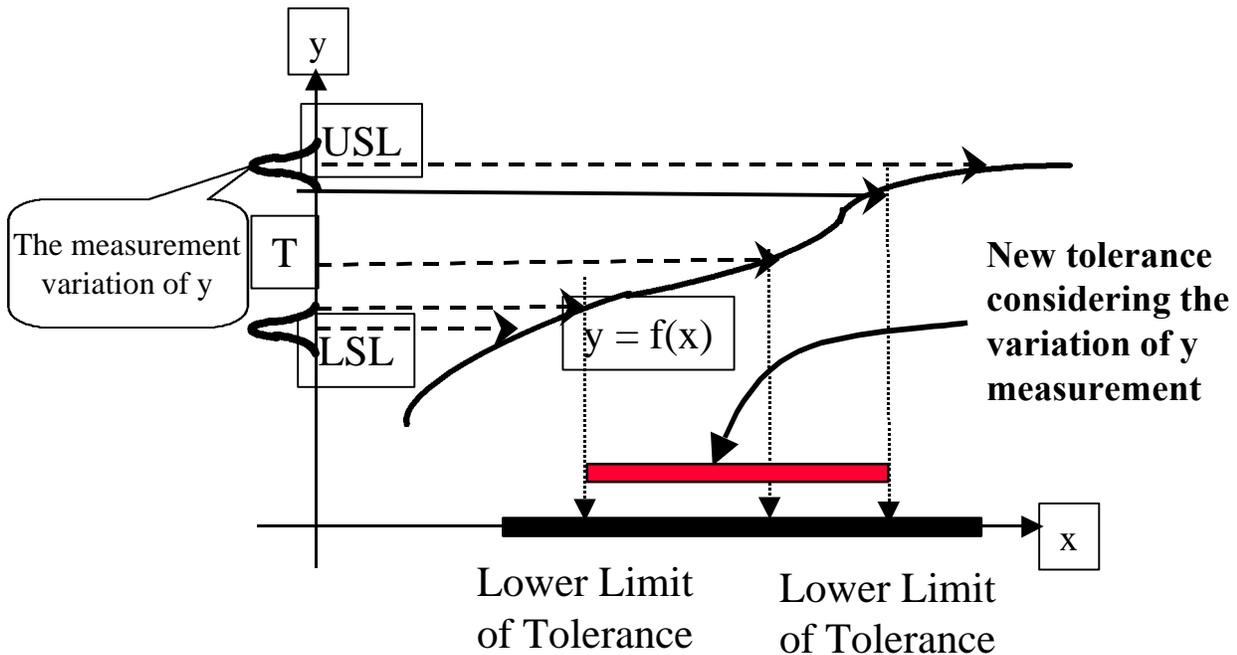
**Tolerance:** the allowable range of variation of  $x$  while still meeting the requirements of  $x$ .

- Establish tolerance of  $x$  based on the requirements of  $y$ , or often the specification limits via the Transfer function.
- The tolerance should allow the project to reach the project objectives established in step 5
- When more than one CTQ are involved, be aware of the trade offs among CTQs.
- Be aware the variations due to the measurement of  $x$ 's and  $y$ 's.



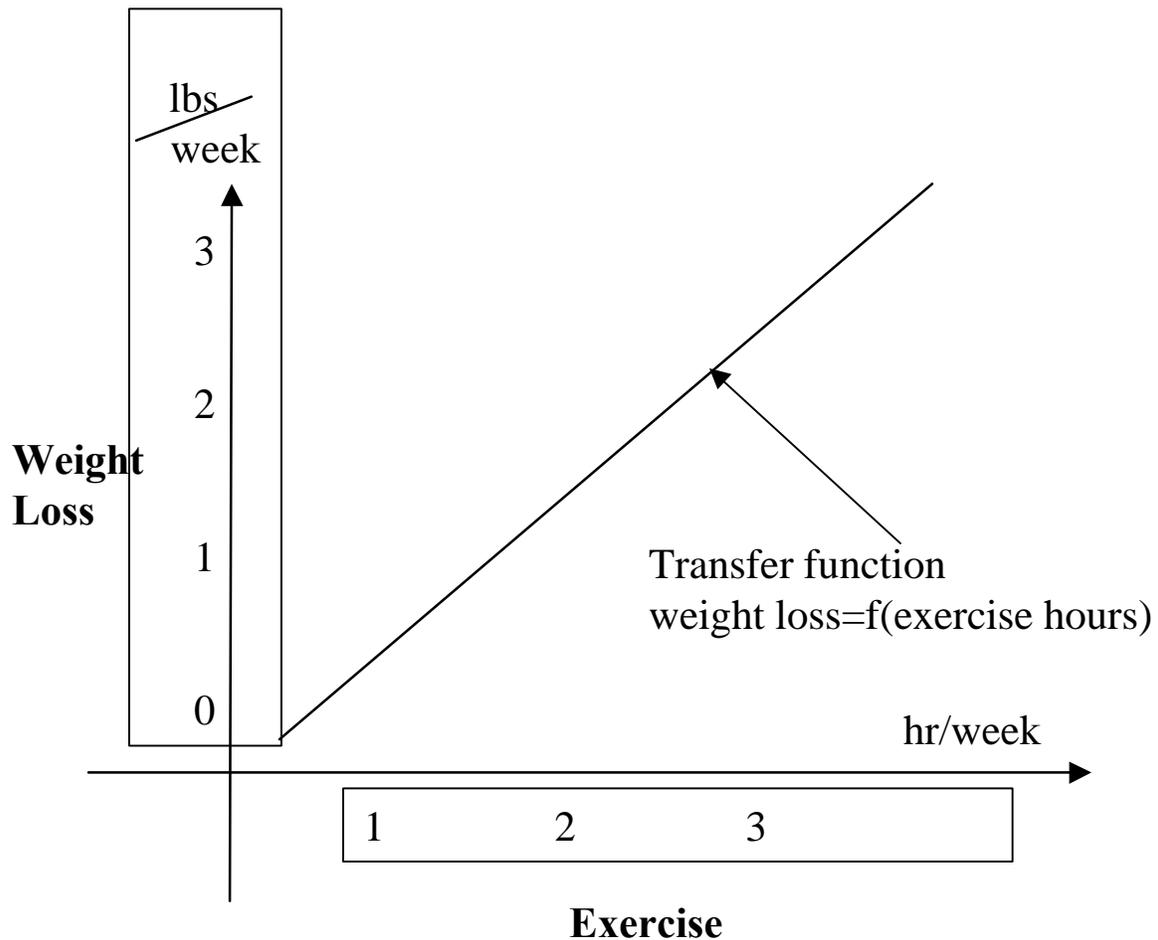


# Consideration of Measurement Variations





## Example: Weight loss wellness program

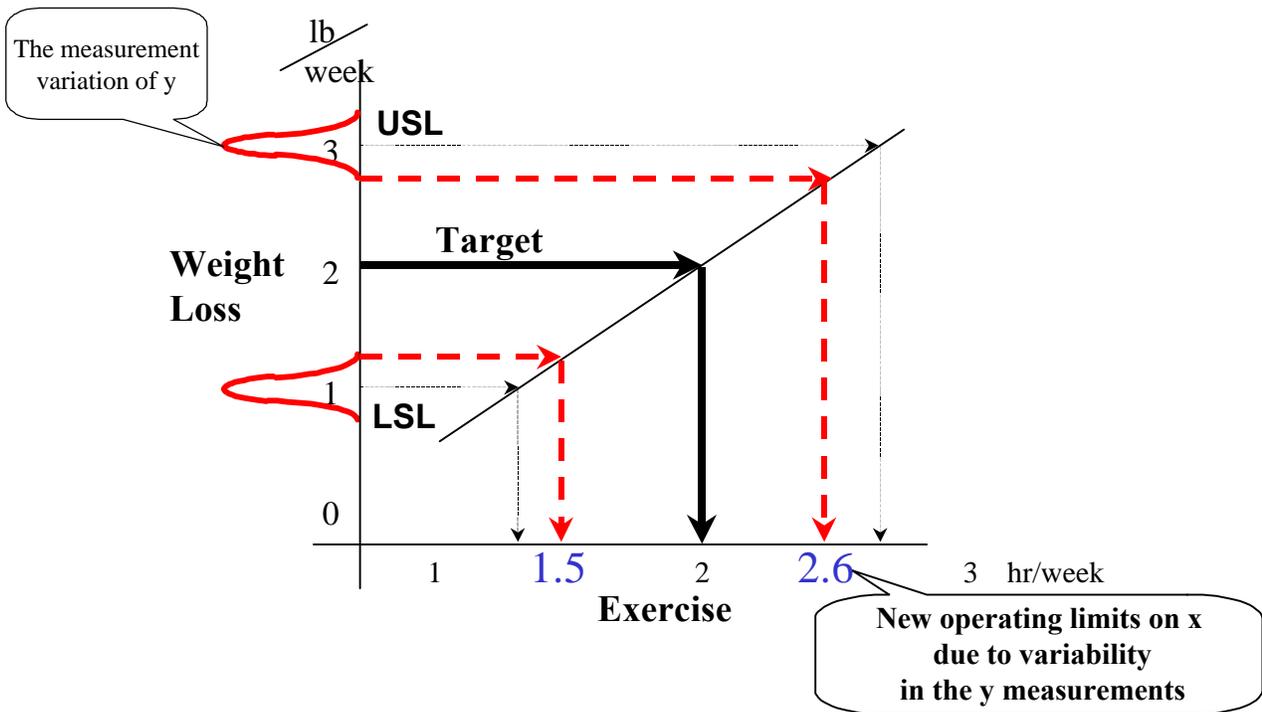
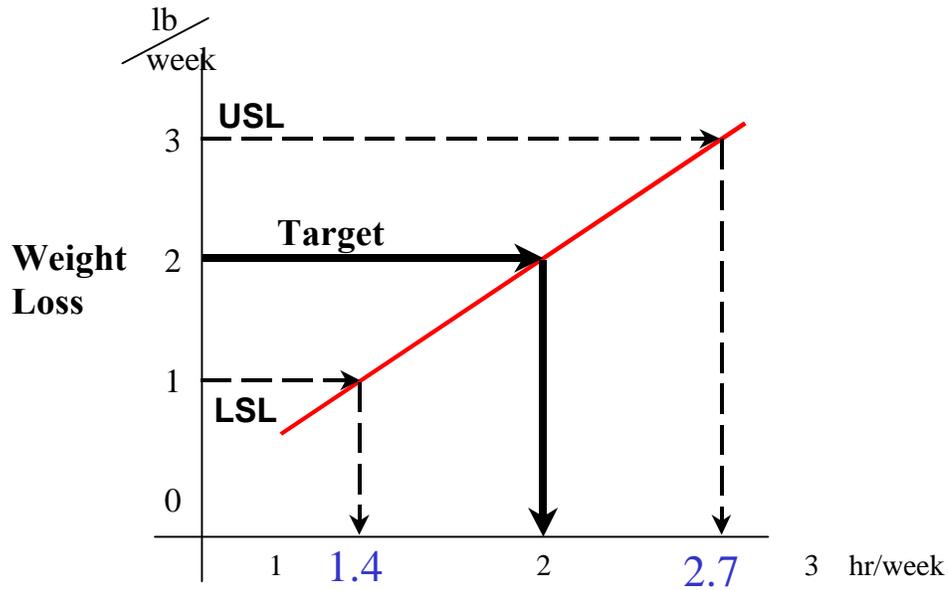


### CTQ requirements:

- Target to lose 2 lb/week
- At least need to lose 1 lb/wk
- No more than 3 lb/wk due to health concerns

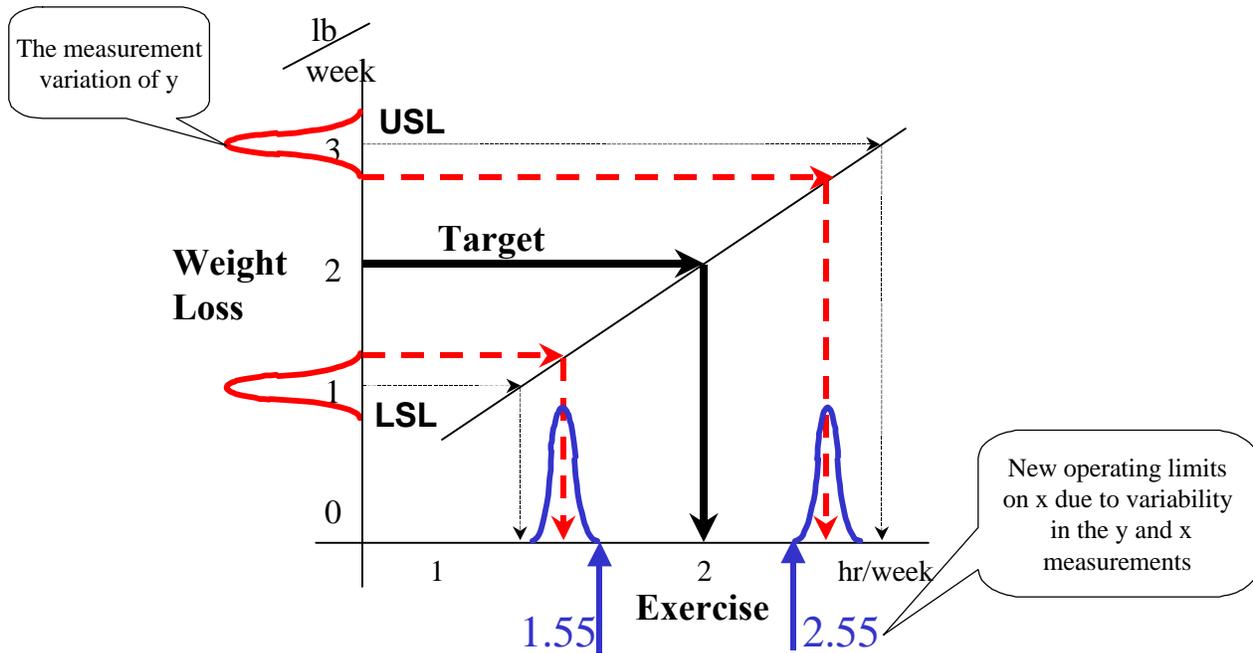


# Example: Weight loss wellness program





# Example: Weight loss wellness program





## Summary

- Set the process tolerances based on product specifications (Exercise time needed to achieve desired weight loss goal.).
- Adjusted our process tolerances to account for product measurement variation ( our scales vary by 1/4 lb).
- The tolerance principle is relatively simple and the applications is relatively straightforward when

(1) the measurement variation of  $y$  and  $x$  are sufficiently small to be ignored.

(2) only one  $x$  factor involved.

However, when measurement variations of  $y$  and  $x$  needs to be considered and multiple  $x$  factors are involved, the math techniques could be more complicated. Courses on DFSS Statistical Design Method or DFSS Statistical Tolerance are offered in various GE businesses.



---

# *Simulation Overview*



## *Objectives*

---

*Simulation can be thought of as the use of a model of a system for the purpose of evaluating the system's behavior under various conditions*

- *Provides general insight into the nature and performance of a business process*
- *“Easy” method of understanding performance of many interacting variables*
- *Helps identify specific problem(s) within a process*
- *Supports development/justification of process designs or changes (costs, quality, risk)*



## *An Introduction to Simulation*

---

- *Basics of simulation*
- *How simulation fits into GE's six sigma efforts*
- *How to manage a simulation project*



## *Where Is It Used?*

### *Examples*

- *Wind tunnel testing of aircraft model*
- *Weather*
- *Traffic patterns*
- *Predicting time needed to travel from Cincinnati to Heathrow airports from the map*
- *Solving the differential equation for circuit current, voltage drops*

*But you may be concerned with Process simulation: mimicking the arrival of calls, customers, etc., by using processing times and costs to predict overall system cycle times, costs, bottlenecks and productivity*



## *Why Might You Use Simulation?*

*Simulations permit inferences to be drawn about systems*

- *Without building the system*
- *Without disturbing the system*
- *Without destroying (or causing breakdown of) the system*

### *Advantages*

- *Relatively cheap laboratory for evaluating decisions before implementation*
- *Provides usable solutions where other methods fail*
- *Computer simulation:*
  - *Visually documents process behavior*
  - *Allows user to see what otherwise might be missed*

### *Disadvantages*

- *Requires large amounts of accurate data to provide a reliable model.*



## *Certain Conditions Make Simulation Appropriate*

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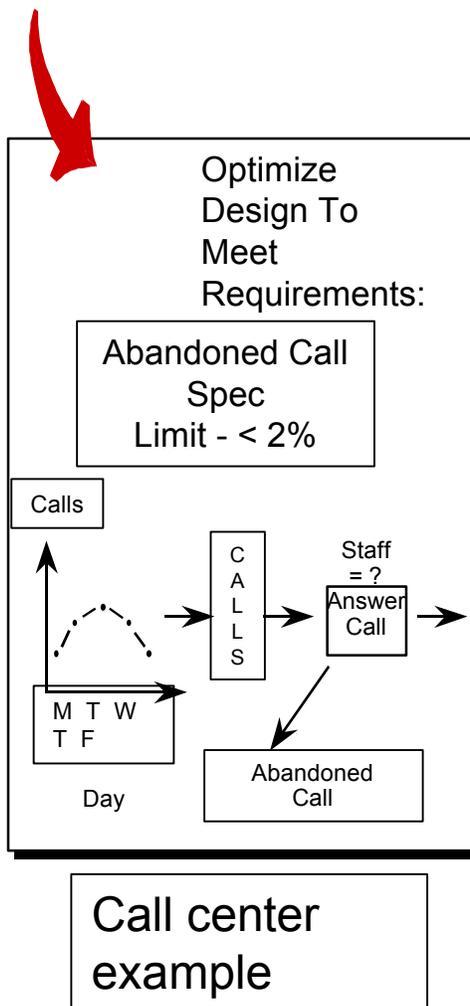
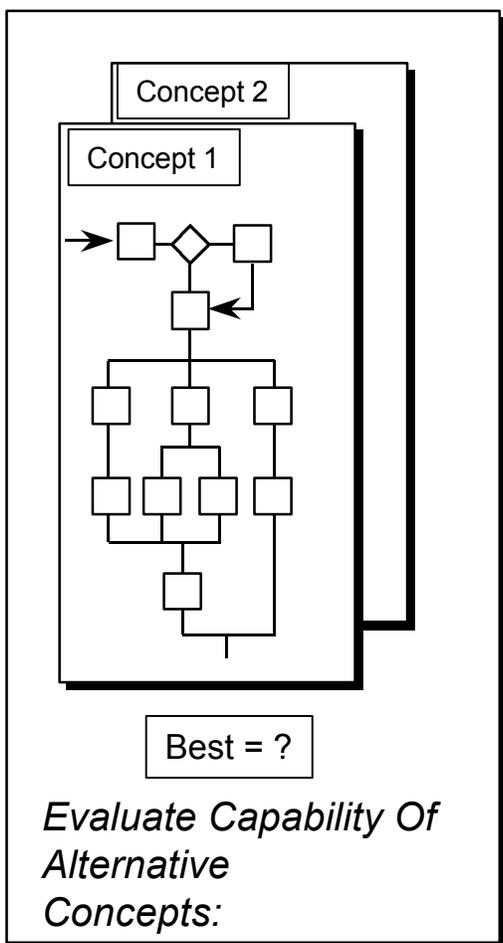
- ***No formula or “good” analytical solution exists for the problem***
  - *for example, complex inventory and scheduling problems.*
- ***The assumptions made in existing analytical models are too restrictive, or otherwise inappropriate for the problem***
- ***Examples***
  - *call center*
  - *determining inventory for service levels*
  - *order fulfillment process*
  - *payment process*
  - *job shop model*



# Modeling a Proposed Process From a DMAIC

*Project Is a Great Way to Test the Viability Prior to Implementation*

**New Process Design**





## *When to Use Crystal Ball?*

### ■ *Advantages*

- *Easy to use*
- *Wide variety of distributions*
- *Graphical display*

### ■ *Disadvantages*

- *Cumbersome to model complex, multi-step processes in Excel - you have to define the equations before you can model the process*
- *Does not provide a visual flow of the process*

- *Use CB if you can define the equations in a “simple” spreadsheet, or if you feel comfortable doing some visual basic programming*



# *There Are Three General Steps to Simulation*

<b>Step</b>	<b>% of Time</b>	<b>% of Time</b>
Plan	10%	33%
Program	80%	33%
Solve	10%	33%



***Natural  
Inclination***



***Appropriate  
Allocation of  
Time***



## *There Is a Basic Methodology for the Modeling and Simulation Process*

---

- ***Specify - Understand the problem to be studied and objective of doing simulation. Develop a project plan and get customer sign-off***
- ***Develop - Describe model based on expert interviews and observation of process***
- ***Quantify - Collect data needed to define process properties***
- ***Implement - Prepare software model***
- ***Verify - Determine that computer model executes properly***
- ***Validate - Compare model output with real process (if it exists)***
- ***Plan - Establish the experimental options to be simulated***
- ***Conduct - Execute options and collect performance measures***
- ***Analyze - Analyze simulation results***
- ***Recommend - Make recommendations***



## *A Few of the Previous Ten Steps Require Particular Emphasis*

---

- ***DO NOT go directly to preparing the software model - spend plenty of time planning***
- ***Start with a team that has at least the following expertise:***
  - ***Intimate knowledge of the process to be modeled and the associated logic***
  - ***Intimate knowledge of what data is available***
  - ***Knowledge of probability and statistics***
- ***Make sure a process owner has bought in to the simulation effort***
- ***Don't try to model every step of the process, rather only those steps that have a significant impact on the outcome.***
  - ***A model of the entire business would be great, but very difficult to create. Make sure the interdependencies you are modeling are highly relevant, and could not just be "black boxed" instead.***



## *More on the Details*

---

- ***Stay within project boundaries***
- ***Clearly define the project***
  - ***What questions do you want the model to answer? Answer must be very precise.***
  - ***What process variables do you want to be able to control? Again, answer must be very specific.***
- ***Given the questions and parameters from above, what data will you need?***
  - ***Ensure the team understands what data is available, and in what form (electronic, manual, archived, etc.)***
  - ***Be very precise in your data requests - create very detailed data templates so the data gatherers can just fill-in the blanks***



# *Crystal Ball Simulation*



## *Crystal Ball Overview*

- *Crystal Ball is an Excel Add-On*
- *Given that  $Y$  is a function of some  $Xs$ , distributions are assigned to the  $Xs$ , then random samples of the  $Xs$  are drawn to determine the distribution of the  $Y$*
- *Input  $Xs$  can be assigned to a number of established distributions (Normal, Poisson, Uniform), or a user defined distribution*
- *$Y$  must be a result of some equation or routine run in Excel. This relation can be anything that Excel can calculate and is often referred to as the ‘transfer function’*
- *Repeated sampling of the  $Xs$  and calculation of  $Y$  is called a ‘Monte Carlo’ simulation*



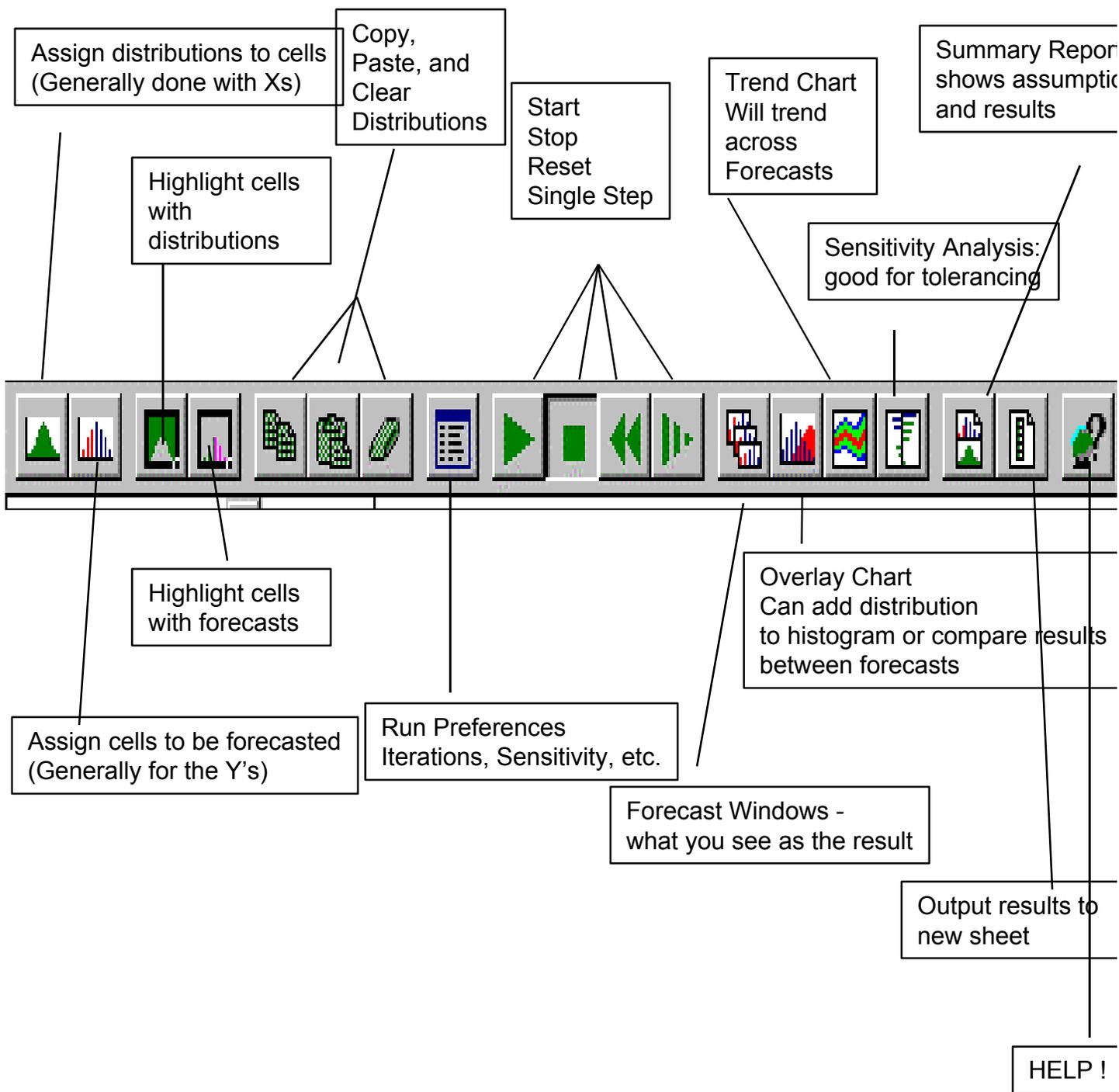
## **WARNINGS !!!**

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- ***Crystal Ball can only make predictions given your assumptions***
  - *poor assumptions yield poor results !*
- ***Crystal Ball should be used for rough order predictions***
  - *'extreme value' estimates should not be relied on*
- ***Rerun Crystal Ball simulations to see if the result is robust***
  - *Certain models may be very sensitive to initial values used in the simulator*
- ***Crystal Ball is only used for processes that can be modeled with a function in Excel***
  - *If the process is more complicated, use another modeling package (e.g.. Process Model)*
- ***Crystal Ball does not track individual objects, people, widgets, etc.. through time or space***



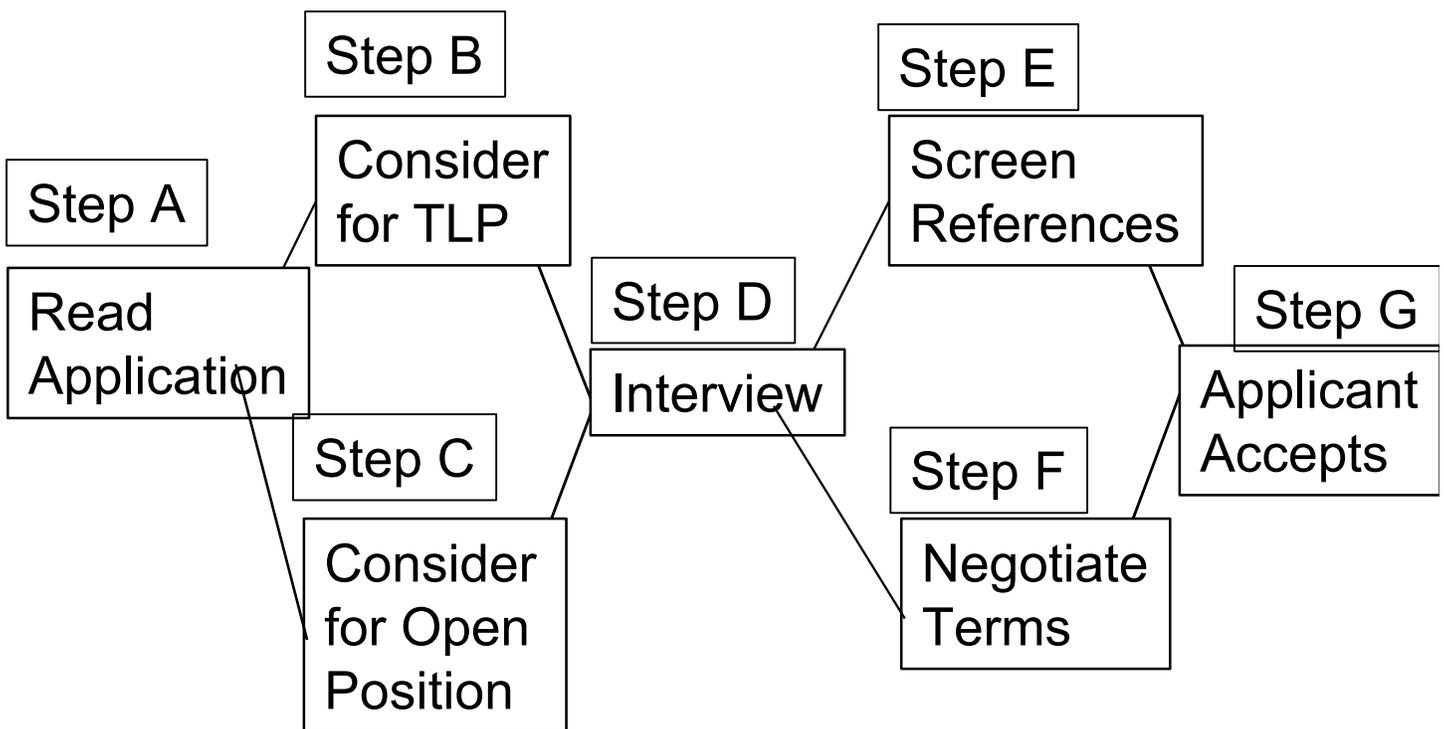
# Crystal Ball Menu





## Crystal Ball Example - 'Hiring2.xls'

- *A simplified hiring process is shown below*
- *We wish to analyze the overall cycle time for this process*



Cycle time adds the *shorter* of these 2 steps

Cycle Time adds the *longer* of these 2 steps



## *Crystal Ball Example - 'Hiring2.xls'*

- *Each step in the process has a certain cycle time, as well as some variation (summarized in the Excel file)*
- *We want to understand the distribution of our total cycle time, and which steps in the process most affect the variation in our response*
- *The following pages step through how to use Crystal Ball to find out*



## Step 1 - Setting up the Spreadsheet

- *It is often useful to set up a sheet where output  $Y$  can be calculated from a set of sample values*
- *These samples values can then be assigned distributions based on fixed cells in other parts of the spreadsheet*

Step	Sample Values	Output Cycle Time
A	3	30
B	5	
C	7	
D	7	
E	10	
F	10	
G	5	

$$\begin{aligned}\text{Cycle time} &= A + \min(B,C) + D + \text{Max}(E,F) + G \\ &= 3 + \min(5,7) + 7 + \text{max}(10,10) + 5 \\ &= 3 + 5 + 7 + 10 + 5 = 30\end{aligned}$$





## *Step 2 - continued*

---

- *Assigning parameters to cell values makes it easier to make changes later*
- *Different distributions require different inputs*
  - *Normal requires mean and standard deviation*
  - *Uniform requires minimum and maximum*
  - *Triangular requires minimum, peak, and maximum*
- *A distribution can be truncated by setting a lower and/or an upper limit (change this in the box where it shows 'infinity')*
- *Each Input step is assigned a distribution as shown in the spreadsheet*



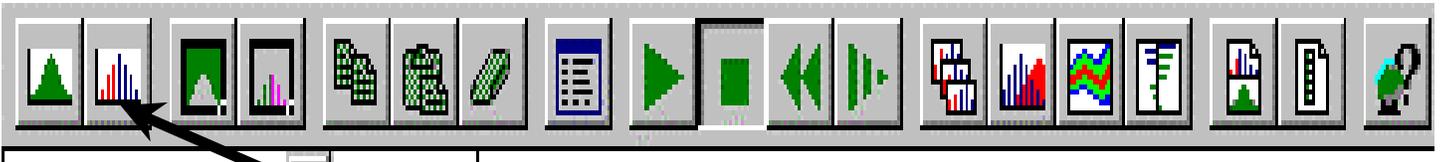
## Exercise

- *Check steps B to E, and assign the correct values to steps F and G*
- *The distributions are shown in the Excel spreadsheet*
- *Use the ‘Sample Values’ cells as the Xs*
  - *refer the inputs directly to the cells in the ‘process map’*

Step	Cell Ref.	
A	D19 D20	
B	G18 G19	
C	G22 G23	
D	K19 K20	
E	N18 N19	
F	N22 N23 N24	
G	R19 R20	



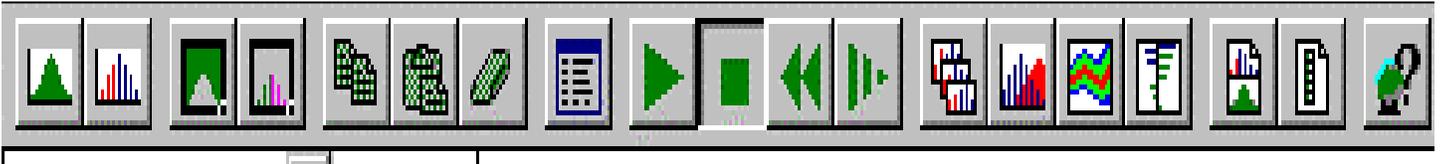
## *Step 3 - Assign the Y to be Forecast*



- *Click on the output Y, then the second button on the toolbar*
- *This tells Excel to run the simulation with that cell as the output*
- *Several Ys can be set up to be forecast simultaneously*
- *The options are usually OK as is*



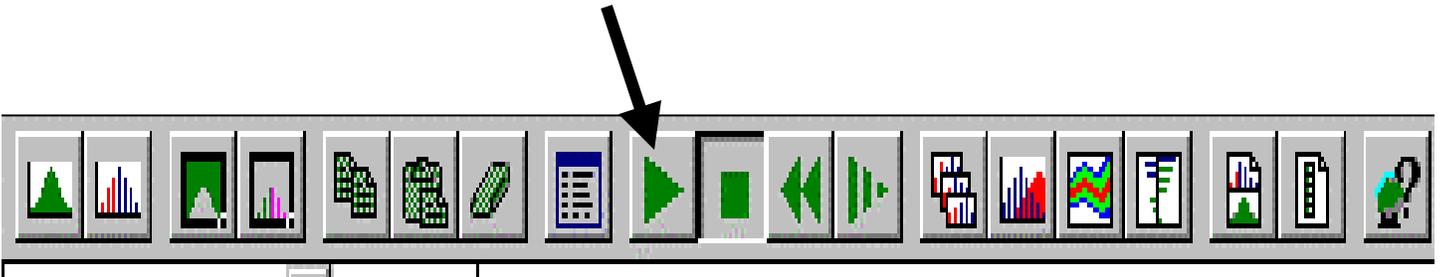
## *Step 4 - Assigning Run Preferences*



- *Run Preferences establishes how long the simulations runs, etc.*



## *Step 5 - Run the Simulation*

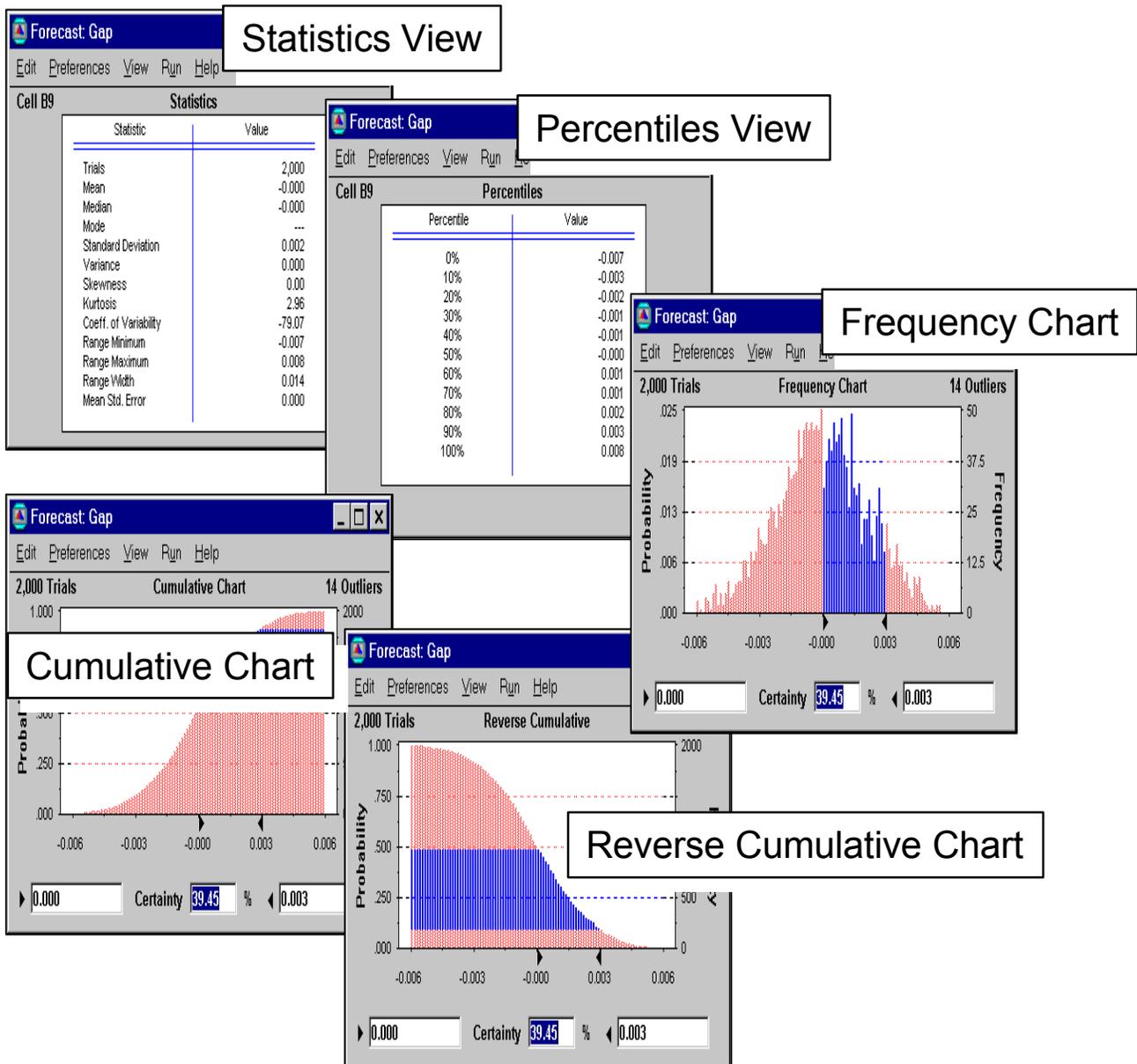


- ***Click the 'forward' key to run the simulation. This will bring up a picture showing the distribution of Y as it is run***
- ***Under 'View', you can change the format of the output***



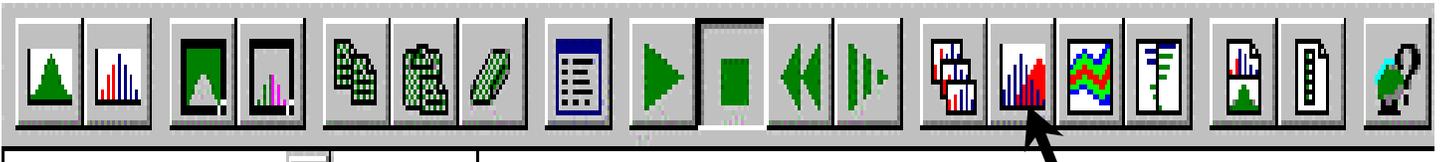
## The Forecast Window (cont.)

- Different views of the forecast window:  
*Views menu*





# Step 6 - Create Overlay Chart



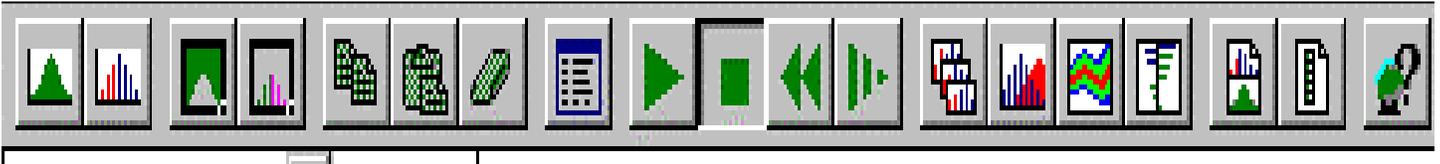
■ Pull in the forecast distribution

The image shows two screenshots of Microsoft Excel. The left screenshot shows the 'Overlay Chart' dialog box with a 'Frequency Comparison' chart area. The right screenshot shows the 'Choose Forecasts' dialog box with 'Available Forecasts' and 'Chosen Forecasts' lists. The 'Chosen Forecasts' list contains 'F28: Y'. The background shows an Excel spreadsheet with a table of data.

Sample	Step	Values	Output Cycle Time
	A	3 days	30 days
	B	5 days	
	C	7 days	
	D	7 days	
	E	10 days	
	F	10 days	
	G	5 days	



# Step 6 - Create Overlay Chart - continued



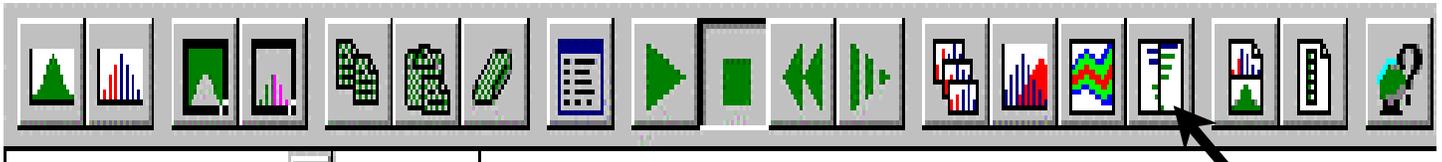
## ■ Add fitted distribution for comparison

The screenshots illustrate the process of adding a fitted distribution to a histogram. The left screenshot shows the 'Add Distribution...' button being clicked. The right screenshot shows a normal distribution curve overlaid on the histogram. A table of sample data is visible in the background of both screenshots.

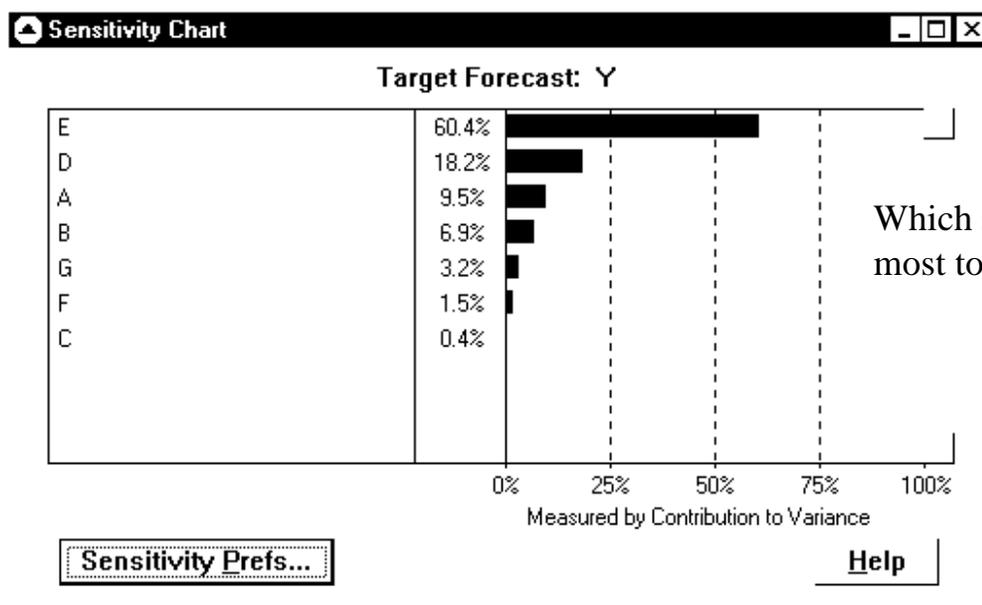
Sample	Step	Values	Output Cycle Time
A	3	days	30 days
B	5	days	
C	7	days	
D	7	days	
E	10	days	
F	10	days	
G	5	days	



## Step 7 - Sensitivity Analysis



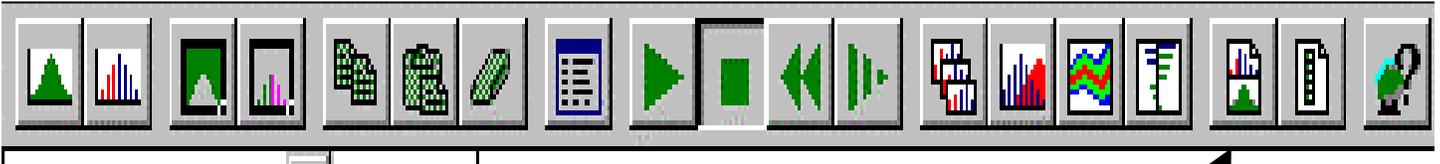
- *A sensitivity analysis tells us which X is most influencing the variation in our Y*
- *This allows us to focus our efforts to reduce variation*



↑ You may need to open this menu to get the 'contribution to variance' pareto.



## Step 8 - Writing Reports



- *Crystal Ball will create a summary report of assumptions and results*
- *This includes graphics / objects which can be copied to Word or Powerpoint*

**Create Report**

<b>Report Sections</b>		<b>Forecasts</b>	<b>Assumptions</b>
<input checked="" type="checkbox"/> Overlay Chart	100 %	<input checked="" type="radio"/> All	<input checked="" type="radio"/> All
<input type="checkbox"/> Trend Chart	100 %	<input type="radio"/> Chosen...	<input type="radio"/> Chosen...
<input checked="" type="checkbox"/> Sensitivity Chart	100 %	<input type="radio"/> Open	
<input checked="" type="checkbox"/> Forecasts		<b>Percentiles</b>	
<input checked="" type="checkbox"/> Summary		<input type="radio"/> Quartiles [25%]	
<input checked="" type="checkbox"/> Statistics		<input type="radio"/> Quintiles [20%]	
<input checked="" type="checkbox"/> Chart	100 %	<input checked="" type="radio"/> Deciles [10%]	
<input checked="" type="checkbox"/> Percentiles		<input type="radio"/> Icosatiles [5%]	
<input type="checkbox"/> Frequency Counts		<input type="radio"/> 2.5, 5, 50, 95, 97.5% - tiles	
<input checked="" type="checkbox"/> Assumptions		<input type="radio"/> 10, 25, 50, 75, 90% - tiles	
<input checked="" type="checkbox"/> Parameters		<b>Charts</b>	
<input checked="" type="checkbox"/> Chart	50 %	<input type="radio"/> Color	<input checked="" type="radio"/> B&W

OK Cancel Help



## *Take Aways - Step 9*

- *Tolerancing helps to define the allowable range of variation of  $x$  while still meeting the requirements of  $x$ .*
- *Simulation can be a way to set tolerances on your  $x$ 's.*
- *Crystal Ball simulates the distribution of an output  $Y$* 
  - *Given  $Y = f(X)$  - also known as a transfer function*
  - *Given the distributions on the  $X$ 's*
- *This simulation randomly samples the  $X$ 's and calculates the  $Y$ .*
  - *This is called a Monte-Carlo Simulation*
- *Crystal Ball is good for processes with direct transfer functions*
  - *Equations which can be coded in Excel*
- *More complicated processes require a more detailed approach*
  - *Attainable in “Process Model” software*



# *Introduction to Control*



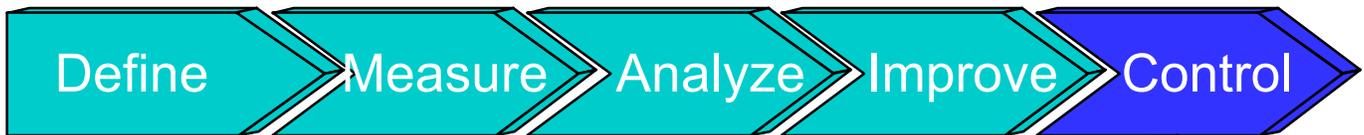
# 12 Step Six Sigma Process

Step	Description	Focus	Tools	Deliverables
<b>Define</b>				
A	Identify Project CTQs		VOC Tools, VOC Data	Project CTQs (1)
B	Develop Team Charter			Approved Charter (2)
C	Define Process Map			High Level Process Map (3)
<b>Measure</b>				
1	Select CTQ Characteristic	Y	Fishbone, FMEA, Pareto Customer, QFD	Project Y (4)
2	Define Performance Standards	Y, X	Customer, Blueprints	Performance Standard for Project Y (5)
3	Establish Data Collection Plan, Validate Measurement System*, & Collect Data	Y, X	Gage Study	Data Collection Plan & MSA (6), Data for Project Y (7)
<b>Analyze</b>				
4	Establish Process Capability	Y	Capability Indices	Process Capability for Project Y (8)
5	Define Performance Objective	Y	Team, Benchmarking	Improvement Goal for Project Y (9)
6	Identify Variation Sources	X	Hypothesis Tests	Prioritized List of all Xs (10)
<b>Improve</b>				
7	Screen Potential Causes	X	DOE-Fractional, Process Map, Fishbone, FMEA	List of Vital Few Xs (11)
8	Discover Variable Relationships & Propose Solution	X	DOE-Full, Prediction Eqns., DFSS, Statistical Tolerancing	Proposed Solution (13)
9	Establish Operating Tolerances & Pilot Solution	Y, X		Piloted Solution (14)
<b>Control</b>				
10	Validate Measurement* System	Y, X	Gage Study	MSA*
11	Determine Process Capability	Y, X	Capability Indices	Process Capability Y, X (15)
12	Implement Process Control System & Project Closure	X	Risk Analysis, Mistake Proof, SPC QPT	Sustained Solution (15), Documentation (16), Leveraged Solution (17), Financial Audit (12)

\* You may want to validate the Measurement System throughout the DMAIC steps.



# Control Phase



**10. Define and Validate Measurement System on X's in Actual Application**

*Deliverable:*  
*Measurement System is Adequate to Measure X's*

*Tools:*

- See Step 3

**11. Determine Process Capability**

*Deliverables:*

- *Determine Post-Improvement Capability and Performance*
- *Confirm the Improvement Goal Has Been Realized*

*Tools*

- See Step 4

**12. Implement Process Control**

*Deliverables:*  
*Develop and Implement Process Control Plan*

*Tools:*

- *Control charts*
- *Mistake Proofing*
- *FMEA*



## *Control*

---

*In the physical world, the law of Entropy explains the gradual loss of order in a system. The same law applies to business processes.*

*Unless we add “energy” (in the form of documentation and ongoing process controls), processes will tend to degrade over time, losing the gains achieved by design and improvement activities.*

*The quality plan is the structure through which we add this “energy” to business processes.*



## *Control: Main Objectives*

---

- *To make sure that our **process stays in control** after the solution has been implemented.*
- *To quickly detect the out of control state and determine the associated **special causes** so that **actions** can be taken to correct the problem before nonconformances are produced.*



## *Maintaining Control*

---

- *Keep Xs within tolerance by using appropriate controls (Risk Management, Mistake Proofing, etc.)*
- *Apply control charts to Xs to monitor and control variation.*
- *Understand implications on existing quality plans due to modification of current control systems.*
- *Establish transition plan for maintaining control of improved process.*



## *Detecting Variation*

- **Common Cause Variation**
  - *natural variability*
  - *random*
  - *inherent in the process*
  
- **Special Cause Variation**
  - *may be caused by operator errors, adjusted machines, or defective raw materials*
  - *generally large when compared to the common cause variation*
  - *considered an unacceptable level of process performance*
  
- **Special causes tend to cause a process to shift out of control where the process output does not meet the desired specifications.**



## *What is a Process Control System?*

---

- *A process control system*
  - *strategy for maintaining the **improved process performance** over time*
  - *identifies the specific actions and tools required for **sustaining the process improvements** or gains*
  
- *A control system may incorporate*
  - *Risk Management*
  - *Mistake-proofing devices*
  - *Statistical process control (SPC)*
  - *Data collection plans*
  - *Ongoing measurements*
  - *Audit plans*
  - *Response plans\**
  - *Product drawings*
  - *Process documentation*
  - *Process ownership*



## *Why is a Process Control System Important?*

- *Defines the **actions, resources, and responsibilities** needed to make sure the problem remains corrected and the benefits from the solution continue to be realized.*
- *Provides the methods and tools needed to **maintain the process improvement**, independent of the current team.*
- *Ensures that the improvements made have been **documented** (often necessary to meet regulatory requirements).*
- ***Facilitates** the solution's **full-scale implementation** by promoting a common understanding of the process and planned improvements.*



## *Effective Process Control System*

---

- *The Process Control System provides the on-going process control and is based on:*
  - *the importance of the requirement*
  - *the production/process method*
  - *the capability of the process*



## *Key Steps in Developing a Process Control System*

---

- *Complete an implementation plan.*
  - *Plan and implement the solution and develop a method to **control** each **vital X** or key sources of variation*
  - *Define all possible areas that may require action in order to control the process X and then determine the appropriate course of **action** to take*
  
- *Develop a data collection plan to confirm that your solution meets your improvement goals.*
  - *Establish **ongoing measurements** needed for the project Y and create a **response plan** to follow in case process performance falls below established standards*



## *Key Steps in Developing a Process Control System*

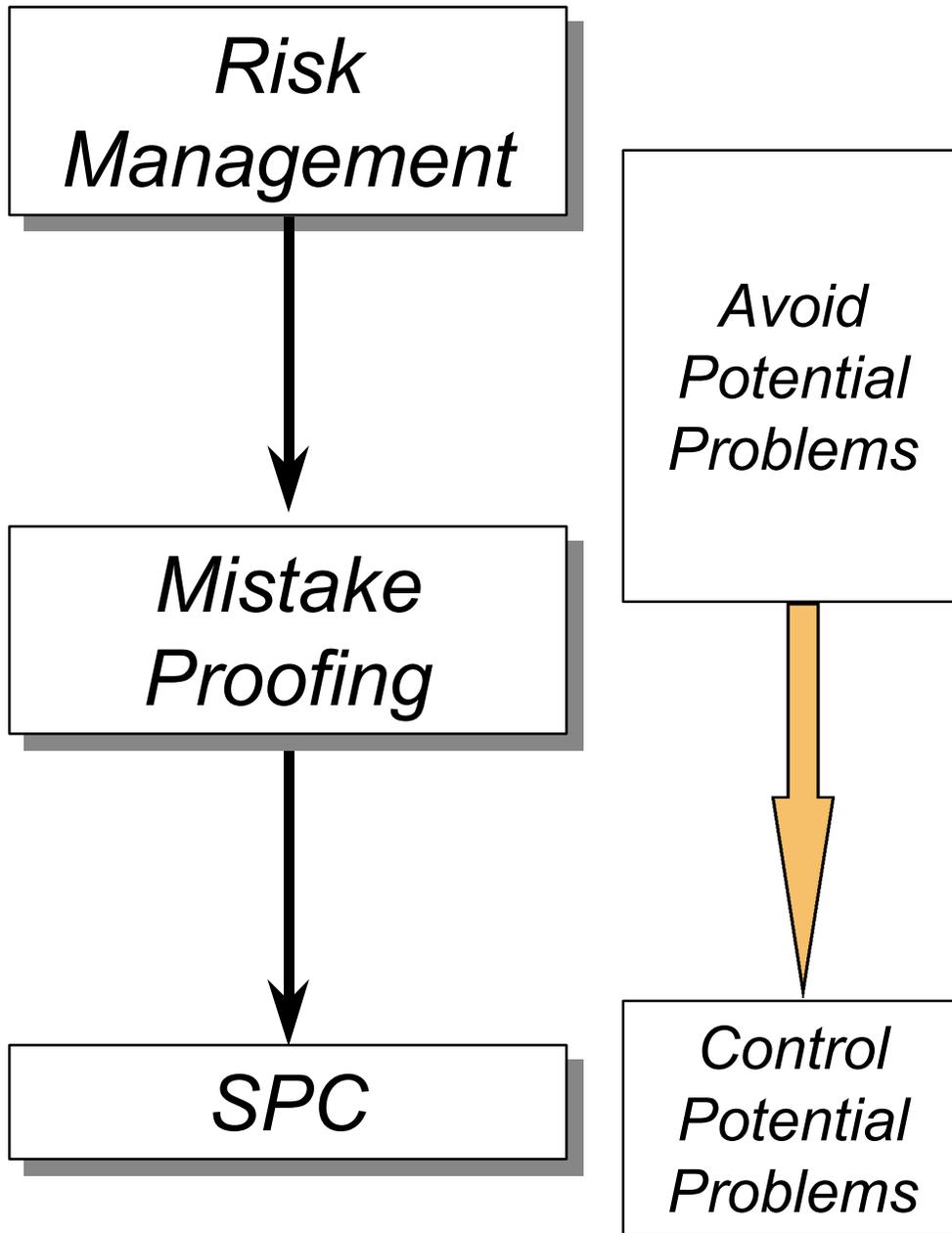
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- *Communicate your strategy.*
  - *Document the process and control plan to ensure **process standardization** and the continuation of the solution's benefits*
  
- *Train Personnel.*
  
- *Run the new process and collect the data to confirm your solution.*



# Control Mechanisms

## Three Main Control Mechanisms





## *Risk Management*

---

- *Determine the probability and impact of each risk presented by the planned process change.*
- *Link the probability and impact of occurrence to the risk, then determine the abatement action.*
- *Assign ownership and determine timing for each abatement action.*



## *Mistake Proofing*

- *Helps to sustain a solution by eliminating the possibility that an  $X$  can be set outside the desired level or configuration...or*
- *Warns the process operator before the  $X$  goes outside limits so preventative action can be taken.*
- *Mistake proofing can be used alone or with either risk management or statistical process control to sustain a solution.*



## *Statistical Process Control*

- *Control charts can be used to monitor  $X$ s and quickly detect a change in the process due to special cause variation.*
- *Very helpful when your  $X$ s cannot be mistake proofed or easily controlled within the required tolerance range.*



## *Controls - Group Discussion*

---

### **What works & what does not work?**

- *Mistake-Proofing Methods*
- *Measurement Methods*
- *Behavioral Methods*

### **Class Exercise:**

*Think of process improvement projects you have been involved with (not necessarily Six Sigma). What controls did you put in place? Which controls worked and which did not?*



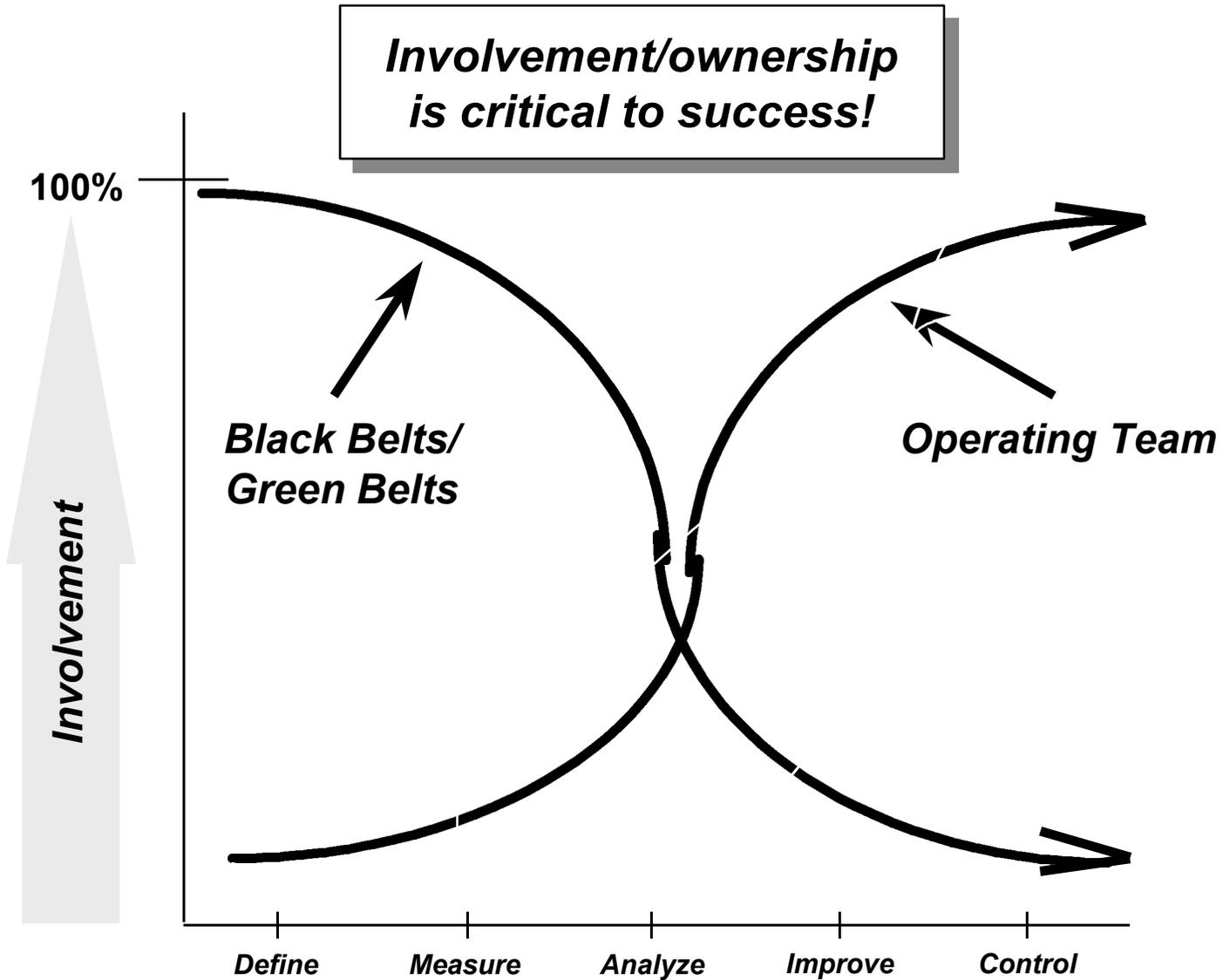
## *Key Steps in Developing a Process Control System*

---

- *Complete an implementation plan.*
- *Develop a data collection plan to confirm that your solution meets your improvement goals.*
- *Communicate your strategy.*
- *Train Personnel.*
- *Run the new process and collect the data to confirm your solution.*



## *For a Successful Project*

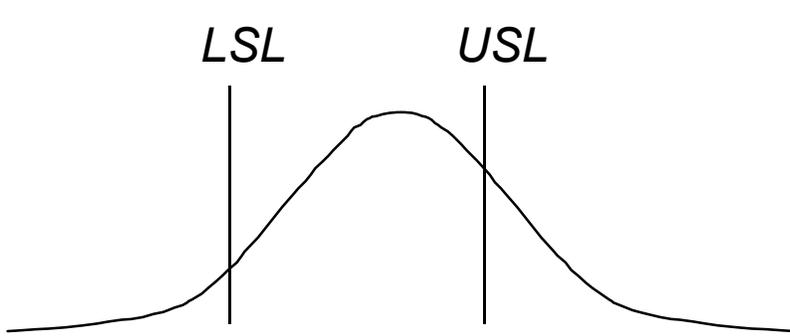


*Plant managers, engineering, and leaders  
are functional champions*

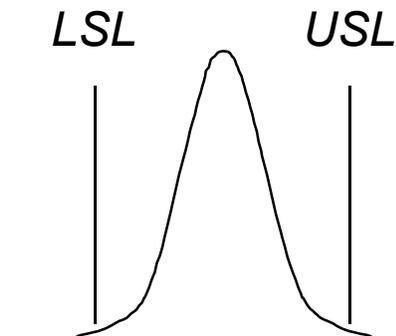


## Confirm the Solution

- Calculate new process capability after we implement the improvement.
- Determine if the new process capability (process sigma, or Z short term) meets your improvement goal.
  - See if you achieved the desired mean shift, variance reduction, or DPMO reduction
  - Use hypothesis testing as demonstrated in the approach introduced in the Analyze phase when you verified your list of Xs with data



*Process Before Improvement*



*Process After Improvement*



## *Assess the Effectiveness of Solution*

- *Was the process improvement successful in satisfying the needs of the customer?*
- *Did the solution result in any additional or unexpected benefits?*
- *Can the solution be leveraged to other projects?*
- *Are there other Green Belt projects that can be started?*

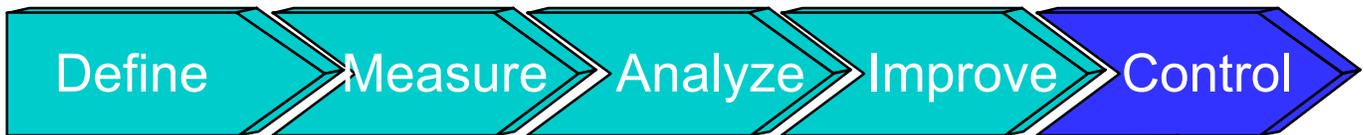


## *Take Aways—Introduction to Control*

- *The objective of control is to see that an intended change or solution remains in place.*
  
- *There are various mechanisms that can be used to control a process:*
  - *Risk Management*
  - *Mistake Proofing*
  - *Statistical Process Control (SPC)*
  - *Control Plans*



# Control Phase



**10. Define and Validate Measurement System on X's in Actual Application**

*Deliverable:*  
*Measurement System is Adequate to Measure X's*

*Tools:*

- See Step 3

**11. Determine Process Capability**

*Deliverables:*

- *Determine Post-Improvement Capability and Performance*
- *Confirm the Improvement Goal Has Been Realized*

*Tools*

- See Step 4

**12. Implement Process Control**

*Deliverables:*  
*Develop and Implement Process Control Plan*

*Tools:*

- *Control charts*
- *Mistake Proofing*
- *FMEA*



## *Step 10*

---

*Define and Validate the Measurement System on X's in the actual application*

*Project Deliverable: Measurement System is adequate to Measure X's*



## *Step 11*

### *Determine Process Capability*

*Project Deliverable: Determine Post-Improvement Capability and Performance ( $Z_{st}$  and  $Z_{lt}$ )*



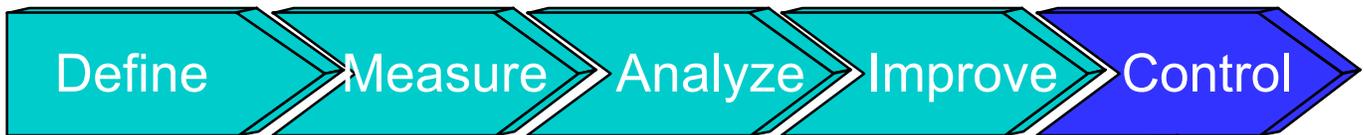
## *Step 11*

---

- *Calculate post improvement capability of performance based on the technique described in Step 4.*
- *Confirm improvement goal established in Step 5 has been realized on the y.*
- *If not, go back to Step 6 to look for additional sources of variation.*



## Control Phase



**10. Define and Validate Measurement System on X's in Actual Application**

*Deliverable:*  
Measurement System is Adequate to Measure X's

*Tools:*  
■ See Step 3

**11. Determine Process Capability**

*Deliverables:*  
■ Determine Post-Improvement Capability and Performance  
■ Confirm the Improvement Goal Has Been Realized

*Tools*  
■ See Step 4

**12. Implement Process Control**

*Deliverables:*  
Develop and Implement Process Control Plan

*Tools:*  
■ Control charts  
■ Mistake Proofing  
■ FMEA



---

# *Quality Planning*



# *The Quality Plan in the Project Lifecycle*

*Define  
Measure  
Analyze  
Improve*

- *Focus on the Right CTQ*
- *Quantify the Problem*
- *Determine the Drivers*  
$$Y = f(X)$$
- *Identify Needed Change*
- *Implement the Change*

*Control  
Close*

- *Validate Measurement System*
- *Determine Process Capability*

- ***Develop/Modify Quality Plan***
  - > ***Process Documentation***
  - > ***Process Controls***

- ***Implement Process Controls***
- ***Audit Plan Established***
- ***Transition to Operating Owners***



## *What is a Quality Plan?*

- **A quality plan is a documented plan** whose purpose is to **ensure** each product characteristic or process requirement stays in **conformance**.
- **A quality plan may include:**
  - *Process documentation and standards:*
    - > *Procedures to follow*
    - > *Operating tolerances or other specifications*
  - *Process controls:*
    - > *Items to be monitored and audited*
    - > *Response planning for process breakdowns*
- *ISO 9000 has strict requirements for creating and maintaining quality plans.*
- *For non-manufacturing functions, other process standards (e.g. Tollgates or local procedures) may provide better structure for creating or maintaining a quality plan.*



## *Quality Plan - Documentation*

---

- *Describes the flow of the process*
  - *process flowcharts*
  - *deployment flowcharts: a chart to show who is responsible for a particular process step*
- *Describes standard operating procedures*
  - *be specific: tell precisely what actions to take and when and where to take them*
  - *keep descriptions at a level so that the job can be performed well by a person who is not fully trained*
  - *describe how to prevent product or process variation--include cause and effect relationships*
  - *provide operating tolerances and other specifications*
  - *provide clear and reasonable instructions*
- *Helpful hints*
  - *test the procedures*
  - *check that all steps are included*
  - *stress the importance of procedures*
  - *provide a method for updating*



## *Quality Plan - Process Control Plan*

---

- *The Process Control Plan provides the on-going control and is based on:*
  - *the importance of the requirement*
  - *the production/process method*
  - *the capability of the process*
  
- *Whenever possible use existing controlled systems to establish a control plan*
  - *quality systems (QS, Quality Operating Procedures, etc)*
  - *workstation instruction books*
  - *process specs*



## *Quality Plan - Process Controls*

---

*The Key to Process Controls is ...*

### ***Monitoring*** (frequent intervals)

- *Ongoing measurements of process **variation** and/or **capability***
- *Responding to and taking action on **non-random variation** and/or **poor performance***
- *Done by those closest to process*

*Maintaining Process Controls Requires ...*

### ***Auditing*** (infrequent intervals)

- *Broad review of entire process to ensure that controls are in place and effective:*
  - *current documentation & standards*
  - *general compliance with procedures*
  - *valid measurements*
  - *proper monitoring and response*
- *Done by those further from process*



## *Process Controls - Monitoring*

---

### **KEY QUESTIONS:**

- 1. Why monitor?*
- 2. What should I monitor?*
- 3. How much data do I collect?*
- 4. How can I detect changes in process variation or capability?*
- 5. What do I do if I detect a change?*
- 6. If the process is in control and capable, are my customers still satisfied?*



## *Monitoring - Key Question #1*

---

### **Why monitor?**

*Because initial verification is not enough...*

- *First Article Inspection (FAI) examines all product or process characteristics on the initial run of a modified process. FAI requirements are found primarily in local manufacturing/engineering procedures.*
- *Information Management methods often call for initial verification to ensure quality of software after it is placed in production.*
- *These methods provide **ONLY INITIAL ASSURANCE** of process performance. Effective monitoring provides **ON-GOING ASSURANCE**.*



## *Monitoring - Key Question #2*

---

### ***What should I monitor?***

- *Output measures*
  - *customer satisfaction*
  - *CTQs*
  - *volumes (sales, throughput)*
- *Process measures*
  - *“upstream” monitoring points*
  - *supplier performance*
  - *volumes (throughput, inventory)*
- *Input measures*
  - *key process variables*

***Monitor what is most likely to help you detect and correct variation before it results in customer dissatisfaction.***



## *Monitoring - Key Question #3*

### **How much data do I collect?**

- *Determine the type and frequency of appraisal required for Control Plan Effectiveness.*
- *Factors to consider:*
  - *Impact of process Y on customer satisfaction*
  - *Impact of X on Y*
  - *Repeatability of measurement*
  - *Capability of process*
  - *Cost to obtain measurement*



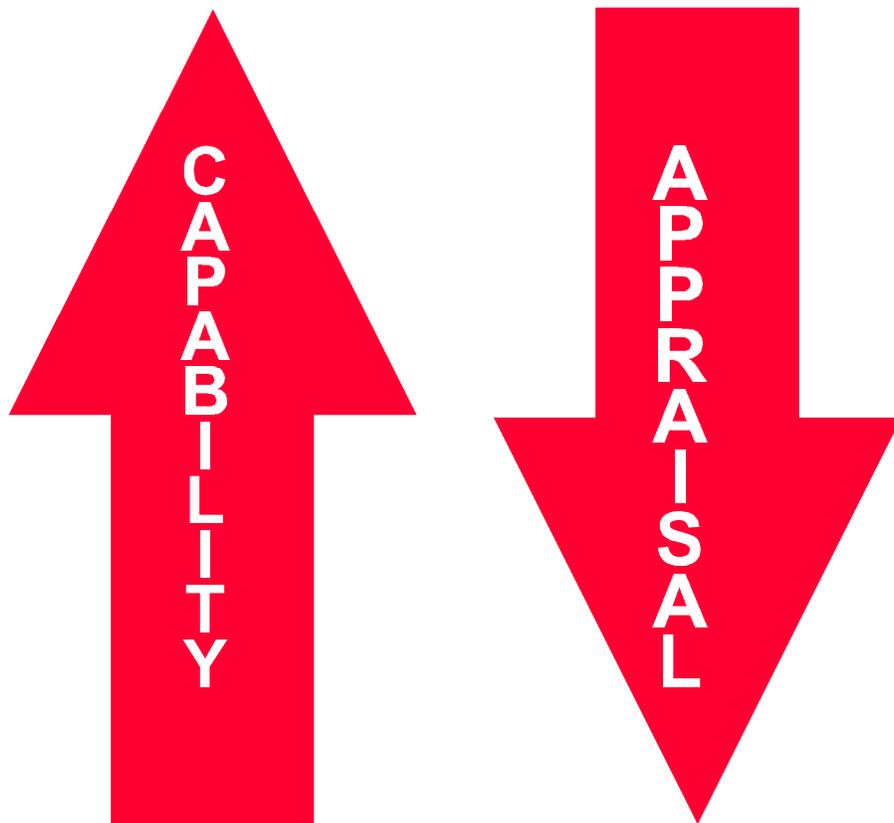
## *Monitoring - Sufficient Appraisal*

<b>Importance of Requirement</b>	<b>Production / Process Method</b>	<b>Process Capability</b>	<b>Method of Primary Control</b>	<b>Appraisal</b>



## *Monitoring - Sufficient Appraisal (cont.)*

- *Better Capability = Less Appraisal*





## *Monitoring - Key Question #4*

### ***How can I detect changes in my process?***

- earliest possible detection*
- acceptable false alarm*
- monitor vital Xs with control chart*
- look for out of control indications*



## *Monitoring - Key Question #5*

### ***What do I do if I detect a change in the process?***

- *take action based upon control plan*
- *investigate for probable special cause*
- *response planning—a list of actions that provide direction when a process has achieved a certain state*



## *Monitoring - Key Question #6*

---

***If the process is in control and capable,  
are my customers still satisfied?***

- customer surveys*
- meeting specification limits*



## *Process Controls - Audit Requirement*

- *Purpose is to **validate** that the controls are still in place and are effective.*
- *Audit failures of the controls can result in re-implementation or modification to the controls to **regain confidence** that the project gains will remain.*
- *If an audit failure shows not only a failure of the controls, but also the project itself, a **re-evaluation** of Project benefits should be done.*
- *Should be done by an **independent party** whenever possible.*
- *Required by QPT: Section “W” in Control/Closure screen.*
- *Subject to ISO auditing. (Red Team, etc.)*



## *Types of Control Plans*

---

- 1. Die Control Plan*
- 2. Fixture Control Plan*
- 3. Computer/Tape/Template/Tool Control*
- 4. Variable Data Charting/SPC*
- 5. Characteristic Verification Plan*
- 6. Process Control Acceptance Plan*
- 7. Assembly Stack-Up Plan*
- 8. Machine/Measuring Center Acceptance Plan*



## *Control Methods*

---

### *Used to Assure Characteristic Accountability*

#### *Die Control*

A quality control technique in which the forming or cutting die which generates a requirement is qualified and controlled to assure that it is generating conforming products. The characteristic is checked on the first piece to assure proper set-up and on the last piece to assure that the die wear has not altered the dimensions. After first piece acceptance, all pieces run on that die set-up are accepted and periodic product audits are conducted for verification.

#### *Fixture Control*

A quality control technique in which the fixture used to generate a requirement is qualified and controlled to assure it is generating conforming product. The characteristic is checked on the first piece to verify that the fixture has been made correctly. After first piece acceptance, all pieces run during the cycle period of the fixture are accepted and additional assurance is provided as required.

#### *Tape Control*

A quality control technique which utilized an approval and controlled Tape Program to assure the process is generating conforming product. The axis of movement of the tool is programmed to ensure that blueprint requirements are being met. The characteristic is checked on the first piece. After first piece acceptance, all parts run on an approved tape are accepted and periodic additional assurance is provided as required.



## *Control Methods (cont.)*

---

### *Tool Control*

A quality control technique which utilizes qualified tooling to assure the process is generating conforming product. The characteristic is checked on the first and the last piece at a minimum following the installation of a new tool. After first piece acceptance, all pieces are accepted until the tool is replaced and periodic product audits are conducted for verification.

### *Characteristic Verification Plan (CVP)*

A quality control acceptance technique which utilizes the operator who generates a requirement to check and accept that requirement. The operator is trained and CVP certified, then audited quarterly to ensure that he/she maintains an acceptable level of performance. The operation sheet specifies the verification plan, which can vary from checking every part to checking randomly selected samples. Periodic product audits are typically conducted for verification.

### *Process Control*

A quality control technique in which characteristics are controlled by the controlling process variables. Process capability studies are conducted to manipulate the process variables in order to minimize variation and produce a consistent part. Once the process variables have been stabilized, it is assured that parts will be within blueprint limits. Periodic product audits are conducted for verification.



## *Variable Data Charting (SPC)*

*Statistical Process Control is a quality control technique in which process generated data is used to help make decisions about the capability of the process to generate conforming hardware. A control chart is utilized to assure that the process is statistically in control, exhibiting only random variation. A process capability study is conducted to assure that the six sigma spread of the process will comfortably fit within the specification limits. When these conditions have been met, a process monitoring plan is developed to assure that the process remains in control and capable. If the process goes out of control, showing evidence of non-random assignable cause variation, the control plan is discontinued, generally replaced by 100% inspection, until the assignable cause is determined and corrected. The need for additional assurance is generally determined by the margin between the tolerance spread and the engineering requirement.*



# *Process Management Chart*

**A process management chart is a flowchart and matrix which helps you manage a process by summarizing:**

## **Standardizing and Documenting**

- *what the steps in the process are*
- *who does these steps and when*
- *where more detailed work instructions can be found*

## **Monitoring**

- *where data is taken on the process and on the product*
- *who takes the data*
- *how (by what methods) measurements are taken and recorded*
- *when (how often) data is collected*

## **Response Planning**

- *who takes action based on the data*
- *where to find troubleshooting procedures*
- *what action to take in the case of process failures*



---

# *Risk Management Process Introduction*



## *Objectives:*

---

- *Provide an understanding of how Risk Management can be applied.*
- *Introduce how to do Risk Management by working through key steps.*
  - *identifying risks, rating risks, abating risks, and executing risk management plans*
- *Introduce the methods now used to rate risk & how to use some of the tools available.*
- *Introduce the structure for holding formal risk reviews.*
  - *initially held with trained facilitators*
- *Understand the criticality of tracking & executing risk abatement plans.*



## *What is the Value of a Risk Management Process?*

- *Systematically identifies risk elements that can interfere with process improvement or cause loss of control.*
- *Prevents risk elements from occurring through risk abatement plans.*
- *Periodically reassess risk.*
- *Communication of risk to management.*
  - *Drives clear decisions on risks*



## ***Risk:***

---

*The probability of an undesirable event occurring & the impact/consequence of that event*

## ***Risk Management:***

- *The process of managing risk using risk abatement plans*
  - *Identify & quantify risk elements technical, cost, scheduling & marketing risk elements*
  - *Reducing risk by means of risk abatement plans integrated into the critical path schedules*
  - *Monitoring progress of abatement plan*
  - *Highlight and manage risk as early as possible*



## *When Do I Use It?*

- *Continuously to assess and abate:*
  - *Cost risks*
  - *Technology risks*
  - *Specification risks*
  - *Marketing risks*
  - *Installation risks*
- *DMAIC & DFSS projects to assure improvement & control is maintained*
  - *Ask what could go wrong? How do I prevent it?*
- *During business process (i.e. Tollgate Process)*
  - *Assess & abate risks of open checklist steps*
  - *Checklist steps closed at formal risk reviews, per tollgate requirements*
- *Can also be used to assess risks of key decisions:*
  - *Not doing a test*
  - *Delaying an analysis*



## *The Key Steps of a Risk Management Process*

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- 1 Identify the risk elements & the risk types*
- 2 Assign risk ratings to the risks: probability & consequence of risk*
- 3 Prioritize the risks*
  - High (Red), Medium (Yellow), Low (Green)*
- 4 Identify the risk abatement plans (high & medium risks)*
- 5 Incorporate the risk abatement plan into the work plans*
- 6 Track the risk score reductions & abatement actions vs. plan*
- 7 Continuously update for new risks & for reduction of old risks*



## *Ways to Identify Risk:*

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- *Brainstorming of knowledgeable individuals*
- *Review of lessons learned*
- *Previous experience*
- *Open checklist items or items with risk*
- *FMEAs*
- *Previous design or producibility issues*
- *Look for:*
  - *Complex design issues*
  - *Cutting edge technology issues*
  - *New manufacturing processes*
  - *Untested assembly techniques*
  - *High cost or cost uncertainties*
  - *Schedule slip potentials*
  - *Specification shortfalls*



## Rating the Risk

<u>Risk Type</u>	<u>Probability</u>	<u>x</u>	<u>Consequence</u>	<u>= Risk Score</u>
Performance	4	x	5	= 20
Design Maturity	4	x	5	= 20
Technology	4	x	5	= 20
-				
-				

1 *Pick the risk categories that apply to the risk:*

- Cost
- Technology
- Specification
- Marketing
- Installation

2 *Rate probability of occurrence (1 to 5)*

- *Reference probability of occurrence rating guide*
- *Use words in guide, not “gut feel”*

3 *Rate consequence of occurrence/impact (1 to 5)*

- *Reference consequence of occurrence/impact of Risk chart — use words in guide*

4 *Risk factor score —*

*consequence x probability = 1 to 25*



# Risk Management Process Training

Typical areas to be assessed	High-5	Significant-4	Moderate-3	Minor-2	Low-1
<b>Costs</b>	No experience with cost for similar project.	Cost estimates and resources extrapolated from prior project with 20-30% similar content.	Cost estimates and resources extrapolated from prior project with 40-60% similar content.	Cost estimates and resources extrapolated from prior project with 70-90% similar content.	Cost estimates, resource allocation and schedule achieved on prior project with greater than 90% similar content.
<b>Performance</b>	Performance estimates based on extrapolation of data outside range of current data base.	Estimates based on similar/empirical data within range of current data base.	Estimates based on analytical models within range of current data base with sub-scale tests to verify extrapolated data.	Extensive data base substantiated by near-to-actual size demonstration tests.	Risk abatement plan successfully completed. Performance level satisfies customer requirements.
<b>Technology</b>	New and unique technology with little analysis. Technology development plan may have been published.	New technology analyzed and basic physical principles demonstrated. Technology is feasible.	New/derivative technology with extensive analysis demonstrated by test.	Technology has been demonstrated on full scale application at actual customer conditions.	Technology is mature. All risk issues identified. Implementation program ready for launch.
<b>Design Maturity</b>	New, innovative or complex design in conceptual stage and no testing completed.	New, innovative or complex design subjected to limited sub-scale or component test.	Design is complex or derived from existing similar design with extensive analysis and verified by sub-scale or component test.	Extensive full scale testing of near-to-actual design at actual conditions. Meets all life requirements.	Extensive full scale testing of operational design at actual conditions complete. Meets life requirements. Product-ready design.
<b>Fabrication and Assembly</b>	No experience with similar fabrication and assembly processes.	Limited experience with similar fabrication and assembly processes in prototyping environment. Target cost being established by an NPI team.	Moderate experience with same fabrication and assembly processes in prototyping environment. Target costs acceptable.	Extensive experience with actual fabrication and assembly processes using methods planned for production and approved by manufacturing.	Risk abatement plan successfully completed. Fabrication and assembly processes are production ready. Target cost achieved.
<b>Materials and Processes</b>	New material based on reasonable set of goal data with an identified plan for full development.	New material with limited physical properties data base. New application of a material in a critical part.	Material with expanded data base and verified by sub-scale or component testing. Approved by the design review team.	Material demonstrated in large or full scale test at actual operating conditions and meets life requirements.	Material development risk abatement plan successfully completed for specific application. Production ready material/process.
<b>Schedule</b>	Greater than 70% chance of significant schedule slip.	Greater than 50% chance of moderate schedule slip.	50/50 chance of moderate schedule slip.	30% chance of moderate schedule slip.	Low probability of any significant schedule slip.
<b>Resources</b>	Critical people missing. No openings to hire.	Critical people identified but not yet members of team.	Critical people available but need training or only partially allocated to team.	Critical people partially allocated to project, not relieved of other jobs.	Critical people dedicated to team and trained.
<b>Tools</b>	No tools available.	Some custom tools must be developed. Depend on vendor.	Largely, commercial tools must be adapted to project.	Commercial tools available, but largely untested.	Only commercial tools required.
<b>Process Capability</b>	No process capability data.	Limited data from sub-scale design or components. Estimate of capability possible.	Process capability data exist for similar processes.	Designed experiments have been performed for the process. CTQs and significant variables have been defined.	Process has been specified with statistical tolerances based on process capability analysis.



# Risk Management Process Training

Typical areas to be assessed	High-5	Significant-4	Moderate-3	Minor-2	Low-1
<b>Sigma Level</b>	Sigma level unknown or estimated to be less than 2.	Sigma level estimated to be between 2 and 4. Means of increasing the level is unknown.	Sigma level estimated to be between 3 and 4. Means of increasing the level is understood.	Sigma level determined to be greater than 4 with plans identified to reach 5-6 Sigma.	Sigma level determined to be greater than 5.
<b>Environment, Health and Safety</b>	EHS issues/opportunities not considered. Impacts to project cost and schedule likely.	EHS issues/opportunities not fully identified or considered. Impact to project cost and schedule is possible.	EHS issues/opportunities identified. Impact to project cost and schedule is possible.	EHS issues/opportunities addressed. Impacts to project costs and schedule minimized but opportunities not fully realized.	EHS goals (e.g., 100% compliance; minimize injuries) achieved while contributing to productivity, quality and cost savings.
<b>Market Issues</b>	Proposed product/process has obvious negative impact	Proposed product/process has potential negative impact.	Proposed product/process has no identified negative impact.	Proposed product/process perceived to exhibit a positive market impact	Similar family of products or processes known to exhibit a very positive market impact
<b>Measurement System</b>	No system exists.	Prototype system exists.	System exists but it exhibits high levels of variability.	System exists and exhibits some variability.	System is in place, has been proven and exhibits minimum variability.
<b>Process Scaling Factor</b>	$\geq 100:1$ Process conceptual or demonstrated in laboratory only. Few parameters defined. No process cost model.	$< 100:1$ but $> 20:1$ Process demonstrated on lab scale or lab pilot only. Process cost model preliminary.	$\leq 20:1$ but $> 5:1$ Pilot scale process demonstrated which mimics future process. Process cost model feasible.	$\leq 5:1$ but $> 2:1$ Process demonstrated on production scale equipment on temporary basis ( $< 1$ week).	$\leq 2:1$ Fully integrated process demonstrated for sustained production run.
<b>Teamwork</b>	Far away sites, major language barrier.	Distant sites linked by phone. Some bilingual team members.	Distant sites linked by e-mail. Largely common language.	Distant sites linked by ethernet, e-mail. One language.	Co-located, one language.
<b>Weight</b>	Estimated weight based on scaling analysis only, or dependent upon material not fully developed and rated as significant or greater risk.	Estimates based on scaling to the operational configuration within 10% of goal. Materials have been selected/rated as moderate risk or less.	Estimates based on a mix of scaling analysis and design models. Results are consistent with production data base within 5% of goal.	Estimates based on the production configuration design models. Results are approved by NPI team within 3% of goal.	Risk abatement plan successfully completed. Current production hardware or actual weight is known within 1/2% of goal.
<b>Software Development Process</b>	Software development is characterized as ad hoc and occasionally chaotic. Few processes are defined and success depends on individual effort and heroics.	Basic project management processes are established to track costs, schedule and functionality. Process discipline is in place to repeat successful software development projects with similar applications.	Processes for management and technical activities are documented, standardized and integrated into everyday software processes for the organization. All projects use an approved, tailored version of the organization's standard software development process.	Detailed software development process and product quality measurements are systematically collected, analyzed and acted upon. Software processes and products are quantitatively understood and controlled.	Continuous process improvement is enabled by quantitative feedback from the software development process and from piloting innovative ideas and technologies.



# Rating Risk: Consequence of Occurrence

Consequence of occurrence/Impact of risk				
High-5	Significant-4	Moderate-3	Minor-2	Low-1
<p>Major degradation or shortfall in technical performance requirement which will <b>jeopardize</b> the program if not mitigated.</p> <p><b>EXAMPLE:</b>            Nearly impossible situation. Resolution requires mods to system interface which will ripple through the product.            Cannot meet product introduction even if all available resources are applied to resolve the issue.            Delay of product introduction will jeopardize market position. "If this is not solved, cost per unit increases 25%."</p>	<p>Significant degradation or shortfall in technical performance requirement which could have a <b>major impact</b> on program objectives if not mitigated.</p> <p><b>EXAMPLE:</b>            Possible, but really tough. Resolution will affect some system interfaces requiring new tests. Product introduction will be delayed but the timing and impact are understood.</p> <p>Can be implemented with significant cost impact.</p>	<p>Reduction in technical performance requirement which could be tolerated with <b>limited impact</b> on program objectives if not mitigated.</p> <p><b>EXAMPLE:</b>            Can be done. Resolution is limited to a specific component. Product introduction may be achieved as scheduled but will require reassignment of resources.</p>	<p>Minor reduction in technical performance requirements which could be tolerated with <b>little impact</b> on program objective if not mitigated.</p> <p><b>EXAMPLE:</b>  <b>Typical issue.</b> Resolution will not result in changes to product introduction. Current personnel can handle the issue without additional funding.</p>	<p><b>No impact</b> on program objectives</p> <p><b>EXAMPLE:</b>            Standard design process. Resolution does not negatively affect quality, target cost or schedule.</p>

$$\text{RISK FACTOR SCORE} = \text{PROBABILITY OF OCCURRENCE} \times \text{CONSEQUENCE OF OCCURRENCE}$$



## *Prioritizing the Risks using the Risk Factor Score*

- *Risks are categorized as follows:*
  - *HIGH Risk = **RED** Risk = Score of 16 to 25*
  - *MEDIUM Risk = **YELLOW** Risk = Score of 9 to 15*
  - *LOW Risk = **GREEN** Risk = Score of 1 to 8*
  
- *Goal: reduce all HIGH & MEDIUM risks to LOW*
  - *Using risk abatement actions*
  - *Watch out for scores of 8*
    - *Review them for possible abatement*

***Identify Risks Early & Create  
Abatement Plans to Reduce Risk***



## *Reducing Risk Through Risk Abatement*

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***Risk abatement actions are planned actions that reduce the probability of risk occurrence***

- *All HIGH (Red) and MEDIUM (Yellow) risks must have an abatement plan.*
- *Track action plans carefully to ensure risks are reduced per plan.*



## *Reducing Risk Through Risk Abatement*

### ■ *Example:*

- *Risk rating before abatement =  
prob x conseq = 5 x 4 = 20 (High)*
- *Risk rating after abatement =  
prob x conseq = 2 x 4 = 8 (Low)*
- *The risk changes to the lower value on the date the abatement actions take affect*

- ### ■ *Normally the consequence of the risk occurring does not change, only the probability of the risk occurring changes.*

***Answers the Question: What Can We Do to Definitely Reduce the Risk?***



## *Examples of Possible Risk Abatements*

- *Complete analysis earlier*
- *Conduct periodic project reviews*
- *Involve customers/suppliers/  
manufacturing/field early in the process*
- *Model or simulate the process*
- *Use robust design*
- *Test*
  - *Model or prototype early, conduct a scaled test or validation, test earlier*
- *Provide computer or technical tools to improve productivity*
- *Negotiate schedule, budget, or price changes*
- *Assure proper resourcing*
  - *Involve right skill base at right time in the project*
  - *Shift critical resources*
  - *Out-source work*



## *Lessons Learned*

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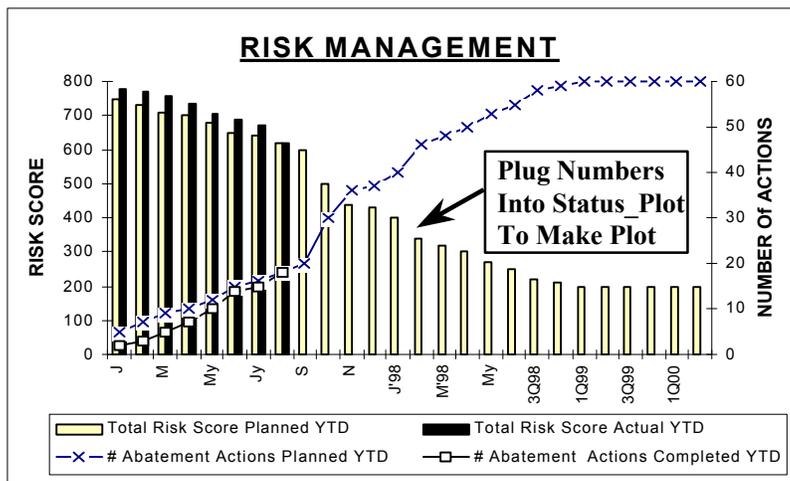
- *Include lessons learned from prior risk management efforts.*
  - *Include broad representation of people in risk assessment*
- *Include realistic actions in work plans & track closely.*
- *Assign ownership and completion dates for all actions.*
- *Risk review report out with appropriate managers is key.*
- *ASK: if test or analysis is the abatement,*
  - *Is there time for a back-up plan if it fails?*
  - *If no time to recover, start the back-up plan NOW.*



# Risk Management Process Training

## TRACKING RISK: RISK STATUS SCORECARD EXAMPLE

### XX GT PROGRAM XX NPI TEAM RISK SCORECARD



**Abatement Action Plan Update**

- Risk Abatement Actions Completed on Schedule
- Following Actions are Overdue:
  - Heat Transfer Analysis System Reliability Study
    - Was 6/97; Now forecast for 11/97
    - Delays Airfoil Risk Reduction, Still Time to Produce on Time
- Etc.

**Show Stoppers/ Barriers**

- Combustor System Low NOX design not verified by test prior to design release
- Compressor 3D aero tests slipped, will delay design release 3 months
- Turbine materials evaluation incomplete, life analysis based on extrapolated data. May require airfoil redesign and two months slip of first Turbine to test
- No Show Stoppers
- Etc.

### Summary

Indicate # of High & Medium Risks From Risk Assessment

Date: J F M A M Jn Jy A S O N D 1Q'98...

#High Risks	10	10	10	7	5	3	2	1				
#Medium	15	15	13	16	17	17	10	9				

- Turbine risk assessment complete
- Gear assessment complete with no risk evaluation above "medium"
- Steam System Assessment Completion Target mm/dd/yy

**Needs/ Recommendations**

- Accelerate Low NOX activity
- Need additional CR&D team member
  - Need additional CR&D team member
  - Get ad hoc member from Aircraft Engines
- Complete compressor tests to define flow characteristics prior to design release
  - Identify required resources to accomplish task by Monday
  - Identify all barriers
- Accept higher risk for turbine material evaluation. Will not affect performance demonstration test. Full life blade will be available for durability tests
- Etc.



## *Risk Management Exercise: 30 mins.*

- *Working in teams, for one or more projects in your team, complete the template on the next page.*
  - *Risk Issues: Identify two or more risk issues.*
  - *Type: Identify the risk type.*
    - Cost, Technology, Specification, Marketing, or Installation*
  - *Prob: Rate probability of occurrence (1 to 5)*
    - *Reference probability of occurrence rating guide — Use words in guide, not “gut feel.”*
  - *Imp: Rate consequence of occurrence/impact (1 to 5).*
  - *Score: Risk factor score — consequence x probability = 1 to 25.*
  - *Prioritize the risks High (Red), Medium (Yellow), Low (Green).*
  - *Remediation Action: Identify the risk abatement plans (high & medium risks).*
  - *Owner: Identify the owner of the abatement action.*
  - *Measure of Success: Identify how you will measure that the abatement was successful.*
  - *Date Done: Estimate when in the project the abatement action will be completed.*
  - *Resid. Risk: Calculate the risk score after the abatement.*
- *One or Two teams will be asked to report on their findings.*



# Risk Management Exercise

Risk Issue (group by Accept? sub-category)	Type	Prob	Imp	Score	Remediation Action	Owner	Measure of Success	Date Done	Resid Risk	Y or N
How to assure employees get properly trained in new "improved" process	Installation	5	4	20	New controlled procedures that require formal training & signoff records for all trained operators - posted at workstations	Mike	Verify posted authorized list	8/98	5	Y



# *Mistake Proofing*



## *Objectives*

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- *Understand difference between **errors** and **defects**.*
- *Understand how defects originate.*
- *Recognize elements of **source inspection** and its role in defect prevention.*
- *Identify key **mistake proofing devices**.*
- *Show mistake proofing as a proactive tool.*
- *Show how mistake proofing fits into the Six Sigma methodology.*



## *Mistake Proofing*

---

- *A technique for eliminating errors.*
- *Making it impossible to make mistakes.*

***“It is good to do it right the first time:  
it is even better to make it impossible  
to do it wrong the first time.”***



## *Principles for Mistake Proofing*

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- **Respect** the intelligence of workers.
- Take over **repetitive** tasks or actions that depend on constantly being alert (vigilance) or memory.
- **Free** a worker's time and mind to pursue more creative and value-adding activities.
- It is **not acceptable** to produce even a small number of defects or defective products.
- The objective is **zero defects**.



## *Exercise*

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### *Mistake Proofing in Everyday Life*

- *Working in small groups, develop a list of examples of mistake proofing that we experience every day.*
- *Be prepared to present your ideas to the rest of the group at: \_\_\_\_\_*
- *Examples:*
  - *Automatic Seatbelts*
  - *Auto-Shut-Off Irons*
  - *Automatic Sinks in Public Facilities*

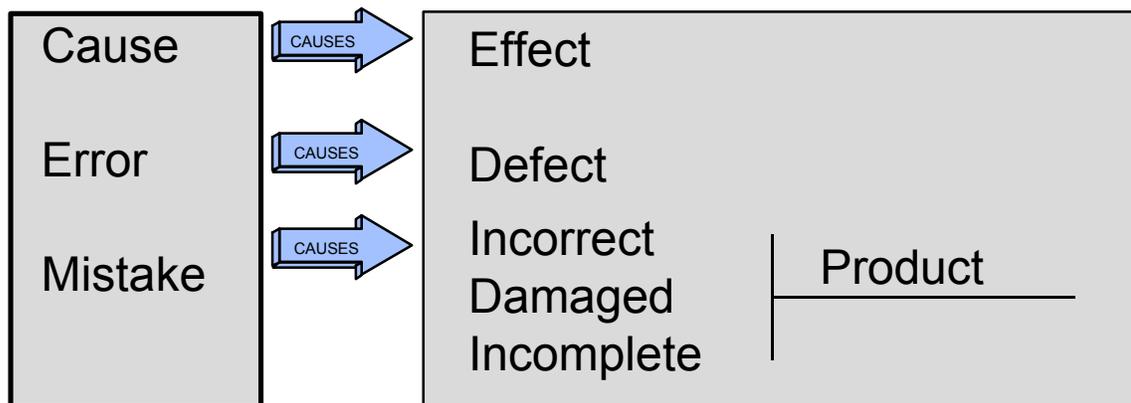


## Defects vs. Errors

***They are not the same thing!***

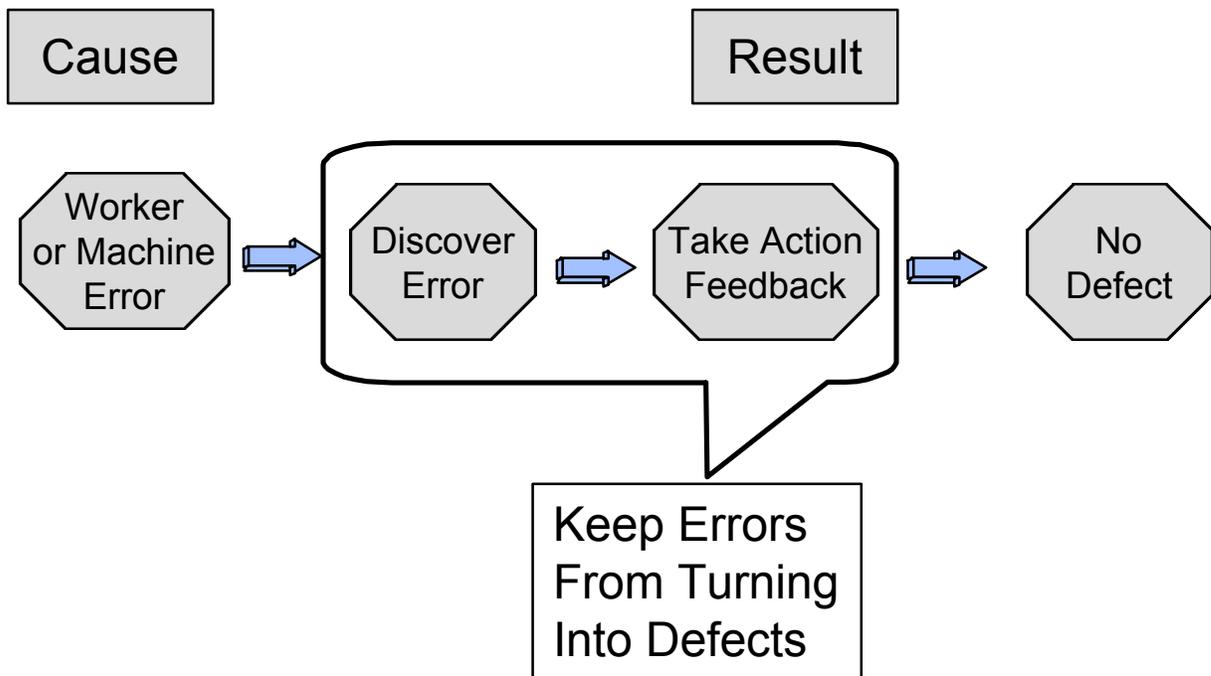
Defects are the result of error.

Error is the cause of defects.





# Preventing Defects





## *Why Do Errors Occur?*

- *Incorrect procedures*
- *Excessive variation in the process*
- *Excessive variation in the inputs*
- *Inaccurate measuring devices*
- *Human error*



## *Ten Types of Human Error*

- 1) **Forgetfulness** (*not concentrating*)
- 2) **Errors in mis-communications** (*jump to conclusions*)
- 3) **Errors in identification** (*view incorrectly...too far away*)
- 4) **Errors made by untrained workers**
- 5) **Willful errors** (*ignore rules*)
- 6) **Inadvertent errors** (*distraction, fatigue*)
- 7) **Errors due to slowness** (*delay in judgment*)
- 8) **Errors due to lack of standards** (*written & visual*)
- 9) **Surprise errors** (*machine not capable, malfunctions*)
- 10) **Intentional errors** (*sabotage - least common*)

**Use Mistake Proofing to Eliminate these  
Human Errors**



# *Human Error-Provoking Conditions*

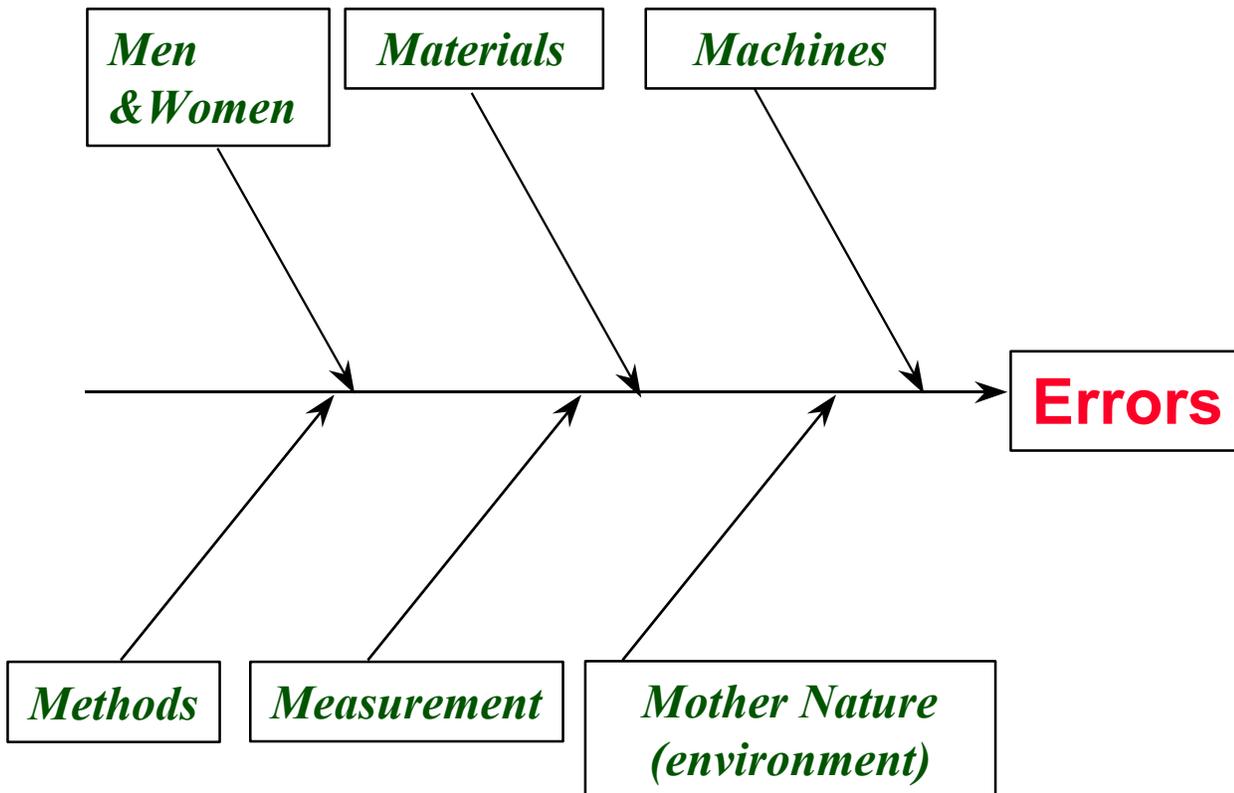
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- 1) *Adjustments*
- 2) *Tooling/tooling change*
- 3) *Dimensionality/specification/critical condition*
- 4) *Many parts/mixed parts*
- 5) *Multiple steps*
- 6) *Infrequent production*
- 7) *Lack of, or ineffective standards*
- 8) *Symmetry*
- 9) *Asymmetry*
- 10) *Rapid repetition*
- 11) *High volume/extremely high volume*
- 12) *Environmental conditions*
  - a. *Material/process handling*
  - b. *Housekeeping*
  - c. *Foreign matter*
  - d. *Poor lighting*



## *Sources of Errors*

- *Variables that determine whether a product is correctly manufactured.*





## *Exercise:*

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### *Types of Errors*

- *Break into groups of 3 to 4 people each.*
- *Spend 15 minutes brainstorming types of errors or mistakes that can be made in your work area or relating to your project.*
- *List your ideas on a flip chart.*
- *Be prepared to make a short presentation to the other groups at:*

\_\_\_\_\_



## *Are Errors Unavoidable?*

- *Traditional view: errors are inevitable.*
  - *People are only human*
  - *There is variation in everything*
  - *Lack of standard operating procedures result in each person having their own way to do things*
  - *Inspection is necessary*
  
- *Six Sigma view: errors can be eliminated.*
  - *Not all errors can be eliminated, but many can and others can be reduced*
  - *The more errors we can eliminate, the better our quality*
  - *The need for inspection can be reduced or eliminated*



## *Is Inspection the Best Method?*

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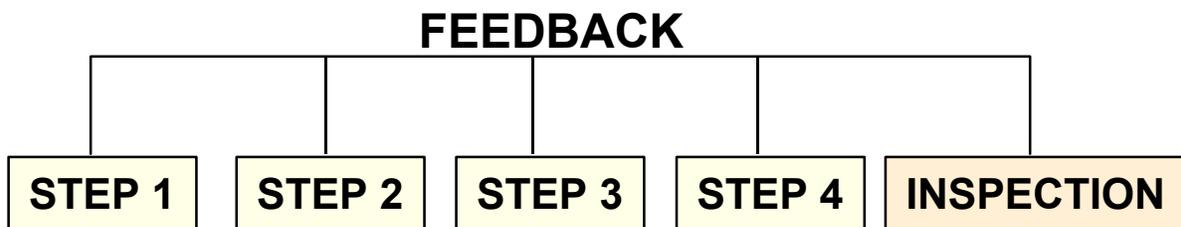
- *Sampling inspection is not 100% effective.*
  - *Sampling helps the manufacturer, but not necessarily the **customer***
- *Traditional 100% inspection is not 100% effective either.*
  - *The **user** is the best inspector*



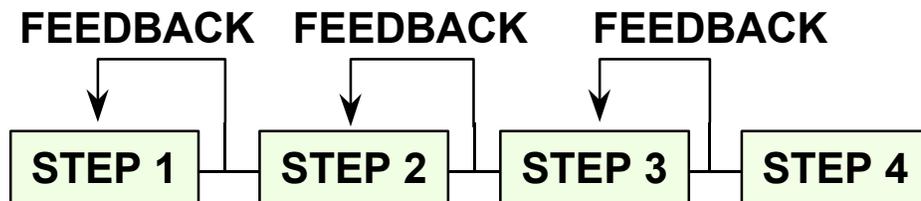
# Mistake Proofing

- 100% error-proofing.
- **Immediate feedback** so action can be taken.

## Traditional Inspection



## Mistake Proofing





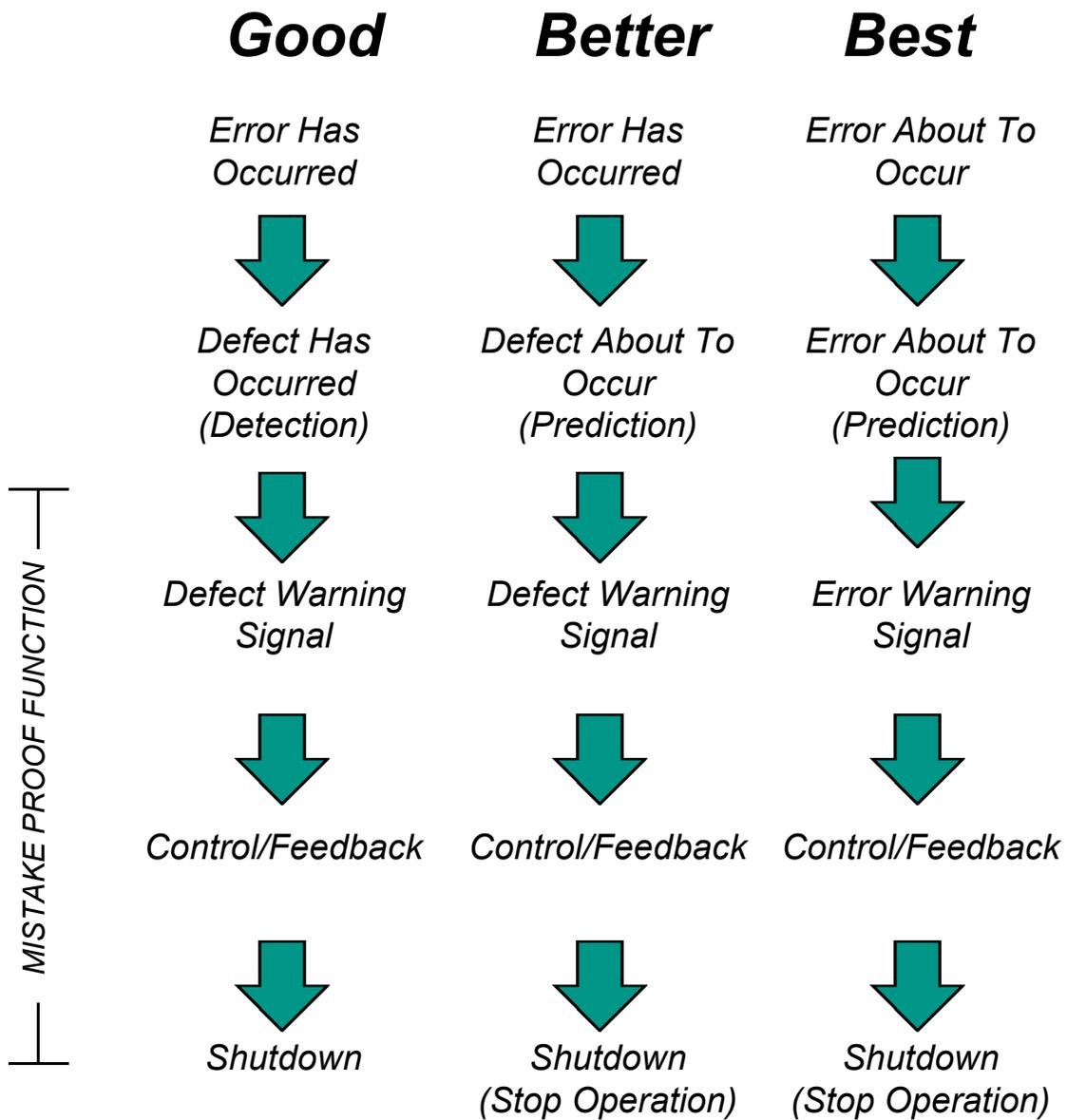
## *When Can We Find Mistakes?*

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- *Before they occur*
  - ***Prediction or Prevention***
  
- *After they occur*
  - ***Detection***



# Elimination of Defects



**Eliminate Defects Through Error Reduction**

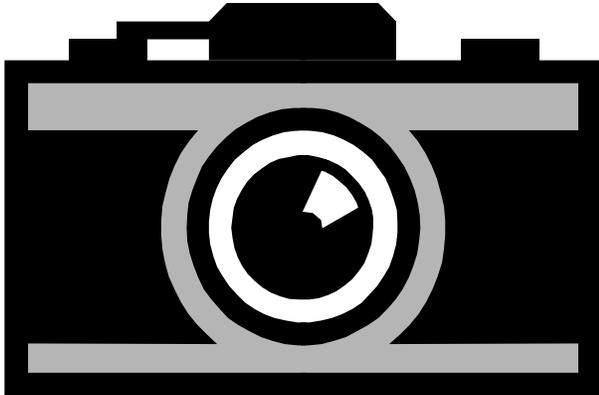


## *Mistake Proofing Techniques*

<b>Technique</b>	<b>Prediction/ Prevention</b>	<b>Detection</b>
<b>SHUTDOWN</b>	<i>When a mistake is about to be made.</i>	<i>When a mistake or defect has been made.</i>
<b>CONTROL</b>	<i>Errors are impossible.</i>	<i>Defective items can not move on to the next step.</i>
<b>WARNING</b>	<i>That something is about to go wrong.</i>	<i>Immediately when something does go wrong.</i>



## *Examples: Shutdown*



### PREDICTION/PREVENTION

*Some cameras will not function when there is not enough light to take a picture.*

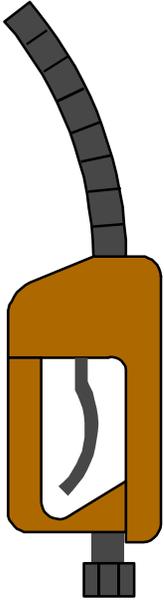
### DETECTION

*Some laundry dryers have a device that shuts them down when overheating is detected.*





## *Examples: Control*

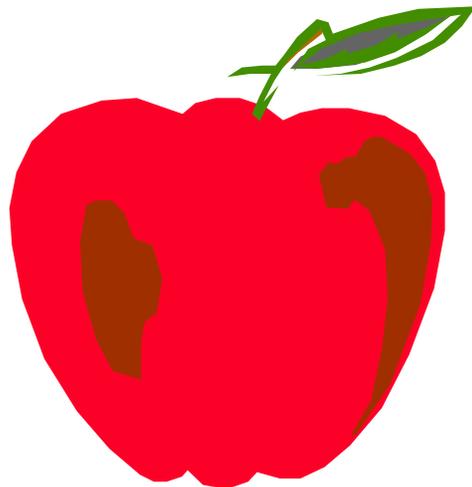


### PREDICTION/PREVENTION

*When gas stations still offered leaded gasoline in addition to unleaded gasoline, the nozzle on the unleaded pump and the hole for the gas tank were smaller than that for leaded gasoline.*

### DETECTION

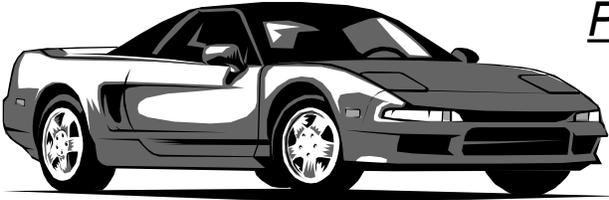
*A fruit orchard that takes great pride in its oversized apples, assures only the biggest apples get to customers by passing all apples through a sizer. Those that do not make it are sent to the discount outlet.*





## *Examples: Warning*

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### PREDICTION/PREVENTION

*Many cars have warning systems to alert the driver that not all seat belts have been fastened.*

### DETECTION

*Smoke detectors provide a warning that smoke has been detected and that there is a possible fire.*





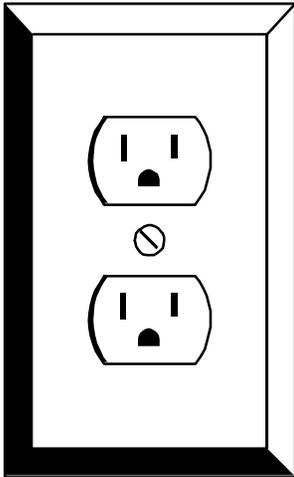
## *Prediction/Prevention and Detection Methods*

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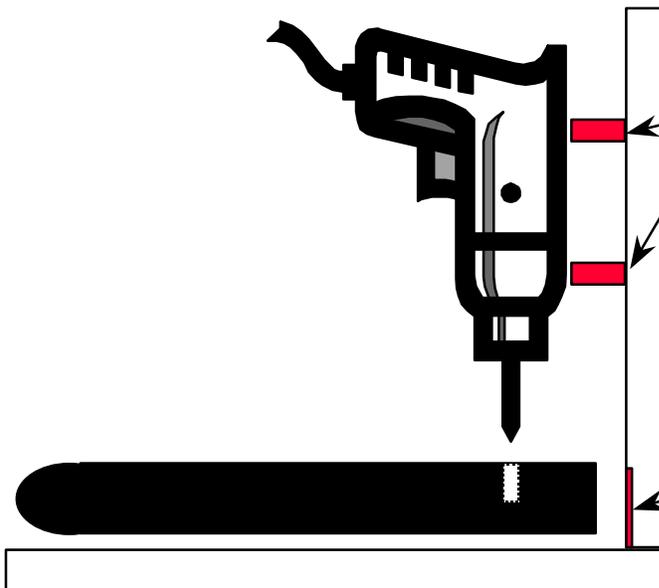
- *Contact methods*
  - *Contact with the part highlights errors*
- *Fixed-value methods*
  - *Errors are detected through **counting***
- *Motion-step methods*
  - *Errors are detected by **motion** or lack of it*



## *Example: Contact Methods*



*U. S. Electrical outlets have been mistake proofed to assure proper polarity. It is impossible to put a plug in the outlet incorrectly.*

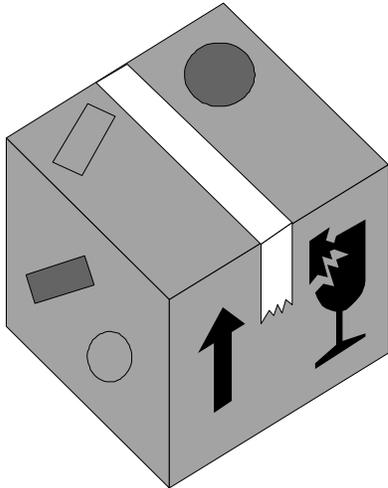


*Guides line up the drill to assure the drilled hole is 90°.*

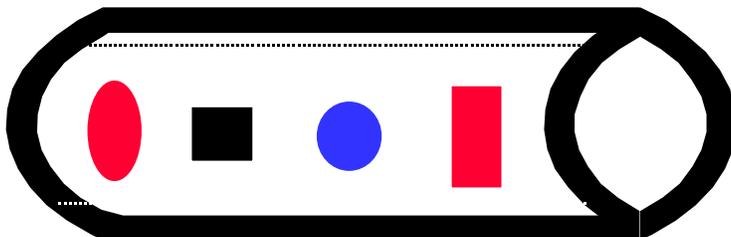
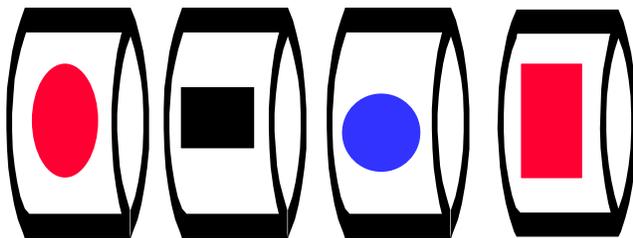
*A magnetic contact assures that the shaft is correctly inserted and seated before drilling.*



## *Example: Fixed Value Methods*



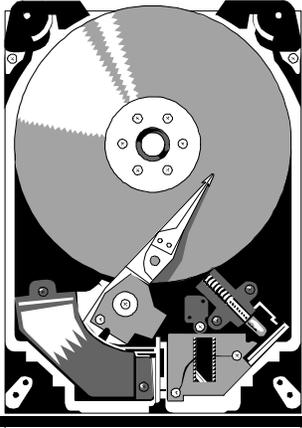
*Four different hazardous material warning labels had to be applied before shipping the product. Boxes without the labels would be returned which would result in delayed shipments.*



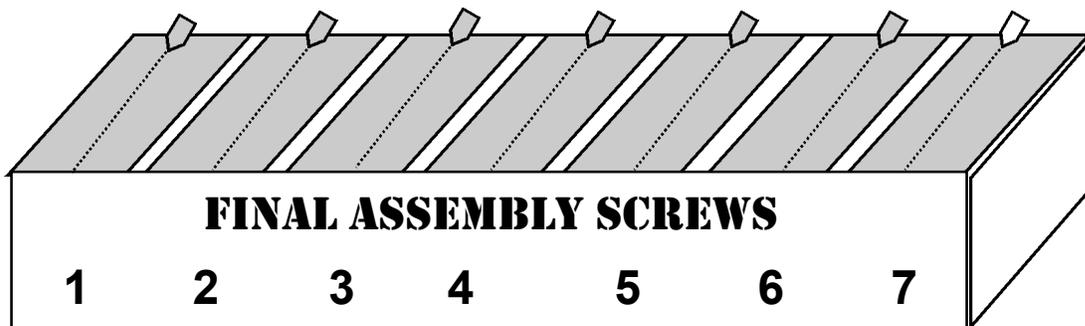
*The labels were on four separate rolls. When all four labels were put onto one roll, it was easy for the worker to know when a label had been missed.*



## *Example: Motion Step Methods*



*Seven screws in various sizes were inserted in the final assembly of a CD ROM drive. Often a screw would be forgotten resulting in high warranty claims.*



*The seven different screws were put into bins with photo-electric switches. When a screw is removed, the beam is broken. The part cannot move on to the next operation until the beam is broken on all seven bins.*

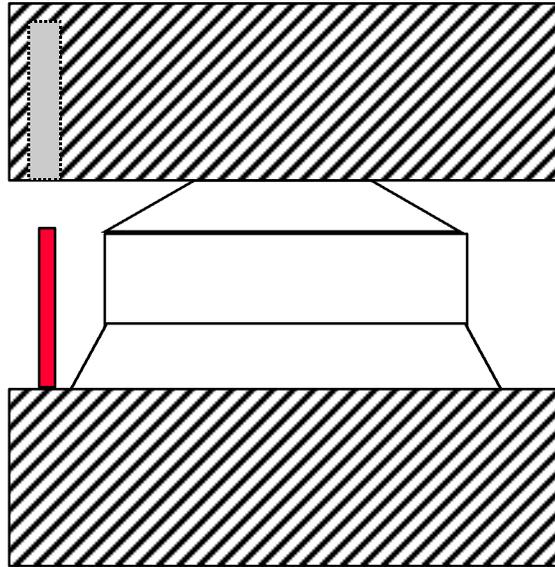


## *Typical Mistake Proofing Tools*

- *Limit Switches*
- *Guide Pins*
- *Counters*



## *Example: Guide Pins*



*Sometimes the set-up operator would set the top and bottom jig incorrectly resulting in defective parts and possible damage to the die. A guide pin prevents the press from closing unless the proper jig is used and it is correctly set up. Each jig has its own unique guide pin.*



## *Example: Limit Switches*

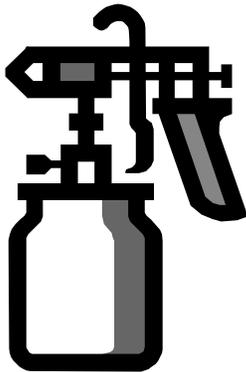
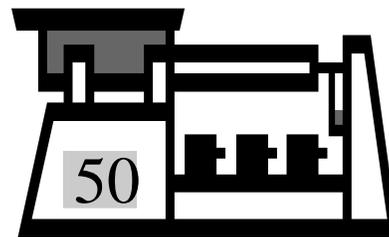
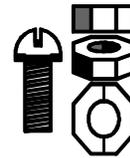


*A forklift truck has a limit switch that will not allow the truck to move forward or in reverse with the mast raised. Another limit switch shuts the forklift down when the load exceeds the maximum weight capacity.*



## Example: Counters

*A scale, specially designed for the screw and bolt packaging operation, digitally displayed the number of screws in the package based on the weight of the screw. The operator no longer had to convert the weight of the screws and bolts into number of pieces.*



*An enamel spraying operation required each part to have short sprays of enamel applied before the part was baked. It was determined that five short sprays gave the optimum coverage. A limit switch was put on the pump that would allow only an 0.5 second spray. A counter was also installed on the spray trigger. Only when the spray trigger was squeezed five times would the part be released to the oven.*



## *Other Examples*

---

- *Brackets/parts that are non-reversible or fully reversible at assembly.*
- *A computer spell checker.*
- *Color coded documents—credit card receipts or invoices (the customer gets the yellow copy, the merchant gets the white copy).*
- *Features that prevent reversed assembly—tabs, slots, etc.*
- *Features that are visibly verifiable after assembly.*
- *Make it **easy** to do it **right**.*
  - *Checklists*
  - *Effective data collection formats*
  - *Work flows with fewer hand-offs*
  - *Symbols*
  - *Color coding*



## *Other Examples*

---

- **Use shapes.**
  - *Store different types of parts in different shaped bins*
  - *Notch a stack of forms so it's easy to tell if the forms are out of order*
- *Hardness check after all thermal processing (prevents material mix-up or missed heat-treat).*
- *For parts with multiple braze cycles—lower braze temperature alloy for each cycle (prevent remelting).*
- *Drop-down box in a data base.*
- *Mutilate scrap hardware immediately.*
- *Keep shipping heights as low as possible but always less than 13'6" (eliminate need for special routing).*



## *Exercise*

---

- *In groups of 4 to 5, develop a way to mistake proof the problem you have been presented with.*
- *Be prepared to present your solution by: \_\_\_\_\_*



## *Mistake Proofing Challenges*

---

- 1. ATM customers complain that they never know the right way to insert their ATM card and usually it takes a couple of tries.*
- 2. A local bottling company wants to make sure that all bottles are filled with exactly the right amount of soda.*
- 3. A large pharmaceutical company is trying to develop a way to assure elderly people living alone take their prescription medicine at the right time and in the correct dosage.*
- 4. A manufacturer of home power tools wants to make sure that customers are wearing eye and hand protection before using their equipment.*
- 5. A consumer electronics company has had several customer complaints lately that their instructions were missing from their product.*



## *Mistake Proofing Challenges cont.*

---

- 6. 9 different tools are needed to change a jig and die. Often, in the middle of the change-over, the set-up operator realizes a tool is missing and has to stop to look for the tool.*
- 7. An easy-to-assemble-furniture manufacturer has received several complaints about holes not being tapped for all screws. Because the furniture is made from hardwood, the customers have to drill the holes themselves.*
- 8. A contact lens solution manufacturer received complaints from distributors that some customers found empty boxes with their shipments. The company wants to make sure no more empty boxes are shipped.*



## *Mistake Proofing Challenges solutions*

---

1. Put a lip on one long side of the card. Standardize the way cards are inserted and then clearly mark the proper installation on the card.
2. Install a photoelectric switch that can measure the amount in the bottle. As part of the nozzle system, include a probe that shuts the nozzle off when the liquid touches the probe.
3. Sell the medication in a dispensing mechanism that helps people keep track of the day and whether or not they've taken their daily medication. Devise a dispenser that releases the proper dosage of medication on the right day at the right time.
4. Have a device on the gloves that must come in contact with the saw handle in order for it to run. Have a special switch on the saw that must be flipped on to confirm that the safety glasses are on.
5. Package the instructions in sets of 25. In final assembly, release boxes 25 at a time. If an instruction sheet remains after the 25 boxes are complete, it can be assumed one box doesn't have instructions.
6. Set up peg boards for each set-up operation. Paint silhouettes of each tool needed on the pegboard. When preparing for the changeover, assemble the tools using the peg board. Missing tools will be obvious.
7. Put a counter on the drill that will not allow the part to be released until the right number of taps have been made.
8. Once the boxes are sealed, as they travel on the conveyor belt to final packaging, blow air at them. Those that get blown off the conveyor belt are empty.



## *Examples in GE*

---

*Think of a situation in your business where mistake proofing can be applied. Describe the following aspects of the concern:*

- *Problem*
  
- *Situation*



## *Mistake Proofing Steps*

---

- 1 *Identify* problems.
- 2 *Prioritize* problems.
- 3 Seek out the *root cause*.
- 4 Create *solutions*.
- 5 *Measure* the results.



## *Identify* problems

- *Brainstorming*
- *Customer returns*
- *Defective parts analyses*
- *Error reports*
- *Failure Mode and Effects Analysis (FMEA)*



## *Prioritize* problems

- *Frequency*
- *Wasted materials*
- *Rework time*
- *Detection time*
- *Detection cost*
- *Overall cost*



## Seek out the *root cause*

- *Do not use mistake proofing to cover-up problems or to treat symptoms.*
- *Use mistake proofing to correct errors at their source.*
- *Other methods to determine the root cause are:*
  - *Ask “why” five times*
  - *Cause & effect diagrams*
  - *Brainstorming*
  - *Stratification*
  - *Scatterplot*



## Create *solutions*

- *Make it impossible to do it wrong.*
- *Cost/benefit analysis.*
  - *How long will it take for the solution to pay for itself?*
- *Thinking outside of the box.*



## *Measure* the results

- *Have errors been **eliminated**?*
  - *Why or why not?*
  
- *What is the **financial impact**?*



## *Mistake Proofing Advantages*

---

- *No formal training programs required.*
- *Eliminates many inspection operations.*
- *Relieves operators from repetitive tasks.*
- *Promotes creativity and value adding activities.*
- *Results in defect-free work.*
- *Provides immediate action when problems arise.*



## *Mistake Proofing Exercise: 20 mins.*

---

- *Working in teams, for one (or more) project(s) per team identify possible areas to mistake proof.*

*One or two teams will be asked to report on their findings.*



## *Appendix\**

---

### *Contact Detection Methods*

- *Limit switches*
- *Microswitches*
- *Touch switches*
- *Differential transformers*
- *Trimetrons*
- *Liquid level relays*

*\*Source: Shingo, Shigeo, Zero Quality Control: Source Inspection and the Poka-Yoke System, Productivity Press, 1986.*



## *Appendix\**

---

### *Non-Contact Methods*

- Proximity detection measures*
- Photoelectric switches*
- Beam sensors*
- Fiber sensors*
- Area sensors*
- Positioning sensors*
- Dimension sensors*
- Displacement sensors*
- Metal passage sensors*
- Color marking sensors*
- Vibration sensors*
- Double-feed sensors*
- Welding position sensors*
- Tap sensors*
- Fluid elements*

*\*Source: Shingo, Shigeo, Zero Quality Control: Source Inspection and the Poka-Yoke System, Productivity Press, 1986.*



## *Appendix\**

---

### *Other Methods*

- *Detecting pressure, temperature, electric current, vibration, number of cycles, timing, and information transmission.*
  - *Pressure gages*
  - *Pressure sensitive switches*
  - *Thermometers*
  - *Thermostats*
  - *Thermistors*
  - *Meter relays*
  - *Current Eyes*
  - *Vibration sensors*
  - *Counters*
  - *PLCs*
  - *Fiber sensors*

*\*Source: Shingo, Shigeo, Zero Quality Control: Source Inspection and the Poka-Yoke System, Productivity Press, 1986.*



# *Control Plans*



## *Control Plan*

---

*A good Control Plan will incorporate at least:*

- Customer-driven Critical To Quality (CTQs)
- Input & Output variables
- Appropriate tolerances (specifications for CTQs)
- Designated control methods, tools and systems
  - SPC
  - Checklists
  - Mistake proofing systems
  - Standard Operating Procedures
  - Manufacturing/Quality/Engineering Standards
- Reaction Plan



## *Control Plan*

---

- This document is an extension of the Current Controls column of the FMEA
- For every CTQ and Input, the measurement system capability (Gage R&R) should be identified
- For Variable data, the Sigma-level should be calculated
- The specifics of the sampling plan with associated Reaction Plan should be listed



# *Control Plan and the relationship to the FMEA*

*The FMEA should be a key source for the identification of key variables to control and for an initial evaluation of the current Control Plan*

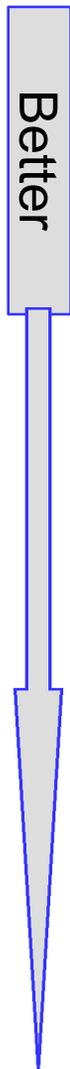
Process Step	Key Process Input	Potential Failure Mode	Potential Failure Effects	S E V	Potential Causes	O C C	Current Controls

DET

Process	Process Step	Input	Output	Process Specification (LS, USL, Target)	Cpk/Dpk (Sample Size)	Measurement Technique	%R&R FT	Current Control Method	Sample Size	Sample Frequency	Reaction Plan



## *Control Plan Methods*



*Inspection and test methods*

*Checklists*

*Standard Operating Procedures  
("SOPs")*

*Statistical Process Control*

*Mistake Proofing Device*

*Automation*



## *Control Plan Methods*

---

*If the method is complicated, refer to a procedure by document number*

*Any changes in the process should consider changes in the Control Method*

*Control methods must include a training plan and process auditing system*

*Control method should identify person responsible for the control of each critical variable*



## ***A good Reaction Plan will incorporate:***

---

***Actions that are the responsibility of people closest to the process***

***References to an SOP and identify the person responsible for the reaction procedure***

***Clear identification and quarantine of suspect or nonconforming product***



## *Additional questions to ask when implementing a Control Plan:*

---

### *Inputs:*

*How are they monitored?*

*How often are they verified?*

*Are optimum target values and specifications known?*

*How much variation is there around the target value?*

*How consistent are they?*

*What causes the variation in the Input?*

*How often is the Input out of control?*

*Which Inputs should have control charts?*



## *Additional questions to ask when implementing a Control Plan:*

### *Standard Operating Procedures*

*Do they exist?*

*Are they understood?*

*Are they being followed?*

*Are they current?*

*Is operator certification performed?*

*Is there a process audit schedule?*

### *System Noise*

*Does it exist?*

*Is it impossible or impractical to control?*

*How robust is the system to noise?*



## ***Additional questions to ask when implementing a Control Plan:***

---

***What special equipment is needed for measurement? What is the measurement capability?***

***Who does the measurement? How often is measurement taken? How are routine data recorded?***

***Who plots the control chart (if one is used) and interprets the information?***

***What key procedures are required to maintain control?***

***What is done with product that is off spec?***

***How is the process routinely audited?***

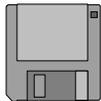
***Who makes the audit? How often? How is it recorded?***

***Is operator training performed and documented?***



# FMEA Template

Six Sigma Control Plan											
Product:		Core Team:						Date (Orig):			
Key Contact:								Date (Rev):			
Phone:											
Process	Process Step	Input	Output	Process Specification (LSL, USL, Target)	Cpk / Cp / or Z Date Performed	Measurement Technique	%R&R P/T	Sample Size	Sample Frequency	Control Method	Reaction Plan



**contplan.xls**



# Control Plan Example

<b>Six Sigma Control Plan</b>												
Product:		Core Team:							Date (Orig):			
Key Contact:									Date (Rev):			
Phone:												
Process	Process Step	Input	Output	Process Specification (LSL, USL, Target)	Cpk / Cp / or Z Date Performed	Measurement Technique	%R&R P/T	Sample Size	Sample Frequency	Control Method	Reaction Plan	
Cable Assy	Connector Attach	Crimp Force		80 - 120 lbs	Z = 4.2/15-Sep-98	Enerpac Load Cell	5%	100%	continuous	Crimping SOP/xbar chart	Call Dimos	
	End Polish	Proper Polish		200-400 microns	Z = 2.1/15-Sep-98	Profilometer	22%	5/lot	each lot	Polish SOP/x bar chart	Call Dimos	
	Measure Attenuation		dB	0 - 0.27 dB/meter	Z = 0.5/15-Sep-98	Fotec Light Meter	?	100%	Continuous	Test Procedure/x bar chart	Call Dimos	



---

# *Variable Control Charts*



## *Statistical Process Control (SPC)*

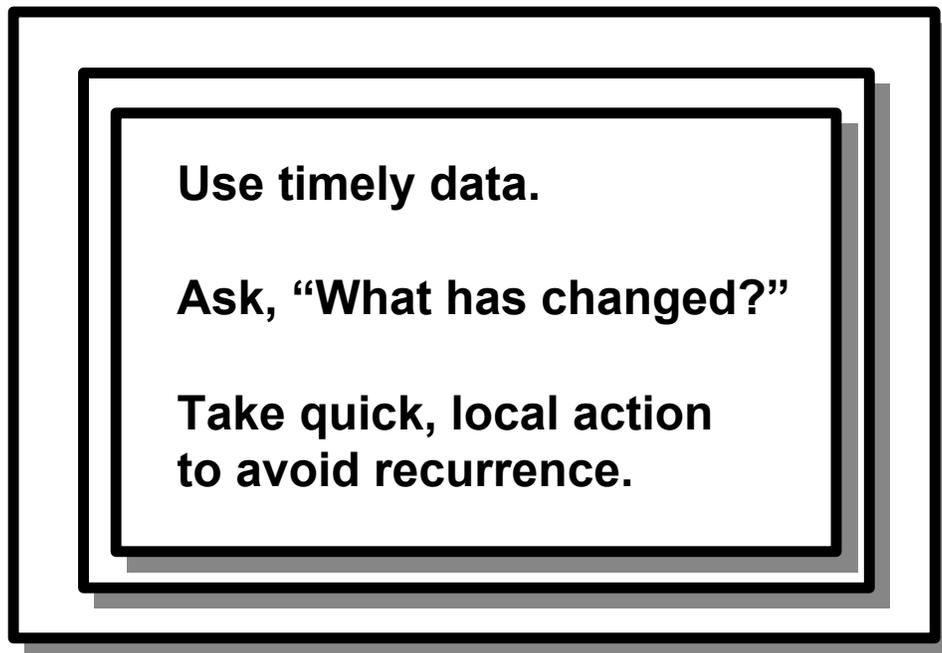
- *The purpose of statistical process control (SPC) is to indicate:*
  - *when a process is working at its intended best (only common cause variation present)*
    - *No corrective action is necessary.*
    - *Unnecessary actions may actually increase process variability.*
  - *when a process is disturbed and needs corrective action of some type (special cause variation present)*
- *Control charts:*
  - *are used to monitor both inputs to process, parameters of a process, or process outputs (Xs and Y)*
  - *are used to recognize when a process has gone out of control*
  - *are used for identifying the presence of special cause variation within a process*
  - *do not tell us if we meet specification limits*
  - *neither identify nor remove special causes*



## *Special Causes*

- *Unusual*
- *Sporadic*
- *Specific*

### *Special Cause Strategy:*



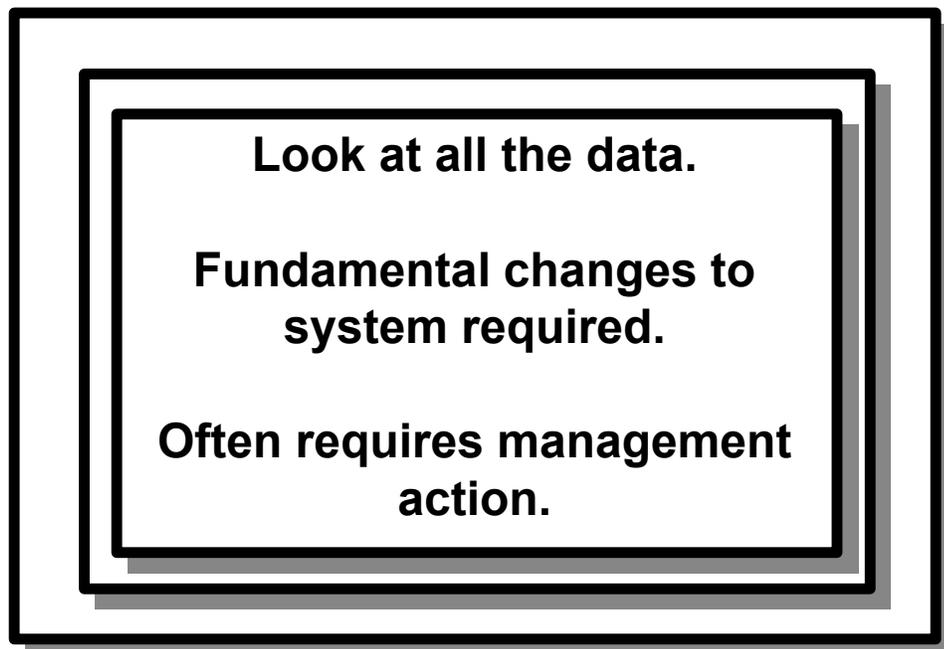
*Use knowledge of the process to eliminate or reduce the assignable causes to reduce variability and to improve the process.*



## *Common Causes*

- *Natural*
- *Random*
- *Requires system changes to improve*

### *Common Cause Strategy:*





## *Definitions*

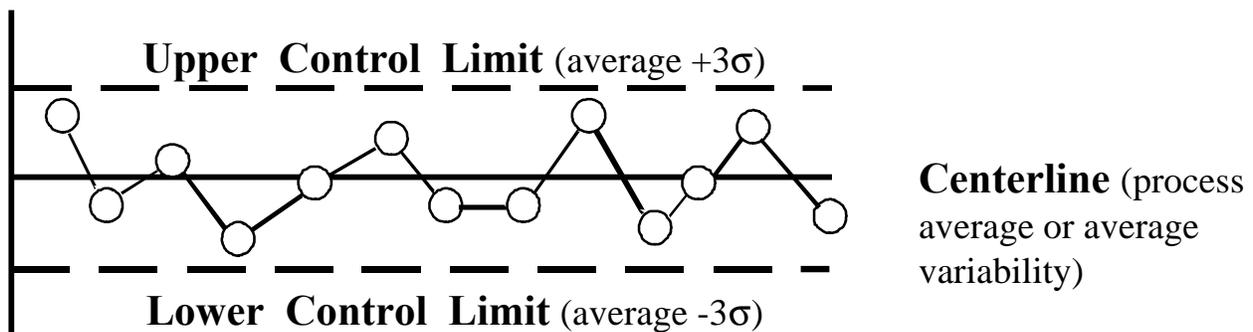
---

- *In Control*
  - *No special cause variation present*
  - *All variation is random*
- *Out of Control*
  - *At least one special cause is present*
  - *Some variation is non-random*



## The Control Chart

- *A time ordered plot of the data.*
- *Reflects the expected range of variation of the data.*
- *Identifies when a special cause is acting on the process.*





## *Some Variable Control Charts*

---

- ***X Bar Chart:*** a plot of the sample means over time.
- ***R Chart:*** a plot of the range (difference between highest and lowest values) of a sample over time.
- ***Individuals Chart:*** a plot of the individual values over time.
- ***Moving Range Chart:*** a plot of the moving range (for two samples  $|X_i - X_{i-1}|$ ) over time.



## *Five Main Uses of Control Charts*

---

- *To **reduce scrap and rework** and for improving productivity.*
- ***Defect prevention.** In control means less chance of nonconforming units produced.*
- ***Prevents unnecessary process adjustments** by distinguishing between common cause variation and special or assignable cause variation.*
- *Provides **diagnostic information** so that an experienced operator can determine the state of the process by looking at patterns within the data. The operator can then make the necessary changes to improve the process performance.*
- *Provides **information** about important process parameters **over time.***



## *How to Collect Data*

---

- *Rational subgroups: collect data so that subgroups contain only common cause variation. The same as in capability analysis.*
- *Choose rational subgroups to gain as much information as possible about the process.*
- *To detect process shifts: each subgroup should consist of measurements taken at approximately the same time.*
  - *Choose a sample so that it maximizes the likelihood of detecting variability between the samples*



## Sampling

- *Sample size: The higher the **process volume** and the easier and cheaper the measurements of the CTQ characteristic, the more likely you are to select an  $\bar{X}$  and R chart (typically 3 - 5 data points per sample) over an Individual and Moving Range chart (I and MR).*
- *Frequency of sampling: Consider hourly, daily, shifts, monthly, annually, lots, and so on. The better your process is performing, the less frequently you will need to sample.*
- *Current industry standard tends to favor smaller, more frequent samples.*



## *Setting Up and Maintaining Control Limits*

---

- *Calculate the control limits with 20 - 25 samples (e.g., for the  $\bar{X}$  and R chart that would mean 20 - 25 samples of size 3 - 5).*
- *If process is in control, go to the last step.*
- *If process is not in control, try to identify special cause.*
- *Remove special cause, recollect data, recalculate control limits, ... until you find the process is in control.*
- *For future monitoring, do not change the limits unless a permanent, desired change has been made to the process.*



## *Types of Errors in Control Charts*

### ■ *3s-level Control Limits*

- *Created by Shewhart to minimize two types of mistakes*
- *Placed empirically because they minimize the two types of mistakes*
- *Are not probability limits*

### *Two types of Mistakes:*

- *Calling a special cause of variation a common cause of variation (Missing an chance to identify a change in the process)*
- *Calling a common cause of variation a special cause of variation (Interfering with a stable process, wasting resources looking for special causes of variation that do not exist.)*



## *Two Types of Control Chart*

---

### **VARIABLE CHART**

- *Uses Measured Values*
  - *Cycle Time, Lengths, Diameters, Drops, etc.*
- *Generally One Characteristic Per Chart*
- *More Expensive, But More Information*

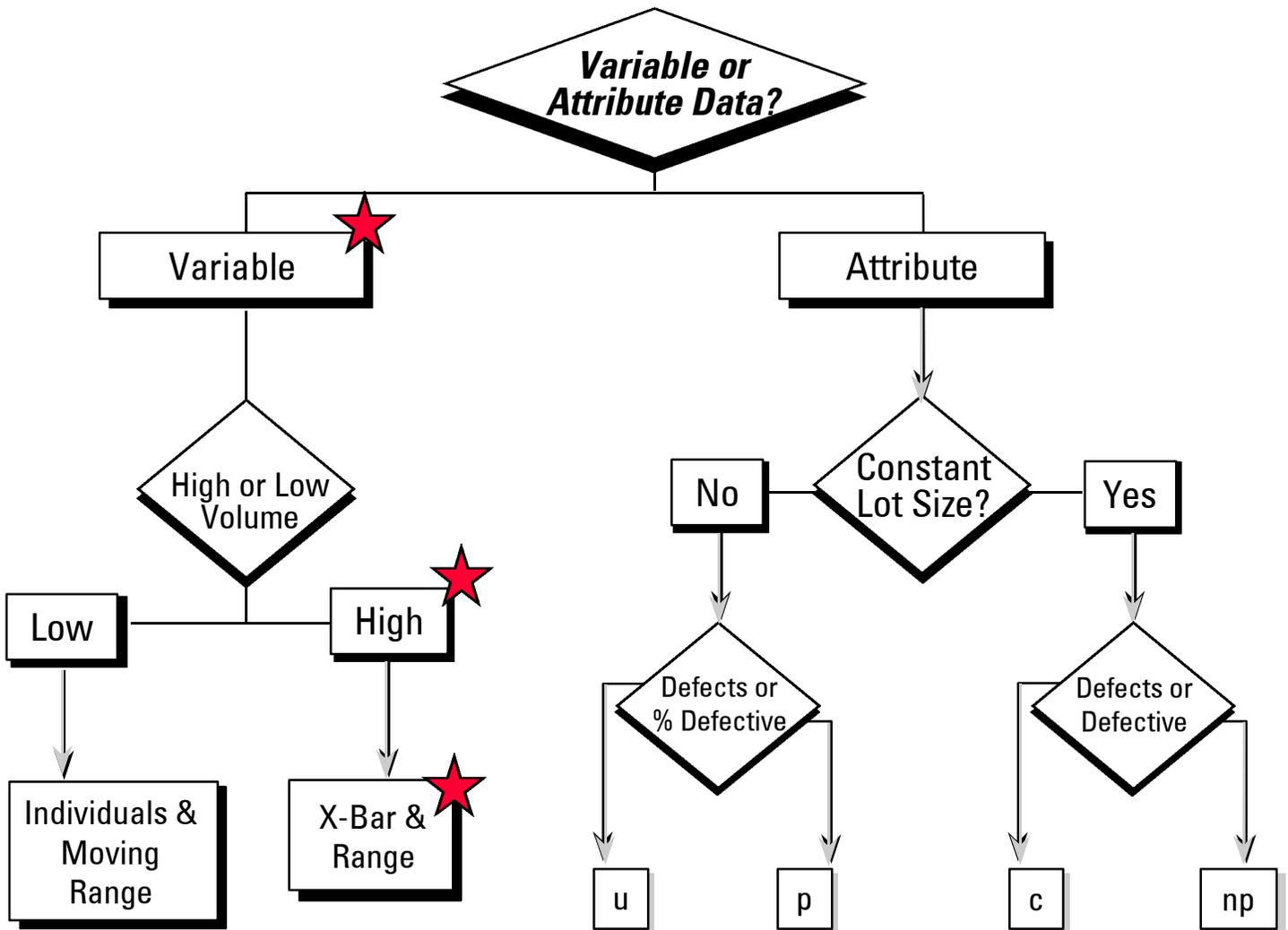
### **ATTRIBUTE CHART**

- *Pass/Fail, Good/Bad, Go/No-Go Information*
- *Can Be Many Characteristics Per Chart*
- *Less Expensive, But Less Information*



# Selecting the Appropriate Control Chart

## X Bar & Range





## *X Bar & R Chart*

Time	0740	0755	0815	0830	0850	0912	0935	1000	1015	1035	1100	1115
1	.056											
2	.060											
3	.058											
Total	.174											
$\bar{X}$	.058											
R	.004											

*Monitor the mean value and the variability of the process.*



## *X Bar & R Chart (cont.)*

Time	0740	0755	0815	0830	0850	0912	0935	1000	1015	1035	1100	1115
1	.056	.057	.058	.057	.059	.057	.055	.060	.060			
2	.060	.063	.060	.065	.063	.059	.059	.062	.066			
3	.058	.060	.059	.061	.061	.058	.057	.061	.063			
Total	.174	.180	.177	.183	.183	.174	.171	.183	.189			
$\bar{X}$	.058	.060	.059	.061	.061	.058	.057	.061	.063			
R	.004	.006	.002	.008	.004	.002	.004	.002	.006			



## *X Bar & R Chart Control Limits*

$\bar{X}$  Chart:

$$\text{UCL} = \bar{\bar{X}} + A_2 \bar{R} \approx \bar{\bar{X}} + 3\mathbf{s}_{\bar{x}}$$

$$\text{LCL} = \bar{\bar{X}} - A_2 \bar{R} \approx \bar{\bar{X}} - 3\mathbf{s}_{\bar{x}}$$

where  $\mathbf{s}_{\bar{x}} = \mathbf{s} / \sqrt{n}$

$R$  Chart:

$$\text{UCL} = D_4 \bar{R} \approx \bar{R} + 3\mathbf{s}_R$$

$$\text{LCL} = D_3 \bar{R} \approx \bar{R} - 3\mathbf{s}_R$$



## *X Bar & R Chart Example*

**Minitab File: Xbar\_r.mtw** contains measured data for a main shaft O.D.—see column 1(C1) = NC\_Lathe. The data is in subgroups of size 3.

The O.D. specifications are .060 +/- .003.

=====

1. Check stability with a run chart. Check for normality.
2. Using Minitab, create an Xbar and R Chart—what are your observations?
3. Do the given specifications (specs) “relate” to the Control Limits on the Xbar Chart? If so, how?
4. Use the given data and the specs to obtain Process Capability estimates for this machining operation.
5. How does Process Control “relate” to Process Capability?



# X Bar & R Chart Example (cont.)

MINITAB FILE: Xbar\_r.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the path 'Control Charts' > 'Xbar-R...' is selected. The worksheet 'Xbar\_r.mtw' is visible, containing data for two variables: 'NC\_LATHE' and 'pH'. The data is as follows:

	C1	C2	C3	C6	C7	C8	C9
↓	NC_LATHE	pH					
1	0.0560000	4.50000					
2	0.0600000	4.20000					
3	0.0580000	4.30000					
4	0.0570000	4.40000					
5	0.0630000	4.20000					
6	0.0600000	4.20000					
7	0.0580000	4.20000					
8	0.0600000	4.30000					

At the bottom of the window, the status bar reads: 'Draw a control chart for subgroup means and ranges'.



# X Bar & R Chart - Output

1. Double Click "C1."

2. Type in a 3 for Subgroup size.

3. Click "OK."

**Xbar-R Chart**

C1	NC_LATHE
C2	pH

Data are arranged as

Single column: 'NC\_LATHE'

Subgroup size: 3

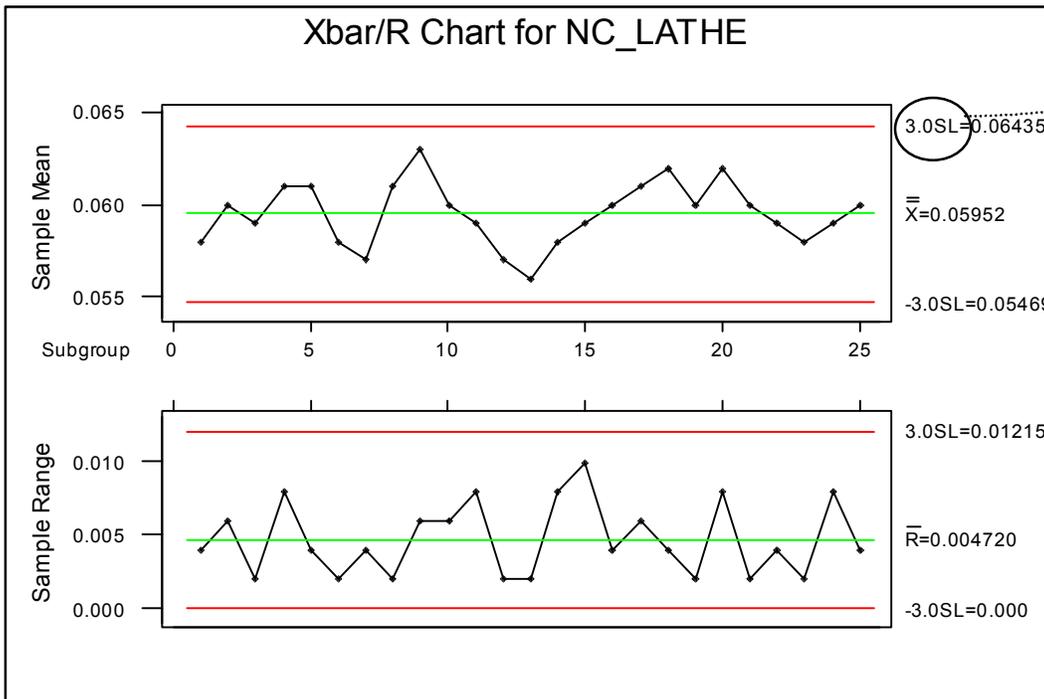
(use a constant or an ID column)

Subgroups across rows of:

Historical mean: [ ] (optional)

Historical sigma: [ ] (optional)

Select Help Estimate Parameters BY Groups in... OK Cancel



Note that 3.0 SL denotes a 3 sigma limit = Control Limit

Do not confuse this with specification limits.



## *X Bar & R Chart - Output*

*CONTINUED:*

-----

- 3. Use the given data and the specs to obtain Process Capability estimates for this machining operation.*
  
- 4. How does Process Control “relate” to Process capability?*

=====

*Use the Minitab Six Sigma Process Capability Tool with:*

*LSL = 0.057*

*Target = 0.060*

*USL = 0.063*



## *X Bar & R Chart Exercise*

---

***Minitab File: Xbar\_r.mtw*** contains pH data in time order from a protective coating bath -- see C2. Five readings are taken every hour during the first shift. The minimum allowable pH value is 4.15.

*Using Minitab, create an Xbar and R chart.*

*What are your observations?*

*What is the process Capability?*

*Comment on process Control vs. Capability.*



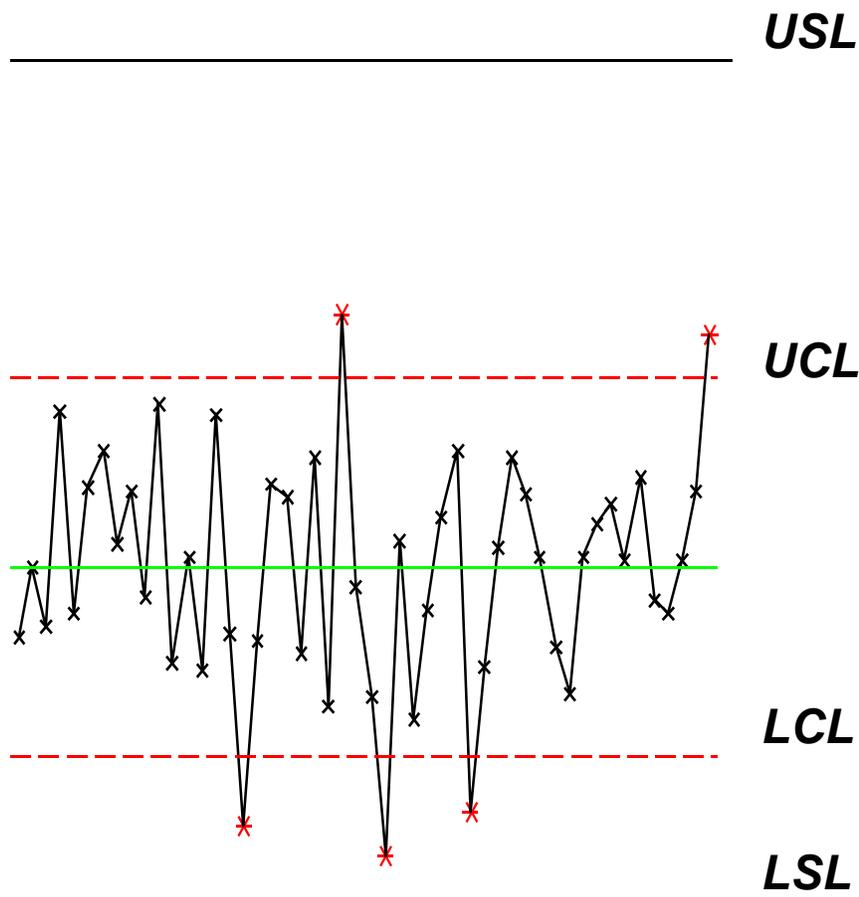
## Note:

- **Do not** confuse the control limits with the specification limits.
- Specification limits are **external** to the process. For instance, they could represent engineering requirements to satisfy a CTQ characteristic.
- Control limits are **internal** to the process, they reflect the expected range of variation for that process.
- Specification limits are for **individual** values, whereas on an X bar chart the control limits are for the **sample averages**.



## Control Limits vs. Specification Limits

*The control limits are for averages, not individual values. Most specifications are for individual values.*





## *Four Western Electric Rules*

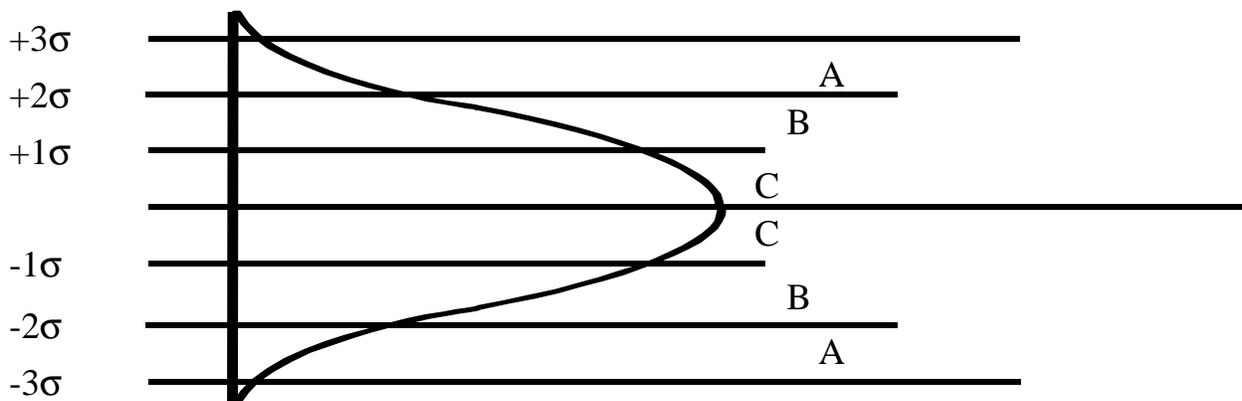
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- *The Process is “Out of Control” if...*
  - ① *A point is outside the control limits.*
  - ② *2 out of 3 consecutive points  $> 2 s$  away from the mean on the same side.*
  - ③ *4 out of 5 consecutive points  $> 1 s$  away from the mean on the same side.*
  - ④ *9 consecutive points are on one side of the mean.*



## *Minitab Rules*

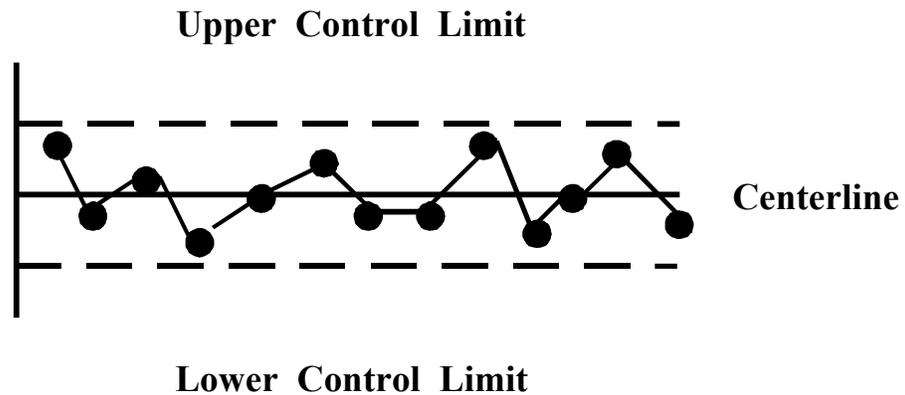
- 1. One point beyond zone A.*
- 2. Nine points in a row in zone C or beyond. (All on one side.)*
- 3. Six points in a row, all increasing or decreasing.*
- 4. Fourteen points in a row, alternating up and down.*
- 5. Two out of three points in a row in zone A or beyond.*
- 6. Four out of five points in a row in zone B or beyond.*
- 7. Fifteen points in a row in zone C, above or below center.*
- 8. Eight points in a row beyond zone C, above or below center.*





## *Process In-Control*

- *Exhibiting random variation around the centerline*

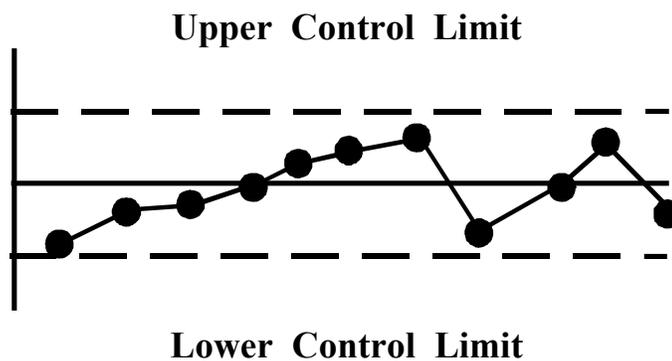
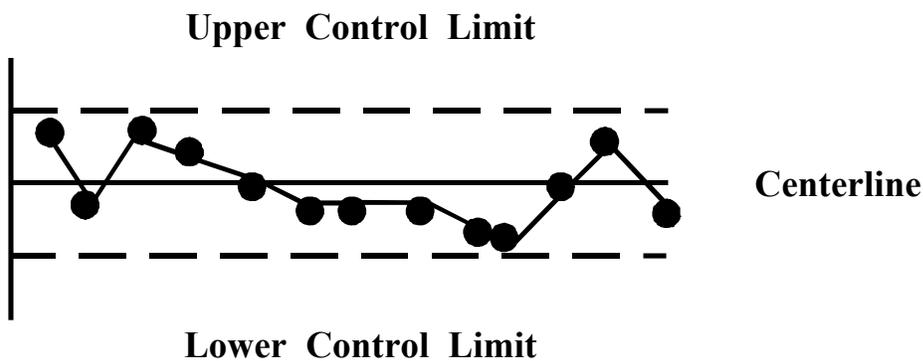


**No Evidence of Assignable Cause**



# Process Out of Control

Data Trending Downward or Upward  
Six or More Data Points in a Row

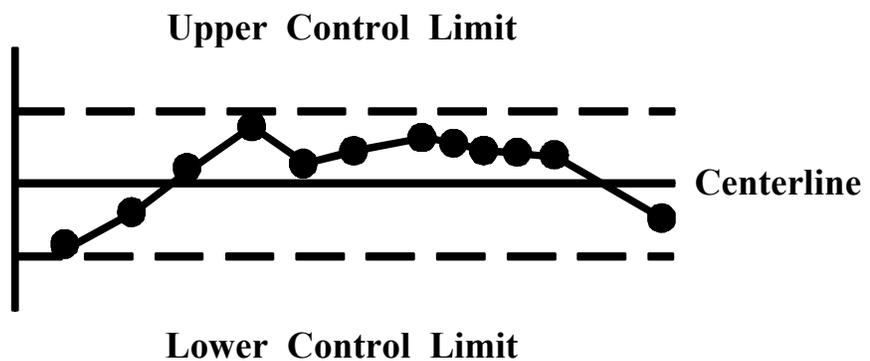
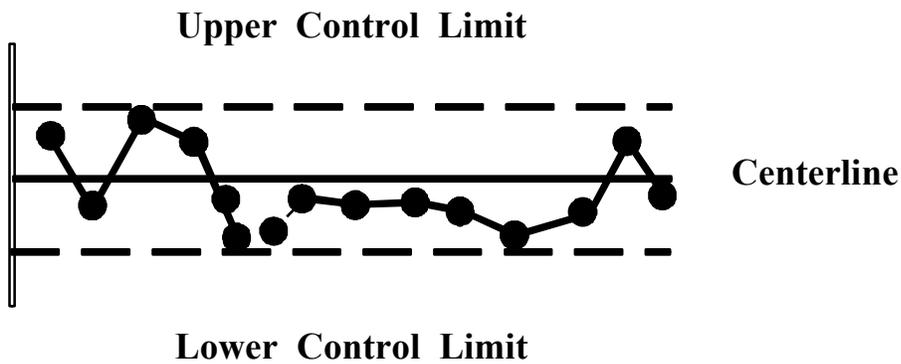


**Assignable Cause Evident**



# Process Out of Control

A Run of Nine or More Data Points in a Row On Either Side of the Centerline

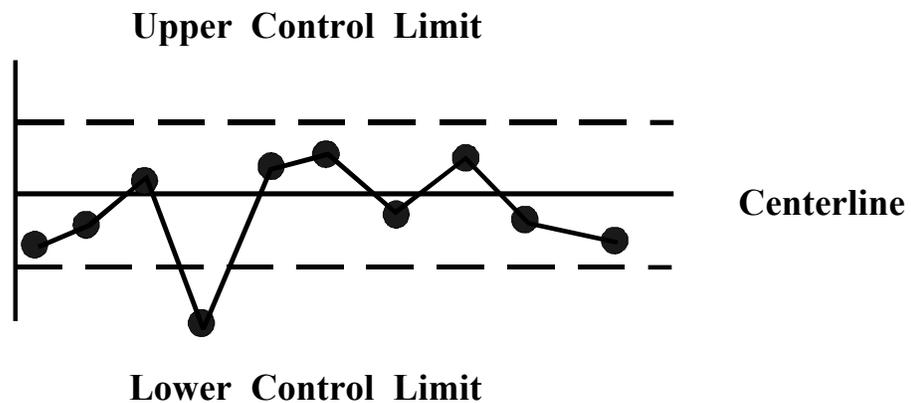
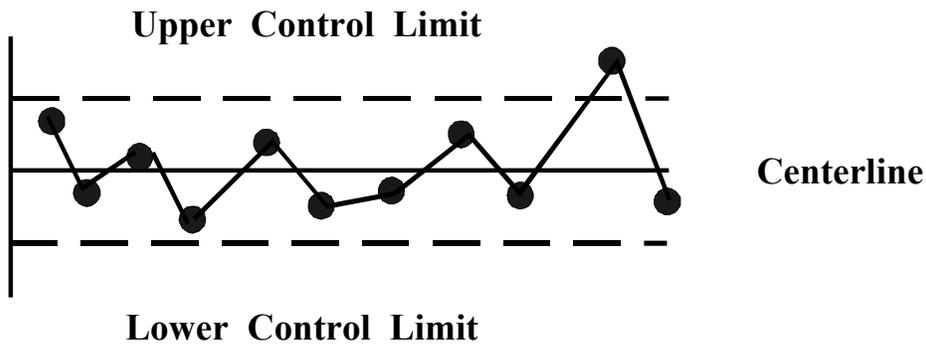


**Assignable Cause Evident**



# Process Out of Control

One or More Data Points  
Above The UCL or Below the LCL



**Assignable Cause Evident**



---

## *Best Uses of X Bar & R Charts*

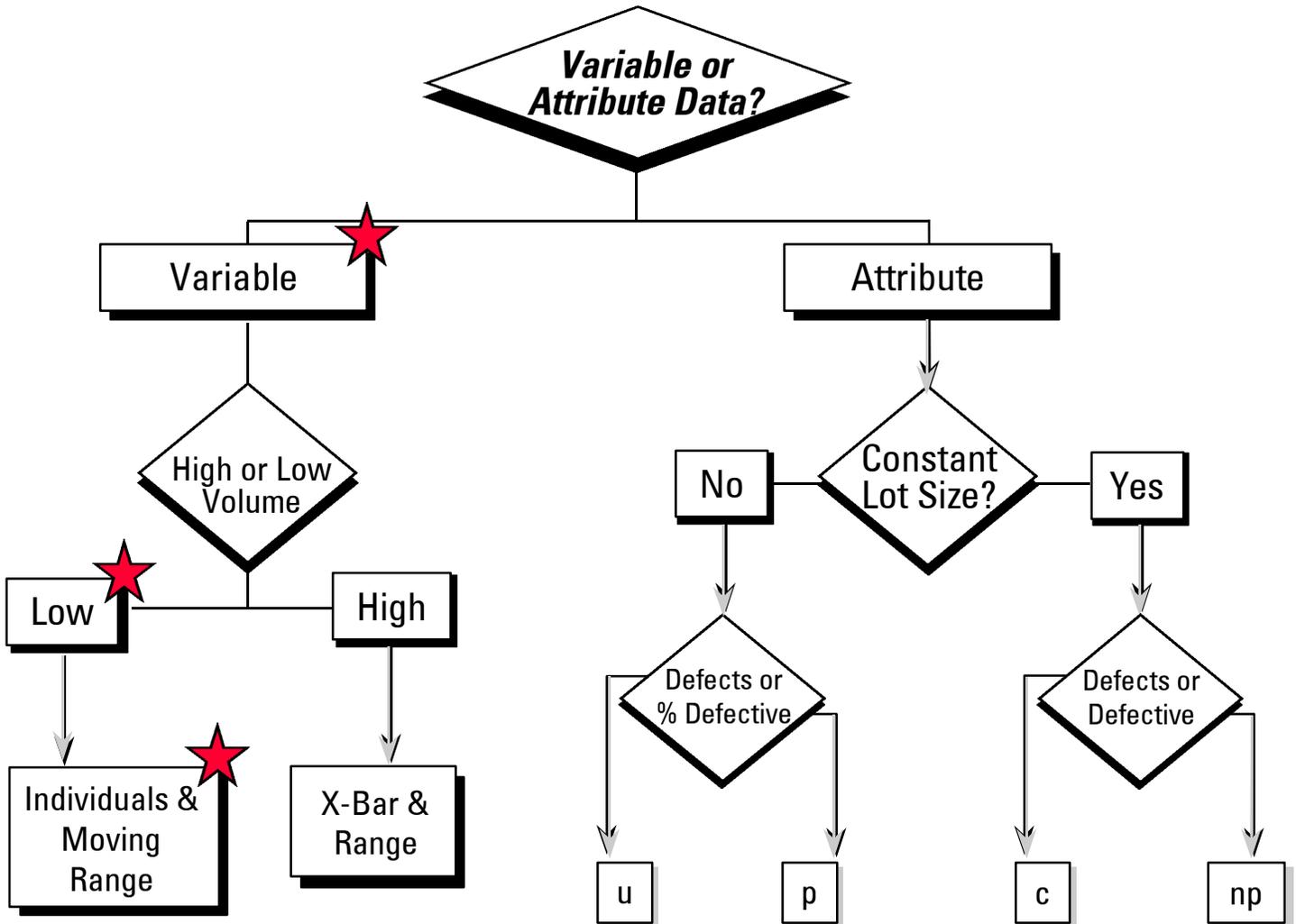
---

- *Medium To High Volume Operations*
- *Continuous vs. Intermittent Operations*
- *Six Ms Are Stable, Repetitious*



# Selecting the Appropriate Control Chart

## Individuals & Moving Range





## *Individuals & Moving Range Charts*

- *More useful in low volume, intermittent operations*
- *Similar to  $\bar{X}$  bar & R Charts, Except...*
  - *Single Values, Not Subgroups*
  - *Range Values Must Be Artificially Constructed*
  - *Somewhat “Noisier” Because Of Loss Of “Damping”*



## *I & MR Chart Control Limits*

I Chart:

$$\text{UCL} = \bar{X} + 3 \frac{\bar{R}}{d_2}$$

$$\text{Center Line} = \bar{X}$$

$$\text{LCL} = \bar{X} - 3 \frac{\bar{R}}{d_2}$$

$$\text{where } \bar{R} = \frac{\sum_{i=1}^{n-m+1} R_i}{n-m+1}$$

$$\text{and } R_i = |X_{\max} - X_{\min}|$$

$n$  = # of observations

$m$  = # of elements used  
to calculate the  
moving range.

MR Chart:

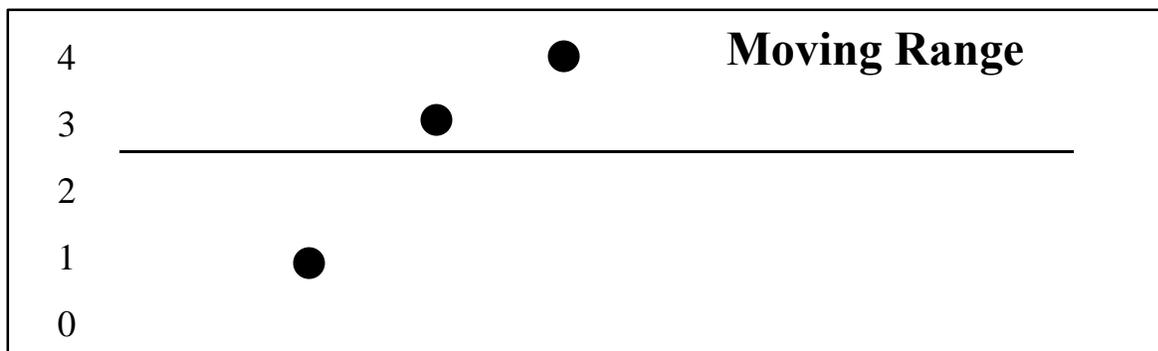
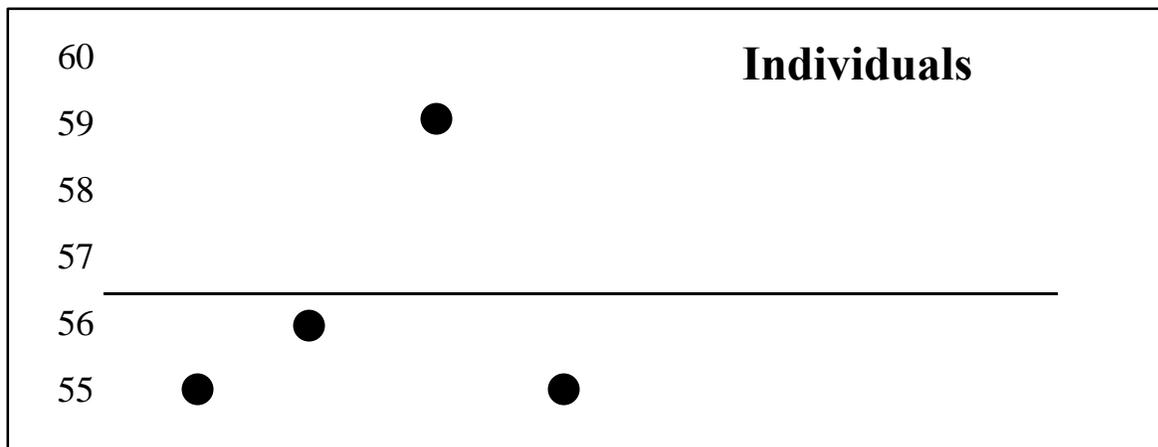
$$\text{Center Line} = \bar{R}$$

Calculate control limits using Minitab



# Building an Individuals and Moving Range Chart

<u>Individual Data</u>	<u>Moving Range</u>
55	N/A
56	$ABS(55-56) = 1$
59	$ABS(56-59) = 3$
55	$ABS(59-55) = 4$





## *Example:*

# *Individuals & Moving Range Chart*

---

- *Data from a shaft diameter turning operation are entered on the control chart form on the next page for 25 consecutive pieces of product, in production sequence.*
- *The data is in **Minitab File: Imr.mtw**, column shaft\_OD. Using Minitab, create the I-MR chart.*
  - *Analyze your results. Are there out-of-control indications? List the indications, if any, by type and by plot point numbers.*
- *What is happening in the process?*



# Example: Individuals & Moving Range Chart

## MINITAB FILE: Imr.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the path 'Control Charts' > 'I-MR...' is selected. The worksheet 'Imr.mtw' is visible, containing data for 'Shaft\_OD' and 'runout'. The status bar at the bottom indicates the action: 'Draw a control chart for observations and a moving range chart'.

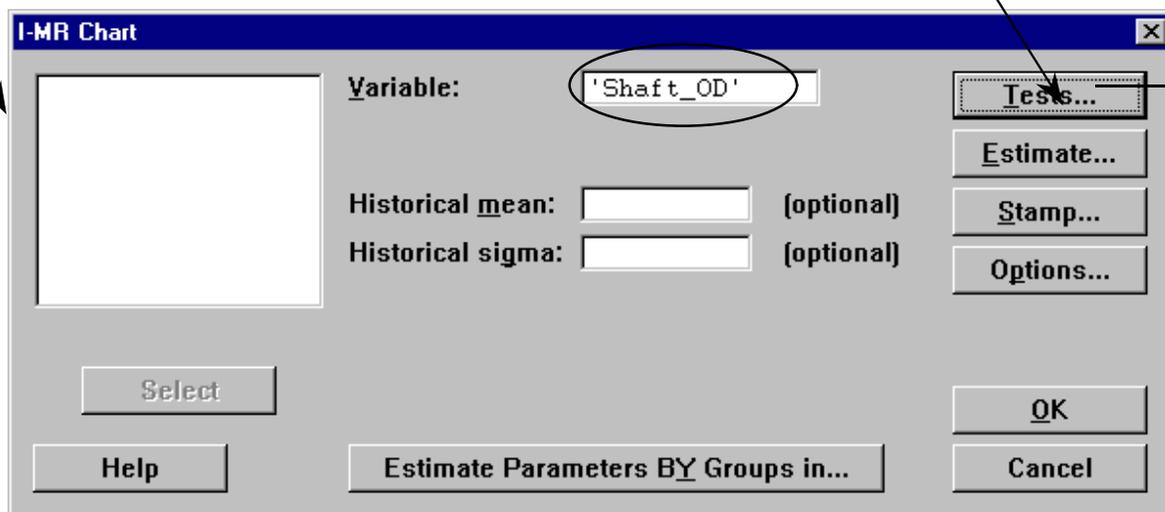
	C1	C2	C3	C6	C7	C8	C9
↓	Shaft_OD	runout					
1	0.249500						
2	0.250000	4					
3	0.250500	4					
4	0.250000	0					
5	0.250500	-4					
6	0.250000	4					
7	0.250500	4					
8	0.250500	2					



# Input: Individuals & Moving Range Chart

1. Double click on  
"Shaft\_OD."

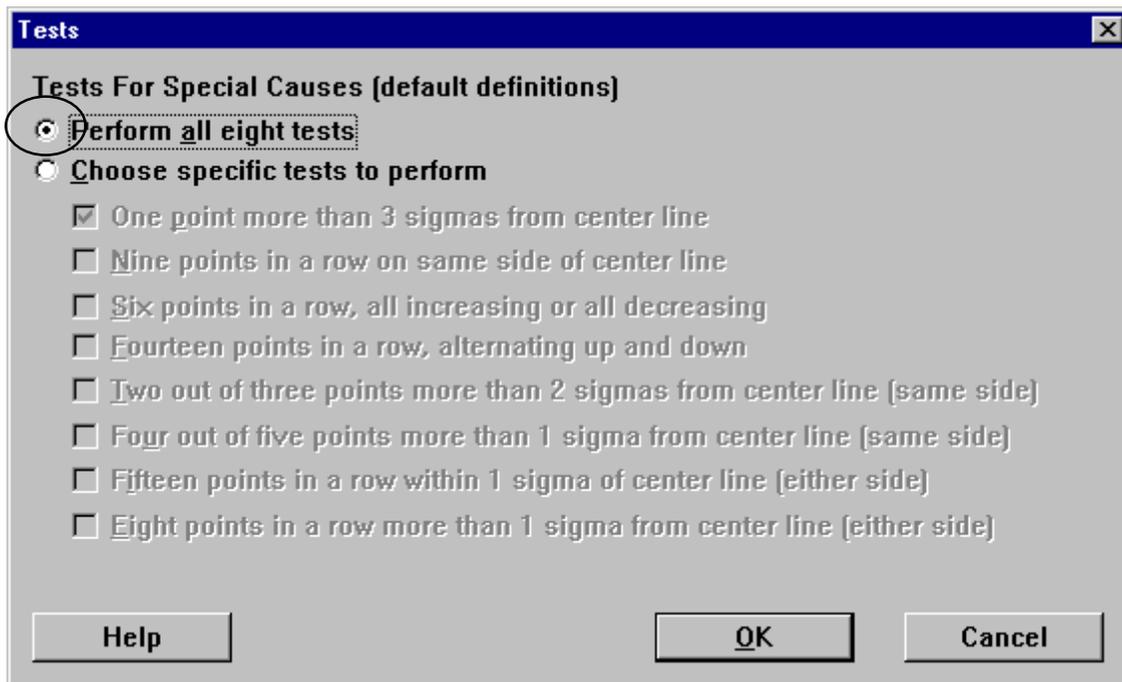
2. Click "Tests."



The I-MR Chart dialog box is shown with the following fields and buttons:

- Variable:** 'Shaft\_OD' (circled)
- Historical mean:** [ ] (optional)
- Historical sigma:** [ ] (optional)
- Buttons:** Select, Help, Estimate Parameters BY Groups in..., Tests... (circled), Estimate..., Stamp..., Options..., OK, Cancel

3. Click on  
Perform all eight  
tests.

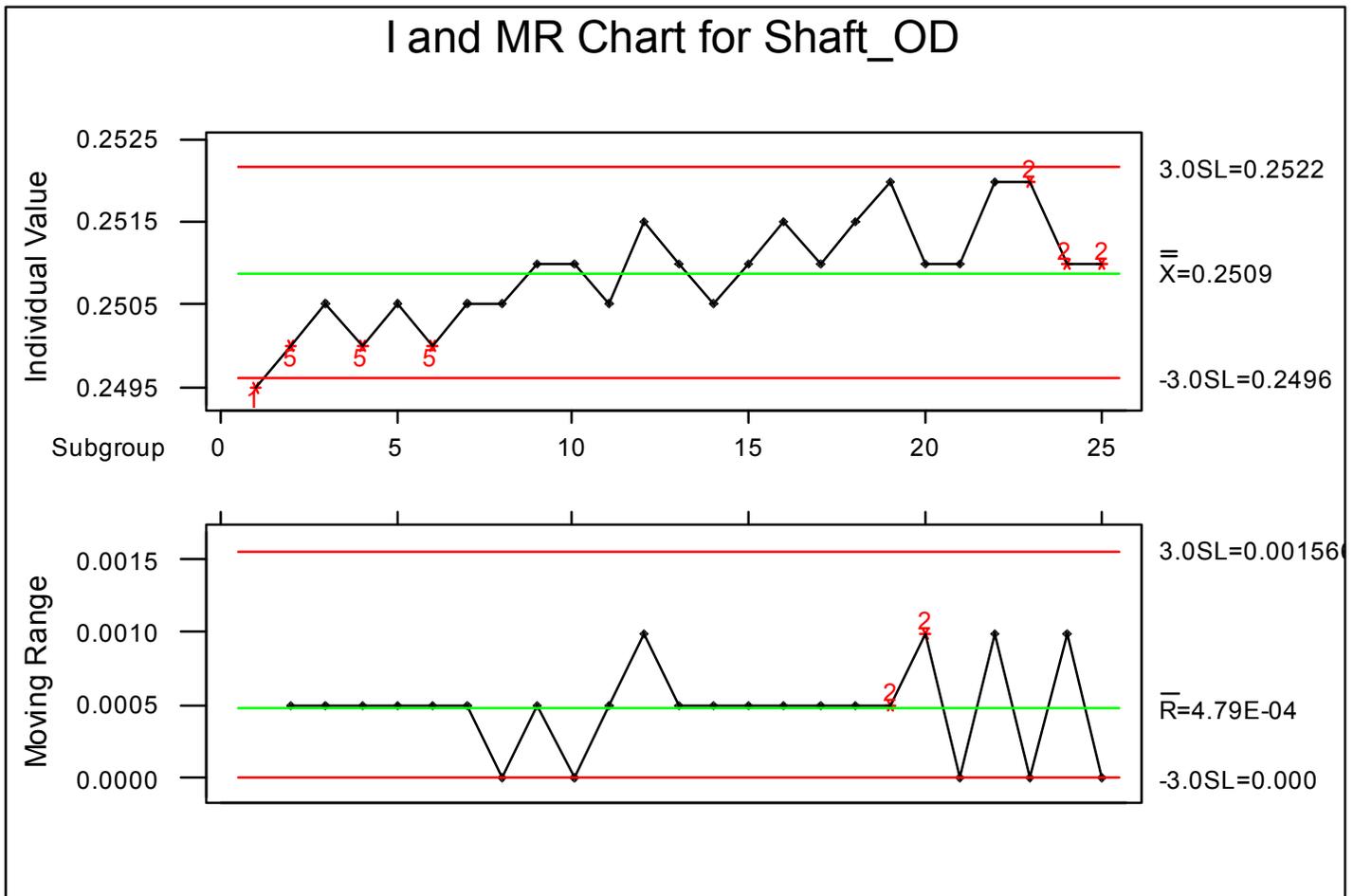


The Tests dialog box is shown with the following options:

- Tests For Special Causes (default definitions)**
- Perform all eight tests** (circled)
- Choose specific tests to perform**
- One point more than 3 sigmas from center line
- Nine points in a row on same side of center line
- Six points in a row, all increasing or all decreasing
- Fourteen points in a row, alternating up and down
- Two out of three points more than 2 sigmas from center line (same side)
- Four out of five points more than 1 sigma from center line (same side)
- Fifteen points in a row within 1 sigma of center line (either side)
- Eight points in a row more than 1 sigma from center line (either side)
- Buttons:** Help, OK, Cancel



# Output: Individuals & Moving Range Chart





## *Exercise: Individuals & Moving Range Chart*

- *The data for 24 consecutive measurements of runout on a machined diameter are contained in **Minitab File: lmr.mtw**, column runout. The data are recorded as plus or minus from zero in whole numbers representing .001”.*
  - *Using Minitab, create an I-MR chart.*
  - *What are your observations?*
  - *What action, if any, should be taken?*



## *X Bar and R Chart*

---

- *Cluster or periodic measurements of characteristic*
- *Frequency depends on line speed and stability*
- *Subgroup averages plotted on X-bar chart*
- *X-bar chart monitors central tendency of a process over time*
- *Subgroup ranges plotted on Range chart*
- *R chart monitors the variability of a process over time*
- *Provides data-smoothing effect*



## *Individuals & Moving Range Chart*

- *Useful in low volume, intermittent processes*
- *Range value artificially constructed from successive readings*
- *Subgroup size ( $n$ ) usually 2*
- *Some correlation between charts is possible*
- *Less damping, more “noise” in chart—tougher to spot true process shift*
- *Displays the variability between individual observations over time*
- *Assumes that past and present data is equally important*



## *A Valid Variable Control Chart Has...*

- *Data in time or production sequence*
  - *to show stability, time-to-time variation*
- *A measure of central tendency*
  - *to portray behavior of process center*
- *A measure of variability*
- *Control limits*
  - *to allow separating common cause from assignable cause*



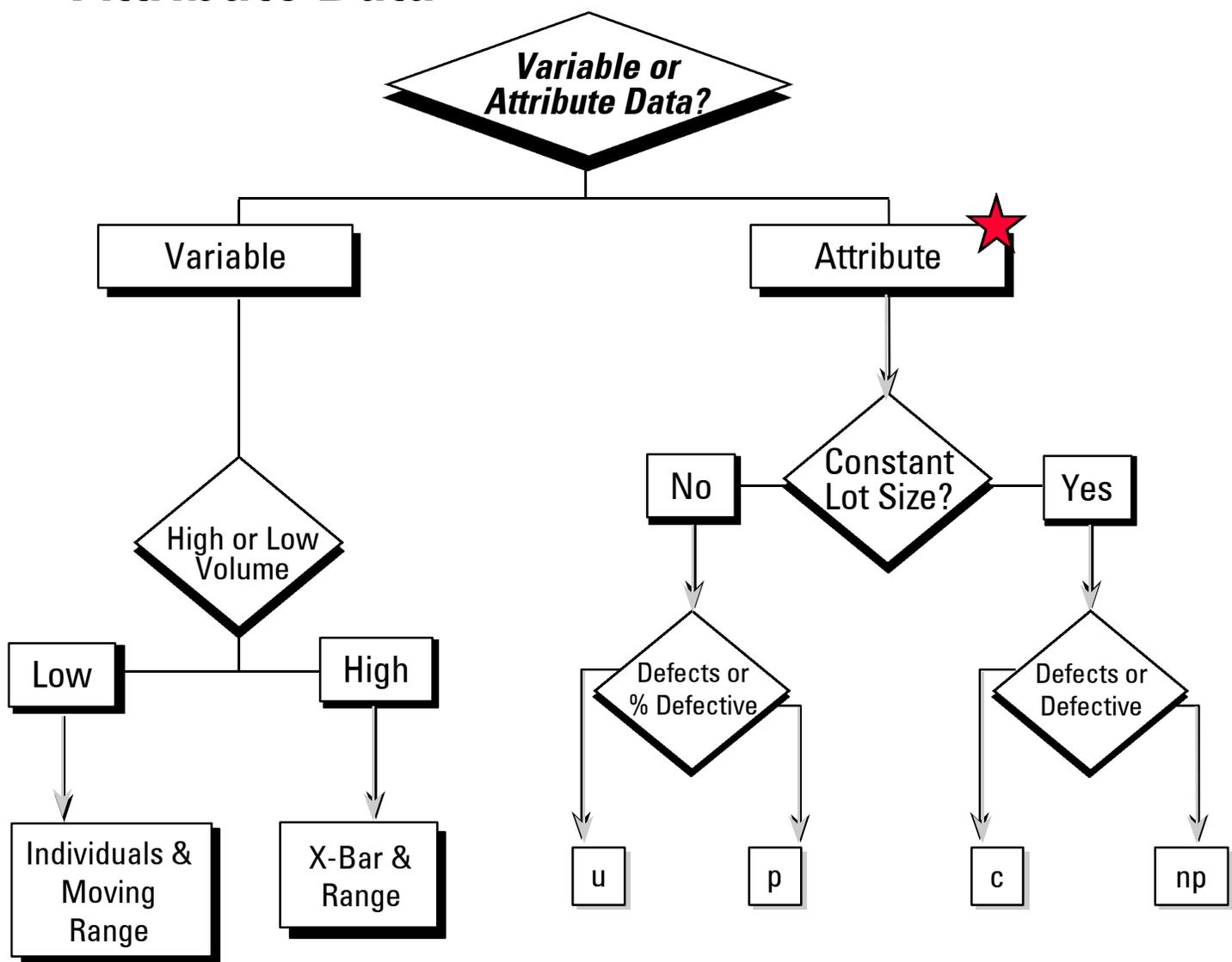
---

# *Attribute Control Charts*



# Selecting the Appropriate Control Chart

## Attribute Data





## *Attribute Control Charts*

- *Useful when characteristic measurements are not available*
- *Based on counting/classifying (Go/No Go, Pass/Fail, Good/Bad)*
- *Based on Poisson or Binomial Distribution statistics*
- *Control limits calculated differently from variable control limits, but interpreted in similar fashion*
- *One chart can cover any number of characteristics, but can be more difficult to analyze signals*
- *One chart instead of two—no range chart*

*Precise operational definitions of a defect are important. Operational definitions can then be uniformly and unambiguously applied by all inspectors.*



## *Important Definitions*

### ■ **A Defect**

- *A single characteristic that does not meet requirements*

### ■ **A Defective**

- *A unit that contains one or more DEFECTS*

***Attribute Charts Can Consider Either Case Depending On The Chart Type Chosen***



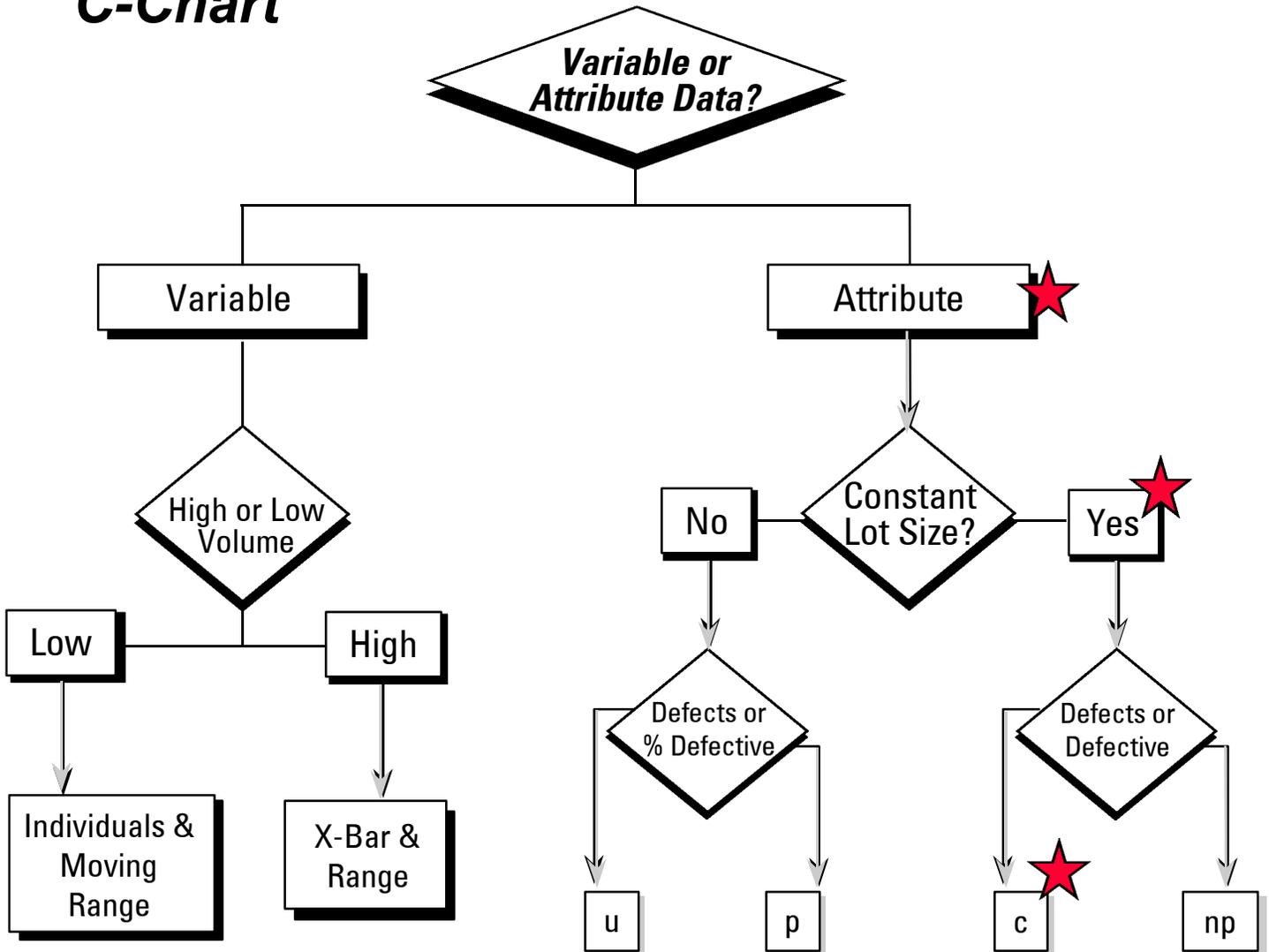
# Classification of Attribute Chart Types

	Constant Lot / Unit Size	Variable Lot / Unit Size	
Defects	<b>c</b>	<b>u</b>	Poisson
Defective	<b>n p</b>	<b>p</b>	Binomial



# Selecting the Appropriate Control Chart

## C-Chart





## C-Chart

- *Chart for defects per unit (subgroup)*
- *Based on Poisson distribution*
  - *High probability of finding defect of some type. Large samples are needed if defect probabilities are low.*
  - *Lower probability of a defect of a given type*
- *Works best on complex unit of product*
- *Constant subgroup/lot size*

$$\bar{C} = \frac{\text{Total number of defects}}{\text{Total number of units or subgroups of constant size}}$$

$$\text{UCL} = \bar{C} + 3 \sqrt{\bar{C}}$$

$$\text{LCL} = \bar{C} - 3 \sqrt{\bar{C}}$$



## C-Chart Example

### Attribute Data

- Manufacturing data indicates that a significant loss occurs from welding nonconformances on part A detected at NDT. The data on the number and general type of nonconformity for each part tested is maintained by serial number in the NDT log books.
- To determine the current performance of the welding process we will plot the number of nonconformities per subgroup of two parts on a C-Chart using the data from the log book:

Date	6/1	6/2	6/3	6/4	6/5	6/8	6/9
Number of Nonconf. per Subgroup	4	2	5	6	10	5	6
		3	2	8	5	7	6
		7	4	9	9		
			7	5	7		
				4	5		
				6	6		
					7		

### Example Part I.

- The file C\_chart, column weld\_I, contains the data given above.
1. Using Minitab, create a C-Chart.
  2. Is the high level of nonconformance we are experiencing due to an assignable cause or random variation?
  3. What are some actions for consideration to reduce the level of nonconformances generated by this process?
  4. What is the Process Capability?



# C-Chart Example - Part I

MINITAB FILE: C\_chart.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the path 'Stat > Control Charts > C...' is highlighted. The data table at the bottom contains the following information:

	C1	C2	C3	C6	C7	C8	C9
↓	Weld_I	Weld_II	Contract:				
1	4	6	15				
2	2	7	15				
3	3	1	15				
4	7	5	25				
5	5	4	15				
6	2	5	15				
7	4	1	16				
8	7	3	26				

Draw a control chart for the number of defects



# C-Chart Example - Input

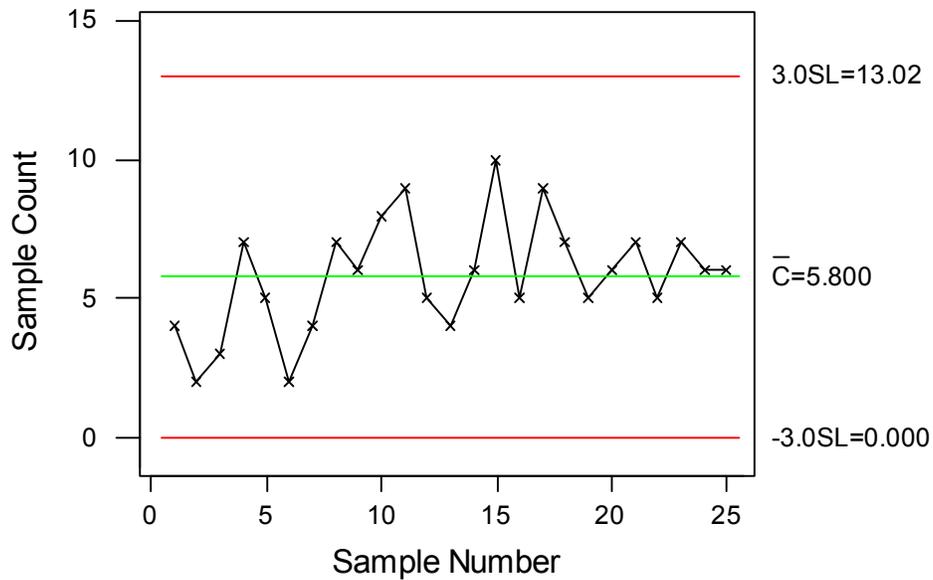
**C Chart** [X]

Variable:

Historical mu:  (optional)

Annotation   
Frame   
Regions

C Chart for Weld\_I





# Capability Analysis

## Report 7: Product Performance

Characteristic	Defs	Units	Opps	TotOpps	DPU	DPO	PPM	ZShift	ZBench
1	4	2	20	40	2.000	0.100000	100000	1.500	2.782
2	2	2	20	40	1.000	0.050000	50000	1.500	3.145
3	3	2	20	40	1.500	0.075000	75000	1.500	2.940
4	7	2	20	40	3.500	0.175000	175000	1.500	2.435
5	5	2	20	40	2.500	0.125000	125000	1.500	2.650
6	2	2	20	40	1.000	0.050000	50000	1.500	3.145
7	4	2	20	40	2.000	0.100000	100000	1.500	2.782
8	7	2	20	40	3.500	0.175000	175000	1.500	2.435
9	6	2	20	40	3.000	0.150000	150000	1.500	2.536
10	8	2	20	40	4.000	0.200000	200000	1.500	2.342
11	9	2	20	40	4.500	0.225000	225000	1.500	2.255
12	5	2	20	40	2.500	0.125000	125000	1.500	2.650
13	4	2	20	40	2.000	0.100000	100000	1.500	2.782
14	6	2	20	40	3.000	0.150000	150000	1.500	2.536
15	10	2	20	40	5.000	0.250000	250000	1.500	2.174
16	5	2	20	40	2.500	0.125000	125000	1.500	2.650
17	9	2	20	40	4.500	0.225000	225000	1.500	2.255
18	7	2	20	40	3.500	0.175000	175000	1.500	2.435
19	5	2	20	40	2.500	0.125000	125000	1.500	2.650
20	6	2	20	40	3.000	0.150000	150000	1.500	2.536
21	7	2	20	40	3.500	0.175000	175000	1.500	2.435
22	5	2	20	40	2.500	0.125000	125000	1.500	2.650
23	7	2	20	40	3.500	0.175000	175000	1.500	2.435
24	6	2	20	40	3.000	0.150000	150000	1.500	2.536
25	6	2	20	40	3.000	0.150000	150000	1.500	2.536
Total	145			1000		0.145000	145000	1.500	2.558

**Roll up Zbench = 2.558**

**DPMO = 145,000**

**Process is:**

- **in Control**
- **not very Capable**



## C-Chart Example - Part II

We will assume for the purpose of our exercise that investigation reveals that improved welder training could substantially reduce the random variation in our stable process. A training program is developed and implemented, but how can we determine if the action we have taken has actually reduced the random variation in the process? Let's gather data from the revised process and construct another C-Chart, again using the constant subgroup size of two parts. If the new process is in control, we can compare the control limits with the earlier chart and determine the impact of our training. Also, if no assignable causes are evident, we can begin to make an estimate of process capability for the new process.

The data from the new process:

Date	7/1	7/2	7/3	7/4	7/5	7/8
Nonconformances	6	1	5	6	2	6
per subgroup	7	5	1	2	5	6
		4	3	3	4	2
			3	3	5	
			2	4	3	
					5	
					6	

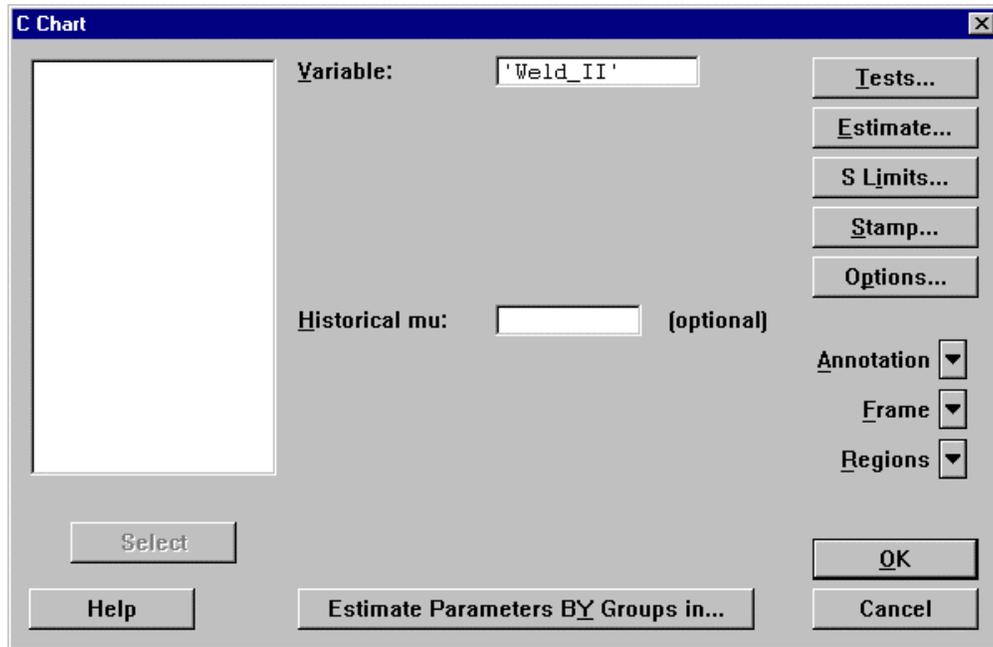
The new data is in file C\_chart.mtw, column weld\_II.

1. Using Minitab, create the C-Chart.
2. Is the new process in control?
3. Has the training improved the process?
4. Can we make an estimate of process capability?
5. How would you state the process capability of the process?



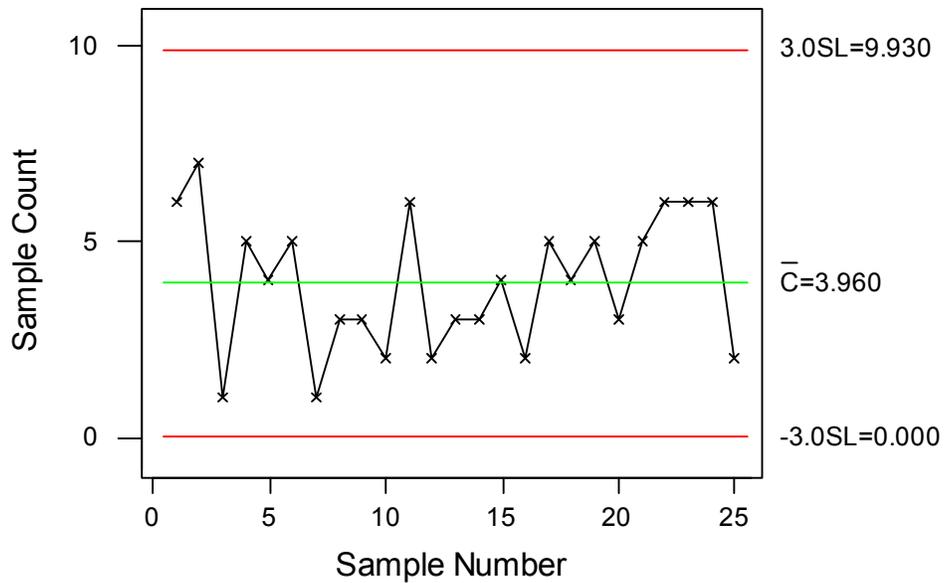
## C-Chart Example - Part II (cont.)

MINITAB FILE: C\_chart.mtw



The image shows the MINITAB 'C Chart' dialog box. The 'Variable' field is set to 'Weld\_II'. The 'Historical mu' field is empty with '(optional)' next to it. On the right side, there are buttons for 'Tests...', 'Estimate...', 'S Limits...', 'Stamp...', and 'Options...'. Below these are dropdown menus for 'Annotation', 'Frame', and 'Regions'. At the bottom, there are buttons for 'Select', 'Help', 'Estimate Parameters BY Groups in...', 'OK', and 'Cancel'.

C Chart for Weld\_II





---

## *C-Chart Exercise*

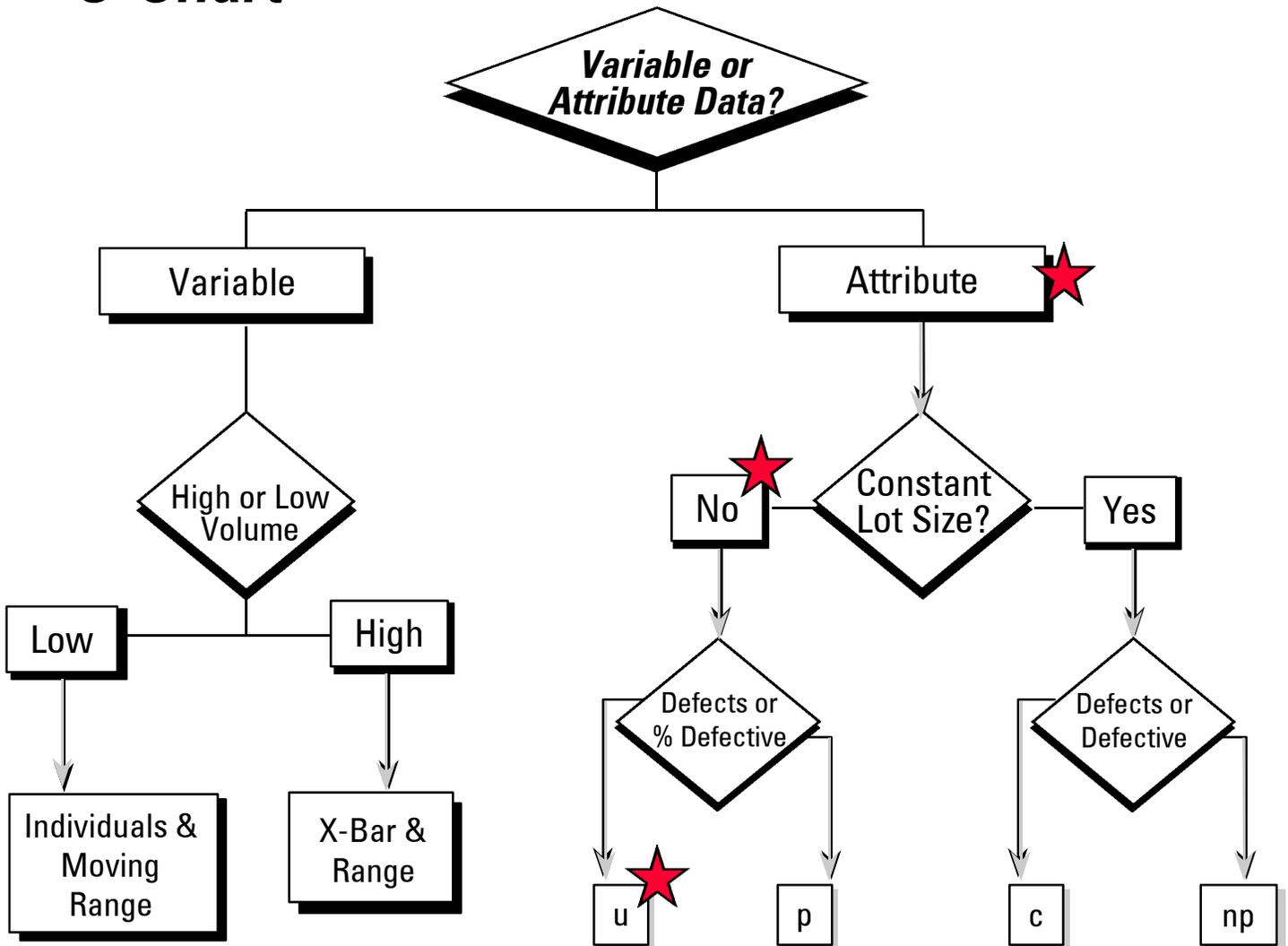
---

- *File C\_chart.mtw contains time order data for Power Plants contracts.*
- *Using Minitab, create a C-Chart of the data.*
- *What are your observations?*



# Selecting the Appropriate Control Chart

## U-Chart





## *U-Chart*

- *U-Chart - defects per unit, variable lot (subgroup) size*
- *Same logic as C-Chart, except variable lot (subgroup) size (n)*

$$\bar{u} = \frac{\text{total number of defects}}{\text{total number of units}}$$

$$\text{UCL} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$

$$\text{LCL} = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$$



## *U-Chart Example*

---

- *In file U\_chart.mtw, column “errors” contains time order data of customer parts order defects found each day. A defect is defined to be inaccurate information found on a parts order requisition. Both the number of defects and the daily number of orders are recorded.*
- *Using Minitab, construct a U-Chart of the data.*
- *Using the Six Sigma Product Report obtain the Process Capability data for this parts ordering operation.*
- *What are your observations?*



# U-Chart Example

## Minitab Menu Commands

MINITAB FILE: U\_Chart.mtw

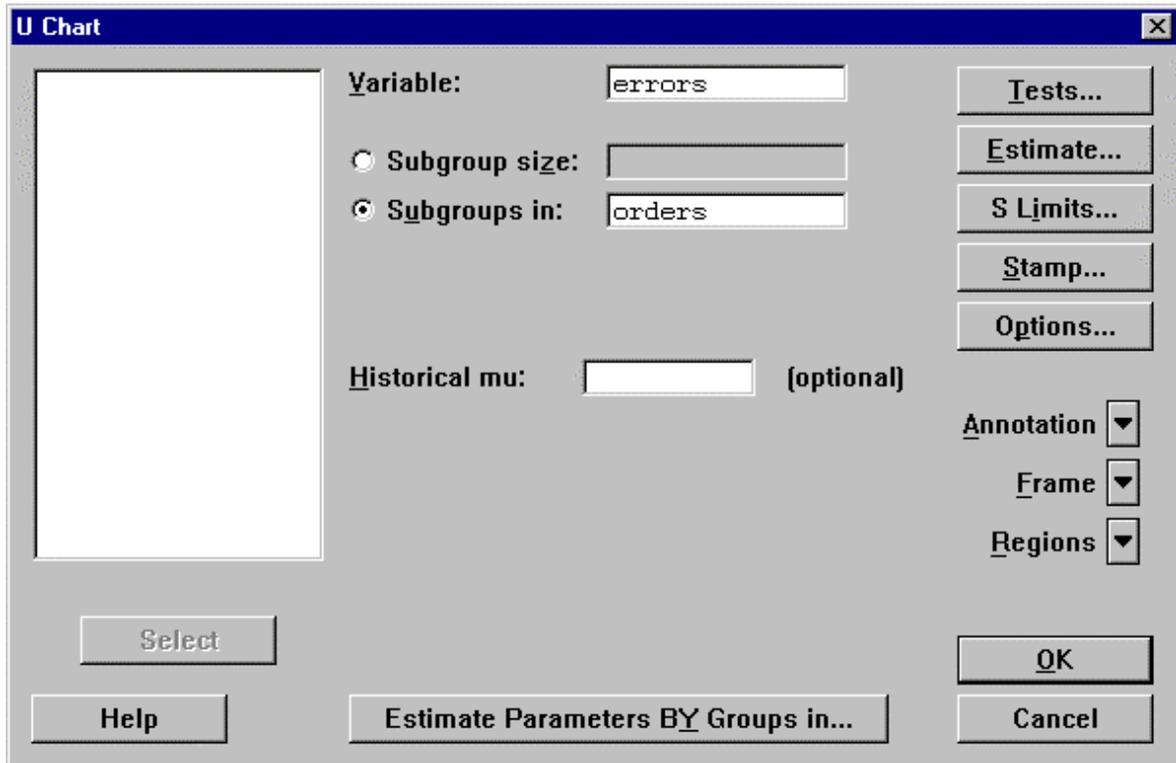
The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the 'Control Charts' option is selected. The 'U-Chart' option is highlighted in the submenu. The worksheet 'U\_chart.mtw' is open, showing data for columns C1 (errors), C2 (orders), and C3 (leaks). The data is as follows:

	C1	C2	C3
→	errors	orders	leaks
1	114	95	1
2	142	95	1
3	146	95	1
4	257	95	1
5	185	95	1
6	228	95	2
7	327	95	4
8	269	95	11

Draw a control chart for the number of defects per unit sampled

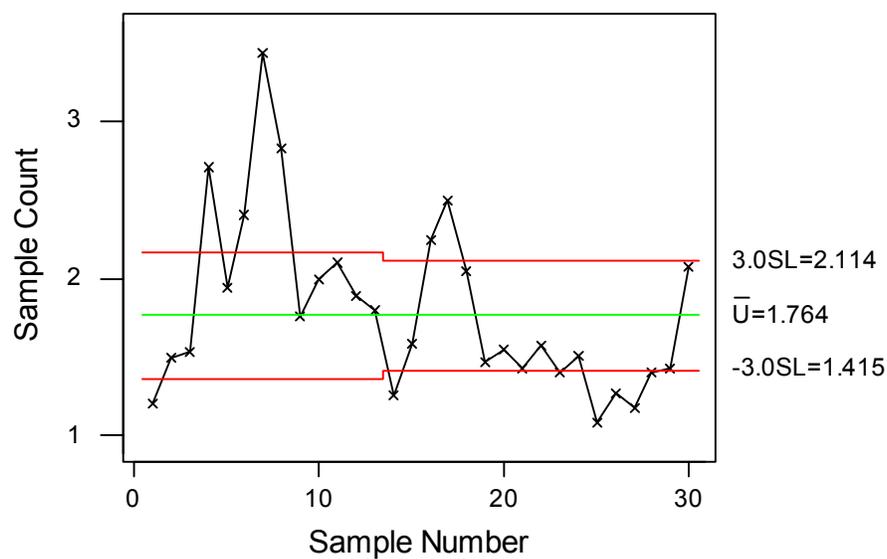


# U-Chart Minitab Input & Output



The image shows the Minitab 'U Chart' dialog box. The 'Variable' field is set to 'errors'. The 'Subgroups in' field is set to 'orders'. The 'Historical mu' field is empty and labeled '(optional)'. On the right side, there are buttons for 'Tests...', 'Estimate...', 'S Limits...', 'Stamp...', and 'Options...'. Below these are dropdown menus for 'Annotation', 'Frame', and 'Regions'. At the bottom, there are buttons for 'Select', 'Help', 'Estimate Parameters BY Groups in...', 'OK', and 'Cancel'.

U Chart for errors





## *U-Chart Exercise*

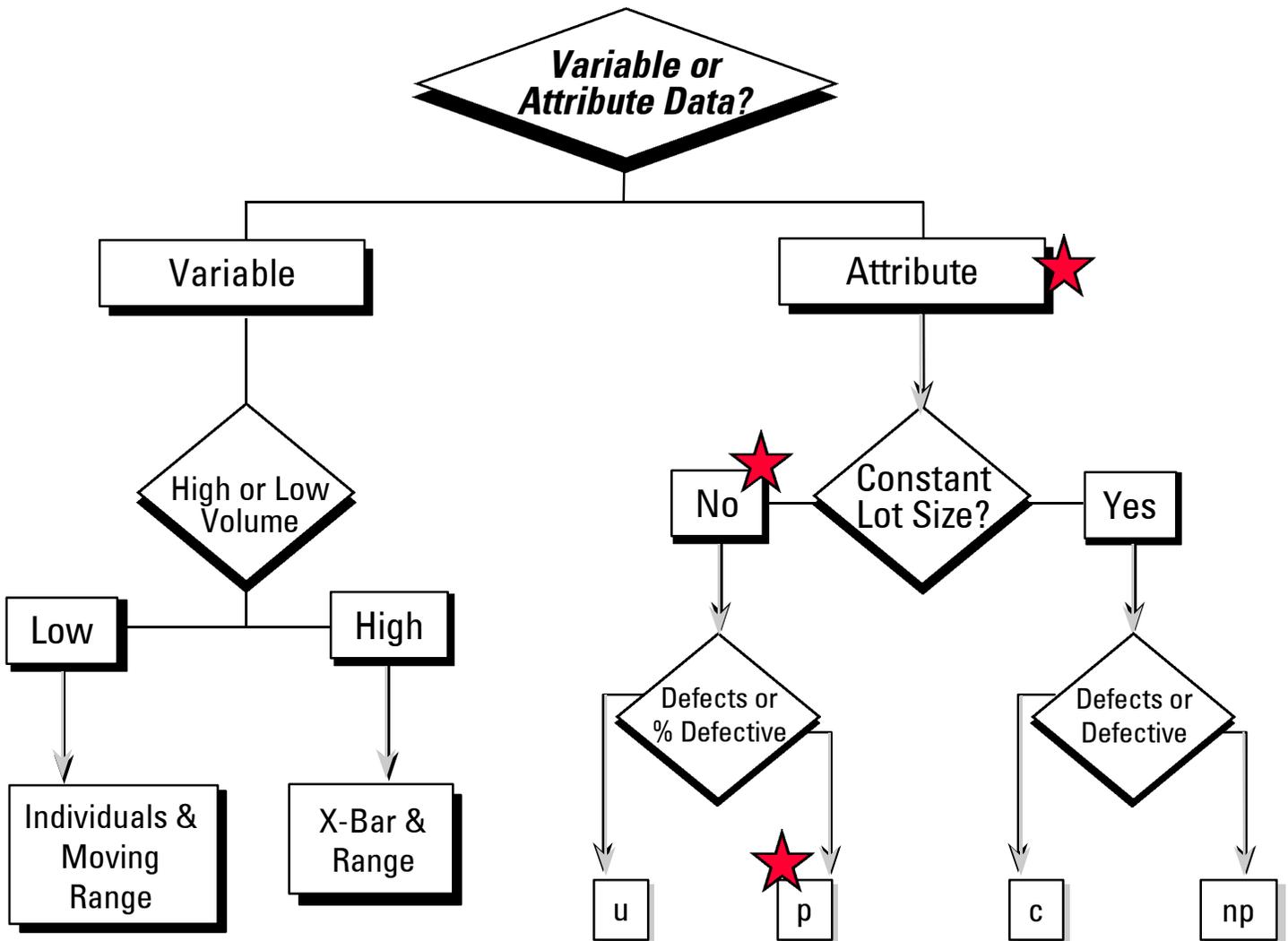
---

- *Two portions of an automobile radiator are assembled together. The number of leaks detected along with the number of assembled radiators were recorded.*
- *The file U\_chart.mtw contains the data.*
- *Using Minitab, construct a U-Chart of the data. Also estimate the Process Capability.*
- *What are your observations?*



# Selecting the Appropriate Control Chart

## P-Chart





## *P-Chart*

- *Chart for fraction/percent nonconforming*
- *Accommodates varying lot sizes*
- *Based on Binomial distribution (pass/fail, good/bad)*

$$\bar{p} = \frac{\text{Total number of defective units}}{\text{total number of units}}$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

### ***P-Chart Example***

*In file P\_chart.mtw, column “voids” contains data for number of parts containing coating voids found at inspection after a coating operation.*

*Using Minitab, create a P-Chart of the data.*

*What are your observations? What is needed to estimate the Process Capability?*



# Commands

## MINITAB FILE: P\_chart.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the 'Control Charts' sub-menu is selected. The 'P...' option is highlighted. The worksheet 'P\_chart.mtw' is visible, showing a table with columns C1 (Voids), C2 (subgrp), and C3-T (month). The data rows are as follows:

	C1	C2	C3-T
↓	Voids	subgrp	month
1	8	968	Jan-95
2	13	1216	Feb-95
3	13	804	Mar-95
4	16	1401	Apr-95
5	14	1376	May-95
6	15	995	Jun-95
7	13	1202	Jul-95
8	10	1028	Aug-95

The status bar at the bottom of the window reads: "Draw a control chart for the proportion of defectives".



# P-Chart Minitab Input & Output

**P Chart**

Variable:

Subgroup size:

Subgroups in:

Historical p:  (optional)

Annotation

Frame

Regions

Select

Help

Estimate Parameters BY Groups in...

Tests...

Estimate...

S Limits...

Stamp...

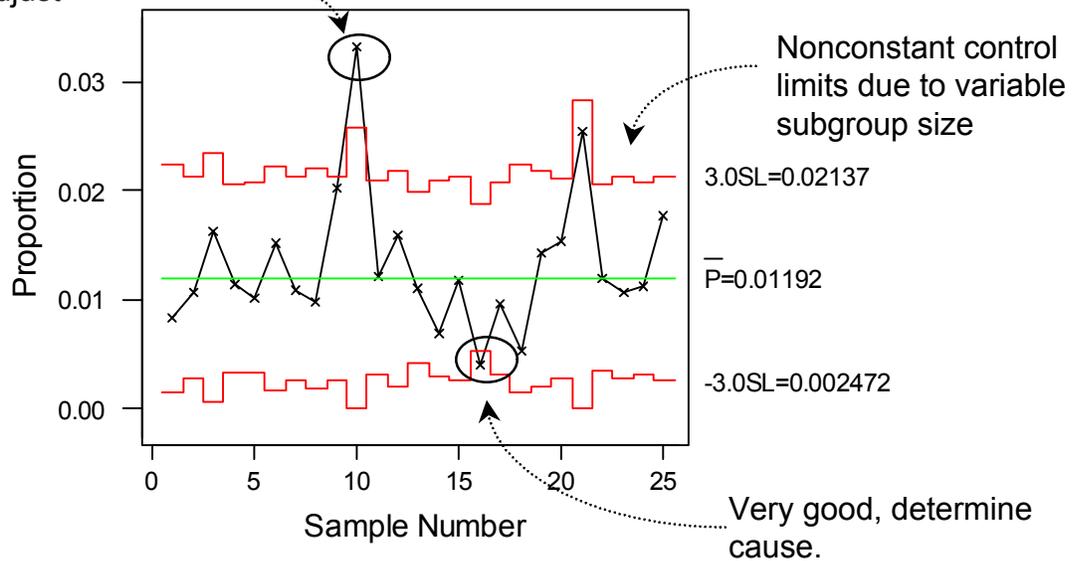
Options...

OK

Cancel

Out of controls: determine cause and adjust

P Chart for Voids





## *P-Chart Exercise*

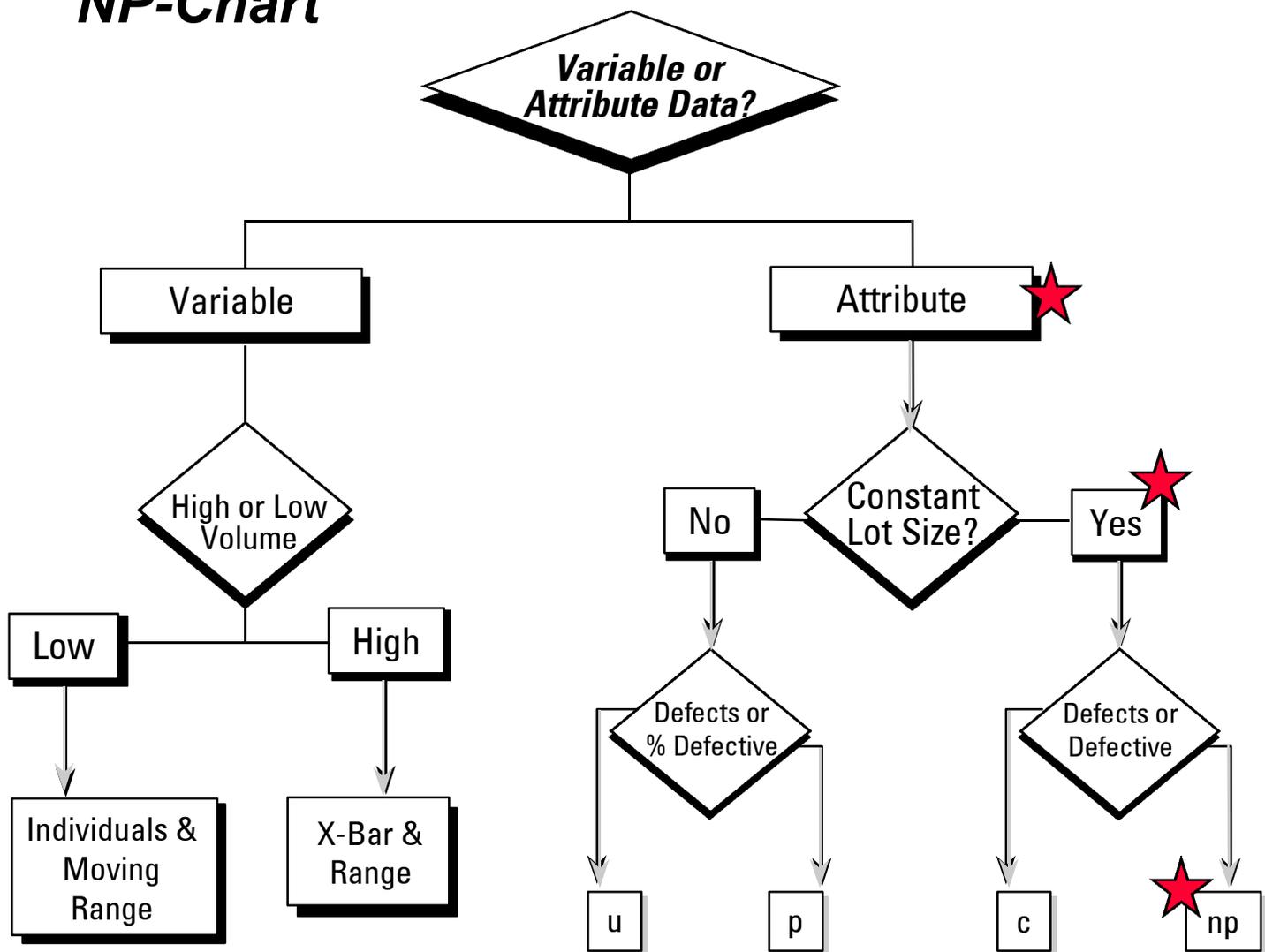
---

- *Customer invoices are checked for entry errors such as a bad P.O. number, a bad price entry, etc. The number of bad customer invoices per month along with the total number of invoices for that month has been recorded for the last 13 months.*
- *File P\_chart.mtw contains the data.*
- *Using Minitab, construct a P-Chart of the data.*
- *What are your observations?*
- *What is needed to estimate Process Capability? Discuss.*



# Selecting the Appropriate Control Chart

## NP-Chart





## *NP-Chart*

- *Np-chart: number non-conforming in subgroup*
- *Same logic as the p-chart, except constant lot size (n)*

$$\bar{np} = \frac{\text{total number of defective units}}{\text{total number of subgroups of } n \text{ units}}$$

$$\text{UCL} = \bar{np} + 3\sqrt{\bar{np}(1 - \bar{p})}$$

$$\text{LCL} = \bar{np} - 3\sqrt{\bar{np}(1 - \bar{p})}$$



## *NP-Chart Example*

---

- *In file `Np_chart.mtw`, column “switches” contains inspection data from 25 consecutive lots of electrical switches.*
- *The lot size is constant at 100 switches per lot.*
- *Using Minitab, create an Np-Chart of the data.*
- *What are your observations?*
- *What is the Process Capability? What do you need to know to answer this?*



# NP-Chart Example

## Minitab Menu Commands

MINITAB FILE: Np\_chart.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and the 'Control Charts' sub-menu is selected. The 'NP...' option is highlighted. The worksheet 'Np\_chart.mtw' is visible, showing data for 'switches' and 'unsatisfied'.

	C1	C2	C3	C6	C7	C8	C9
↓	switches	unsatisfied					
1	11	11					
2	9	20					
3	15	19					
4	11	24					
5	22	19					
6	14	18					
7	7	16					
8	10	12					

Draw a control chart for the number of defectives



# NP-Chart

## Minitab Input & Output

NP Chart

Variable: switches

Subgroup size: 100

Subgroups in:

Historical p: (optional)

Annotation ▾

Frame ▾

Regions ▾

Select

Help

Estimate Parameters BY Groups in...

Tests...

Estimate...

S Limits...

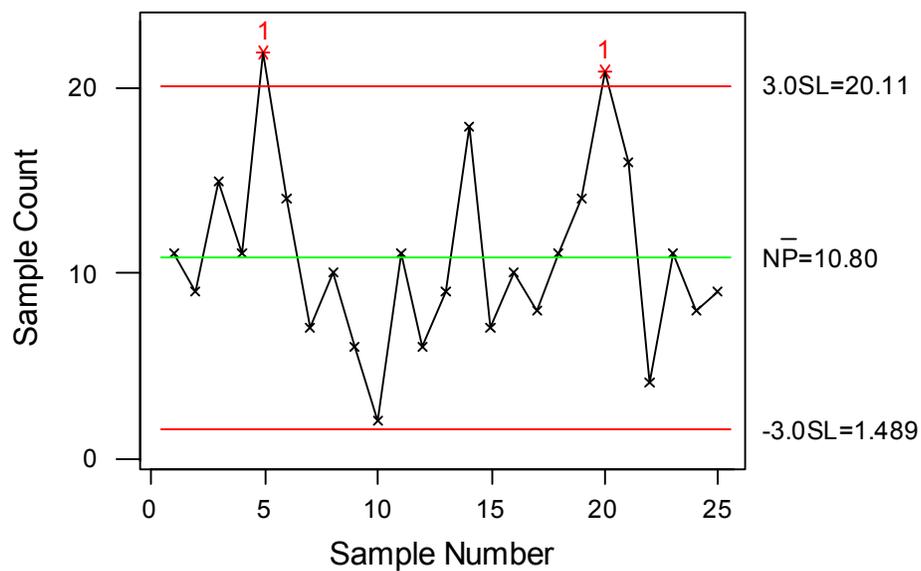
Stamp...

Options...

OK

Cancel

NP Chart for switches





## *NP-Chart Exercise*

---

- *The clinic surveyed 60 patients per day. They were asked to rate the quality of care they received on a 1-5 scale, five being the best and one being the worst.*
- *The number of patients who rated their experience as 3 or lower was recorded for each day. These patients were considered unsatisfied. The data is in file `Np_chart.mtw`, column “unsatisfied.”*
- *Make a NP-Chart of the data using Minitab.*
- *Run a Six-Sigma Product Report.*
- *What are your observations?*



## *Attribute Chart Subgroup Size*

### ■ *Rule of Thumb:*

- *Select a Subgroup Size that will Provide an Average Defect/Defective Count of Approximately*

$$\bar{C}, \bar{U}, N\bar{P} > 5.0$$

*To Make UCL & LCL Nearly Symmetrical  
Around the Mean*

- *For P-charts, to select the appropriate sample size such that 95% of the subgroups will have at least one defective, use the relationship*

$$n = \frac{3}{\bar{p}}$$



## *Summary of Attribute Charts*

- *Useful when variable data not available*
- *Use count/classification data—pass/fail, good/bad*
- *Same general rules for interpretation as variable charts*
- *Useful as end-to-end overview; use variable charts for further study of problems*
- *Can use data gathered for other purposes*
- *Generally less expensive to administer, but tell you less*

### ■ **Shortcomings**

- *Including too many variables makes interpretation difficult*
- *Must fit the parameters you are evaluating to theoretical distribution [Poisson (C, U-Charts), Binomial (P, NP-Charts)]*
- *Need to evaluate whether constant/non-constant lot size will help you with root cause analysis*
- *Sensitivity is dependent on magnitude of defect level*



## *Implementing Attribute Charting (SPC):*

**Applications**—document & drive improvement in processes and products where:

- *measurement (variable data) is not easy, economic, or possible*
- *counting (good/bad, OK/NG = **attribute data**) from “grading/sorting,” is easy and more economical*
- *each unit is considered “unique”—few units are similar*
- *data rates are slow—long periods between samples (attribute charting is also applicable in fast data rate cases)*

**Examples** = *non-conformities in:*

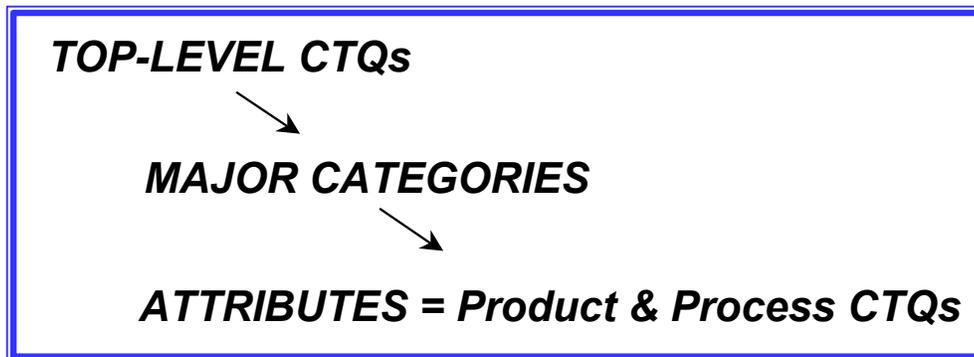
- *fabrications, bases, skids, etc. (medium to large “structures”)*
- *complex assembly*
- *proposal, contract preparation*
- *sales development, ITO actions/activities/processes*
- *design/build items*
- *transactional quality, OTR actions/activities/processes*
- *order processing, bill paying*
- *field service, repair, up-grade contract execution*
- *etc.*



# Attribute Matrix & SPC Charting— Implementation

**Step 1:** Identify top level criteria = customer/  
functional/performance/output driven  
= High Level CTQs.  
[CTQ = Critical to Quality Characteristic]

**Step 2:** Perform criteria decomposition



**Step 3:** Set up **Attribute Matrix**—include estimated  
"Opportunity for Non-Conformity" ... and  
estimated **Rework Cost \$**, or **Rework Hours**,  
per Nonconformity.



**Step 4:** Data Gathering

**Step 5:** Chart and Analysis—see Examples

**Attribute Matrix drives Control Chart**



## *Steps & Link to Six Sigma Tools*

---

### **Step**

- 1: Identify top level criteria (CTQs)
- 2: Perform **criteria decomposition**
- 3: Set up **Attribute Matrix**
- 4: Data Gathering
- 5: Chart and Analysis

### **Link:**

Steps 1 & 2 = Quality Function Deployment (QFD), House of Quality

Step 3 = Design-for-Manufacturability, Design for Six Sigma (DFSS)—of product and/or process

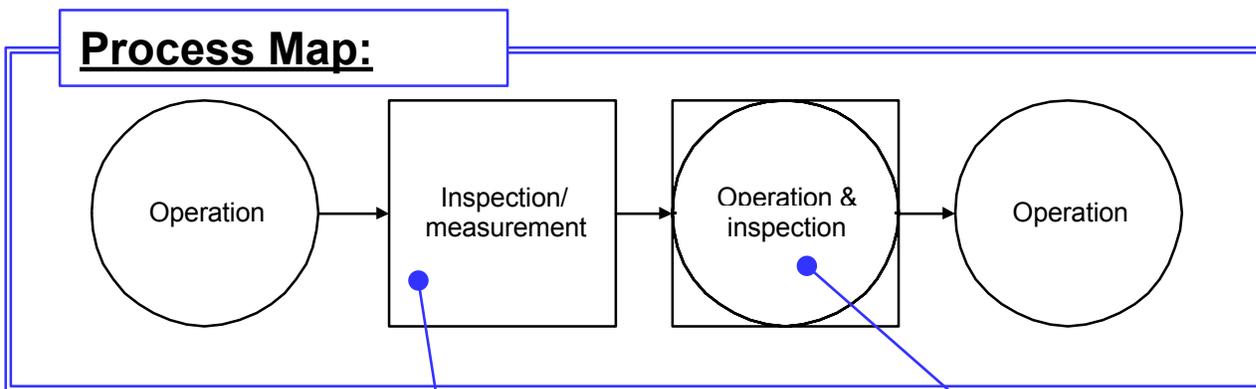
Steps 4 & 5 = Control: Statistical Process Control (SPC/SQC), Control Charts and Capability Reporting



# Attribute Matrix "Structure"

**TOP LEVEL:** PERFORMANCE, FUNCTIONAL, STRUCTURAL, SAFETY, INSTALLATION, MAINTAINABILITY, APPEARANCE, ECONOMIC, PACKAGING, etc.

[Each Top Level criteria can be a separate matrix and chart application for a particular production area: Welding, Paint Shop, Fit-Up, Final Assembly, etc.]



**ATTRIBUTE MATRIX:**

TOP LEVEL																														
MAJOR CATEGORIES																														
ATTRIBUTE (KQC)																														
Opportunity for non-conform																														
Rework \$ per non-conformity																														
Unit Number	***** Set up for charting *****																													
	1 = Non-Conforming 0 = OK																													
TOTAL [for Pareto]																														

Tie "Inspections"/ Tollgates to Matrix Categories

← Attributes →

Tie Matrix to Process Map -- check attributes against in-process inspection, hold-points, QC check lists, work instructions, job tickets, followers, manufacturing plan (MPP), ..etc.



## *Attribute Matrix Structure*

### *Example 1: Fabrication Shop — Attribute Matrix Structure*

#### **MAJOR CATEGORIES:**

##### **PERFORMANCE**

- Cubic foot per minute (CFM)
- Gallons per Hour (GPH) = Flow rate
- PSI
- Start torque
- Temperature
- etc.

##### **STRUCTURAL**

- Materials specs
- Weld quality
- Connection points
- Foundation points
- Pass-throughs
- Hardware specs
- etc.

##### **APPEARANCE/ PRESERVATION:**

- Surface prep
- Paint quality = prep & application
- Coating—Dry Film Thickness (DFT)
- Visual inspection
- Documentation
- etc.

**Criteria Decomposition may be applied to any business or production process—Transactional and/or Product Quality  
(CQ example = ITO Proposal Preparation)**





# Fab Shop

## Example 1: Fabrication Shop—continued

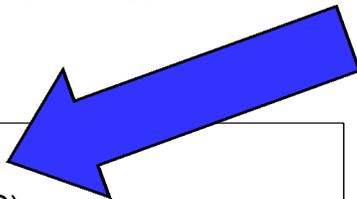
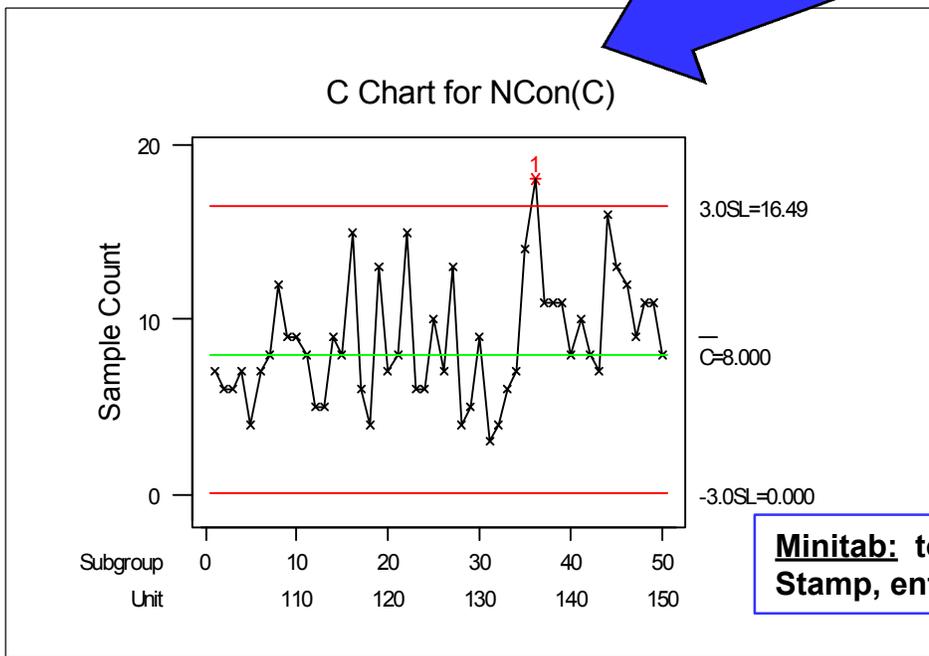
Background: fabrication 100% inspected for non-conformities (attribute errors). Inspection results, in Errors per Unit[c], are entered for 50 units -- see Attribute Matrix.

*Note:* each unit is approximately the same, produced per similar drawings, plans, processes = 52 Opportunities per Unit.

Initial control limits calculated from first 25 units—nos. 101-125



Unit	c	Unit	c
101	7	126	7
102	6	127	13
103	6	128	4
104	7	129	5
105	4	130	9
106	7	131	3
107	8	132	4
108	12	133	6
109	9	134	7
110	9	135	14
111	8	136	18
112	5	137	11
113	5	138	11
114	9	139	11
115	8	140	8
116	15	141	10
117	6	142	8
118	4	143	7
119	13	144	16
120	7	145	13
121	8	146	12
122	15	147	9
123	6	148	11
124	6	149	11
125	10	150	8



**Minitab:** to label with Unit No.—select Stamp, enter Tick Labels = Unit

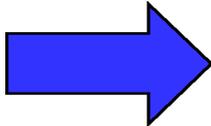


# Fab Shop

## Example 1: Fabrication Shop—continued

### Control data drives Capability Reporting

Unit	c
101	7
102	6
103	6
104	7
105	4
106	7
107	8
108	12
109	9
110	9
111	8
112	5
113	5
114	9
115	8
116	15
117	6
118	4
119	13
120	7
121	8
122	15
123	6
124	6
125	10

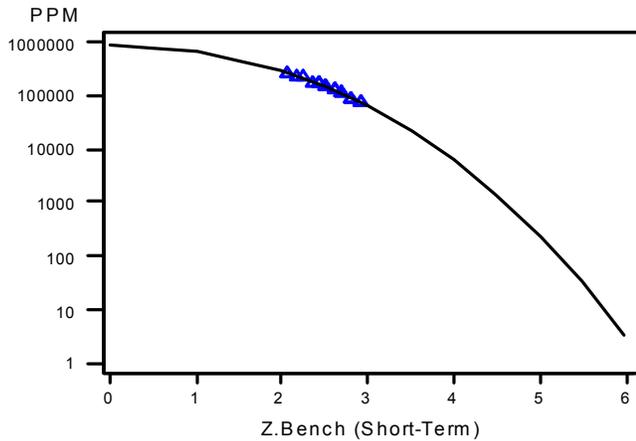


#### Report 7: Product Performance

Characteristic	Defc	Units	Opps	TotOpps	DEU	DCO	PPM	ZShift	ZBench
1	7	1	52	52	7.000	0.134615	134615	1.500	2.605
2	6	1	52	52	6.000	0.115385	115385	1.500	2.698
3	6	1	52	52	6.000	0.115385	115385	1.500	2.698
4	7	1	52	52	7.000	0.134615	134615	1.500	2.605
5	4	1	52	52	4.000	0.076923	76923	1.500	2.926
6	7	1	52	52	7.000	0.134615	134615	1.500	2.605
7	8	1	52	52	8.000	0.153846	153846	1.500	2.520
8	12	1	52	52	12.000	0.230769	230769	1.500	2.236
9	9	1	52	52	9.000	0.173077	173077	1.500	2.442
10	9	1	52	52	9.000	0.173077	173077	1.500	2.442
11	8	1	52	52	8.000	0.153846	153846	1.500	2.520
12	5	1	52	52	5.000	0.096154	96154	1.500	2.804
13	5	1	52	52	5.000	0.096154	96154	1.500	2.804
14	9	1	52	52	9.000	0.173077	173077	1.500	2.442
15	8	1	52	52	8.000	0.153846	153846	1.500	2.520
16	15	1	52	52	15.000	0.288462	288462	1.500	2.058
17	6	1	52	52	6.000	0.115385	115385	1.500	2.698
18	4	1	52	52	4.000	0.076923	76923	1.500	2.926
19	13	1	52	52	13.000	0.250000	250000	1.500	2.174
20	7	1	52	52	7.000	0.134615	134615	1.500	2.605
21	8	1	52	52	8.000	0.153846	153846	1.500	2.520
22	15	1	52	52	15.000	0.288462	288462	1.500	2.058
23	6	1	52	52	6.000	0.115385	115385	1.500	2.698
24	6	1	52	52	6.000	0.115385	115385	1.500	2.698
25	10	1	52	52	10.000	0.192308	192308	1.500	2.369
Total	200		1300			0.153846	153846	1.500	2.520



#### Report 8A: Product Benchmarks





# Variable Opportunities

## Example 2: Units with Variable Opportunities—Matrix:

UNIT NO.	SPECS				SURFACE PREPARATION										COATINGS				COATING APPLICATION						Inspection Measurements					Final Inspect.			TOTAL NON-COMFORMANCES per unit [ for charting ]												
	Surface preparation specification	Coating application specification	Manufacturer's latest printed product data sheets (MSDS) & applic. instructions.	Latest revision -- all drawings	Weld splatter	Weld flux	Sharp corners	Laminations	Rough welds	Skip welds sealed or caulked, type of caulk	Compressed air dry and clean	Conditions suitable for abrasive blasting	Oil, grease, dirt, salt and contaminant free	Anchor pattern as specified, Testex PFO-Film	All dust and debris removed	Surface as called for in spec SP6, SP10, etc.	Coatings are those specified	Coatings correctly mixed and agitated	Coatings thinned correctly	Coatings have not exceeded pot life	Batch No recorded	Storage and shelf life not exceeded	Ambient conditions OK for painting	Clean, dry surface	Correct DFT	runs	dry spray	voids, holidays, ghosts	other & miscell.	Brush over welds, corners, edges(dbl. cover)	Difficult to reach areas coated sufficiently	Defects properly marked and repaired		Blast profile with tape	DFT each coat and final DFT	Wet bulb temperature	Dry bulb temperature	Relative humidity	Dew point	Metal surface temperature	Smooth, no dryspray, runs	Complete coverage	DFT as specified	Other	
Opportunity for non-conform	Varies from unit to unit				Varies from unit to unit										Varies from unit to unit						Varies from unit to unit																								
101	1	1	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	2	1	52	
102	1	1	1	1	1	1	1	1	1	12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	56	
103	1	1	1	1	1	1	1	1	1	5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	2	1	49	
104	1	1	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	2	1	52	
105	1	1	1	1	1	1	1	1	1	36	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	2	1	80	
106	1	1	1	1	1	1	1	1	1	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	2	1	52		
107	1	1	1	1	1	1	1	1	1	40	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	2	1	84		
108	1	1	1	1	1	1	1	1	1	21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	2	1	65			
<p><b>Case of Units with VARYING OPPORTUNITY per Unit</b>            Attribute Matrix --&gt; p-charting            1. Set up "GRAY BAR" for each Unit.            2. Use p-chart to track process control &amp; performance.            3. Reset UCL, Centerline, &amp; LCL as improvement is reflected in data.            4. Set DPU, DPO, and DPMO values based on data -- L1 analysis.</p>																																													
TOTAL [for Pareto]	0	0	8	0	3	0	0	0	13	0	0	3	0	0	0	0	0	1	2	0	0	0	0	2	4	0	0	0	3	0	1	0	1	5	0	0	0	1	0	0	0	3	8	490	
																					58	0.12																							



# Variable Opportunities

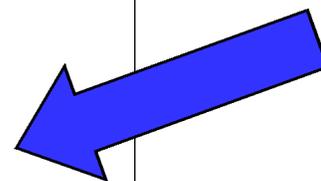
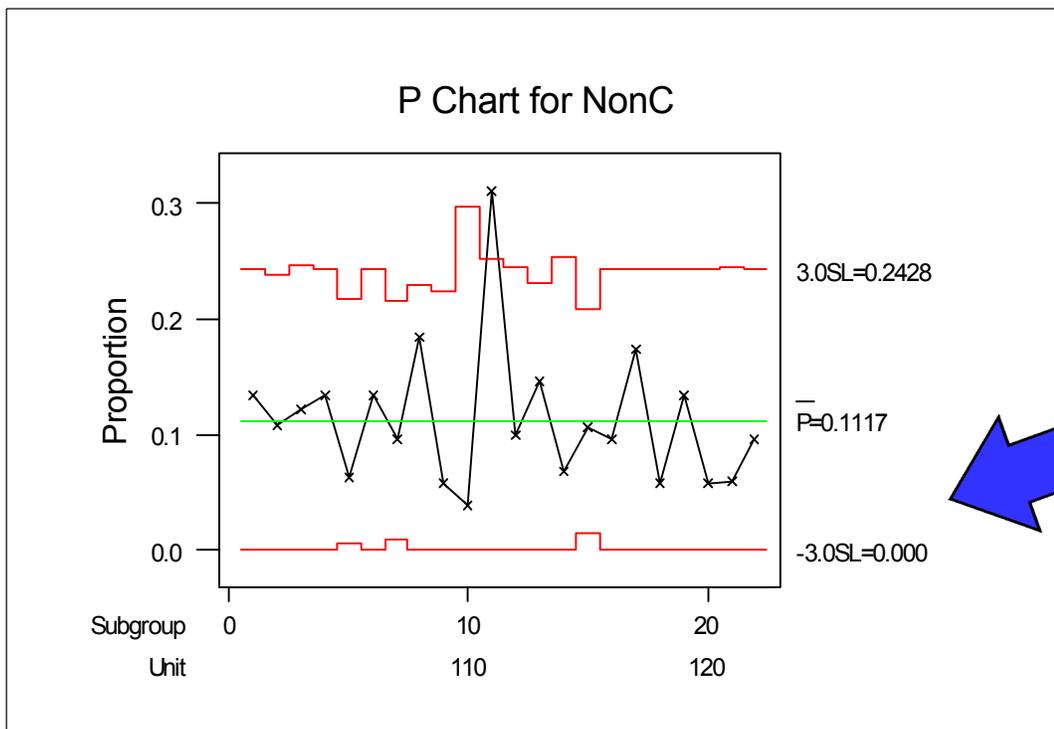
Example 2: Variable Opportunity—Continued:

**Background:** “fabrication” 100% inspected for non-conformities (attribute errors). Unit “Complexity” varies—see Attribute Matrix.

Data for 22 Units is available = P-Chart for CONTROL.



	Unit	Oppt	NonC
1	101	52	7
2	102	56	6
3	103	49	6
4	104	52	7
5	105	80	5
6	106	52	7
7	107	84	8
8	108	65	12
9	109	70	4
10	110	26	1
11	111	45	14
12	112	50	5
13	113	62	9
14	114	44	3
15	115	95	10
16	116	52	5
17	117	52	9
18	118	52	3
19	119	52	7
20	120	52	3
21	121	50	3
22	122	52	5
		<u>1244</u>	<u>139</u>





# Example 2

## Example 2: Capability

**Estimating Process Capability = DPMO & Zbench:**  
[ 22 Units in example ]

**1. Data:**

Defects = D = 139  
Units = U = 22  
Total Opp. = TOP = 1244

**2. Calculate:**

$DPMO = (D/TOP) * 1,000,000 =$   
 $DPO * 1,000,000$   
DPMO = 111,736

$\sigma_{LT} = 1.22$   
 $\sigma_{ST} = \sigma_{LT} + 1.5 =$   
ZBench = 2.72

**Also:**

Defects per Unit = DPU = 6.32

Defects per Opportunity = DPO =  
0.111736334

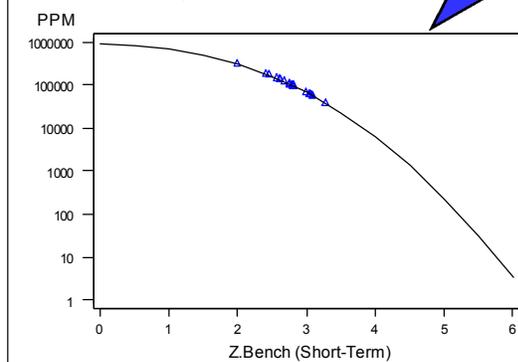
OR -- Minitab Six Sigma Product (L1) tool

Report 7: Product Performance

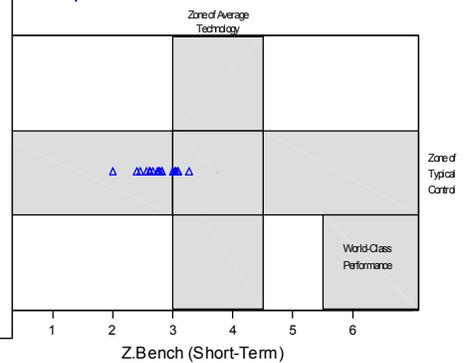
Characteristic	Defcs	Units	Opps	TotOpps	DPU	DPO	PPM	ZShift	ZBench
1	7	1	52	52	7.000	0.134615	134615	1.500	2.605
2	6	1	56	56	6.000	0.107143	107143	1.500	2.742
3	6	1	49	49	6.000	0.122449	122449	1.500	2.663
4	7	1	52	52	7.000	0.134615	134615	1.500	2.605
5	5	1	80	80	5.000	0.062500	62500	1.500	3.034
6	7	1	52	52	7.000	0.134615	134615	1.500	2.605
7	8	1	84	84	8.000	0.095238	95238	1.500	2.809
8	12	1	65	65	12.000	0.184615	184615	1.500	2.398
9	4	1	70	70	4.000	0.057143	57143	1.500	3.079
10	1	1	26	26	1.000	0.038462	38462	1.500	3.269
11	14	1	45	45	14.000	0.311111	311111	1.500	1.993
12	5	1	50	50	5.000	0.100000	100000	1.500	2.782
13	9	1	62	62	9.000	0.145161	145161	1.500	2.557
14	3	1	44	44	3.000	0.068182	68182	1.500	2.989
15	10	1	95	95	10.000	0.105263	105263	1.500	2.752
16	5	1	52	52	5.000	0.096154	96154	1.500	2.804
17	9	1	52	52	9.000	0.173077	173077	1.500	2.442
18	3	1	52	52	3.000	0.057692	57692	1.500	3.074
19	7	1	52	52	7.000	0.134615	134615	1.500	2.605
20	3	1	52	52	3.000	0.057692	57692	1.500	3.074
21	3	1	50	50	3.000	0.060000	60000	1.500	3.050
22	5	1	52	52	5.000	0.096154	96154	1.500	2.804
Total	139			1244		0.111736	111736	1.500	2.717

Roll Up

Report 8A: Product Benchmarks



Report 8B: Product Benchmarks



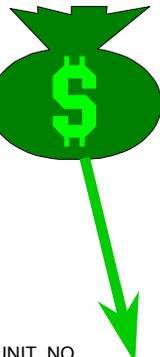


# Example 3

## Example 3: "Rework Cost" charting = \$\$

UNIT NO.	SURFACE PREPARATION												COATING APPLICATION								TOTAL NON-COMFORMANCES per unit [ for charting ]	REWORK COST per unit (\$\$)		
	Surface defects correct/remvd												No flaws											
	Weld splatter	Weld flux	Sharp corners	Laminations	Rough welds	Skip welds sealed or caulked, type of caulk	Compressed air dry and clean	Conditions suitable for abrasive blasting	Oil, grease, dirt, salt and contaminant free	Anchor pattern as specified, Testex Pr-O-Film	All dust and debris removed	Surface as called for in spec SP6, SP10, etc.	Ambient conditions OK for painting	Clean, dry surface	Correct DFT	runs	dry spray	voids, holidays, ghosts	other & miscell.	Brush over welds, corners, edges(dbl. cover)	Difficult to reach areas coated sufficiently	Defects properly marked and repaired		
Rework Cost(\$)	4	2	20	10	30	5	10	100	25	8	30	4	15	5	50	5	5	8	6	10	25	6		
Opportunity for non-conform	1	1	1	1	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	29	\$ Rework per Unit
101					1									1								1	3	60
102	1				1										1								3	84
103	1																	1					5	100
104																							2	55
105																							1	100
106	1																						4	160
107																							3	110
108																			1				8	286
TOTAL [for Pareto]	3	0	0	0	5	0	0	3	0	0	0	0	0	2	4	0	0	0	3	0	1	0	29	955

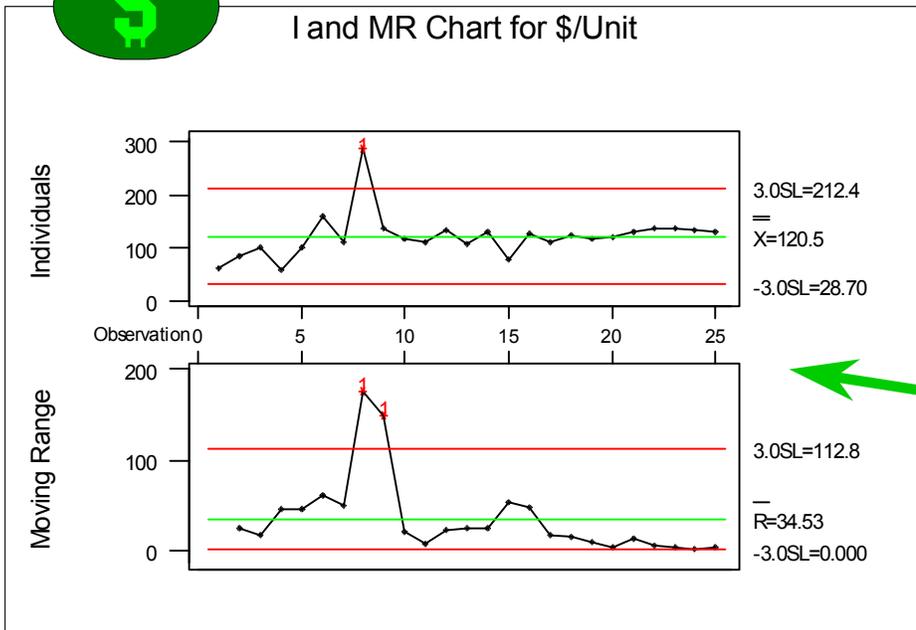
**Add "Cost Bar" to Attribute Matrix - track Rework \$ per Unit**





## Example 3

### Example 3: “Rework Cost” Chart— use Control Charts of Rework \$\$



Unit	c	\$/Unit
101	7	60
102	6	84
103	6	100
104	7	55
105	4	100
106	7	160
107	8	110
108	12	286
109	9	136
110	9	116
111	8	109
112	5	132
113	5	107
114	9	130
115	8	78
116	15	124
117	6	108
118	4	123
119	13	114
120	7	118
121	8	130
122	15	136
123	6	134
124	6	133
125	10	130

**In this case: Rework \$ per Unit not improving**





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# *Process Focused Control Charts*



## *Objectives*

---

- *To apply the theory of control charting to several parts within the **same process**.*
- *To plot the deviation from nominal/target for **each part** on the **same control chart** for easy monitoring of the process.*
- *To monitor **variation** within a process by examining several characteristics of many parts (I & MR Chart).*



## *Why Use Process Focused Control Charts?*

### **VARIABLE CHART**

- ✓ *Uses Measured Values*
  - ✓ *Cycle Time, Lengths, Diameters, Drops, etc.*
- ✓ *Generally One Characteristic Per Chart*
- ✓ *More Expensive, But More Information*

### **ATTRIBUTE CHART**

- ✓ *Pass/Fail, Good/Bad, Go/No-Go Information*
- ✓ *Can Be Many Characteristics Per Chart*
- ✓ *Less Expensive, But Less Information*

### **PROCESS FOCUSED CHART**

- ✓ *Monitors Several Parts From Same Process*
- ✓ *Measures Deviation From Nominal/Target*
- ✓ *Typically an I & MR Chart Monitoring Several Characteristics of Several Parts*



## *Process Focused: Steps 1 & 2*

- 1. Define the process (general is better than specific)*
- 2. Identify the parameters that measure performance*
- 3. Gather data in production sequence*
- 4. Record variables data as a deviation from nominal/target*
- 5. Analyze for patterns*

**Note: A process is one set of the 6Ms**



## *Processes to Monitor*

<b><i>Process</i></b>	<b><i>Parameter</i></b>
<i>Welding</i>	<i>% Penetration Voids per foot</i>
<i>Turning</i>	<i>Diameter, Runout</i>
<i>Milling</i>	<i>Flatness</i>
<i>Grinding</i>	<i>Dimension, Finish</i>
<i>Stamping</i>	<i>Dimension, Burrs</i>
<i>Banking</i>	<i>Posting Errors</i>
<i>Typing</i>	<i>Errors per Page</i>
<i>Drafting</i>	<i>Errors/100 Drawings issued on time</i>



## *Process Focused: Step 3*

- 1. Define the process (general is better than specific)*
- 2. Identify the parameters that measure performance*
- 3. Gather data in production sequence*
- 4. Record variables data as a deviation from nominal/target*
- 5. Analyze for patterns*

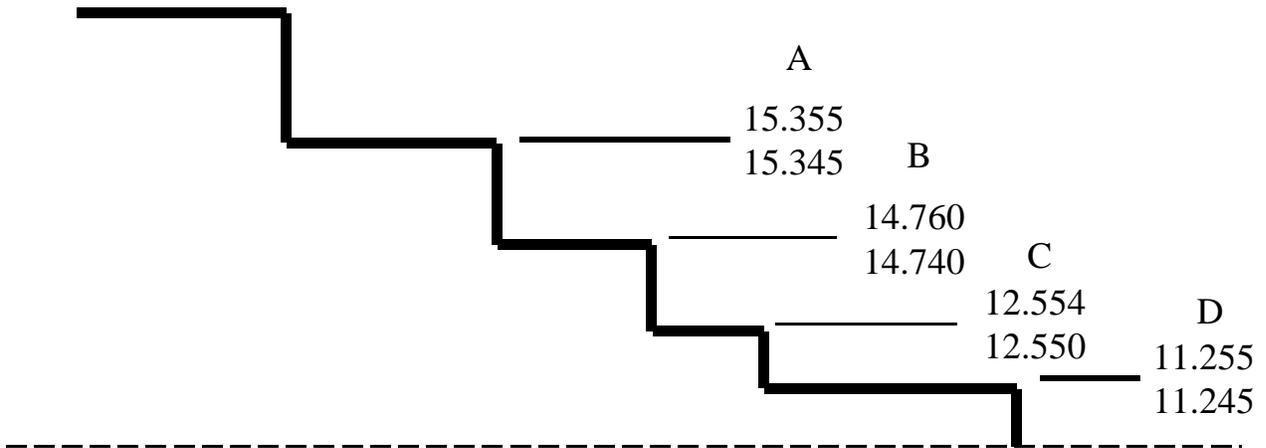


## *Process Focused: Step 4*

- 1. Define the process (general is better than specific)*
- 2. Identify the parameters that measure performance*
- 3. Gather data in production sequence*
- 4. Record variables data as a deviation from nominal/target*
- 5. Analyze for patterns*



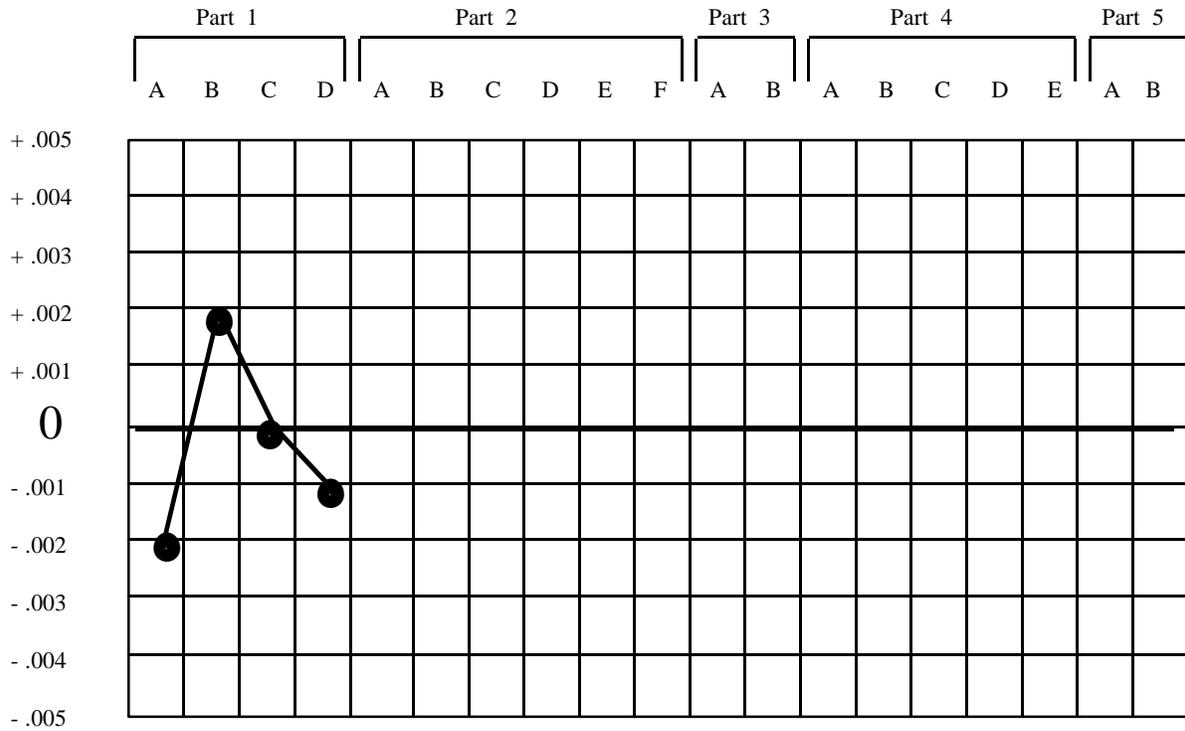
## Deviation from Nominal



<i>Dimension</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
NOMINAL	15.350	14.750	12.552	11.250
ACTUAL	15.348	14.752	12.552	11.249
VAR. FROM NOM.	- .002	+ .002	0	- .001

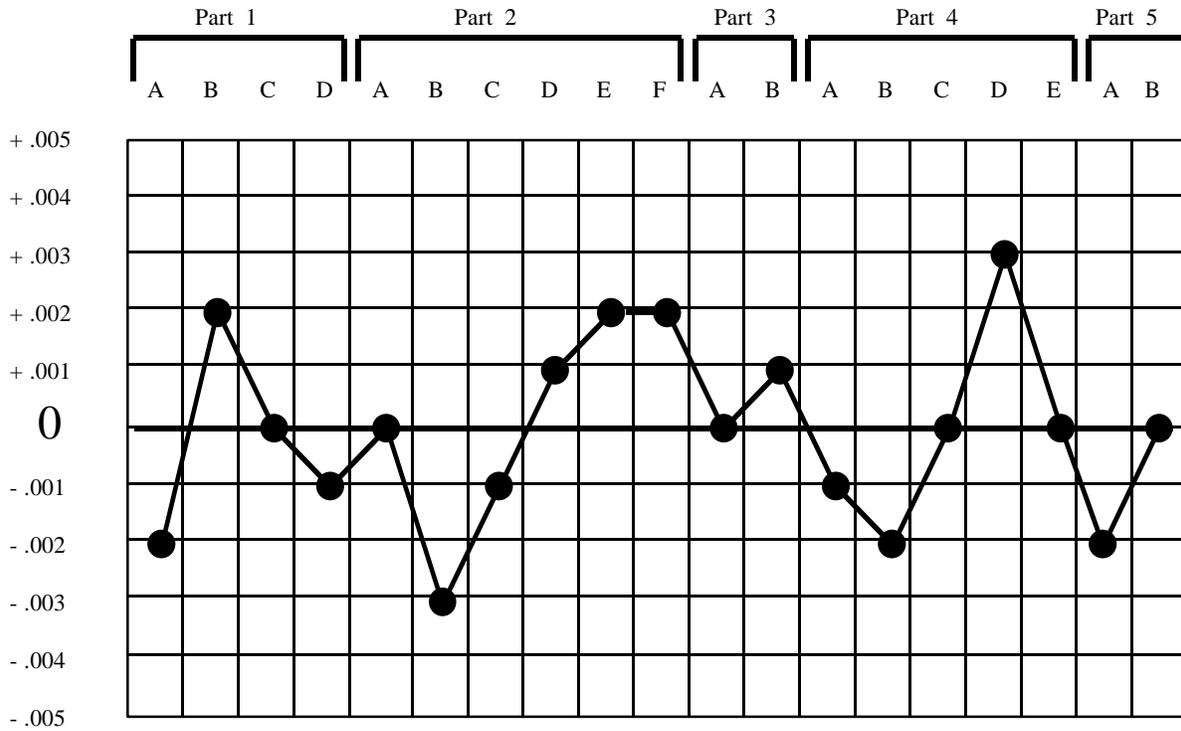


# Deviation from Nominal





# Deviation from Nominal





## Control Chart Calculations

### ■ I & MR Chart

- *I Chart: Plots the individual values of deviation from nominal over time*
- *MR Chart: Plots the moving range (typically  $|X_i - X_{i-1}|$ ) over time*

Time - to - Time:  $(\bar{X})$

$$UCL = \bar{X} + E_2 \bar{R}$$

$$LCL = \bar{X} - E_2 \bar{R}$$

$$E_2 = 2.660$$

$$n = 2$$

Feature - to - Feature: (R)

$$UCL = D_4 \bar{R}$$

$$LCL = D_3 \bar{R}$$

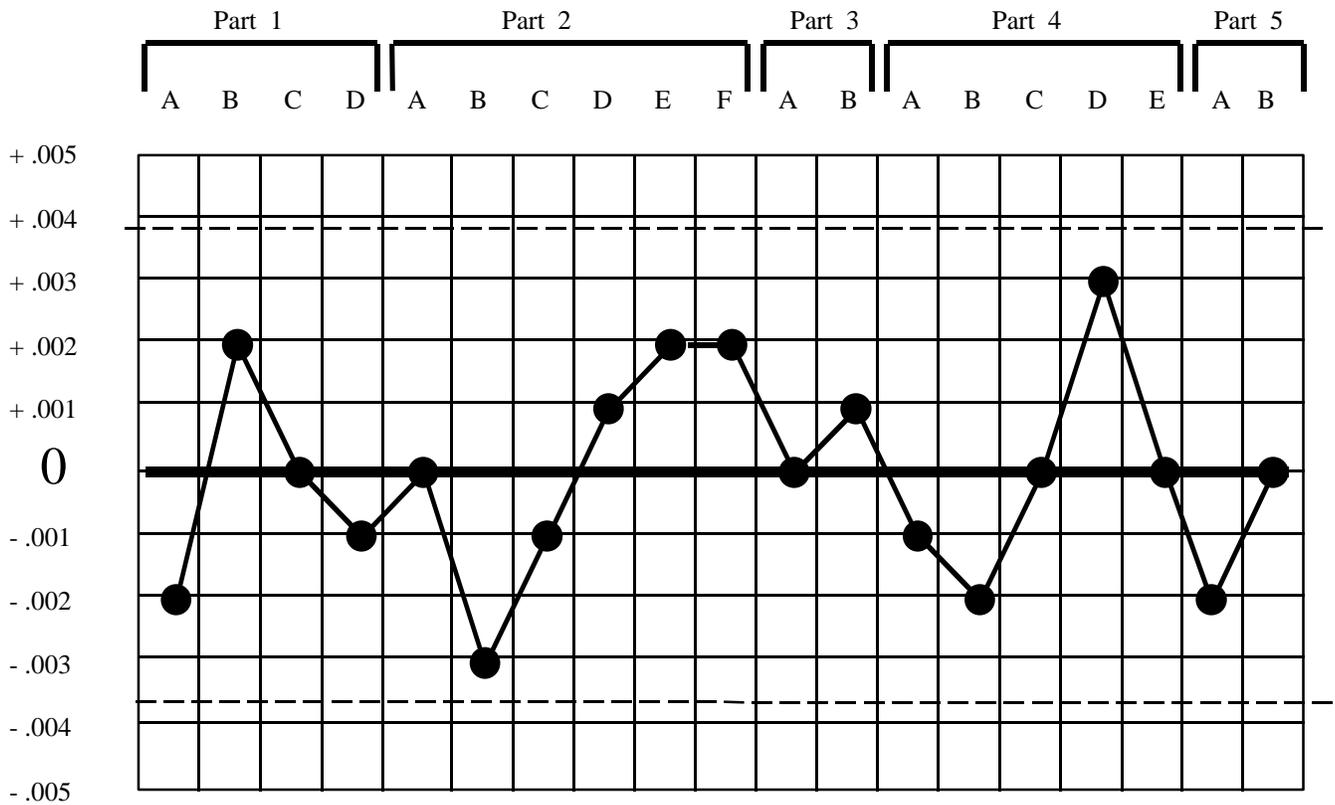
$$D_4 = 3.267$$

$$D_3 = 0$$

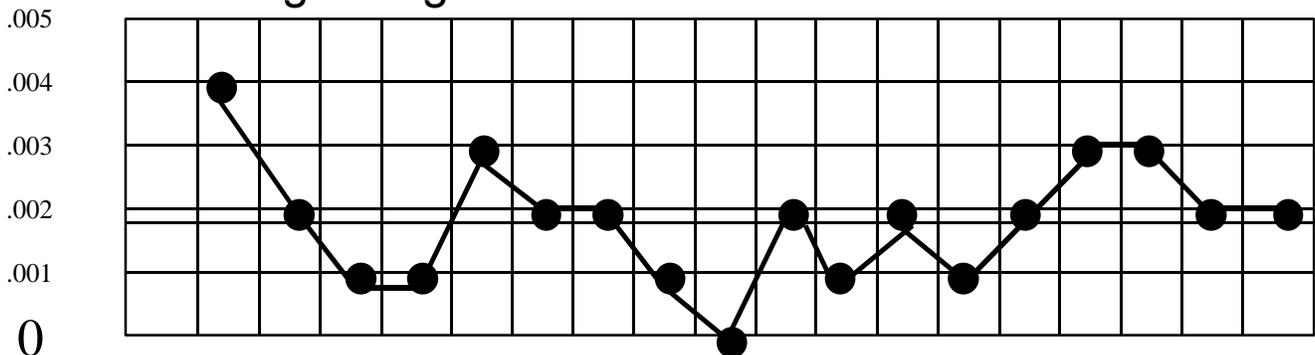


# Individual & Moving Range Charts

## Individual



## Moving Range





## *Step 5*

---

- 1. Define the process (general is better than specific)*
- 2. Identify the parameters that measure performance*
- 3. Gather data in production sequence*
- 4. Record variable data as a deviation from nominal/target*
- 5. Analyze for patterns*



## Processed Focused Control Chart Example

The data below represents one week's output of an NC lathe, and consists of measurements taken by the operators in the sequence in which they were machined. In each case, the operator was instructed to come as close to nominal as he could before recording the final dimension. The data is in file *Low\_vol.mtw*. Use Minitab to construct a "variation from nominal" chart.

What is your analysis of the control chart?

What do you suppose was happening between Dimension E on Part 2 and Dimension D on Part 4?

Part ID	Dimension	Nominal	Actual Variation
1	A	13.570	13.567
1	B	12.012	12.008
1	C	22.125	22.124
1	D	20.652	20.652
2	A	6.878	6.881
2	B	6.275	6.278
2	C	2.175	2.179
2	D	2.005	2.004
2	E	1.750	1.745
3	A	16.846	16.843
3	B	14.116	14.113
4	A	25.125	25.124
4	B	24.000	24.000
4	C	27.375	27.377
4	D	26.625	26.630
4	E	21.174	21.175
5	A	4.375	4.378
5	B	4.125	4.122
5	C	3.890	3.890
6	A	27.445	27.442
6	B	26.565	26.562
6	C	24.188	24.189
6	D	21.010	21.010
6	E	18.750	18.753
6	F	16.915	16.917



# Process Focused Control Chart Example

MINITAB FILE: Low\_vol.mtw

The screenshot shows the Minitab software interface. The 'Stat' menu is open, and 'Control Charts' is selected. The 'I-MR...' option is highlighted. The data table below shows the following values:

	C1	C2	C3	C6	C7	C8	C9
	nominal	actual	var	OD_act	OD_nom	OD_Var	
1	13.570	13.567	-0.00300	0.595000	0.597000	0.0020000	
2	12.012	12.008	-0.00400	0.650000	0.649000	-0.0010000	
3	22.125	22.124	-0.00100	0.597000	0.597000	0.0000000	
4	20.652	20.652	0.00000	0.599000	0.597000	-0.0020000	
5	6.878	6.881	0.00300	0.864000	0.865000	0.0010000	
6	6.275	6.278	0.00300	0.865000	0.865000	0.0000000	
7	2.175	2.179	0.0040000	0.648000	0.649000	0.0010000	
8	2.005	2.004	-0.0010000	0.649000	0.649000	0.0000000	



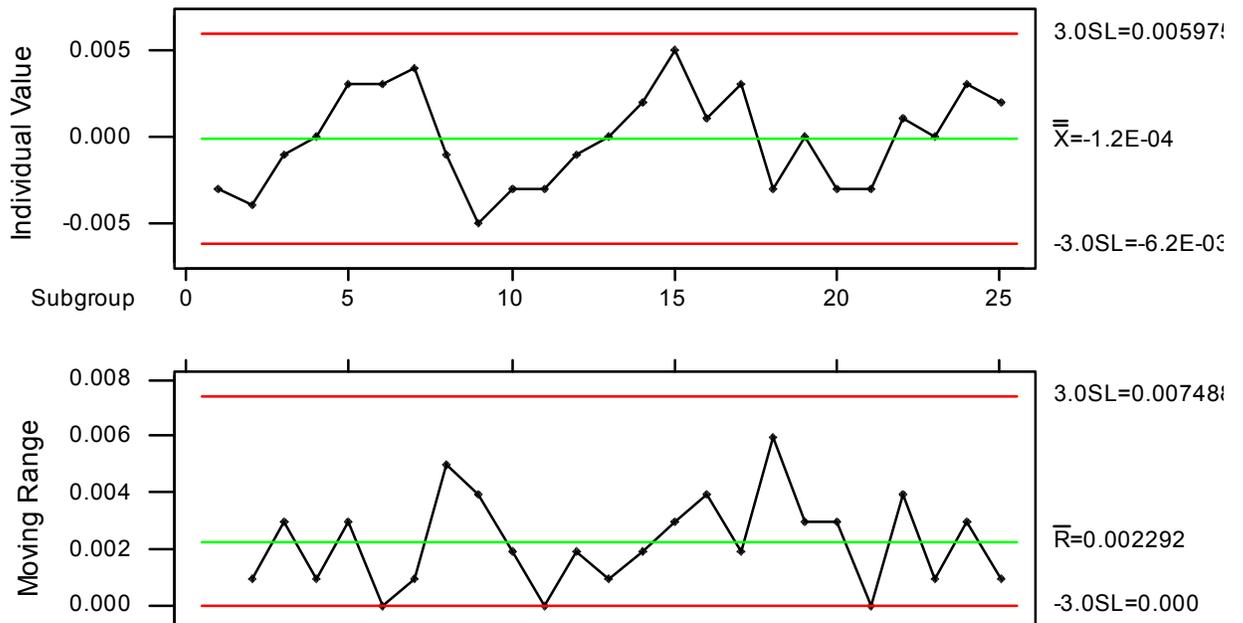
# Process Focused Control Chart Minitab Input and Output

1. Double Click  
on C3.

2. OK.

The dialog box for creating an I-MR chart is shown. The 'Variable' field contains 'var'. There are fields for 'Historical mean' and 'Historical sigma', both marked as optional. On the right side, there are buttons for 'Tests...', 'Estimate...', 'Stamp...', and 'Options...'. At the bottom, there are buttons for 'Select', 'Help', 'Estimate Parameters BY Groups in...', 'OK', and 'Cancel'.

I and MR Chart for var





## Parts Delivery Example

Customers routinely order parts to support seasonal outages. One key customer requirement is that parts be delivered on time (too early and the customer may not be ready, too late and the parts may hold up an outage). In general, orders have several parts and each part has its own distinct cycle time. Therefore, some orders may have multiple partial shipments as parts become available. We can monitor the overall delivery to want date with a process focused control chart which tracks the deviation from promised to actual delivery date for each part in each order.

What is your analysis of the control chart?

What do you suppose happened to part A in Order 5?

The data is contained in **MINITAB FILE: Del\_time.mtw**

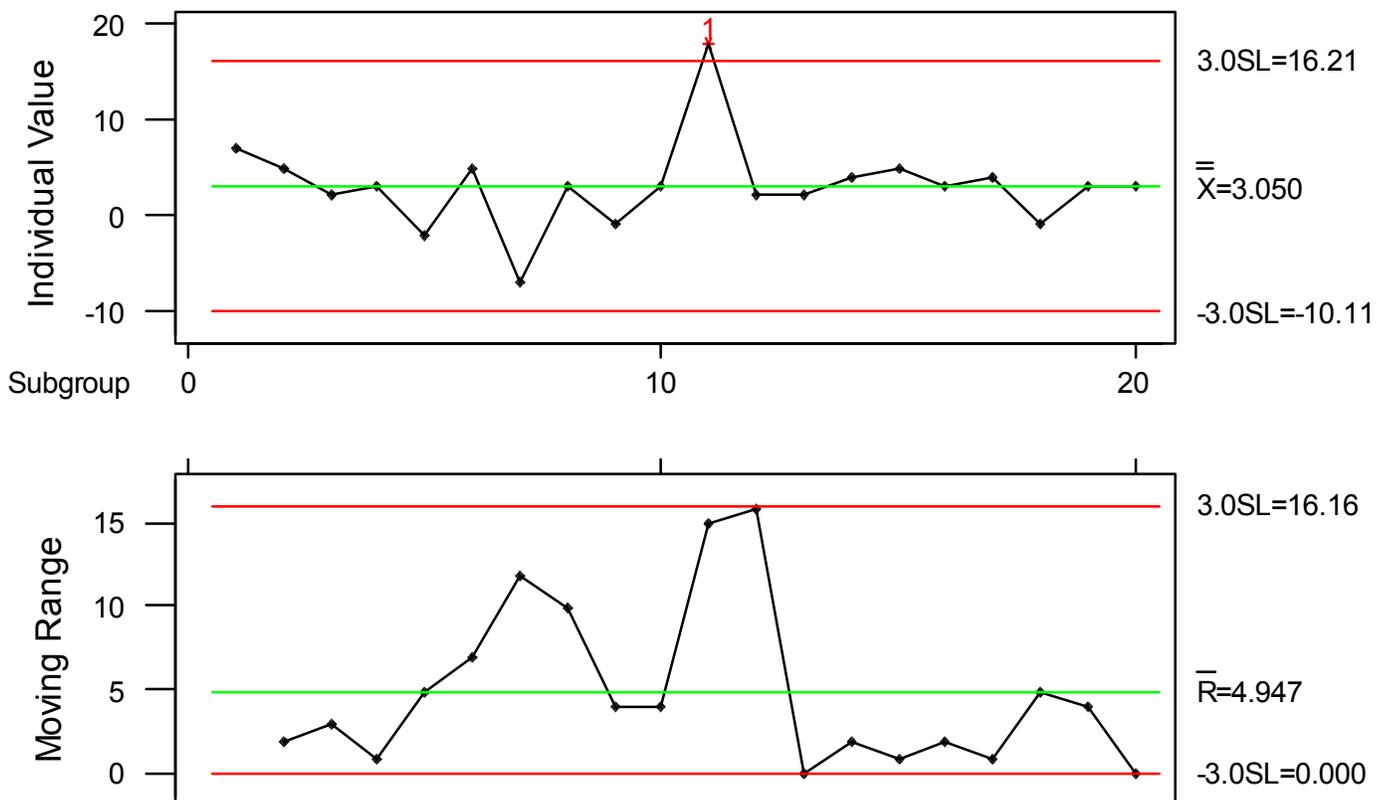
Order	Part	Promised date	Actual Date	Deviation
1	A	06/25/98	07/02/98	7
1	B	06/25/98	06/30/98	5
1	C	06/25/98	06/27/98	2
2	A	06/24/98	06/27/98	3
2	B	06/24/98	06/22/98	-2
3	A	06/22/98	06/27/98	5
3	B	06/22/98	06/15/98	-7
3	C	06/22/98	06/25/98	3
3	D	06/22/98	06/21/98	-1
4	A	06/15/98	06/18/98	3
5	A	06/10/98	06/28/98	18
5	B	06/10/98	06/12/98	2
5	C	06/10/98	06/12/98	2
5	D	06/10/98	06/14/98	4
5	E	06/10/98	06/15/98	5
6	A	06/05/98	06/08/98	3
6	B	06/05/98	06/09/98	4
7	A	06/02/98	06/01/98	-1
7	B	06/02/98	06/05/98	3
7	C	06/02/98	06/05/98	3



## Minitab Results

Use the same Minitab commands as on the previous pages to produce the I & MR chart for the **Minitab File: Del\_time.mtw**

I and MR Chart for Deviation





## *Exercise*

# *Process Focused Control Charts*

- *An NC Lathe is set up to turn an outer diameter on a family of outer rings for the CF6-80 engine. Three rings in the family have dimensions*

<i>Ring A</i>	<i>40.865 +/- .002</i>
<i>Ring B</i>	<i>38.649 +/- .002</i>
<i>Ring C</i>	<i>48.597 +/- .002.</i>
- *The rings are the same material, and differ only in size. Fixturing is universal, so that all three rings turn on the same fixture. Planning is identical except for dimensional callouts.*
- *The data is contained in **Minitab File: Low\_vol.mtw**, columns “OD\_act” and “OD\_nom.”*
  1. *Use Minitab to construct a “low volume” chart.*
  2. *Is the process in a state of statistical control? Why/Why Not?*
  3. *Can you estimate process capability? If you think you can, use the Minitab capability macro. The tolerances are +/- .002.*



## *Summary of Process Focused Control Charts*

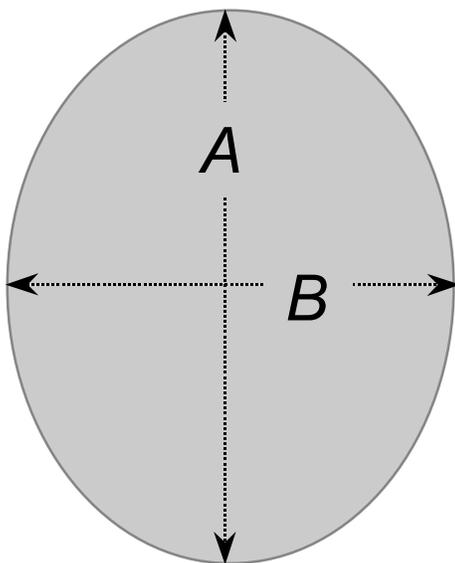
---

- *Allows merging data from multiple parts*
- *Not tied to one specific characteristic*
- *Adaptable to families of like parts*
- *Leads naturally to machine capability study*
- *Rapid multiplication of data points*
- *Conventional Control Chart interpretation*
- *Operator must aim for nominal and come as close as possible*
- *Should use same gage resolution*
- *Measurements should be same order of magnitude*
- *Double-check for normality before predicting process capability*



## 2-R Control Charts Example

- A journal diameter on a shaft has a requirement of 4.763/4.768, and  $T.I.R. = .003$ "
- Two equally spaced diametral readings,  $A$  and  $B$ , are used to evaluate for an out-of-round condition ( $T.I.R.$ ).
  - $A$  denotes the maximum diametral reading
  - $B$  denotes the minimum diametral reading
- How do you analyze the data?



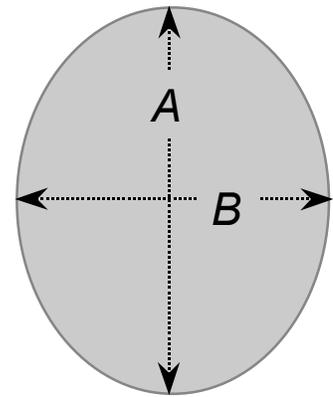
$$T.I.R. = |A - B|$$



# Calculations

## Average Diameter:

$$\frac{A + B}{2} = \text{Time-to-Time Variation } (\bar{X})$$

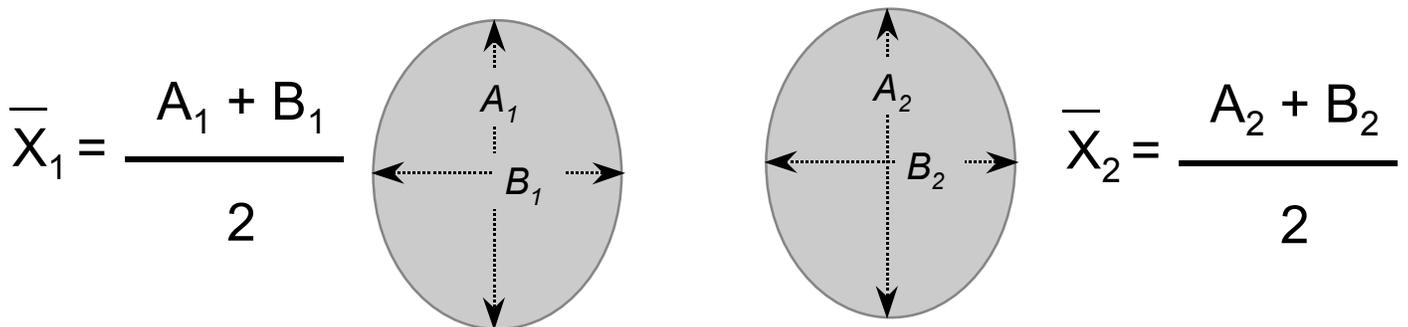


## T.I.R (Total Indicator Reading):

$$|A - B| = \text{Within Piece Variation } (R_w)$$

## Variation Between Parts:

$$|\bar{X}_2 - \bar{X}_1| = \text{Piece-to-Piece Variation } (R_p)$$

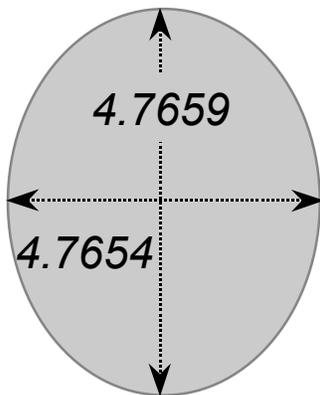




## Calculations

For the first 7 shafts,

Max: A	Min: B	$\bar{X}$	$R_p$	$R_w$
4.7665	4.7658	4.76615	--	.0007
4.7659	4.7654	4.76565	.00050	.0005
4.7667	4.7661	4.76640	.00075	.0006
4.7659	4.7655	4.76570	.00070	.0004
4.7668	4.7663	4.76655	.00085	.0005
4.7664	4.7660	4.76620	.00035	.0004
4.7659	4.7657	4.76580	.00040	.0002



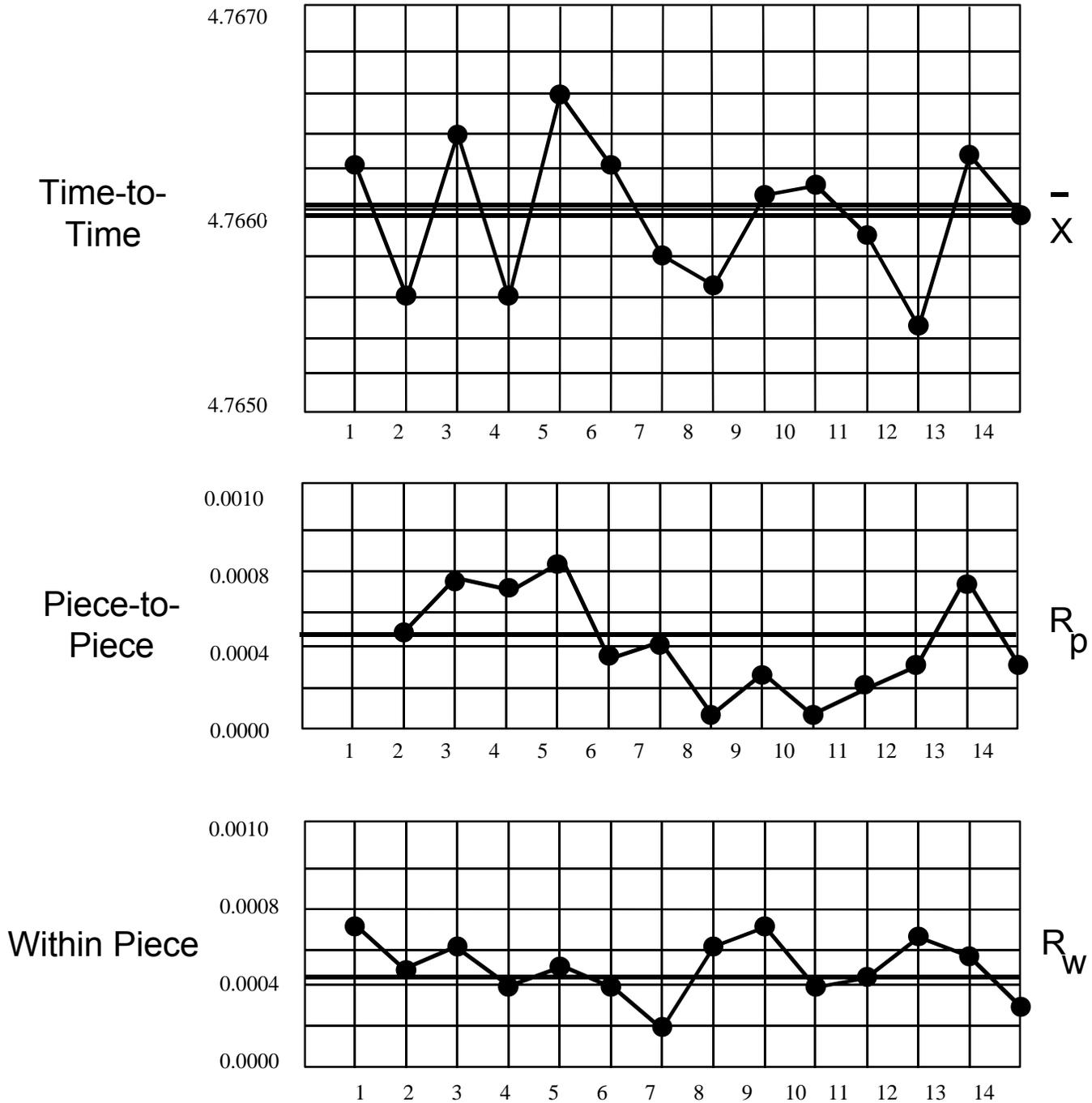
$$\bar{X} = \frac{4.7659 + 4.7654}{2} = 4.76565$$

$$R_p = |4.7659 - 4.7654| = .00050$$

$$R_w = |4.76565 - 4.76615| = .00050$$



# Control Charts





## Control Chart Calculation

### ■ Same as I-MR Chart

Time-to-Time:  $\bar{X}$

$$UCL = \bar{X} + E_2 \bar{R}_p$$

$$LCL = \bar{X} - E_2 \bar{R}_p$$

Within Piece:

$$UCL = D_4 \bar{R}_w \quad LCL = 0$$

Piece-to-Piece:

$$UCL = D_4 \bar{R}_p \quad LCL = 0$$

} Typically  
n = 2



## 2-R Chart Example

An NC lathe produces a shaft on a Finish Turn operation. A journal on the shaft has an engineering spec requirement of 4.763/4.768. The journal is measured at two places 90 degrees apart with an OD micrometer reading to .0001 inch. In the table below the data has been coded from 4.76XX for ease of calculation.

Piece #	Max	Min	(Max + Min)/2	Rp	Rw
1.	4.76 (65)	4.76 (58)	4.76(61.5)	-----	.000(7)
2.	59	54	56.5	5	5
3.	67	61	64	7.5	6
4.	59	55	57	7	4
5.	68	63	65.5	8.5	5
6.	64	60	62	3.5	4
7.	59	57	58	4	2
8.	60	54	57	1	6
9.	64	57	60.5	3.5	7
10.	63	59	61	0.5	4
11.	61	56	58.5	2.5	5
12.	59	52	55.5	3	7
13.	66	60	63	7.5	6
14.	61	58	59.5	3.5	3
15.	63	60	61.5	2	3
16.	59	52	55.5	6	7
17.	58	53	55.5	0	5
18.	62	58	60	4.5	4



## *Example (cont.)*

<b>Piece #</b>	<b>Max</b>	<b>Min</b>	<b>(Max + Min)/2</b>	<b>Rp</b>	<b>Rw</b>
19.	66	60			
20.	59	55			
21.	62	58			
22.	61	58			
23.	65	60			
24.	64	58			
25.	60	55			

- 1. Use Minitab and file 2r\_chart to complete the data table above.*
- 2. Use Minitab to construct an individuals chart on the averages, a moving range chart on the averages, and a range chart on the subgroups.*
- 3. Analyze the data for stability and out-of-control indications. What is your evaluation of the process?*



# 2-R Chart Example, $\bar{X}$ - Minitab Menu Commands

MINITAB FILE: 2r\_chart.mtw

Stat > Control Charts > I-MR-R (Between/Within)...

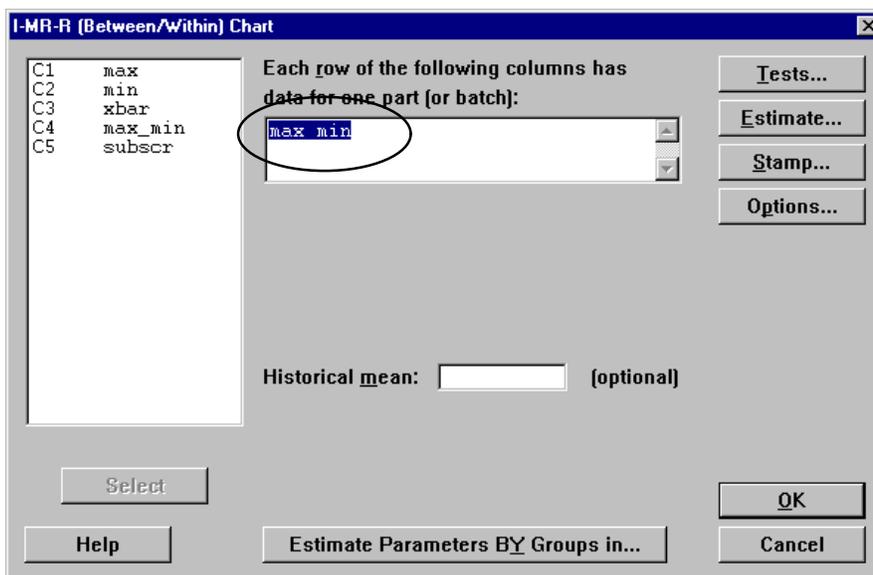
	C2	C3	C4	C7	C8	C9	C10
↓	min	xbar	max_min				
1	58	61.5	61				
2	54	56.5	58				
3	61	64.0	59				
4	55	57.0	58				
5	63	65.5	61				
6	60	62.0	61				
7	57	58.0	59	4			
8	54	57.0	55	4			

Draw three charts to assess both between and within subgroup variability

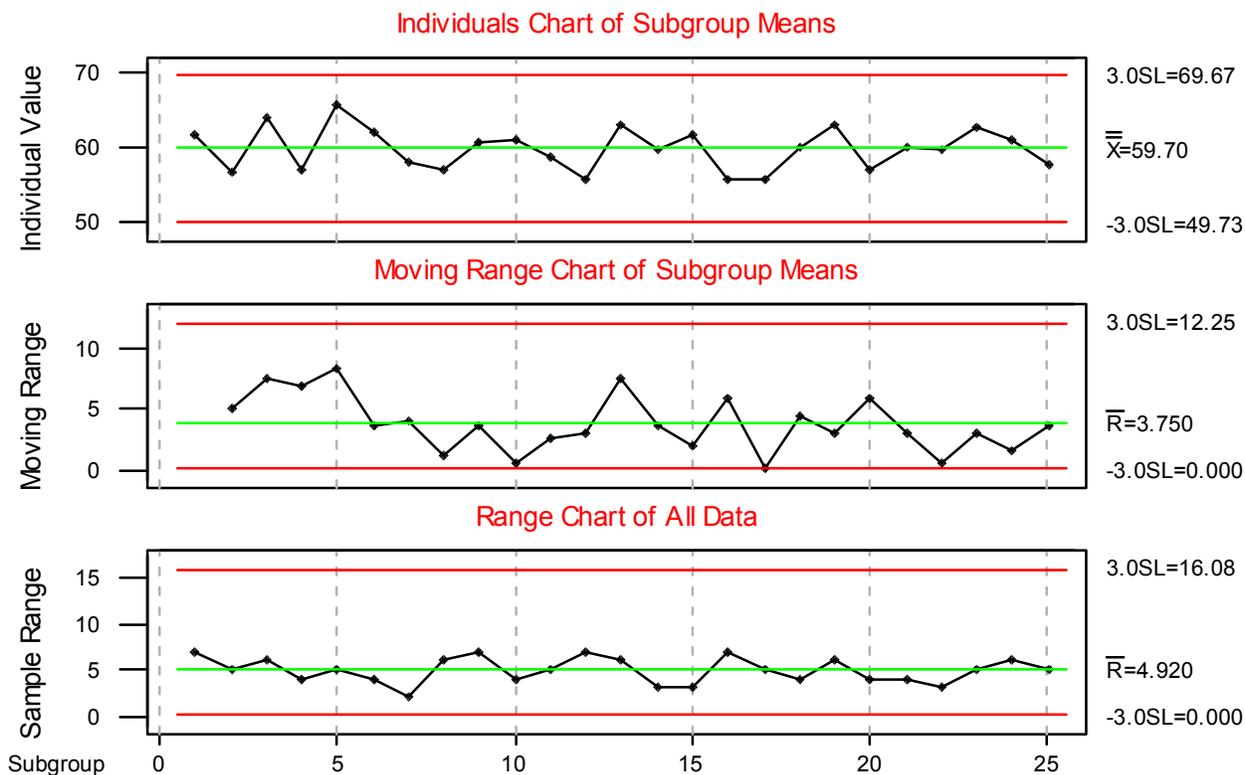


# 2-R Chart Example, $\bar{X}$ , $R_p$ , $R_w$ - Minitab Input & Output

1. Double Click on "max" and "min."



WB (within and between) Chart for max...min





## *Hints*

---

- 1. Plotting the data will always tell you more than not plotting the data.*
- 2. Even a little data will identify very good and very bad processes.*
- 3. Swift response to assignable cause indications has very beneficial effects.*
- 4. Be aware of the assumptions you are making.*



---

# *Control Chart Summary*



# *SPC Concepts*

---

## Shewhart

### **“CONTROLLED VARIATION”**

*Stable and consistent  
Random chance  
Predictable*

### **“UNCONTROLLED VARIATION”**

*Unstable, Inconsistent  
“Assignable Causes”  
Unpredictable*

## Deming

### **“COMMON CAUSES”**

*Inherent in “System”  
Management controls  
Only Management can fix*

### **“SPECIAL CAUSES”**

*May be local in nature  
Abnormal to system  
May be locally fixed*



*Understand the Process  
Search Out Causes of Variation and Remove*



## *Five Main Uses of Control Charts*

---

- *To reduce scrap and rework and for improving productivity.*
- *Defect prevention. In control means less chance of nonconforming units produced.*
- *Prevents unnecessary process adjustments by distinguishing between common cause variation and special or assignable cause variation.*
- *Provides diagnostic information so that an experienced operator can determine the state of the process by looking at patterns within the data. The operator can then make the necessary changes to improve the process performance.*
- *Provides information about important process parameters over time.*



## *Hints*

---

- 1. Plotting the data will always tell you more than not plotting the data.*
- 2. Even a little data will identify very good and very bad processes.*
- 3. Swift response to assignable cause indications has very beneficial effects.*
- 4. Be aware of the assumptions you are making.*



## Control Charts

---

### VARIABLE CHART

- ✓ *Uses Measured Values*
  - ✓ *Cycle Time, Lengths, Diameters, Drops, etc.*
- ✓ *Generally One Characteristic Per Chart*
- ✓ *More Expensive, But More Information*

### ATTRIBUTE CHART

- ✓ *Pass/Fail, Good/Bad, Go/No-Go Information*
- ✓ *Can Be Many Characteristics Per Chart*
- ✓ *Less Expensive, But Less Information*

### PROCESS FOCUSED CHART

- ✓ *Monitors Several Parts From Same Process*
- ✓ *Measures Deviation From Nominal/Target*
- ✓ *Typically an I & MR Chart Monitoring Several Characteristics of Several Parts*



## Control Limits

- *In general, control charts will use control limits which are + or - three standard deviation units from the center line.*
  - *UCL = Average + 3 standard deviations*
  - *LCL = Average - 3 standard deviations*
- *In computing three-sigma control limits, one must always use an average dispersion statistic.*
- *For example, in the  $\bar{X}$ - $\bar{R}$  chart,  $\bar{R}$  is an average dispersion statistic.*
  - *UCL = average +  $A_2\bar{R}$*
  - *LCL = average -  $A_2\bar{R}$*

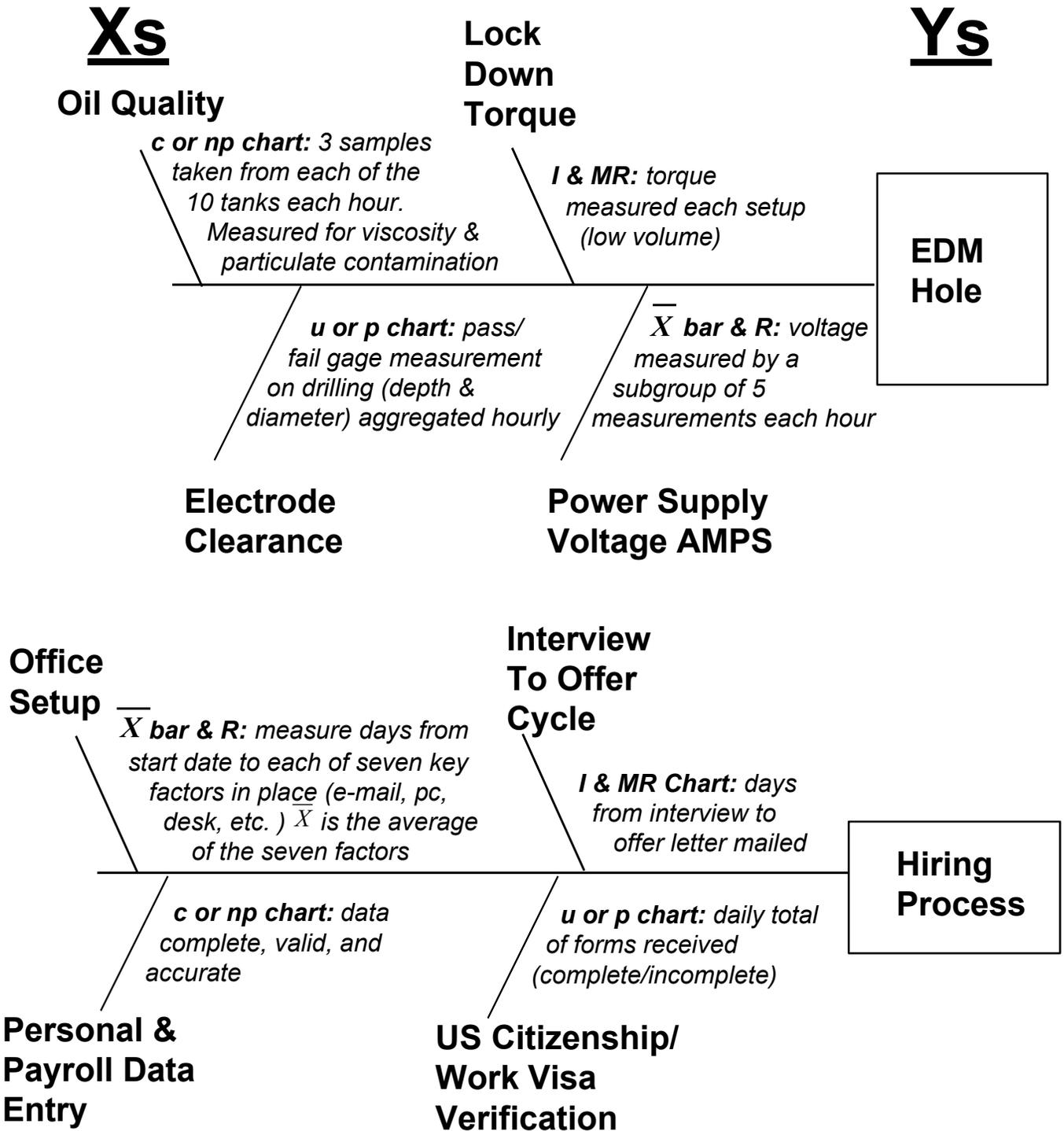


## *Control Limits vs. Specification Limits*

- **Do not** confuse the control limits with the specification limits.
- Specification limits are **external** to the process. For instance, they could represent engineering requirements to satisfy a CTQ characteristic.
- Control limits are **internal** to the process, they reflect the expected range of variation for that process.
- Specification limits are for **individual** values, whereas on an X bar chart the control limits are for the **sample averages**.

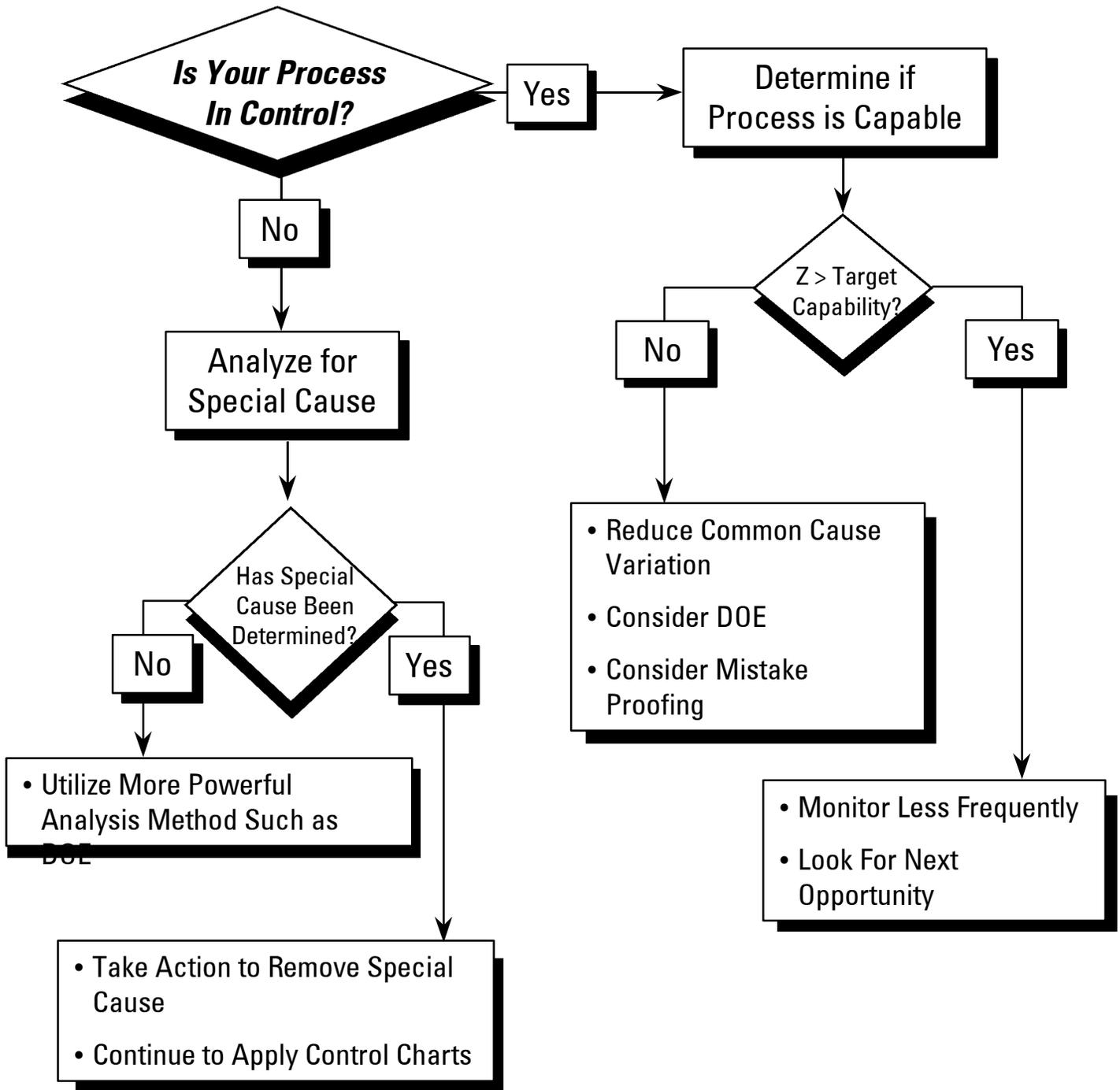


# Examples: Applying Control Processes to the Xs





## What Action Should You Take?





## *Process States*

### *The Four States of a Process*

#### **1. Chaos**

- *Process Out-Of-Control, Producing Non-Conforming Product*
- *Even the Level of Nonconformance Is Unstable*
- *Assignable Causes Dominate the Output*
- *“Fixes” Don’t Work For Very Long*

***How to Begin to Sort Out the Problems to be Solved?***



## *Process States (cont.)*

### *The Four States of a Process*

#### **2. *The Brink of Chaos***

- *Process Unstable, Some Nonconforming Product Is Being Produced*
- *Instability Will Continually Change Product Characteristics*
- *Process Output Is Influenced By Assignable Causes*
- *No Assurance the Next Piece Produced Will Be Conforming*

***How to Determine Existence of Assignable Causes?***



## *Process States (cont.)*

### *The Four States of a Process*

#### **3. *The Threshold State***

- *Process Inherently Stable Over Time, But Producing Some Nonconforming Product*
- *Proportion Nonconforming Predictable*
- *Some Nonconformances Will be Shipped*
- *If Process Natural Spread Greater Than the Spec, Common Causes Must be Reduced/Removed*
- *Process Must Be Monitored to Assure Desired Effect Is Achieved*

***How to be Certain That the Process Has Improved?***



## *Process States (cont.)*

### *The Four States of a Process*

#### **4. *The Ideal State***

- *The Process is Inherently Stable Over Time*
- *Operating Conditions are not Changed Arbitrarily (Follow the Process Plan)*
- *The Process Average is Set and Kept at the Proper Level*
- *The Natural Spread of the Process is Less Than the Specified Tolerance*

***How to be Certain That None of These Conditions Change or Degrade?***



## *Control Deliverables*

- *Establish how your process improvement requires monitoring*
- *Review current measurement system (e.g., switch to continuous data)*
- *If mistake proof solution - no chart is normally required*
- *Attribute (u, p, c, np) or Variable (individuals, moving range, x-bar, range)*
- *Establish LCL/UCL insuring that they are within the LSL/USL*
- *Establish control charts with process capability*
- *Establish owner of control charts & process*
- *Review risk of going out of control*



## *Control Deliverables (cont.)*

---

- *Establish periodic auditing requirements*
- *Review mistake proof solution*
- *Insure team members understand control requirements*
- *Finalize financial savings*
- *Update Six Sigma Quality Project Tracking database*



## *Exercise: 20 mins.*

---

- *For one or more projects in your term, brainstorm possible ways to control your process improvement.*
- *What control techniques will you use?*
  - *Control charts--variable or attribute*
  - *Documentation*
  - *Checklists*
  - *Standardization*

*One or Two groups will be asked to report their findings.*



## *Take Aways—Step 12*

---

- *Understand the phases of a quality plan:*
  - *Adequate Customer Requirement Review*
  - *Producible Design*
  - *Adequate Manufacturing Method*
  - *Effective Control Plan*
  - *Sufficient Appraisal*
  
- *Capability has an inverse relationship with the need for appraisal.*



## *Take Aways—Step 12*

---

- *Risk management is the process of managing risk through risk abatement plans.*
- *Risk management can be used to:*
  - *Systematically identify risk elements that can interfere with a process*
  - *Prevent risk elements from occurring through risk abatement plans*
  - *Communicate risk to management*
- *There are different types of risks such as cost risks, technology risks, specification risks, marketing risks, and installation risks.*
- *Identify risk, calculate risk scores, and take action to mitigate or eliminate the risk.*



## *Take Aways—Step 12*

- *Variable control charts can be used with continuous data to tell when a process is:*
  - *experiencing only common cause variation and working at its intended best*
  - *when the process is disturbed and needs corrective action*
- *Control charts:*
  - *time ordered plot of data*
  - *reflect the expected range of variation of the data*
  - *identifies when a special cause appears to be influencing the data*
- *X-Bar & R charts are used for plotting means and ranges of subgroups over time.*
- *I & MR charts are used for plotting individual values and moving ranges over time.*



## *Take Aways—Mistake Proofing*

- *Mistake proofing is a **proactive** tool used to eliminate or reduce errors.*
- *There are many types of human error.*
- *There are many error-provoking conditions.*
- *Inspection is limited in improving quality.*
- *Mistakes can be Predicted, Prevented, or Detected.*
- *There are five mistake proofing steps:*
  - *Identify Problems*
  - *Prioritize Problems*
  - *Seek Out the Root Cause*
  - *Create Solutions*
  - *Measure the Results*



## *Take Aways—Step 12*

---

- *Control limits are typically calculated as 3 standard deviations away from the mean of the process.*
  
- *Control limits and specification limits are not the same.*
  - *Control limits are calculated from the sample data; they are internal to the process*
  - *Specification limits are determined by your performance standard; they are external to the process*
  
- *Know when a process is out of control: Western Electric Rules.*
  
- *Control charts are only as good as the actions that you take to keep the process under control.*



## *Take Aways—Step 12*

---

- *Attribute control charts are used to monitor the level of nonconformance of a process.*
- *Select the appropriate attribute control chart based upon*
  - *constant vs. variable lot size*
  - *defects vs. defectives*
- *Defect*
  - *A single characteristic that does not meet requirements*
- *Defective*
  - *A unit that contains one or more DEFECTS*



## *Take Aways—Step 12*

---

- *Process Focused Control Charts allow us to monitor several parts within the **same process**.*
- *Data is recorded as variation from a target.*
- *2-R control charts can be used when trying to monitor an out-of-round, flat, etc. condition.*



## *Take Aways—Step 12*

---

- *Control charts are effective only to the extent that the organization uses the knowledge the charts provide.*
- *The validity of Control charts is built upon the proper use of rational sampling and rational subgrouping.*
- *Different subgroups answer different questions.*